



Higgspllosion: Solving the hierarchy problem via rapid decays of heavy states into multiple Higgs bosons

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Abstract

We introduce and discuss two inter-related mechanisms operative in the electroweak sector of the Standard Model at high energies. Higgspllosion, the first mechanism, occurs at some critical energy in the 25 to 10^3 TeV range, and leads to an exponentially growing decay rate of highly energetic particles into multiple Higgs bosons. We argue that this is a well-controlled non-perturbative phenomenon in the Higgs-sector which involves the final state Higgs multiplicities n in the regime $n\lambda \gg 1$ where λ is the Higgs self-coupling. If this mechanism is realised in nature, the cross-sections for producing ultra-high multiplicities of Higgs bosons are likely to become observable and even dominant in this energy range. At the same time, however, the apparent exponential growth of these cross-sections at even higher energies will be tamed and automatically cut-off by a related Higgsperion mechanism. As a result, and in contrast to previous studies, multi-Higgs production does not violate perturbative unitarity. Building on this approach, we then argue that the effects of Higgspllosion alter quantum corrections from very heavy states to the Higgs boson mass. Above a certain energy, which is much smaller than their masses, these states would rapidly decay into multiple Higgs bosons. The heavy states become unrealised as they decay much faster than they are formed. The loop integrals contributing to the Higgs mass will be cut off not by the masses of the heavy states, but by the characteristic loop momenta where their decay widths become comparable to their masses. Hence, the cut-off scale would be many orders of magnitude lower than the heavy mass scales themselves, thus suppressing their quantum corrections to the Higgs boson mass.

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1. Introduction

The recent discovery of a light Higgs boson at the Large Hadron Collider (LHC) [1,2] constitutes an outstanding success of the Standard Model (SM) of particle physics. Before its discovery, the presence of a light scalar boson with a mass within the reach of the LHC was predicted, to ensure unitarity in scattering processes between longitudinal gauge bosons. While the SM is certainly an incomplete theory of nature, it fails to explain the observed matter-antimatter asymmetry and it does not provide a cold dark matter candidate, it is widely believed that the Higgs boson interactions with all other SM particles renders it a self-consistent theory, up to very high energy scales. As such it is currently arguably impossible to point to a specific energy scale at which the SM has to be augmented by new physics to explain fundamental questions in nature.

Yet, the Higgs boson, as a light elementary scalar particle, suffers from the so-called fine-tuning problem. Quantum corrections are involuntarily dragging the Higgs boson mass to the new physics mass scale m_{new} , such that $m_h^2 \simeq m_0^2 + \delta m_{\text{new}}^2$. In order to obtain the observed physical mass of $m_h \simeq 125$ GeV the bare parameter of the theory m_0 has to be increasingly precisely tuned, depending on how widely the electroweak scale is separated from the new physics scale. The guiding principle that parameters of our quantum field theory should not have to be unnaturally precisely tuned is currently our strongest argument for the existence of a new physics scale, not too far away from the electroweak scale. Popular ways to avoid the Hierarchy problem altogether are supersymmetric and composite Higgs models, which each however have their own so-called little Hierarchy problems.

Looking beyond $2 \rightarrow 2$ scattering processes, which are unitarised due to tree-level cancellation effects between gauge and Higgs boson interactions, the SM might still run into problems at energy scales as low as $\mathcal{O}(100)$ TeV, as perturbative unitarity might be violated in $2 \rightarrow nh$ multi-Higgs boson production processes. At sufficiently high energies it becomes kinematically possible to produce high multiplicity final states with $n \gg 1$ particles in a weakly interacting theory. It was pointed out already more than a quarter of a century ago in Refs. [3,4] that the factorial growth in n can arise from the large numbers of Feynman diagrams contributing to the scattering amplitude \mathcal{M}_n at large n . This reasoning works in any quantum field theory where there is no destructive interference between Feynman diagrams in computations of on-shell quantities. This is indeed the case in a scalar field theory with $\lambda\phi^4$ -type interactions [5–7], where tree graphs all have the same sign, and the leading-order high-multiplicity amplitudes acquire the factorial behaviour, $\mathcal{M}_n \sim \lambda^{n/2} n!$. This observation, assuming that the amplitudes do not decay rapidly in moving off the multi-particle thresholds, leads to the factorial growth of the decay rates, $\Gamma_n \sim \lambda^n n! \times f_n(E)$, of highly energetic states and signals that perturbation theory becomes effectively strongly coupled for $n > 1/\lambda$ [8–12] and can result in sharply growing with energy high-multiplicity observables. For example, it was shown recently in Refs. [13,14] that such high multiplicity production processes may be within reach of a future hadron collider at 100 TeV. Already at 50 TeV the perturbative cross-sections for 140 Higgs bosons are at picobarn level.

Using the Higgspllosion mechanism, we will simultaneously address both short-comings of the SM stated above: the Hierarchy problem, and the possibility of a breakdown of perturbative unitarity in high multiplicity processes. We will show that the sharply growing cross-sections actually prevent the violation of perturbative unitarity in multi-Higgs processes and further naturally tame the effect of quantum corrections to the Higgs boson mass. The key point here is that the decay width is the imaginary part of the 2-point correlator, with the LSZ-amputated external lines. In a physical process, for example when the highly virtual Higgs boson is produced as an

intermediate state in the gluon fusion process before decaying into a high-multiplicity final state, the amplitude is of the form,

$$\mathcal{M}_{gg \rightarrow h^*} \times \frac{i}{p^2 - M_h^2 + i M_h \Gamma(p^2)} \times \mathcal{M}_{h^* \rightarrow n \times h}, \quad (1.1)$$

where M_h is the Higgs mass and $\Gamma(s)$ is the energy-dependent total width of the Higgs at the scale s . It is of course the same rate as the result of computing the phase-space integral in the cross-section,

$$\int d\Pi_n |\mathcal{M}_{h^* \rightarrow n \times h}|^2 = 2M_h \times \Gamma_n(s), \quad \Gamma = \sum_n \Gamma_n. \quad (1.2)$$

Hence the cross-section of the single-Higgs-exchange process (1.1) at high energies is schematically of the form

$$\sigma_n \sim \frac{\sqrt{s} \Gamma_n(s)}{s^2 + M_h^2 \Gamma^2(s)}, \quad (1.3)$$

and at asymptotic energies, where $\Gamma_n \rightarrow \infty$, is in fact consistent with unitarity.

The occurrence of sharply growing decay rates of highly energetic (or super-massive) initial states into high-multiplicity states of relatively soft Higgs bosons (and in all likelihood other massive vector bosons) will be called the Higgspllosion effect. As it effectively amounts to an exploding multi-particle decay width $\Gamma_n(s)$ of super-massive heavy states X , Higgspllosion must affect their propagators,

$$\Delta_X(p) = \frac{i}{p^2 - M_X^2 - i \text{Im} \Sigma_X(p^2)} = \frac{i}{p^2 - M_X^2 + i M_X \Gamma_X(p^2)}, \quad (1.4)$$

appearing in the loops contributing to the quantum corrections to the Higgs mass. If, due to the Higgspllosion mechanism, the decay width Γ_X of the heavy particle into n Higgs bosons exceeds the heavy mass M_X at the scale $\sqrt{s_*}$, which is much smaller than M_X , then it will be the scale $\sqrt{s_*}$ rather than M_X , which will provide the cut-off of the loop integrals in the self-energy contributions to the Higgs mass. Our central point is that purely on dimensional grounds, the Hierarchy problem for the Higgs mass is reduced by a positive power of the factor of $\frac{s_*}{M_X^2} \ll 1$.

This article is organised as follows: In Sec. 2 we review briefly how off-shell momenta enter the propagator and decay width of a scalar particle. We derive the scaling behaviour for the dimensionless quantity \mathcal{R} due to Higgspllosion in Sec. 3. In Sec. 4 we introduce the Higgspllosion mechanism, showing that perturbative unitarity is not violated in multi-Higgs production processes in the Standard Model. The connection between Higgspllosion and the dynamical taming of the Hierarchy problem we discuss in Sec. 5. In Sec. 6 we offer our conclusions.

2. Propagators and partial decay widths of massive fields

We are interested in investigating quantum effects caused by steeply growing multi-particle decay rates of a highly virtual (or highly energetic) degree of freedom in the initial state above a certain critical energy. The decay widths enter the propagators of the relevant states, thus we start in this section with a brief review of the full propagator for a massive scalar. In subsequent sections this will be used in our discussion of two cases: the Higgs propagators appearing as intermediate states in high-energy cross-sections, and the ultra-heavy states contributing to the Higgs mass through loop effects.

Consider a simple quantum field theory of a single real scalar field ϕ described by the Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m_0^2 \phi^2 - \mathcal{L}_{\text{int}}(\phi), \quad (2.1)$$

where m_0 denotes the bare mass parameter and the interaction term $\mathcal{L}_{\text{int}}(\phi)$ includes the usual renormalisable self-interactions of ϕ , for example $\mathcal{L}_{\text{int}} = \frac{\lambda}{4!} \phi^4$ or $\mathcal{L}_{\text{int}} = \frac{\lambda}{4} (\phi^2 - v^2)^2$. The Feynman propagator of ϕ is the Fourier transformation of the 2-point Green function, and reads

$$\Delta_\phi(p) = \int d^4x e^{ip \cdot x} \langle 0 | T(\phi(x) \phi(0)) | 0 \rangle = \frac{i}{p^2 - m_0^2 - \Sigma(p^2) + i\epsilon}, \quad (2.2)$$

where $\Sigma(p^2)$ is the self-energy of ϕ , i.e. $-i\Sigma(p^2)$ is the sum of all one-particle-irreducible (1PI) diagrams contributing to the two-point function. It is related to the amplitude for a $1 \rightarrow 1$ particle scattering, $\mathcal{M}(p \rightarrow p)$ via the LSZ reduction formalism, so that

$$\mathcal{M}(p \rightarrow p) = -Z_\phi \Sigma(p^2), \quad (2.3)$$

and Z_ϕ is the wave-function renormalisation constant. What we have on the right hand side of Eq. (2.2) is the resummed or dressed propagator since it can be Taylor expanded in terms of the bare propagators and the self-energy insertions,

$$\frac{i}{p^2 - m_0^2 - \Sigma(p^2)} = \frac{i}{p^2 - m_0^2} + \frac{i}{p^2 - m_0^2} \sum_{n=1}^{\infty} \left(-i\Sigma(p^2) \frac{i}{p^2 - m_0^2} \right)^n. \quad (2.4)$$

For simplicity, from now on, we are dropping the $i\epsilon$ factor in the propagators.

The physical (or pole mass) mass m is then defined as the location of the pole in the full propagator of Eq. (2.2). It is the solution of the equation,¹

$$m^2 - m_0^2 - \Sigma(m^2) = 0, \quad \text{or} \quad m^2 = m_0^2 + \text{Re} \Sigma(m^2). \quad (2.5)$$

The meaning of the self-energy at the fixed scale $p^2 = m^2$ is that it provides the shift to the bare mass, $\text{Re} \Sigma(m^2) = \delta m^2$, in order to obtain the observable and finite physical mass $m^2 = m_0^2 + \delta m^2$.

Using the equation (2.5) for the physical mass we can represent the dressed propagator Eq. (2.2) in the form,

$$\Delta_\phi(p) = \frac{i}{p^2 - m^2 - [\Sigma(p^2) - \Sigma(m^2)]} = \frac{i}{p^2 - m^2} \left(\frac{1}{1 - \frac{d\Sigma}{dp^2} \Big|_{p^2=m^2} + \mathcal{O}(p^2 - m^2)} \right),$$

which in the limit $p^2 \rightarrow m^2$ results in the well-known pole form of the propagator,

$$\Delta_\phi(p) \Big|_{p^2 \rightarrow m^2} \simeq \frac{iZ_\phi}{p^2 - m^2}, \quad \text{where} \quad Z_\phi = \left(1 - \frac{d\Sigma}{dp^2} \Big|_{p^2=m^2} \right)^{-1}. \quad (2.6)$$

Z_ϕ is the field renormalisation constant which has already appeared in Eq. (2.3).

¹ In our toy-model the particles are absolutely stable near their mass-shell. The model contains only self-interactions of the field ϕ and the decays become kinematically allowed only at energies above the multi-particle mass-thresholds, i.e. $p^2 > (2m)^2$. Hence the self-energy Σ at $p^2 = m^2$ contains no imaginary part as it is below the multi-particle mass-thresholds. Hence $\Sigma(p^2 = m^2) = \text{Re} \Sigma(p^2 = m^2)$ and the pole in (2.5) is on the real axis.

In this paper we will be mostly interested in the kinematic regime(s) far away, i.e. far above or far below, from the single-particle mass shell region $p^2 \simeq m^2$ of the propagator in Eq. (2.6). In the case of the light stable field ϕ we are considering at present, the regime of interest is such that multi-particle decays with ultra-high multiplicities $n \gg 1/\lambda \gg 1$ can contribute to the propagator, and hence $p^2 \gtrsim (nm)^2 \gg m^2$. In this case the propagator is described by the full expression of Eq. (2.2), and the self-energy contains a non-vanishing imaginary part. Specifically we will concentrate on the scenarios where multi-particle decays of a virtual ϕ into n -particle states, with ultra-high multiplicities n lead to decay widths which grow sharply with energy $E = \sqrt{s}$ above some critical value E_{crit} . If this scenario is realised in nature, one can enter the energy regime where $\text{Im}\Sigma(s) \gg m^2$. This is the regime of interest we will concentrate on in this work.

In the single-field toy model of Eq. (2.1) we are discussing at present, the particles described by the field ϕ are well-defined asymptotic states of mass m and they are absolutely stable not too far above their single-particle mass threshold, $m^2 \leq p^2 < (2m)^2$. Indeed, we have assumed that ϕ interacts only with itself, and there are no interactions with lighter states in the Lagrangian. This results in multi-particle thresholds at $p^2 \geq (nm)^2$ for $n = 2, 3, \dots$ corresponding to $\phi \rightarrow n \times \phi$ decays at energies $s \geq (nm)^2$ for $n \geq 2$. Thus, at around the single-particle mass-shell the decay width is zero, the propagator is real-valued and contains only the pole term – as indicated by Eq. (2.6). However, at higher energy scales, the multi-particle mass thresholds are reached resulting in the appearance of the imaginary part of $\Sigma(p^2)$ in the full propagator on the right hand side of Eq. (2.2). For the full propagator we have

$$\begin{aligned} \Delta_\phi(p) &= \frac{i}{p^2 - m^2 - \text{Re}[\Sigma(p^2) - \Sigma(m^2)] - i\text{Im}\Sigma(p^2)} \\ &= \frac{iZ_\phi}{p^2 - m^2 - iZ_\phi \text{Im}\Sigma(p^2)} + \dots \end{aligned} \quad (2.7)$$

In deriving this expression we Taylor-expanded the quantity

$$\text{Re}[\Sigma(p^2) - \Sigma(m^2)] = \text{Re} \left. \frac{d\Sigma}{dp^2} \right|_{m^2} (p^2 - m^2) + \mathcal{O}((p^2 - m^2)^2), \quad (2.8)$$

and used the definition of the wave-function renormalisation constant (2.6). The dots on the right hand side of Eq. (2.7) denote the contributions of higher order terms in the Taylor expansion of $\text{Re}\Sigma(p^2)$ which will always assume to be subleading to the effects we want to study here and that they can be treated as higher-order corrections in perturbation theory.

We will thus use the following expression for the scalar field propagator

$$\Delta_\phi(p) \simeq \frac{iZ_\phi}{p^2 - m^2 - iZ_\phi \text{Im}\Sigma(p^2)} = \frac{iZ_\phi}{p^2 - m^2 + i m \Gamma(p^2)}, \quad (2.9)$$

where we traded the imaginary part of the self-energy for the energy-dependent decay width $\Gamma(p^2)$, cf. Eq. (2.3),

$$-Z_\phi \text{Im}\Sigma(p^2) = \text{Im}\mathcal{M}(p \rightarrow p) = m \Gamma(p^2), \quad (2.10)$$

with the decay width being determined by the partial widths of n -particle decays at energies $s \geq (nm)^2$,

$$\Gamma(s) = \sum_{n=2}^{\infty} \Gamma_n(s), \quad \Gamma_n(s) = \frac{1}{2m} \int d\Pi_n |\mathcal{M}(1 \rightarrow n)|^2. \quad (2.11)$$

\mathcal{M} is the amplitude for the $1^* \rightarrow n$ process and the integral is over the n -particle Lorentz-invariant phase space.

In summary, for the UV-renormalised propagator $\Delta_R(p) = Z_\phi^{-1}$, we will use the following expression in terms of the pole mass m^2 , the renormalised self-energy $\Sigma_R(p^2) = Z_\phi \Sigma(p^2)$, or the physical width $\Gamma(p^2)$, and the renormalised coupling constant(s),

$$\Delta_R(p) = \frac{i}{p^2 - m^2 - i \operatorname{Im} \Sigma_R(p^2)} = \frac{i}{p^2 - m^2 + i m \Gamma(p^2)}. \quad (2.12)$$

All quantities in the expression above are UV-finite. The framework of using the propagator for the Higgs boson with the energy-dependent width as the correct description, applicable for all kinematic regions is widely used in the literature, see e.g. Refs. [15,16], and is consistent with our treatment.² In the following section we will concentrate on the decay width $\Gamma(s)$.

3. Multi-particle decay width of the Higgs boson

We now consider the ultra-high multiplicity Higgsplisions of highly energetic virtual particles in the Standard Model. Specifically, we will describe the main features of the mechanism using a simplified model for the Standard Model Higgs boson in terms of a QFT of a single real scalar field $h(x)$ with non-vanishing vacuum expectation value (VEV) $\langle h \rangle = v$,

$$\mathcal{L} = \frac{1}{2} \partial^\mu h \partial_\mu h - \frac{\lambda}{4} (h^2 - v^2)^2. \quad (3.1)$$

This theory is a reduction of the SM Higgs sector in the unitary gauge to a single scalar degree of freedom, $h(x)$ which for our purposes we take to be stable, so there are no decays into fermions, and we have also decoupled all vector bosons etc. The physical VEV-less scalar $\varphi(x) = h(x) - v$, describes the Higgs boson of mass $M_h = \sqrt{2\lambda} v$ and satisfies the classical equation arising from Eq. (3.1),

$$-(\partial^\mu \partial_\mu + M_h^2) \varphi = 3\lambda v \varphi^2 + \lambda \varphi^3. \quad (3.2)$$

The first step in our programme is to determine the multi-particle amplitudes describing the $1^* \rightarrow n$ transitions of the highly virtual Higgs boson into n non-relativistic Higgses at the leading order (i.e. tree-level) in perturbation theory. We take the bosons in the final state to be non-relativistic because we are interested in keeping the number of particles n in the final state as large as possible, that is, near the maximum number allowed by the phase space, $n \lesssim n_{\max} = E/M_h$. Such n -point amplitudes were studied in detail in scalar QFT in [6,9] and were derived for the theory of Eq. (3.1) with spontaneous symmetry breaking in Ref. [12],

$$\mathcal{A}_{1^* \rightarrow n}(p_1 \dots p_n) = n! (2v)^{1-n} \exp \left[-\frac{7}{6} n \varepsilon \right], \quad n \rightarrow \infty, \quad \varepsilon \rightarrow 0, \quad n\varepsilon = \text{fixed}. \quad (3.3)$$

Note that the expression above is for the $1^* \rightarrow n$ current, and the conventionally-normalised amplitude $\mathcal{A}_{1^* \rightarrow n}$ is obtained from it by the LSZ amputation of the single off-shell incoming line,

$$\mathcal{M}_{1 \rightarrow n} := (s - M_h^2) \cdot \mathcal{A}_{1^* \rightarrow n}(p_1 \dots p_n). \quad (3.4)$$

² In this paper we focus exclusively on multi-Higgs decays and are not concerned with the decays of the Higgs into lighter SM particles below its mass threshold. These can be readily incorporated.

As indicated, these tree-level amplitudes are computed in the double-scaling limit with large multiplicities $n \gg 1$ and small non-relativistic energies of each individual particle, $\varepsilon \ll 1$, where

$$\varepsilon = \frac{\sqrt{s} - nM_h}{nM_h} = \frac{1}{nM_h} E_n^{\text{kin}} \simeq \frac{1}{n} \frac{1}{2M_h^2} \sum_{i=1}^n \vec{p}_i^2, \quad (3.5)$$

so that the total kinetic energy per particle mass $n\varepsilon$ in the final state is fixed. The first factor on the right-hand side of Eq. (3.3) corresponds to the tree-level amplitude (or more precisely a current with one incoming off-shell leg) computed on the n -particle threshold,

$$\mathcal{A}_{1 \rightarrow n}^{\text{thr.}} = n! (2v)^{1-n} = n! \left(\frac{\lambda}{M_h^2} \right)^{\frac{n-1}{2}}, \quad (3.6)$$

or, equivalently, after the LSZ reduction of the incoming line,

$$\mathcal{M}_{1 \rightarrow n}^{\text{thr.}} = n! (n^2 - 1) \frac{\lambda^{\frac{n-1}{2}}}{M_h^{n-3}}, \quad (3.7)$$

which is an exact expression for tree-level amplitudes valid for any value of n [6]. The kinematic dependence in Eq. (3.3) then produces in the non-relativistic limit an exponential form-factor which has an analytic dependence on the kinetic energy of the final state $n\varepsilon$. But, importantly, the factorial growth $\sim \lambda^{n/2} n!$ characteristic to the multi-particle amplitude on mass threshold remains. Its occurrence can be traced back to the factorially growing number of Feynman diagrams at large n [17–19] and the lack of destructive interference between the diagrams in the scalar theory. We refer the reader to Refs. [6,9,12] for more detail about these amplitudes.

The next step is to integrate the amplitudes in Eq. (3.3) over the n -particle phase-space at large n (in the approximation where the outgoing particles are non-relativistic). The relevant dimensionless quantity describing the multi-particle processes is

$$\mathcal{R}_n(s) := \frac{1}{2M_h^2} \int d\Pi_n |\mathcal{M}(1 \rightarrow n)|^2, \quad (3.8)$$

and the decay rates $\Gamma_n(s)$ and the cross-sections $\sigma_n(s)$ are obtained from $\mathcal{R}_n(s)$ after an appropriate overall rescaling with M_h and s . Following in the steps of Refs. [10,12], we obtain the characteristic exponential expression for the $1 \rightarrow n$ particles rate \mathcal{R} in the high-energy, high-multiplicity limit:

$$\mathcal{R}(\lambda; n, \varepsilon) = \exp \left[n \left(\log \frac{\lambda n}{4} - 1 \right) + \frac{3n}{2} \left(\log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} n\varepsilon \right], \quad (3.9)$$

$$\Gamma_n(s) \propto \mathcal{R}(\lambda; n, \varepsilon), \quad \text{and} \quad \sigma_n(s) \propto \mathcal{R}(\lambda; n, \varepsilon).$$

In particular, note that the ubiquitous factorial growth of the large- n amplitudes translates into the $\frac{1}{n!} |\mathcal{M}_n|^2 \sim n! \lambda^n \sim e^{n \log(\lambda n)}$ factor in the rate \mathcal{R} above.

To summarise our discussion so far, let us consider the multi-particle limit $n \gg 1$ and scale the centre-of-mass energy $\sqrt{s} = E$ linearly with n , $E \propto n$, keeping the coupling constant small at the same time, $\lambda \ll 1$. It was pointed out first in Refs. [9,10], and then argued for extensively in the literature, that in this limit the multi-particle rates have a characteristic exponential form,

$$\mathcal{R} = e^{nF(\lambda n, \varepsilon)}, \quad \text{for } n \rightarrow \infty, \lambda \rightarrow 0, \varepsilon = \text{fixed}, \quad (3.10)$$

where it is assumed that the high-multiplicity, weak-coupling limit above, the factor λn is held fixed, while the fixed value can be small or large (with the former case allowing for a perturbative treatment, while the latter one requiring a large λn resummation of perturbation theory, somewhat reminiscent to the large $g^2 N_c$ 't Hooft coupling limit in gauge theories). The quantity ε is the average kinetic energy per particle per mass in the final state of Eq. (3.5), and $F(\lambda n, \varepsilon)$ is a certain a priori unknown function of two arguments. At tree-level, the dependence on λn and ε , factorises into individual functions of each argument,

$$F^{\text{tree}}(\lambda n, \varepsilon) = f_0(\lambda n) + f(\varepsilon), \quad (3.11)$$

and the two independent functions are given by the following expressions in the Higgs model of Eq. (3.1), in complete agreement with the expression Eq. (3.9),

$$f_0(\lambda n) = \log\left(\frac{\lambda n}{4}\right) - 1, \quad (3.12)$$

$$f(\varepsilon)|_{\varepsilon \rightarrow 0} \rightarrow f(\varepsilon)_{\text{asympt}} = \frac{3}{2} \left(\log\left(\frac{\varepsilon}{3\pi}\right) + 1 \right) - \frac{25}{12} \varepsilon. \quad (3.13)$$

One can further come up with various improvements in the understanding and control of the exponential behaviour of the multi-particle rate. In particular, at tree-level the function $f_0(\lambda n)$ is fully determined, but the second function, $f(\varepsilon)$, characterising the energy-dependence of the final state, is determined by Eq. (3.13) only at small ε , i.e. near the multi-particle threshold. This point was addressed recently in Ref. [13] where the function $f(\varepsilon)$ was computed numerically in the entire range $0 \leq \varepsilon < \infty$.

What about the inclusion of loop corrections to the tree-level multi-particle rates above? This has been achieved at the leading order in λn in Ref. [9] by resumming the one-loop correction to the amplitude on the multi-particle mass threshold computed in Refs. [20,21]. The result is that the 1-loop correction in the Higgs theory under consideration does not affect the factorial growth, but provides an exponential enhancement to the rate (though strictly speaking it is valid only at small values of λn) and results in the modified expression for f_0 ,

$$f_0(\lambda n)^{1\text{-loop}} = \log\left(\frac{\lambda n}{4}\right) - 1 + \sqrt{3} \frac{\lambda n}{4\pi}. \quad (3.14)$$

Of phenomenological interest is whether the multi-particle rates can become observable at certain energy scales and, at even higher energies, exponentially large – in the limit of near maximal kinematically allowed multiplicities. To answer this, it is required to resum the perturbation theory and address the large λn limit. Very recently, we have computed the exponential rate in the $\lambda n \gg 1$ limit using the Landau WKB-based formalism, following the approach of Ref. [10]. These results will be reported in a forthcoming publication [22]. The correction to the tree-level rate in the non-relativistic regime is found to be of the form $\approx +3.02 n \sqrt{\frac{\lambda n}{4\pi}}$.

As a result, the non-perturbatively corrected multi-particle rate in Eq. (3.9) becomes [22]

$$\mathcal{R} = \exp \left[\frac{\lambda n}{\lambda} \left(\log \frac{\lambda n}{4} + 3.02 \sqrt{\frac{\lambda n}{4\pi}} - 1 + \frac{3}{2} \left(\log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \varepsilon \right) \right]. \quad (3.15)$$

This expression is derived at small ε and thus is supposed to hold in the non-relativistic limit.

The resulting rates have a sharp exponential dependence on n and, consequently, on energy. In order to be able to probe sufficiently high multiplicities, they have to be kinematically allowed, i.e. in our single-field example, $n < n_{\text{max}} = E/M_h$. The amplitudes grow with n , as

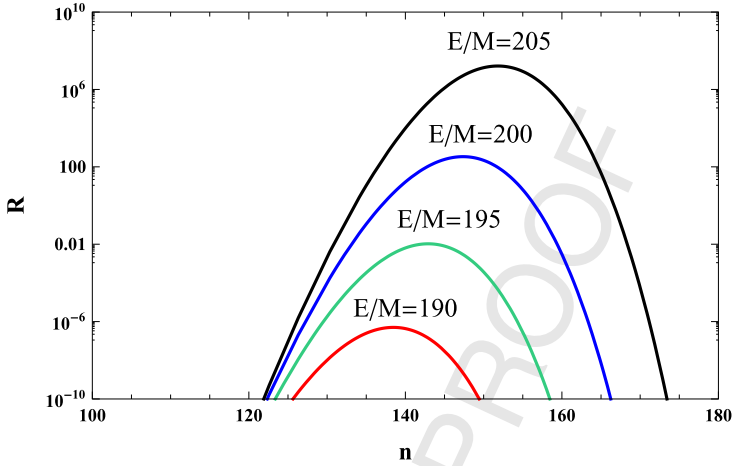


Fig. 1. Partial decay widths (in units of mass M_h) of a highly-energetic single-particle state into n Higgs bosons h plotted as function of n . The four lines correspond to the energies of the initial state equal $190M_h$, $195M_h$, $200M_h$ and $205M_h$, as indicated. There is a sharp exponential dependence of the peak rate on the energy varying from $\mathcal{R} \lesssim 10^{-6}$ at $E = 190M_h$ (red line) to $\mathcal{R} \gtrsim 10^7$ at $E = 205M_h$ (black line). The peak multiplicities $n_* \sim 150$ in these examples are not far from the maximally allowed values at the edge of the phase space $n_{\max} \sim E/M_h$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$\exp[n \log \lambda n]$, reaching their maximal values in the soft limit where n is maximal, but this effect is counter-acted by the diminishing phase-space volume near the edge of the kinematically accessible region. The competition between the two effects is clearly seen in the expressions for \mathcal{R} already at tree-level in Eq. (3.9) and similarly in the re-summed perturbation theory expression in Eq. (3.15). The growth of the exponent in \mathcal{R} with increasing λn is counteracted at the edge of the phase-space by the $\log \varepsilon$ factor where $\varepsilon = (E - M_h n)/(M_h n) \rightarrow 0$ when $n \rightarrow n_{\max}$. As a result we expect that the rate will peak at a non-perturbatively large value of $n \gg 1/\lambda$ but before the edge of the phase-space at $\varepsilon = 0$ is reached.

The relevant parameters are the energy $\sqrt{s} = E$ in the units of the elementary scalar mass, in our case M_h , and the number of particles in the final state n rescaled by the (small) coupling constant, λn . In the regime of relatively low-energies, $E/M_h \lesssim 10^2$, the multi-particle rates and cross-sections are exponentially small (essentially zero). But above the critical energy E_{crit} in the region of $\simeq 200 M_h$, using the plots in Fig. 1 as a guide, and for large values of n towards the edge of the allowed phase-space, the exponential growth in the rates starts competing with the exponential suppression, the rates become of the order 1 and then blow up exponentially. In Fig. 1 we sketch the behaviour of the rate \mathcal{R} in Eq. (3.15) at fixed energies $E = 190M_h$, $195M_h$, $200M_h$ and $205M_h$ as the function of the number of particles in the final state. For concreteness we have set $\lambda = 1/8$. The values of E are chosen to illustrate the sharp rise in the rate from the exponentially suppressed to the exponentially enhanced level – the transition which occurs very sharply with energy as it changes by just a few percent.

The structure of the peak in n is easy to understand. Starting at n_{\max} at the right of the plot, we are at the end of the phase-space and the rate is zero. Then by decreasing the values of n to the left of n_{\max} , the phase-space volume starts to grow and so does the rate \mathcal{R} . On the other hand, in the opposite limit, at low values of n , the rate is exponentially small again. Hence there must exist a local maximum, which clearly prefers as large as possible values of n but before the edge of the phase-space is reached.

In summary, we conclude that at sufficiently high energies $E > E_{\text{crit}} \sim 2 \times 10^2 M_h$ (the precise value would depend on the robustness of the model used³) the multi-particle decay rates of an initial state into Higgs bosons develop a non-perturbative peak centred at $n = n_\star \gg 1/\lambda$ which tends to be near the edge of the kinematically accessible multi-particle phase-space, $n_\star = n_{\text{max}} - \Delta n = E/M_h - \Delta n$. The peak occurs in a non-perturbative regime, $n \gg 1/\lambda$, and the width of the peak $2\Delta n$ is roughly of the order $1/\lambda$. Most of the energy available in the initial state is used to maximise the multiplicity n_\star of the final state bosons produced near the edge of the phase space, as such they correspond to relatively soft modes.

It is tempting to interpret this peak as a creation of a semi-classical object – a classicalon – which then decays into soft modes with ultra-high multiplicities. There are apparent parallels with the classicalisation phenomenon [23–25] in which the theory prevents itself from probing shorter and shorter distances at very high energies by redistributing the energy of the initial state into many weakly interacting soft quanta.

4. Higgspersion, cross-sections and perturbative unitarity

The scattering cross-sections for producing multiple Higgs bosons in the high-multiplicity limit $n \gtrsim 100$ at collider energies in the regime of 100 TeV were addressed and computed recently in Refs. [13,14] (with certain simplifying assumptions). These calculations consider the gluon fusion process where intermediate highly energetic Higgs bosons are produced before subsequently branching into high-multiplicity multi-Higgs final states. The results of Ref. [14] are based on the computation of the leading polygons – the triangles, boxes, pentagons and hexagons – to the gluon fusion production processes, further combined with the subsequent branchings to reach high final state multiplicities. This can be represented as

$$\mathcal{M}_{gg \rightarrow n \times h} = \sum_{\text{polygons}} \mathcal{M}_{gg \rightarrow k \times h^*}^{\text{polygons}} \sum_{n_1 + \dots + n_k = n} \prod_{i=1}^k \mathcal{A}_{h_i^* \rightarrow n_i \times h}, \quad (4.1)$$

where the final partonic amplitude $\mathcal{M}_{gg \rightarrow k \times h^*}$ is convoluted with the gluon PDFs to obtain the collider cross-section. The factors of $\mathcal{A}_{h_i^* \rightarrow n_i \times h}$, after being squared and integrated over the multi-particle phase-space, result in a factor of $\mathcal{R}_n(s)$ appearing in the cross-section. It was found that the characteristic energy and multiplicity scales where these cross-sections become observable are within the 50 and 100 TeV regime with of order of 130 Higgses, or more, in the final state. We refer the reader to Fig. 2 (taken from Ref. [14], and to the above reference for more details).

Fig. 2 indicates that at collider energies below approximately 50 TeV, the processes are completely unobservable. However at higher energies, from 50 to a 100 TeV, the cross-sections reach a picobarn level and become observable for 130 to 150 Higgs bosons produced. This is the regime where a dramatic change away from the usual weakly-coupled perturbative description of the electro-weak physics takes place. We note that the multiplicity range where the slope of the cross-section in Fig. 2 changes so that the cross-section starts to increase with the multiplicity,

³ It is important to note that the overall structure of the peaks observed in Fig. 1 does not depend critically on the detailed form of the expression in (3.15). All that is required is that the factorial growth of the tree-level amplitudes – manifested as the $n \log \lambda n$ term in the exponent of (3.15) – is not erased by the higher-order quantum corrections. The main points of the Higgsplosion and Higgspersion mechanisms discussed in this paper can be understood by simply assuming the scaling behaviour of the type sketched in Fig. 1.

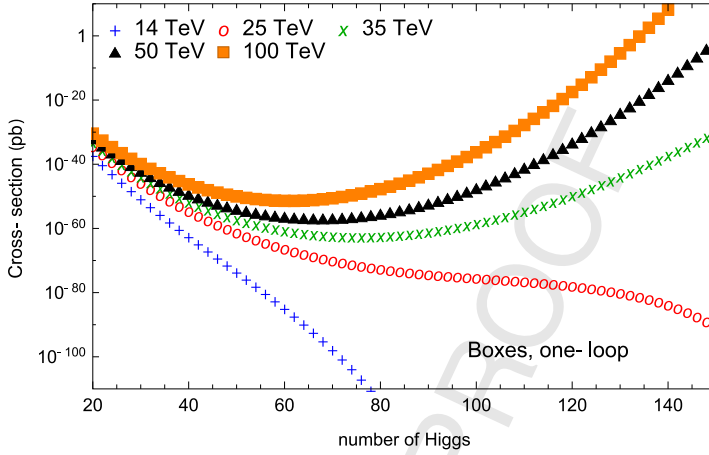


Fig. 2. Cross-sections for multi-Higgs production at proton colliders including the PDFs for different energies of the proton-proton collisions plotted as the function of the Higgs multiplicity. Only the contributions from the boxes are included. The Figure is taken from [14].

corresponds to the left-hand-side of the peaks in \mathcal{R} shown in Fig. 1. The plot range in Fig. 2 is cut-off before the cross-sections for 35, 50 and 100 TeV reach their local maxima.

4.1. Unitarity

We will now argue that as soon as the cross-sections have reached the observable level, any subsequent increase in the available energy will not result in the unbounded growth of the rates. Instead, the cross-sections will actually decrease, and there will be no violation of perturbative unitarity. For concreteness, consider the simplest process with a single intermediate off-shell Higgs propagator.⁴ The amplitude for this process reads (*cf.* Eq. (4.1)):

$$\mathcal{M}_{gg \rightarrow h^*} \times \frac{i}{p^2 - M_h^2 + i M_h \Gamma(p^2)} \times \mathcal{M}_{h^* \rightarrow n \times h}, \quad (4.2)$$

where $\Gamma(s)$ is the energy-dependent total width of the Higgs at the scale s , and it will lead to the Higgsperion of the total cross-section at asymptotically high energies. In other words, the off-shell current $\mathcal{A}_{h^* \rightarrow n \times h}$ in Eq. (4.1) includes the full dressed propagator times the amplitude $\mathcal{M}_{h^* \rightarrow n \times h}$.

In the limit $s \gg M_h^2, m_t^2$, the corresponding parton-level cross-section becomes,

$$\sigma_{gg \rightarrow n \times h}^{\Delta} \sim y_t^2 m_t^2 \log^4 \left(\frac{m_t}{\sqrt{s}} \right) \times \frac{1}{s^2 + M_h^4 \mathcal{R}^2} \times \mathcal{R}_n, \quad (4.3)$$

and asymptotes to $1/\mathcal{R}$ in the limit $\mathcal{R} \rightarrow \infty$. The inclusion of the decay width is of course only relevant when $\Gamma(s)$ becomes comparable to s/M_h . This conclusion is general and applies to higher-order polygons with more than one internal Higgs propagator.

In summary the multi-particle high-energy cross-section has the behaviour of the type,

⁴ This corresponds to the contribution of triangle diagrams to the gluon fusion production. The processes from all higher-order polygons, with more than one intermediate Higgs propagator can be dealt with in a similar fashion.

$$\sigma_{gg \rightarrow n \times h} \sim \begin{cases} \mathcal{R} & : \text{for } \mathcal{R} \lesssim 1 \\ 1/\mathcal{R} \rightarrow 0 & : \text{for } \mathcal{R} \gg 1 \text{ at } s \rightarrow \infty. \end{cases} \quad (4.4)$$

The first line in the equation above is the result of Higgspllosion and the second line is the consequence of the Higgspersion mechanism.

4.2. A comment on the Källén–Lehmann formula

It can also be helpful to address potential unitarity violations in the theory [26,27] using the Källén–Lehmann representation of the propagator for a scalar field ϕ ,

$$\Delta_\phi(p) = \int_0^\infty \frac{ds}{2\pi} \frac{i}{p^2 - m^2} \rho(s), \quad (4.5)$$

where $\rho(s)$ is the spectral density function, see e.g. [28],

$$\begin{aligned} \rho(s) &= \sum_n 2\pi \delta\left(\sqrt{s} - \sum_{i=1}^n p_i\right) |\langle 0|\phi(0)|n\rangle|^2 \\ &= 2\pi Z_\phi \delta(s - m_\phi^2) + \sum_{n \geq 2} \int d\Pi_n |\mathcal{A}(1^* \rightarrow n)|^2(s) \\ &= 2\pi Z_\phi \delta(s - m_\phi^2) + \frac{1}{(s - m^2)^2} \sum_{n \geq 2} \int d\Pi_n |\mathcal{M}(1 \rightarrow n)|^2(s), \end{aligned} \quad (4.6)$$

and in the last line we have pulled out the external line propagators to represent the expression in terms of the conventionally normalised scattering amplitudes $\mathcal{M}(1 \rightarrow n)$. With this we find,

$$\Delta_\phi(p) = \frac{i Z_\phi}{p^2 - m^2} + i \sum_{n \geq 2} \int_{(nm)^2}^\infty \frac{ds}{2\pi} \frac{1}{p^2 - s} \frac{\int d\Pi_n |\mathcal{M}(1 \rightarrow n)|^2(s)}{(s - m^2)^2}. \quad (4.7)$$

For $|p^2| < 4m^2$ the second term on the right hand side gives a non-singular contribution to the propagator and the residue of the propagator pole is entirely determined by the first term. The probability rates for $1 \rightarrow n$ processes thus appear in Eq. (4.7) as the additive order corrections to the propagator, and, importantly, they are integrated over s . Thus it appears from this formula that if the multi-particle decay rates Γ_s are exponentially divergent at large $s \gtrsim s_{\text{crit}}$, upon integration over s , these corrections will blow up even at low values of p^2 , i.e. $|p^2| < 4m^2 \ll \sqrt{s_{\text{crit}}}$. Thus, it is tempting to say that to guarantee unitarity, the higher order terms in n on the right hand side of Eq. (4.7) should not be too large [26].

This conclusion, however, depends on the validity of the above expression in (4.7). Let us examine it and start by re-writing the term on the right hand side in terms of the imaginary part of the self-energy (cf. (2.10)–(2.11)),

$$-Z_\phi \text{Im } \Sigma(p^2) = \frac{1}{2} \int d\Pi_n |\mathcal{M}(1 \rightarrow n)|^2, \quad (4.8)$$

so that,

$$(-i)\Delta_\phi(p) = \frac{Z_\phi}{p^2 - m^2} + \sum_{n \geq 2} \int_{(nm)^2}^{\infty} \frac{ds}{2\pi} \frac{1}{s - p^2} \frac{2 Z_\phi \operatorname{Im} \Sigma(s)}{(s - m^2)^2}. \quad (4.9)$$

Note that the imaginary part of the self-energy is proportional to the discontinuity of the self-energy on the cut along the real axis of the complex variable s ,

$$\operatorname{disc} \Sigma(s)|_{s \geq (nm)^2} = \Sigma(s + i\epsilon) - \Sigma(s - i\epsilon) = 2i \operatorname{Im} \Sigma(s), \quad (4.10)$$

hence the second term in (4.9) is,

$$\frac{1}{2\pi i} \sum_{n \geq 2} \int_{(nm)^2}^{\infty} ds \frac{1}{s - p^2} \frac{Z_\phi \operatorname{disc} \Sigma(s)}{(s - m^2)^2}.$$

By adding to it an integral over the circular contour at $|s| \rightarrow \infty$ with the counter-clockwise orientation, we have,

$$\begin{aligned} & \frac{1}{2\pi i} \sum_{n \geq 2} \int_{(nm)^2}^{\infty} ds \frac{1}{s - p^2} \frac{Z_\phi \operatorname{disc} \Sigma(s)}{(s - m^2)^2} + \frac{1}{2\pi i} \oint_{|s| \rightarrow \infty} ds \frac{1}{s - p^2} \frac{Z_\phi \Sigma(s)}{(s - m^2)^2} \\ &= \frac{1}{2\pi i} \oint_{s=p^2} ds \frac{1}{s - p^2} \frac{Z_\phi \Sigma(s)}{(s - m^2)^2} = \frac{Z_\phi \Sigma(p^2)}{(p^2 - m^2)^2}, \end{aligned} \quad (4.11)$$

where on the last line we have used Cauchy's theorem. Hence, we conclude that if the contour integral at $|s| \rightarrow \infty$ is negligible and can be added (that is if the integrand goes to zero at infinite s), the Källén–Lehmann formula for the renormalised propagator takes a familiar form:

$$\Delta_\phi(p) = \frac{i}{p^2 - m^2} + \frac{i}{p^2 - m^2} (-i \Sigma_R(p^2)) \frac{i}{p^2 - m^2}, \quad (4.12)$$

which is essentially a single perturbation in terms of the self-energy.

This derivation, however, breaks down completely when the $\operatorname{Im} \Sigma(s)$ explodes rather than falls off at $s \rightarrow \infty$, which is precisely the case of interest for our consideration. In this case the contour in (4.11) cannot be closed up at infinity and the dispersion relation (4.11) is invalid. We thus conclude that the formal justification of the perturbative Källén–Lehmann representation for the propagator in (4.7) or equivalently (4.9) is meaningful only for a sufficiently well-behaved imaginary part of the self-energy expression at large s . In the scenario which is of main interest to us in this work, the decay rates (or equivalently, the imaginary part of Σ) happen to grow rather than vanish at infinity. In this case one cannot use the dispersion relation to restore the real part from the imaginary part of the self-energy by closing up the contour, and the Källén–Lehmann representation in the form (4.7), (4.9) simply becomes invalid. Hence the growing multi-particle decay rates do not necessarily imply the breakdown of unitarity of the theory. In the previous sub-section we have already argued that the relevant physical cross-sections in this case do not blow up and hence do not destroy unitarity either.

5. Higgsplosion of heavy states below their mass-threshold

To outline the Higgsplosion approach as a solution to the Hierarchy problem in the Standard Model, let us consider a contribution of a hypothetical heavy scalar X of mass M_X to the Higgs

boson mass parameter. This obviously requires that X and the Higgs boson h can interact with each other and we further assume that X is not absolutely stable. This picture is captured by a simple Lagrangian,

$$\mathcal{L}_X = \frac{1}{2} \partial^\mu X \partial_\mu X - \frac{1}{2} M_X^2 X^2 - \frac{\lambda_P}{4} X^2 h^2 - \mu X h^2. \quad (5.1)$$

The Higgs-portal interactions $\lambda_P X^2 h^2$ ensure that X -loops contribute to the Higgs boson mass while the interaction $\mu X h^2$ lifts the $X \rightarrow -X$ parity symmetry and ensures that X can decay into multiple Higgs bosons $X \rightarrow hh$.

Calculating the contribution to the Higgs boson mass from the scalar X , we find

$$\begin{aligned} \Delta M_h^2 &\sim \lambda_P \int \frac{d^4 p}{16\pi^4} \frac{1}{M_X^2 - p^2 + i \operatorname{Im} \Sigma_X(p^2)} \\ &= \lambda_P \int \frac{d^4 p}{16\pi^4} \left(\frac{M_X^2 - p^2}{(M_X^2 - p^2)^2 + (\operatorname{Im} \Sigma_X(p^2))^2} - \frac{i \operatorname{Im} \Sigma_X(p^2)}{(M_X^2 - p^2)^2 + (\operatorname{Im} \Sigma_X(p^2))^2} \right). \end{aligned}$$

Now, due to the Higgspllosion effect the multi-particle contributions to the width of X explode at the values of the loop momenta $p^2 = s_*$, where $\sqrt{s_*} \simeq \mathcal{O}(25)$ TeV according to Fig. 1. This is much below the masses of the hierarchically heavy states which we can assume to be at the GUT scale ± 10 orders of magnitude. Because of the sharp exponential growth of the width $\operatorname{Im} \Sigma_X(s) \propto \mathcal{R}_n(s)$ with the energy, it provides a sharp UV cut-off in the integral over the loop momenta at $p^2 = s_*$. Hence the integral in the expression above amounts to

$$\Delta M_h^2 \propto \lambda_P \frac{s_*}{M_X^2} s_*. \quad (5.2)$$

This is suppressed by the factor of $\left(\frac{\sqrt{s_*}}{M_X}\right)^4 \simeq \left(\frac{25 \text{ TeV}}{M_X}\right)^4$ relative to the normal expectations without the Higgspllosion-driven disintegration of the heavy particles.

$$\text{For } \Gamma(s_*) \simeq M_X \text{ at } s_* \ll M_X^2 \implies \Delta M_h^2 \propto \lambda_P \frac{s_*}{M_X^2} s_* \ll \lambda_P M_X^2. \quad (5.3)$$

The reasoning above equally applies to any heavy modes, as far as they can decay and have a non-vanishing interaction with the Higgs boson. These modes could be the heavy 10^{12} GeV sterile neutrinos which are important for the standard thermal Leptogenesis [29–31], a heavy inflaton [32,33], GUT-scale particles [34,35], flavons [36,37], or the heavy degrees of freedom that would appear at the $f_a \simeq 10^{11}$ GeV scale relevant for the axion [38–41].

At one-loop level, one can always estimate the contributions to the Higgs mass from the heavy states of any spin with generic interactions with the Higgs boson using the Coleman–Weinberg effective potential,

$$M_h^2 = \frac{\partial^2 V_{\text{eff}}}{\partial h^2}, \quad (5.4)$$

where

$$V_{\text{eff}} = \frac{1}{64\pi^2} \int^{\sqrt{s_*}} d^4 p \operatorname{STr} \log(p^2 + M_X(h)^2). \quad (5.5)$$

$\text{STr} = \text{Tr}(-1)^F$ is the supertrace and $M_X(h)$ denotes the Higgs-field-dependent contribution to the heavy field mass in the $h(x)$ background. The main point, as above, is that the integral over the loop momenta is cut-off at the relatively low scale $\sqrt{s_\star}$ where the Higgspllosion of the heavy states takes place.

It is remarkable that the Hierarchy problem introduced into the Standard Model by the existence of a microscopic light Higgs boson is addressed in this approach by Higgsploding the heavy states into the original light Higgs bosons. The underlying cause of the apparent problem provides its own solution.

6. Conclusions

The discovery of the Higgs boson, roughly 50 years after its prediction, marked one of the greatest successes of the SM. While its interactions with all other particles ensures the restoration of perturbative unitarity in $2 \rightarrow 2$ scattering processes, it was long argued that the presence of a scalar particle in the theory could lead to unitarity violation in multi-Higgs production processes already at energies of $\mathcal{O}(100)$ TeV. Further, the Higgs boson, as an elementary scalar particle, suffers from the well-known Hierarchy problem. We have reexamined and connected both issues, thereby providing a simultaneous solution to both questions: We introduced the Higgspllosion mechanism, arguing that the rapid increase of the decay rate of very heavy or highly energetic particles is a physical effect, but that this effect leads to Higgspsersion, i.e. it restores perturbative unitarity in multi-Higgs boson production processes. While the cross section of multi-Higgs production processes can still reach observable levels, its exponential growth is avoided and the SM retains self-consistency to highest energies. Quantum corrections of heavy particles to the Higgs boson's mass are driving the Hierarchy problem. If however, the heavy particle's width increases rapidly beyond a certain energy threshold, these contributions are tamed and the Hierarchy problem can be avoided.

Our discussion of the Higgspllosion mechanism in Sec. 3 was based on a simplified model of the Higgs sector in terms of a purely scalar field theory, Eq. (3.1), with a non-vanishing VEV. In a more complete theory, e.g. the Standard Model, the effects of the interactions of the Higgs doublet with the $SU(2) \times U(1)$ vector bosons as well as with the Standard Model fermions should be included. We have not attempted to account for these effects in the present study. We expect that these effects are important; for example it is known that the inclusion of the longitudinal vector bosons into the final state lead to additional factorially growing contributions at tree level [11,12]. A more detailed understanding of the effects of Higgs interactions with vectors and fermions, especially including the loop corrections is currently lacking and clearly deserves a separate investigation. Our aim here was to motivate the Higgspllosion phenomenon and its applications in the context of a simple Higgs-like scalar model.

In summary, the existence of a microscopic light Higgs boson introduces arguably two fundamental issues into the Standard Model: one is the fine-tuning of the Higgs boson mass, and the other is an apparent breakdown of perturbativity for high-multiplicity processes. However, we find that self-interactions of the Higgs boson provide the mechanisms to heal the Standard Model and retain self-consistency of the theory. In case these mechanisms are realised in nature, it would be interesting to study their implications on explanations of fundamental questions in nature, e.g. the nature of dark matter or the underlying mechanism of the matter-antimatter asymmetry. One way of probing the Higgspllosion picture experimentally could be to look for observable multi-Higgs and/or multi-vector-boson processes at a future high-energy hadron collider.

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