On the ultimate precision of meson mixing observables

Thomas Jubb [*,1], Matthew Kirk [†,1], Alexander Lenz [‡,1], Gilberto Tetlalmatzi-Xolocotzi[§,1]

[1] IPPP, Department of Physics, Durham University, Durham DH1 3LE, UK

Version of November 22, 2016

Abstract

Meson mixing is considered to be an ideal testing ground for new physics searches. Experimental precision has greatly increased over the recent years, exceeding in several cases the theoretical precision. A possible limit in the theoretical accuracy could be a hypothetical breakdown of quark-hadron duality. We propose a simple model for duality violations and give stringent phenomenological bounds on its effects on mixing observables, indicating regions where future measurements of $\Delta\Gamma_d$, a_{sl}^d and a_{sl}^s would give clear signals of new physics. Finally, we turn our attention to the charm sector, and reveal that a modest duality violation of about 20% could explain the huge difference between HQE predictions for D mixing and experimental data.

*thomas.jubb@durham.ac.uk

†m.j.kirk@durham.ac.uk

‡alexander.lenz@durham.ac.uk

§gilberto.tetlalmatzi-xolocotz@durham.ac.uk

1 Introduction

Despite having passed numerous tests, the standard model of particle physics leaves many fundamental questions unanswered which might be resolved by extensions of this model. Flavour physics is an ideal candidate for general indirect new physics searches, as well as for dedicated CP-violating studies, which might shed light on the unsolved problem of the baryon asymmetry in the Universe. For this purpose hadronic uncertainties on flavour observables have to be under control. Various flavour experiments have achieved a high precision in many observables, in several cases challenging the precision of theory calculations. LHCb in particular, as an experiment designed to study beauty and charm physics, contributes to the currently impressive status of experimental precision. As we attempt to test the SM to the highest level of precision, the question of how sure we can be about any deviations from the current theoretical predictions being evidence of new physics comes to the fore. Such a question is the subject we tackle in this paper.

Many current theory predictions rely on the Heavy Quark Expansion (HQE), and we will examine how the idea of quark-hadron duality – which is assumed by the HQE – can be tested. We will use current data from B mixing, the dimuon asymmetry, and B meson lifetimes to constrain violations of quark-hadron duality, and then see how this affects the predicted values of other observables. We also investigate how the current trouble with inclusive predictions of mixing in the charm sector can be explained through a mild violation of quark-hadron duality.

We discuss what improvements could be made in both theory and experiment in order to further constrain duality violating effects, and what level of precision would be necessary to properly distinguish between genuine new physics and merely a non-perturbative contribution to the SM calculation. In this spirit, we also provide a first attempt at improving the theory prediction, using the latest results and aggressive error estimates to see how theory uncertainties could reduce in the near future.

Our paper is organised as follows: in Sec. 2 we explain the basic ideas of duality violation in the HQE. We introduce in Sec. 2.1 a simple parameterisation for duality violation in B mixing and we derive bounds on its possible size. The dimuon asymmetry and the lifetime ratio $\tau(B_s^0)/\tau(B_d^0)$ can provide complementary bounds on duality violation, which is discussed in Sect. 2.2 and Sect. 2.3. The bounds in the B system depend strongly on the theory uncertainties, hence we present in Sect. 3 a numerical update of the mixing observables with an aggressive error estimate for the input parameters. In Sec. 4 we study possible effects of duality violation in D-mixing. We conclude in Sec. 5 with a short summary and outlook.

2 Duality violation

In 1979 the notion of duality was introduced by Poggio, Quinn and Weinberg [1] for the process $e^+ + e^- \to hadrons$.[1] The basic assumption is that this process can be well approximated by a quark level calculation of $e^+ + e^- \to q + \bar{q}$. In this work we will investigate duality in the case of decays of heavy hadrons, which are described by the heavy quark expansion (see e.g. [4–11] for pioneering papers and [12] for a recent review). The HQE is a systematic

---

[1]The concept of duality was already used in 1970 for electron proton scattering by Bloom and Gilman [2,3].

expansion of the decay rates of b hadrons in inverse powers of the heavy quark mass.

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2}\Gamma_2 + \frac{\Lambda^3}{m_b^3}\Gamma_3 + \frac{\Lambda^4}{m_b^4}\Gamma_4 + ... \, , \tag{2.1}$$

with Λ being of the order of the hadronic scale.[2] One finds that there are no corrections of order Λ/m_b and that some corrections from the order Λ^3/m_b^3 onwards are enhanced by an additional phase space factor of $16\pi^2$. The HQE assumes quark hadron duality, i.e. that the hadron decays can be described at the quark level. A violation of duality could correspond to non-perturbative terms like $\exp[-m_b/\Lambda]$, which give vanishing contributions, when being Taylor expanded around $\Lambda/m_b = 0$ (see e.g. [14] and also [15] for a detailed discussion of duality, its violations and some possible models for duality violations). To estimate the possible size of these non-perturbative terms we note first that the actual expansion parameter of the HQE is not Λ/m_b but the hadronic scale Λ normalised to the momentum release $\sqrt{M_i^2 - M_f^2}$, where M_i is the mass of the initial state and M_f the sum of the final state masses. This can be shown by an explicit derivation of the HQE. Hence the expansion parameter for the quark-level decay $b \to c\bar{c}s$, $\Lambda/\sqrt{m_b^2 - 4m_c^2}$, is considerably larger than for the decay $b \to u\bar{u}u$, where it is Λ/m_b. In other words: the less phase space is accessible in the final state, the worse is the convergence property of the HQE for this class of decays and the larger might be the hypothetical duality violating terms. The remaining phase space for B_s^0 decay into light mesons (e.g. $B_s^0 \to K^-\pi^+$, via $b \to u\bar{u}d$), due to the dominant quark level decay (e.g. $B_s^0 \to D_s^-\pi^+$, via $b \to c\bar{u}d$) and into the leading contribution to $\Delta\Gamma_s$ (e.g. $B_s^0 \to D_s^+ D_s^-$, via $b \to c\bar{c}s$) reads

$$M_{B_s^0} - M_K - M_\pi = 4.73\,\text{GeV} \, , \tag{2.2}$$

$$M_{B_s^0} - M_{D_s^+} - M_\pi = 3.26\,\text{GeV} \, , \tag{2.3}$$

$$M_{B_s^0} - 2M_{D_s^{(*)+}} = 1.43(1.15)\,\text{GeV} \, . \tag{2.4}$$

The crucial question is, whether the phase space in $B_s^0 \to D_s^{(*)+}D_s^{(*)-}$ is still large enough to ensure quark hadron duality.

To get some idea for the possible values of the expansion parameter and the non-perturbative terms in inclusive b-quark decays, we vary Λ within 0.2 and 2 GeV,[3] m_b within 4.18 and 4.78 GeV and m_c within 0.975 and 1.67 GeV:

Channel	Expansion parameter x	Numerical value	$\exp[-1/x]$
$b \to c\bar{c}s$	$\frac{\Lambda}{\sqrt{m_b^2 - 4m_c^2}} \approx \frac{\Lambda}{m_b}\left(1 + 2\frac{m_c^2}{m_b^2}\right)$	$0.054 - 0.58$	$9.4 \cdot 10^{-9} - 0.18$
$b \to c\bar{u}s$	$\frac{\Lambda}{\sqrt{m_b^2 - m_c^2}} \approx \frac{\Lambda}{m_b}\left(1 + \frac{1}{2}\frac{m_c^2}{m_b^2}\right)$	$0.045 - 0.49$	$1.9 \cdot 10^{-10} - 0.13$
$b \to u\bar{u}s$	$\frac{\Lambda}{\sqrt{m_b^2 - 4m_u^2}} = \frac{\Lambda}{m_b}$	$0.042 - 0.48$	$4.2 \cdot 10^{-11} - 0.12$

$$\tag{2.5}$$

From this simple numerical exercise one finds that duality violating terms could easily be of a similar size as the expansion parameter of the HQE. Moreover decay channels like $b \to c\bar{c}s$

---

[2]One gets different values of Λ for different observables. The numerical value of Λ has to be determined by an explicit calculation. For the case of $\Delta\Gamma_s$ one gets e.g. $\Lambda/m_b \approx 1/5$ [13] and thus $\Lambda \approx 1$ GeV.

[3]This is twice the scale one finds in $\Delta\Gamma_s$ [13].

might be more strongly affected by duality violations compared to e.g. $b \to u\bar{u}s$. Obviously duality cannot be proved directly, because this would require a complete solution of QCD and a subsequent comparison with the HQE expectations, which is clearly not possible. To make statements about duality violation in principle two strategies can be performed:

a) Study simplified models for QCD, e.g. the t'Hooft model (a two-dimension model for QCD, see e.g. [14–19]) and develop models for duality violations, like instanton-based and resonance-based models (see e.g. [14, 15]).

b) Use a pure phenomenological approach, by comparing experiment with HQE predictions.

In this work we will follow strategy b) and use a simple parameterisation for duality violation in mixing observables and lifetime ratios, which will be most pronounced for the $b \to c\bar{c}s$ channel. At this stage it is interesting to note that for many years there have been problems related to applications of the HQE for inclusive b-hadron decays and most of them seemed to be related to the $b \to c\bar{c}s$ channel (see e.g. [12] for a discussion): the Λ_b lifetime, the inclusive semi-leptonic branching ratio of B mesons as well as the average number of charm quarks per b-decay (missing charm puzzle), the B_s^0 lifetime (see Fig. 1 for a time evolution of the experimental value) and the dimuon asymmetry. All of these problems (except the dimuon
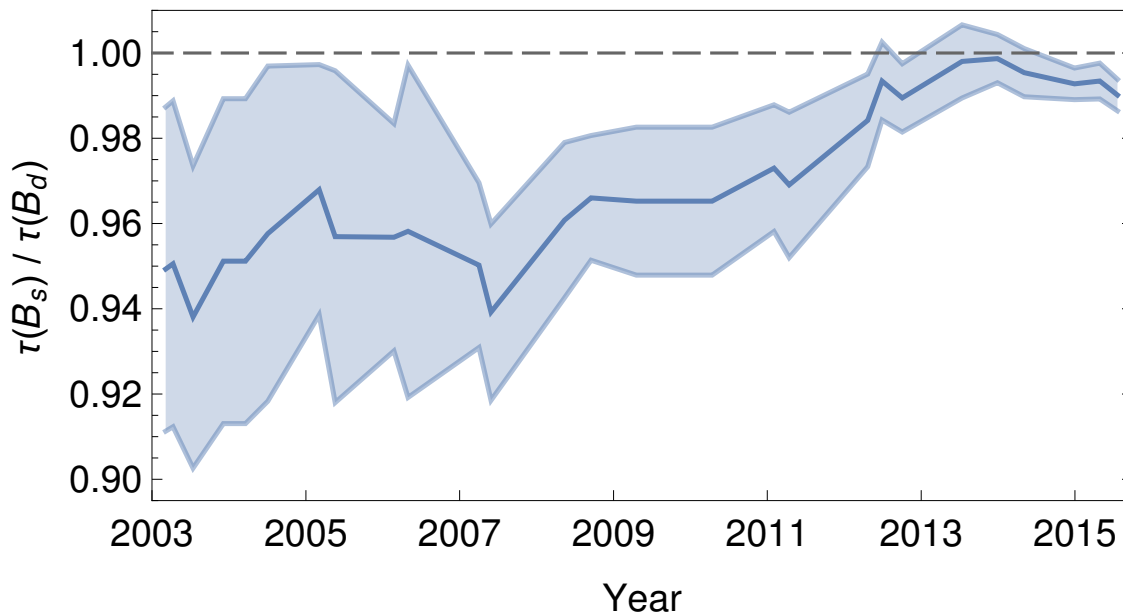

Figure 1: Historical values of the lifetime ratio $\tau(B_s^0)/\tau(B_d^0)$ as reported by HFAG [40] since 2003. The solid line shows the central value and the shaded line indicates the 1σ region, the dotted line corresponds to the theory prediction, which is essentially one, with a tiny uncertainty.

asymmetry, that will be discussed later on) are currently considerably softened and huge duality violations are now ruled out by experiment [20], in particular by the measurement of the decay rate difference of neutral B_s^0 mesons, $\Delta\Gamma_s$, which is to a good approximation a $b \to c\bar{c}s$ transition. But there is still space for a small amount of duality violation - which will be quantified in this work.

We will thus investigate the decay rate difference $\Delta\Gamma_s = 2|\Gamma_{12}^s| \cos\phi_{12}^s$ (see e.g. [21] for the basic mixing formulae) in more detail. The off diagonal matrix element, Γ_{12}^s, of the B_s^0 meson mixing matrix can be determined as a double insertion of the effective weak Hamiltonian describing weak decays of B_s^0 mesons:

$$\Gamma_{12}^s = \frac{1}{M_{B_s^0}} \Im \left[\frac{i}{2} \int d^4x \langle B_s^0 | T\left\{ \mathcal{H}(x)\mathcal{H}(0) \right\} | \bar{B}_s^0 \rangle \right] . \tag{2.6}$$

According to the HQE this expression can be expanded in powers of Λ/m_b

$$\Gamma_{12}^s = \frac{\Lambda^3}{m_b^3} \left(\Gamma_3^{s,(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{s,(1)} + ... \right) + \frac{\Lambda^4}{m_b^4} \left(\Gamma_4^{s,(0)} + ... \right) + ... \tag{2.7}$$

The leading term $\Gamma_3^{s,(0)}$ has been calculated quite some time ago by [22–27], NLO-QCD corrections $\Gamma_3^{s,(1)}$ have been determined in [28–30] and sub-leading mass corrections were done in [31–33]. Corresponding lattice values were determined by [34–37].
The most recent numerical update for the mixing quantities is given in [21] (superseding the numerical predictions in [38,39]) and can be compared to the experimental values from e.g. HFAG [40]. The theory prediction uses conservative ranges for the input parameters - we will present a more aggressive estimate in Sec. 3.

$$
\begin{array}{|c||c|c|}
\hline
\text{Observable} & \text{SM} & \text{Experiment} \\
\hline\hline
\Delta M_s & (18.3 \pm 2.7)\,\text{ps}^{-1} & (17.757 \pm 0.021)\,\text{ps}^{-1} \\
\hline
\Delta\Gamma_s & (0.088 \pm 0.020)\,\text{ps}^{-1} & (0.082 \pm 0.006)\,\text{ps}^{-1} \\
\hline
a_{sl}^s & (2.22 \pm 0.27) \cdot 10^{-5} & (170 \pm 300) \cdot 10^{-5} \\
\hline
\Delta\Gamma_s/\Delta M_s & 48.1\,(1 \pm 0.173) \cdot 10^{-4} & 46.2\,(1 \pm 0.073) \cdot 10^{-4} \\
\hline
\Delta M_d & (0.528 \pm 0.078)\,\text{ps}^{-1} & (0.5055 \pm 0.0020)\,\text{ps}^{-1} \\
\hline
\Delta\Gamma_d & (2.61 \pm 0.59) \cdot 10^{-3}\,\text{ps}^{-1} & 0.66(1 \pm 10) \cdot 10^{-3}\,\text{ps}^{-1} \\
\hline
a_{sl}^d & (-4.7 \pm 0.6) \cdot 10^{-4} & (-15 \pm 17) \cdot 10^{-4} \\
\hline
\Delta\Gamma_d/\Delta M_d & 49.4\,(1 \pm 0.172) \cdot 10^{-4} & 13\,(1 \pm 10) \cdot 10^{-4} \\
\hline
\end{array}
\tag{2.8}
$$

The experimental average for a_{sl}^s has been taken from [41]. Experiment and theory agree very well for the quantities ΔM_q and $\Delta\Gamma_s$. The semileptonic asymmetries and the decay rate difference in the B_d^0 system have not been observed yet. More profound statements about the validity of the theory can be made by comparing the ratio of $\Delta\Gamma_s$ and ΔM_s, where sometheoretical uncertainties cancel and we get

$$\frac{\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\text{Exp}}}{\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\text{SM}}} = 0.96 \pm 0.22 \ (\text{at } 95\% \text{ C.L.}). \tag{2.9}$$

The central value shows a very good agreement of experiment and HQE predictions. The remaining uncertainty leaves some space for new physics effects or for violations of duality. We have taken here the $2\,\sigma$ range of the experimental value, while we consider the theory range to cover all allowed values. Thus we conclude that in the most sensitive decay channel, $b \to c\bar{c}s$, duality seems to be violated by at most 22%. In the next chapter we try to investigate these possibilities a little more in detail.

Alternatively to Eqs.(2.6),(2.7), $\Delta\Gamma_s$ can in principle be determined from exclusive decays, avoiding thus the expansion in Λ/m_b:

$$\Delta\Gamma_s = \Gamma_{B_L} - \Gamma_{B_H} = \sum_f |\langle f|\mathcal{H}|B_L\rangle|^2 - \sum_f |\langle f|\mathcal{H}|B_H\rangle|^2 = 4\Re\left[pq^* \sum_f \langle \bar{B}_s^0|\mathcal{H}\ f\rangle\langle f|\mathcal{H}|B_s^0\rangle\right],$$

(2.10)

where f denotes final states common to B_s^0 and $\bar{B}_s^0$. The coefficients p and q describe the basis change from B_s^0, $\bar{B}_s^0$ to B_L, B_H (see e.g. [21] for the basic mixing formulae). However, the theoretical determination of decay rates for exclusive B_s^0 decays is in general an unsolved hadronic problem, but for certain cases (not including e.g. $B_s \to D_s^{+(*)}D_s^{-(*)}$!) factorisation seems to be applicable, see e.g. [42–44]. Using Eq.(2.10), $\Delta\Gamma_s$ can be estimated by summing up exclusive branching ratios, assuming naive factorisation. This approach has been followed e.g. in [45] and [46]. Using LO-QCD expressions and taking into account a certain number of 2- and 3-body decays the authors of [46] obtained

$$\Delta\Gamma_s^{\text{exclusive}} = (0.111 \pm 0.057)\,\text{ps}^{-1}\,,$$

(2.11)

which is consistent with the direct measurement and with the HQE determination, but suffers from much larger uncertainties.

From a theoretical point of view there is an interesting limiting case, which should, however, not be used for phenomenological applications: $m_c \to \infty$, $m_b \to 2m_c$ (Shifman-Voloshin limit [47]) and neglecting certain terms of order $1/N_c$ one gets [45, 48]:

$$2Br(D_s^{(*)+}D_s^{(*)-})^{\text{SV}-\text{limit}} = \Delta\Gamma_s^{\text{SV}-\text{limit}} \frac{\tau(B_s^0)}{\cos(\phi_{12}^s)}\,,$$

(2.12)

which is, however, not full-filled to a high precision by experiment. Using the experimental result for the branching ratios [49] as an input in Eq.(2.12) we get a decay rate difference of (see also [45, 48])

$$\Delta\Gamma_s^{\text{SV}-\text{limit}} \le (0.060 \pm 0.019)\,\text{ps}^{-1}\,,$$

(2.13)

which is considerably lower than the direct determination. On the other hand this result shows that $D_s^{(*)+}D_s^{(*)-}$ final states give the dominant contribution to $\Delta\Gamma_s$.

Using the above limit and setting further $\alpha_s = 0$, Aleksan et al. [45] could show that both the HQE approach and the exclusive approach yield analytically the following result

$$\Delta\Gamma_s^{\text{Aleksan}} = \frac{G_F^2 m_b^2 M_{B_s^0} f_{B_s}^2}{4\pi} |V_{cb}^* V_{cs}|^2 \sqrt{1 - 4\frac{m_c^2}{m_b^2}} \approx 0.13\,\text{ps}^{-1}\,.$$

(2.14)

This value is now considerably above the direct measurement. Despite looking like a proof of duality, we would like to add some critical comments: we can reproduce Eq.(2.14) from the full HQE expressions for $\Delta\Gamma_s$ by taking only the leading CKM structure into account, by setting α_s to zero, by setting the bag parameters to one (for the $S - P$ operator we also set $M_{B_s} = m_b + m_s$) and we neglect some terms of order m_c^2/m_b^2, while keeping it in the square root in Eq.(2.14). These approximations lead to the effect that the result of Eq.(2.14) is more than 50% larger than the full theory prediction. This deviation is much larger than the current experimental and theoretical uncertainties in $\Delta\Gamma_s$. Thus we conclude that the result of [45] has no practical relevance for our aim of constraining possible sizes of duality violation.

5

2.1 B mixing

The off-diagonal elements Γ_{12}^s and M_{12}^s of the mixing matrix for B_s^0 mesons can be expressed as

$$\Gamma_{12}^s = -\sum_{x=u,c}\sum_{y=u,c}\lambda_x\lambda_y\Gamma_{12}^{s,xy} \ , \quad M_{12}^s = \lambda_t^2\tilde{M}_{12}^s \ . \tag{2.15}$$

Here we have separated the CKM dependence, $\lambda_q = V_{qs}^*V_{qb}$. $\Gamma_{12}^{s,xy}$ describes the on-shell part of a B_s^0 mixing diagram with internal x and y quarks, $x,y \in \{u,c\}$, and $\tilde{M}_{12}^s$ describes the off-shell part without CKM factors.

For simplicity we give only the expressions for B_s^0 mesons when modifications for B_d^0 mesons are obvious, and we will explicitly present expressions for the B_d^0 sector only when they are non-trivial. The physical observables ΔM_s, $\Delta\Gamma_s$ and a_{sl}^s are related to the ratio Γ_{12}^s/M_{12}^s - there several theory uncertainties are cancelling - via

$$\frac{\Delta\Gamma_s}{\Delta M_s} = -\Re\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right) \ , \qquad a_{sl}^s = \Im\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right) \ . \tag{2.16}$$

Using the unitarity of the CKM matrix we can simplify Γ_{12}^s/M_{12}^s.

$$-\frac{\Gamma_{12}^s}{M_{12}^s} = \frac{\Gamma_{12}^{s,cc}}{\tilde{M}_{12}^s} + 2\frac{\lambda_u}{\lambda_t}\frac{\Gamma_{12}^{s,cc} - \Gamma_{12}^{s,uc}}{\tilde{M}_{12}^s} + \left(\frac{\lambda_u}{\lambda_t}\right)^2\frac{\Gamma_{12}^{s,cc} - 2\Gamma_{12}^{s,uc} + \Gamma_{12}^{s,uu}}{\tilde{M}_{12}^s} \tag{2.17}$$

$$= -10^{-4}\left[c + a\frac{\lambda_u}{\lambda_t} + b\left(\frac{\lambda_u}{\lambda_t}\right)^2\right] \ . \tag{2.18}$$

Eq.(2.18) introduces the a, b and c notation of [30]. The way of writing Γ_{12}^s/M_{12}^s in Eq.(2.17) and Eq.(2.18) can be viewed as a Taylor expansion in the small CKM parameter λ_u/λ_t, for which we use the same the CKM input as [21] (the values were taken in 2015 from CKMfitter [50], similar values can be obtained from UTfit [51]). In addition to the CKM suppression a pronounced GIM-cancellation [52] is arising in the coefficients a and b in Eq.(2.18). With the input parameters described in [21] we get for the numerical values of a, b and c:

	B_s^0	B_d^0
c	-48.0 ± 8.3	-49.5 ± 8.5
a	$+12.3 \pm 1.4$	$+11.7 \pm 1.3$
b	$+0.79 \pm 0.12$	$+0.24 \pm 0.06$

$$\tag{2.19}$$

From this hierarchy we see, that $\Delta\Gamma_q/\Delta M_q$ is given to a very good approximation by $-0.0001c$ and a_{sl}^q by $0.0001a\Im(\lambda_u/\lambda_t)$.

Next we introduce a simple model for duality violation. Motivated by the observations in Section 2 we write to a first approximation[4]:

$$\Gamma_{12}^{s,cc} \to \Gamma_{12}^{s,cc}(1 + 4\delta) \ , \tag{2.20}$$

$$\Gamma_{12}^{s,uc} \to \Gamma_{12}^{s,uc}(1 + \delta) \ , \tag{2.21}$$

$$\Gamma_{12}^{s,uu} \to \Gamma_{12}^{s,uu}(1 + 0\delta) \ . \tag{2.22}$$

---

[4]Similar models have been used in [53–55] for penguin insertions with a $c\bar{c}$-loop.

The $c\bar{c}$ contribution is affected by a correction of 4δ, $c\bar{u}$ by δ and $u\bar{u}$ is not affected at all. Already at this stage ones sees that such a model is softening GIM cancellations in the ratio Γ^s_{12}/M^s_{12}; we get

$$\frac{\Gamma^s_{12}}{M^s_{12}} = 10^{-4} \left[c(1+4\delta) + \frac{\lambda_u}{\lambda_t}(a + \delta(6c+a)) + \frac{\lambda^2_u}{\lambda^2_t}(b + \delta(2c+a)) \right] . \tag{2.23}$$

Studying this expression, we find that the decay rate difference is mostly given by the first term on the r.h.s., so we expect $\Delta\Gamma_s/\Delta M_s \approx -c(1+4\delta) \cdot 10^{-4}$, which is equivalent to our naive starting point of comparing experiment and theory prediction for $\Delta\Gamma_s$. The semi-leptonic CP asymmetries will be dominantly given by the second term on the r.h.s., $a^s_{sl} \approx \Im(\lambda_u/\lambda_t)[a + \delta(6c+a)] \cdot 10^{-4}$. Now the duality violating coefficient δ is GIM enhanced by $(6c+a)$ compared to the leading term a. Having an agreement of experiment and theory for semileptonic CP asymmetries could thus provide very strong constraints on duality violation. We get for the observables ΔM_q, $\Delta\Gamma_q$ and a^q_{sl} the following dependence on the duality violating parameter δ:

Observable	B^0_s	B^0_d
$\frac{\Delta\Gamma_q}{\Delta M_q}$	$48.1(1+3.95\delta) \cdot 10^{-4}$	$49.5(1+3.76\delta) \cdot 10^{-4}$
$\Delta\Gamma_q$	$0.0880(1+3.95\delta) \, \mathrm{ps}^{-1}$	$2.61(1+3.76\delta) \cdot 10^{-3} \, \mathrm{ps}^{-1}$
a^q_{sl}	$2.225(1-22.3\delta) \cdot 10^{-5}$	$-4.74(1-24.5\delta) \cdot 10^{-4}$

$$\tag{2.24}$$

As expected we find that the duality violating parameter δ has a decent leverage on $\Delta\Gamma_q$ and a sizeable one on a^q_{sl}. The expressions for $\Delta\Gamma_q$ were obtained by simply multiplying the theory ratio $\Delta\Gamma_q/\Delta M_q$ with the theoretical values of the mass difference, as given in Eq.(2.8). Comparing experiment and theory for the ratio of the decay rate difference $\Delta\Gamma_s$ and the mass difference ΔM_s we found (see Eq.(2.9)) an agreement with a deviation of at most 22%. Thus the duality violation - i.e. the factor $1+3.95\delta$ in Table 2.24 - has to be smaller than this uncertainty:

$$1 + 3.95\delta \le 0.96 \pm 0.22 \Rightarrow \delta \in [-0.066, +0.046] . \tag{2.25}$$

Equivalently this bound tells us that the duality violation in the cc-channel is at most $+18.2\%$ or -26.3%, if the effect turns out to be negative. If there would also be an 22% agreement of experiment and theory for the semileptonic asymmetry a^s_{sl}, then we could shrink the bound to δ down to 0.01. Unfortunately experiment is still far away from the standard model prediction, see Eq.(2.8). However, we can turn around the argument: even in the most pessimistic scenario - i.e. having a duality violation that lifts GIM suppression - the theory prediction of a^s_{sl} can be enhanced/diminished at most to

$$a^s_{sl} = [-0.06, 5.50] \cdot 10^{-5} . \tag{2.26}$$

In the B^0_d system a comparison of experiment and theory for the ratio of decay rate difference and mass difference turns out to be tricky, since $\Delta\Gamma_d$ is not yet measured, see Eq.(2.8). If we would use the current experimental bound on the decay rate difference $\Delta\Gamma_d$, we would get artificially large bounds on δ. Looking at the structure of the loop contributions necessary to calculate Γ^d_{12} and Γ^s_{12}, one finds very similar $c\bar{c}$-, $u\bar{c}$-, $c\bar{u}$- and $u\bar{u}$-contributions. Our duality violation model is based on the phase space differences of decays like $B^0_s \to D_s D_s$ $(c\bar{c})$, $B^0_s \to D_s K$ $(u\bar{c}), (c\bar{u})$ and $B^0_s \to \pi K$ $(u\bar{u})$, which are very pronounced. On the other hand we find that the phase space differences of B^0_s and B^0_d decays are not very pronounced, i.e. the

difference between e.g. $B_s^0 \to D_s D_s$ vs. $B_d^0 \to D_s D$ is small - compared to the above differences due to different internal quarks. Hence we conclude that the duality violation bounds from the B_s^0 system can also be applied to a good approximation to the B_d^0 system. With the B_s^0 bound we get that the theory prediction of a_{sl}^d and $\Delta\Gamma_d$ can be enhanced/diminished due to duality violations at most to

$$a_{sl}^d \in [-12.4, -0.6] \cdot 10^{-4} \ , \tag{2.27}$$

$$\Delta\Gamma_d \in [1.96, 3.06] \cdot 10^{-3} \ \text{ps}^{-1} \ . \tag{2.28}$$

These numbers can be compared to the SM values obtained in [21], see Eq.(2.8). In principle any measurement of these observables outside the ranges in Eq.(2.26), Eq.(2.27) and Eq.(2.28) would be a clear indication of new physics. New physics in $\Delta\Gamma_d$ could have the very interesting effect of reducing [56] the still existing discrepancy of the dimuon asymmetry measured at D0 [57–60]. Currently a sizeable enhancement of $\Delta\Gamma_d$ is not excluded by theoretical or experimental bounds [61]. Thus it is clearly important to distinguish hypothetical duality violating effects in $\Delta\Gamma_d$ from new physics effects.

Since our conclusions (new physics or unknown hadronic effects) are quite far-reaching, we try to be as conservative as possible and we will firstly use a more profound statistical method, a likelihood ratio test. The second modification to ensure that our estimates are conservative concerns our ad-hoc ansatz in Eqs.(2.20), (2.21), (2.22), where we assumed that the cc-part is affected by duality violations four times as much as the cu-part and the uu-part is not affected at all; we can obtain more general results with the following modification

$$\Gamma_{12}^{s,cc} \to \Gamma_{12}^{s,cc}(1 + \delta^{cc}) \ , \tag{2.29}$$

$$\Gamma_{12}^{s,uc} \to \Gamma_{12}^{s,uc}(1 + \delta^{uc}) \ , \tag{2.30}$$

$$\Gamma_{12}^{s,uu} \to \Gamma_{12}^{s,uu}(1 + \delta^{uu}) \ , \tag{2.31}$$

with $\delta^{cc} \geq \delta^{uc} \geq \delta^{uu}$ and the requirement that all δs must have the same sign. Now we get for the observables

$$\frac{\Delta\Gamma_s}{\Delta M_s} = 48.1(1 + 0.982\delta^{cc} + 0.0187\delta^{uc} - 0.000326\delta^{uu}) \cdot 10^{-4} \ , \tag{2.32}$$

$$\Delta\Gamma_d = 26.1(1 + 0.852\delta^{cc} + 0.350\delta^{uc} - 0.202\delta^{uu}) \cdot 10^{-4}\text{ps}^{-1} \ , \tag{2.33}$$

$$a_{sl}^s = 2.225(1 - 7.75\delta^{cc} + 8.67\delta^{uc} + 0.0780\delta^{uu}) \cdot 10^{-5} \ , \tag{2.34}$$

$$a_{sl}^d = -4.74(1 - 8.52\delta^{cc} + 9.60\delta^{uc} - 0.0787\delta^{uu}) \cdot 10^{-4} \ . \tag{2.35}$$

In the case of $\Delta\Gamma_s$, which will be used to determine the size of the duality violating δs, the coefficients of the uu component are suppressed by more than three orders of magnitude compared to the rest and therefore neglected. For the semileptonic CP asymmetries the uu duality violating component is about two orders of magnitude lower than the rest, thus we neglect the uu component in the following. This might lead to an uncertainty of about 20% in the duality bounds for $\Delta\Gamma_d$, which we will keep in mind.

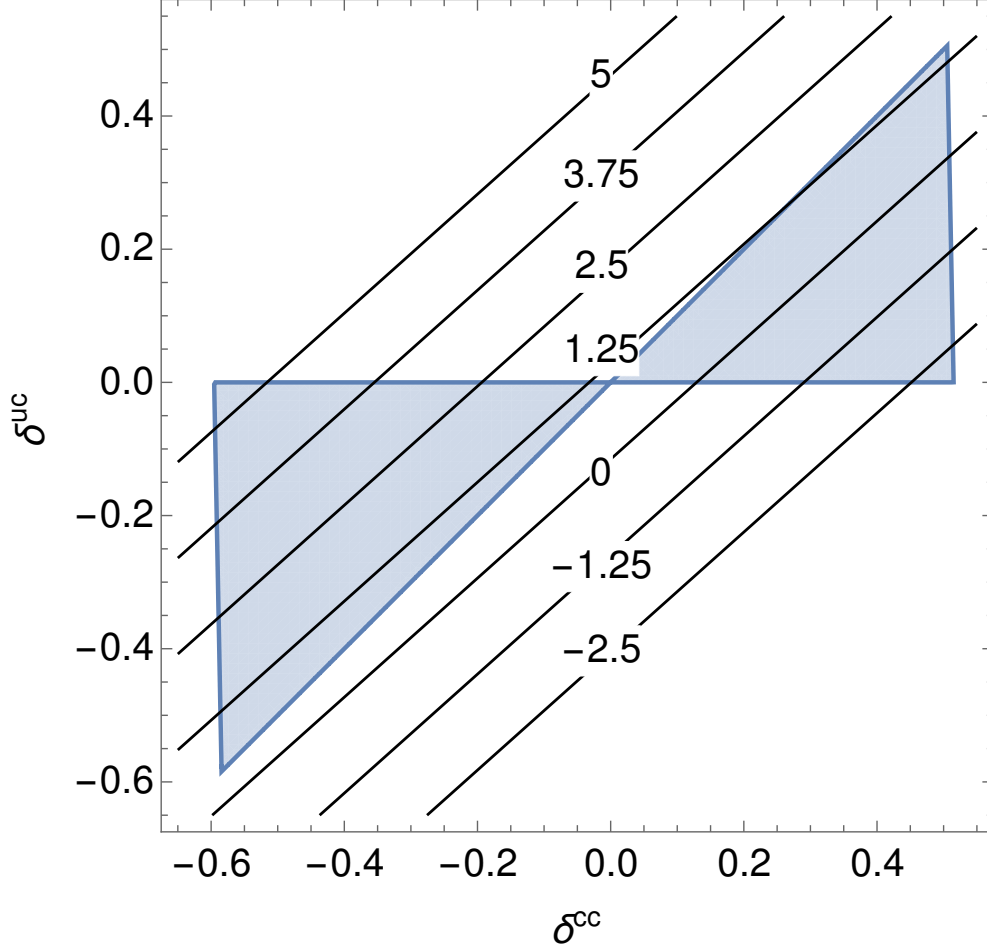Considering only δ^{cc} and δ^{uc} we get with the likelihood ratio test the bounds depicted in

Figure 2: 95% confidence limits on δ^{cc} and δ^{uc} for the B_s^0 system from a comparison of the experimentally allowed region of $(\Delta\Gamma_s/\Delta M_s)$ with the theory expression in Eq.(2.32). The allowed regions for the δs are shaded blue(grey). A deviation of the δs from zero will also affect the theory prediction of a_{sl}^s in Eq. (2.35). The modification factors of $a_{sl}^s/a_{sl}^{s,\mathrm{SM}}$ are denoted by the black lines.

Fig. 2 at a 95% confidence level. Fig. 2 shows that a duality violation of no more than 60% is allowed in either Γ_{cc}^s or in Γ_{uc}^s. We also see that it is in principle possible to see duality violation in $\Delta\Gamma_s$ but not in a_{sl}^s and vice versa. Moreover we find from the functional form of a_{sl}^s, that this quantity achieves a maximum (minimum) when $\delta^{uc} = 0$ and $\delta^{cc} < 0$ or (> 0). Our generalised parameterisation of duality violation gives now the most conservative bounds on the mixing observables

$$a_{sl}^s \in [-6.7, 12.5] \cdot 10^{-5} \,, \tag{2.36}$$

$$a_{sl}^d \in [-29, 16] \cdot 10^{-4} \,, \tag{2.37}$$

$$\Delta\Gamma_d \in [0.7, 4.2] \cdot 10^{-3}\, \mathrm{ps}^{-1} \,. \tag{2.38}$$

The duality bound on a_{sl}^d overlaps largely with the current experimental bound on this observable, here a future improvement in the measurement of a_{sl}^d will give an additional bound on duality violation.

We are now in a position to make a strong statement: any measurement outside this range, cannot be due to duality violation and it will be a strong signal for new physics.

Since the ranges in Eq.(2.36), Eq.(2.37) and Eq.(2.38) are considerably larger than the uncertainties of the corresponding standard model prediction given in Eq.(2.8) the question of how to further shrink the duality bounds is arising. Currently the bound on the duality violating parameters δ come entirely from $\Delta\Gamma_s$, where the current experimental and theoretical uncertainty adds up to $\pm22\%$. Any improvement on this uncertainty will shrink the allowed regions on δ. In Section 3 we will discuss a more aggressive estimate of the theory predictions for the mixing observable, indicating that a theory uncertainty of about $\pm10\%$ or even $\pm5\%$ in $\Delta\Gamma_s/\Delta M_s$ might come into sight. Including also possible improvements in experiment this would indicate a region for δ that is considerably smaller than the ones given in Eq.(2.36), Eq.(2.37) and Eq.(2.38). The current (and a possible future) situation is summarised in Fig. 3. On the l.h.s. of Fig. 3 $\Delta\Gamma_d$ is investigated. The current experimental bound is given by the blue region, which can be compared to the standard model prediction (green). As we have seen above, because of still sizeable uncertainties in $\Delta\Gamma_s$ duality violation of up to 60% can currently not be excluded - this would lead to an extended region (brown) for the standard model prediction including duality violation. If in future $\Delta\Gamma_s$ will be known with a precision of about 5% both in theory and experiment, then the brown region will shrink to the orange one - here also the intrinsic precision of the SM value will be reduced. In other words, currently any measurement of $\Delta\Gamma_d$ outside the brown region will be a clear signal of new physics; in future any measurement outside the orange region can be a signal of new physics. The same logic is applied for the r.h.s. of Fig. 3, where a_{sl}^d and a_{sl}^s are investigated simultaneously. For a_{sl}^s still any measurement outside the bounds in Eq.(2.36) would be a clear indication of new physics. This bound is in Fig. 3 denoted by the tiny brown region. For a_{sl}^d the current experimental region is given by the green area, which is slightly smaller than the orange region, which is indicating the theory prediction including duality violation. Future improvement in experiment and theory for $\Delta\Gamma_s$ will reduce the orange region to the dark blue one and then any measurement outside the dark blue region will be a clear signal of new physics.

In addition we can ask if there are more observables that will be affected by the above discussed duality violations. An obvious candidate is the dimuon asymmetry, which depends on a_{sl}^d, a_{sl}^s and $\Delta\Gamma_d$. This will be discussed in Sec. 2.2. Another candidate is the the lifetime
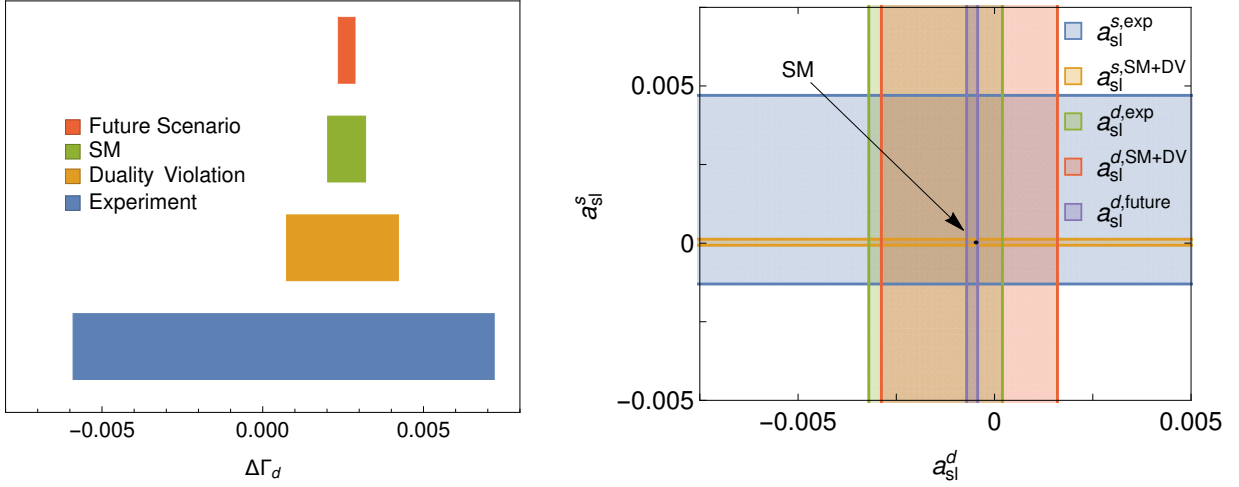
Figure 3: Comparison of SM prediction (green), SM + duality violation (brown), SM + duality violation in future (orange) and current experimental (blue) bound for $\Delta\Gamma_d$ (l.h.s.). On the r.h.s. the experimental bounds on a_{sl}^d (green) and a_{sl}^s (blue) are shown in comparison to their theory values. The uncertainties of the SM predictions are too small to be resolved, the regions allowed by duality violation are shown in brown (a_{sl}^s) and orange (a_{sl}^d). Any measurement outside these duality allowed theory regions will be a clear indication for new physics. For a_{sl}^d the duality allowed region (orange) has a pronounced overlap with the experimental one, in future this region could be shrinking to the dark blue region. The theory uncertainties for the future duality region of a_{sl}^s are so small, that they cannot be resolved in the plot.

ratio $\tau(B_s^0)/\tau(B_d^0)$, where the dominant diagrams are very similar to the mixing ones, this observable will be studied further in Sec. 2.3.

2.2 Duality bounds from the dimuon asymmetry

The D0 collaboration has measured the like-sign dimuon asymmetry finding consistently deviations with the expected value from the Standard Model [57–60]. The most recent experimental determination found a discrepancy of 3.0 σ when interpreted as the result of CP violation in mixing and interference given in terms of the semileptonic asymmetries a_{sl}^s, a_{sl}^d and the life time difference $\Delta\Gamma_d$ respectively, as suggested by [56] and further improved by [62].

Thus we want to investigate the possibility of explaining the discrepancy between theory and experiment as an effect of duality violation. The residual like-sign dimuon charge asymmetry A_{CP} reads

$$A_{CP} = C_{sl}^s a_{sl}^s + C_{sl}^d a_{sl}^d + C_{int} \frac{\Delta\Gamma_d}{\Gamma_d}, \tag{2.39}$$

with coefficients that can be determined using the information provided in [60], we also include a further correction factor in the interference contribution C_{int}, as suggested by [62]. We obtain a standard model estimate for A_{CP} of

$$A_{CP}^{\rm SM} = (-2.61 \pm 0.637) \cdot 10^{-4}. \tag{2.40}$$

Using our simple model for duality violation, see Eq. (2.24), we get for the theory prediction of A_{CP} after including duality violating effects

$$A_{CP} = -2.61(1 - 7.17\delta) \cdot 10^{-4}. \tag{2.41}$$

This can be compared to the experimental result provided by D0 [60]

$$A_{CP} = (-2.35 \pm 0.84) \cdot 10^{-3}. \tag{2.42}$$

We find that there is an agreement between experiment and theory if δ lies in the following region (95 % confidence level)

$$-2.01 \leq \delta \leq -0.23. \tag{2.43}$$

This is clearly out of the range found in Eq. (**??**) from the direct constraints of mixing observables. On the other hand we find with the allowed δ-regions given in Eq. (**??**), that A_{CP} can be at most enhanced to

$$-4.52 \cdot 10^{-4} \leq A_{CP} \leq -1.06 \cdot 10^{-4}, \tag{2.44}$$

which is considerably smaller than the experimental result. This excludes the possibility of explaining the current value for A_{CP} as an effect of duality violation at the 2σ level.

Figure 4: Diagrams contributing to the Γ_{12}^q (l.h.s.) and diagrams contributing to the lifetime of a B_q^0-meson (r.h.s.).

2.3 Duality bounds from lifetime ratios

Very similar diagrams to the ones in Γ_{12}^q arise in the lifetime ratio $\tau(B_s^0)/\tau(B_d^0)$, see Fig. 4. The obvious difference between the two diagrams is the trivial exchange of b and q lines at the right end of the diagrams. A more subtle and more important difference lies in the possible intermediate states, when cutting the diagrams in the middle. In the case of lifetimes all possible intermediate states that can originate from a $x\bar{y}$ quark pair, can arise. In the case of mixing, we have only the subset of all intermediate states into which both B_q^0 and $\bar{B}_q^0$ can decay. Thus one would expect that duality works better in the lifetimes than in mixing. Independent of this observation, our initial argument that the phase space for intermediate $c\bar{c}$-states is smaller than the one for intermediate $u\bar{c}$-states, which is again smaller that the $u\bar{u}$-case, still holds. Hence we assume that the $x\bar{y}$-loop for the lifetime ratio, has the same duality violating factor δ^{xy} as the $x\bar{y}$-loop for Γ_{12}^q. It turns out that the largest weak annihilation contribution to the B_s^0 lifetime is given by a cc-loop, while for the B_d^0 lifetime a uc-loop is dominating. This observation tells us that duality will not drop out in the lifetime ratio, because the dominating contributions for B_s^0 and B_d^0 are affected differently. Using our above model and modifying the cc-loop with a factor $1 + 4\delta$ and the uc-loop with a factor $1 + \delta$, we get with the expressions in [12, 63–65]

$$\frac{\tau(B_s^0)}{\tau(B_d^0)} = 1.00050 \pm 0.00108 - 0.0225\,\delta\,. \tag{2.45}$$

Unfortunately the standard model prediction relies strongly on lattice calculations that are already 15 years old [66] and no update has been performed since then. For a more detailed discussion of the status of lifetime predictions, see [12]. Nevertheless, one finds a big impact of the duality violating factor δ on the final result. A value of $\delta = 1$ would have huge effects, compared to the central value within the standard model and its uncertainty.

Our theory prediction can be compared to the current experimental value for the lifetime ratio [40]

$$\frac{\tau(B_s^0)}{\tau(B_d^0)} = 0.990 \pm 0.004\,. \tag{2.46}$$

If the tiny deviation between theory and experiment is attributed to duality violation, then we get an allowed range for δ of

$$\delta \in [+0.13, +0.80] \text{ (likelihood ratio 95\%)}\,. \tag{2.47}$$

There is currently a discrepancy of about 2.5σ between experiment (Eq.(2.46)) and theory (Eq.(2.45)) and this difference could stem from new physics or a sizeable duality violation

of $\delta \approx 0.5$ in lifetimes, on the other hand we would like to remind the reader to the time evolution of the lifetime measurements shown in Fig.1. The allowed region of the duality
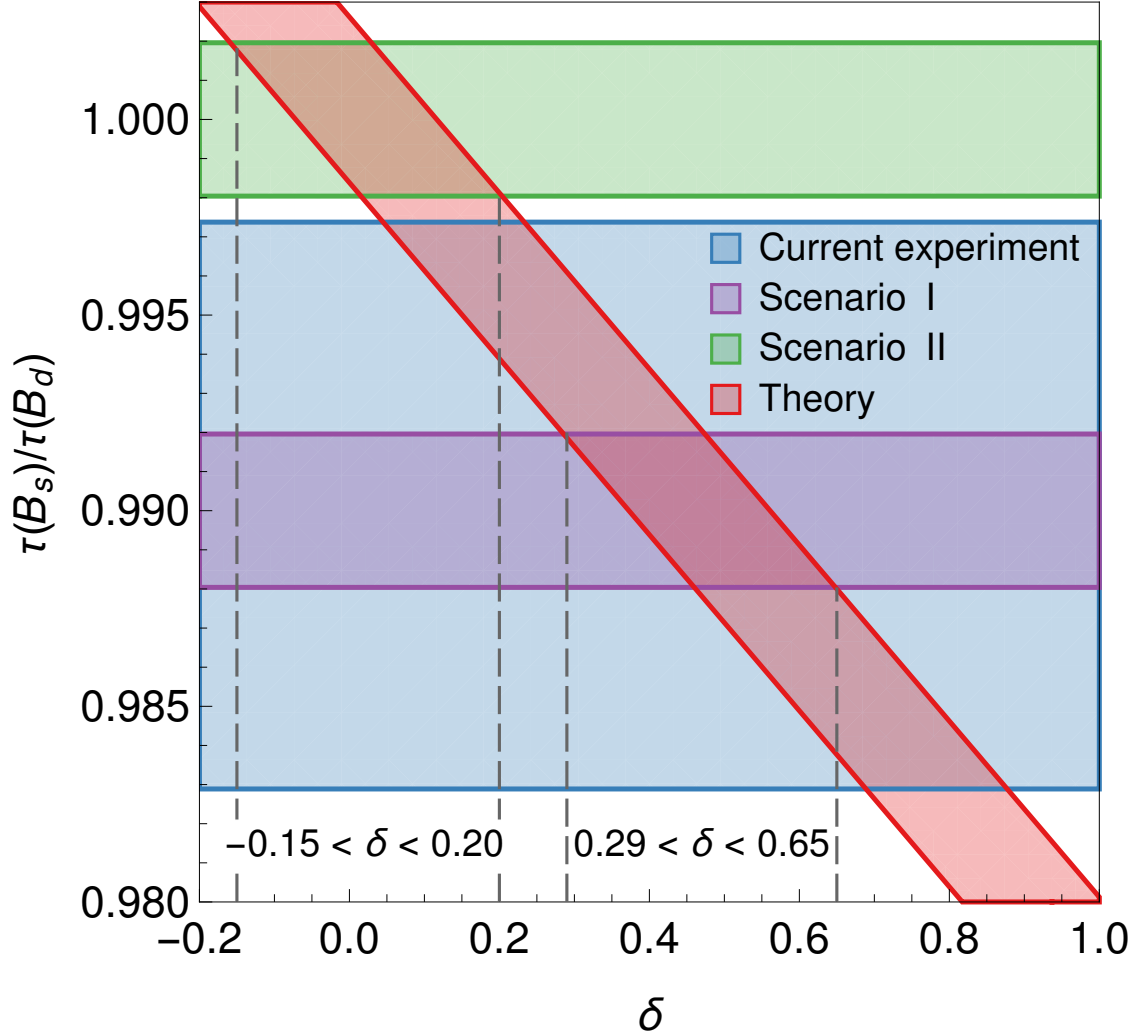

Figure 5: Duality bounds extracted from the lifetime ratio $\tau(B_s^0)/\tau(B_d^0)$. The red band shows the theoretical expected value, see Eq.(2.45) of the lifetime ratio in dependence of the δ. The current experimental bound is given by the blue region and the overlap of both gives the current allowed region δ, indicated in Eq.(2.47). The future scenarios are indicated by the violet band (Scenario I) and the green band (Scenario II). Again the overlap of the future scenarios with the theory prediction gives future allowed regions for δ - in this figure the naive overlap of both regions is shown, this corresponds to a linear addition of uncertainties and leads thus to slightly bigger ranges of δ compared to the text.

violating parameter δ can be read off Fig. 5, where the current experimental bound from Eq.(2.46) is given by the blue region and theory prediction including hypothetical duality violation by the red region. It goes without saying that 2.5 standard deviations is much too little to justify profound statements, thus we consider next future scenarios where the experimental uncertainty of the lifetime ratio will be reduced to ± 0.001.

- Scenario I: the central value will stay at the current slight deviation from one:

$$\frac{\tau(B_s^0)}{\tau(B_d^0)}^{\text{Scenario I}} = 0.990 \pm 0.001 \,. \qquad (2.48)$$

This scenario corresponds to a clear sign of duality violation or new physics in the lifetime ratio. Assuming the first one, we get a range of δ of (see the violet region in Fig. 5)

$$\delta \in [0.34, 0.60] \text{ likelihood ratio } 95\% \,. \qquad (2.49)$$

Thus the lifetime ratio requires large values of δ. Our final conclusions depend now on the future developments of $\Delta\Gamma_s$. Currently $\Delta\Gamma_s$ requires small values of δ, which is in contrast to scenario I. Thus we have to assume additional new physics effects - either in mixing or in lifetimes - that might solve the discrepancy. If in future the theory value of $\Delta\Gamma_s$ will go up sizeable or the experimental value will go down considerably, then mixing might also require a big value of δ and we then would have duality violation as a simple solution for explaining discrepancies in both lifetimes and B_s^0 mixing.

- Scenario II: the central value will go up to the standard model expectation:

$$\frac{\tau(B_s^0)}{\tau(B_d^0)}^{\text{Scenario II}} = 1.000 \pm 0.001 \,, \qquad (2.50)$$

In that case we will find only a small allowed region for δ around zero, see the green region in Fig. 5

$$\delta \in [-0.11, 0.15] \text{ likelihood ratio } 95\% \,. \qquad (2.51)$$

The above region is, however, still larger than the one obtained from $\Delta\Gamma_s$. New lattice determinations of lifetime matrix elements might change this picture and in the end the lifetime ratio might also lead to slightly stronger duality violating bounds than $\Delta\Gamma_s$. Again our final conclusion depends on future developments related to $\Delta\Gamma_s$. If both experiment and theory for mixing stay at their current central values, we simply get very strong bounds on δ. If theory or experiment will change in future, when we could have indications for deviations in mixing, which have to be compared to the agreement of experiment and theory for lifetimes in Scenario II.

In Section 3 we will discuss a possible future development of future theory predictions for mixing observables.

3 Numerical Updates of Standard Model Predictions

We have already pointed out that more precise values of $\Delta\Gamma_s$ are needed to derive more stringent bounds on duality violation in the B system. Very recently the Fermilab MILC collaboration presented a comprehensive study of the non-perturbative parameters that enter B-mixing [67].[5] A brief summary of their results reads:

---

[5]A first numerical analysis with this new inputs was already performed in [68]; but the authors put their emphasis on the implications for the correlation between $\Delta M_{s,d}$ and ε_K in models with constrained MFV and implications for $\Delta\Gamma_{s,d}$ have not been analysed.

- Improved numerical values for the non-perturbative matrix elements $\langle Q \rangle$, $\langle Q_S \rangle$, $\langle \tilde{Q}_S \rangle$, $\langle R_0 \rangle$, $\langle R_1 \rangle$ and $\langle \tilde{R}_1 \rangle$ that are necessary for $\Delta\Gamma_q$ and ΔM_q. Hence we have numerical values for all operators that are arising up to dimension seven in the HQE, up to R_2 and R_3, which are still unknown and can only be estimated by assuming vacuum insertion approximation.

- The results provide a very strong confirmation of vacuum insertion approximation. All their bag parameters turn out to be in the range of 0.8 to 1.2. Sometimes in the literature different normalisations of the matrix elements are used, that lead to values of the bag parameters which differ from one in vacuum insertion approximation, see e.g. the discussion in [21]. The definitions in [67] are all consistent with $B = 1 \pm 0.2$ in vacuum insertion approximation.

- The numerical values of $f_{B_q}^2 B$ are larger than in most previous lattice calculations.

Based on these new results we perform a more aggressive - compared to the recent study in [21] - numerical analysis of the SM predictions, where we try to push the current theory uncertainties to the limits. In particular we will modify the predictions in [21] by using

- Most recent values of the CKM parameter from CKMfitter [50] (similar values can be obtained from UTfit [51]).

- New Fermilab MILC results for the bag parameters of Q, $\tilde{Q}_S$, R_0, R_1 and $\tilde{R}_1$. We do not try to average with other lattice results, e.g. the values given by FLAG [69].

- Assume vacuum insertion approximation for R_2 and R_3 with a small uncertainty of $B = 1 \pm 0.2$. We note that this is not clearly justified yet and it has to be confirmed by independent determinations of the corresponding bag parameters.

- Use results derived from equations of motion $\tilde{B}_{R_3} = 7/5 B_{R_3} - 2/5 B_{R_2}$ and $\tilde{B}_{R_2} = -B_{R_2}$ [31].

We get with the new parameters the following predictions for the two neutral B systems, which are compared with the more conservative theory predictions [21] and the experimental values from HFAG [40], that were already given in Eq.(2.8).

Observable	SM - conservative	SM - aggressive	Experiment
ΔM_s	$(18.3 \pm 2.7)\,\mathrm{ps}^{-1}$	$(20.11 \pm 1.37)\,\mathrm{ps}^{-1}$	$(17.757 \pm 0.021)\,\mathrm{ps}^{-1}$
$\Delta\Gamma_s$	$(0.088 \pm 0.020)\,\mathrm{ps}^{-1}$	$(0.098 \pm 0.014)\,\mathrm{ps}^{-1}$	$(0.082 \pm 0.006)\,\mathrm{ps}^{-1}$
a_{sl}^s	$(2.22 \pm 0.27) \cdot 10^{-5}$	$(2.27 \pm 0.25) \cdot 10^{-5}$	$(-7.5 \pm 4.1) \cdot 10^{-3}$
$\frac{\Delta\Gamma_s}{\Delta M_s}$	$48.1\,(1 \pm 0.173) \cdot 10^{-4}$	$48.8\,(1 \pm 0.125)$	$46.2\,(1 \pm 0.073) \cdot 10^{-4}$
ΔM_d	$(0.528 \pm 0.078)\,\mathrm{ps}^{-1}$	$(0.606 \pm 0.056)\,\mathrm{ps}^{-1}$	$(0.5055 \pm 0.0020)\,\mathrm{ps}^{-1}$
$\Delta\Gamma_d$	$(2.61 \pm 0.59) \cdot 10^{-3}\,\mathrm{ps}^{-1}$	$(2.99 \pm 0.52) \cdot 10^{-3}\,\mathrm{ps}^{-1}$	$0.66(1 \pm 10) \cdot 10^{-3}\,\mathrm{ps}^{-1}$
a_{sl}^d	$(-4.7 \pm 0.6) \cdot 10^{-4}$	$(-4.90 \pm 0.54) \cdot 10^{-4}$	$(-1.5 \pm 1.7) \cdot 10^{-3}$
$\frac{\Delta\Gamma_d}{\Delta M_d}$	$49.4\,(1 \pm 0.172) \cdot 10^{-4}$	$49.3\,(1 \pm 0.149)$	$13\,(1 \pm 10) \cdot 10^{-3}$

$$(3.1)$$

The new theory values for ΔM_q and $\Delta\Gamma_q$ are larger by about 20% than the ones presented in [21] and they are further from experiment. The dominant source for this enhancement is the new value of $\langle Q \rangle$. For the ratios $\Delta\Gamma_q/\Delta M_q$ and a_{sl}^q the central values are only slightly

enhanced. The overall error shrinks by about a factor of two for ΔM_s and also sizeably for ΔM_d, $\Delta \Gamma_q$ and the ratios $\Delta \Gamma_q / \Delta M_q$. For the semileptonic asymmetries the effect is less pronounced.

Taking the deviations above seriously, we can think about several possible interpretations:

1. Statistical fluctuations in the experimental results of the order of three standard deviations might explain the deviation in $\Delta \Gamma_s$, while the deviation in ΔM_s cannot be explained by a fluctuation in the experiment.

2. Duality violations alone cannot explain these deviations, because they have no visible effects on ΔM_q.

3. The lattice normalisation for $f_B^2 B$ is simply too high, future investigations will bring down the value and there is no NP in mixing. Currently there is no foundation for this possibility, but we try to leave no stone unturned. Since $f_B^2 B$ cancels in the ratio of mass and decay rate difference, we can use the new values to give the most precise SM prediction of $\Delta \Gamma_s$ via

$$\frac{\Delta \Gamma_s}{\Delta M_s} \cdot 17.757 \, \text{ps}^{-1} (\equiv \Delta M_s^{\text{exp}}) = 0.087 \pm 0.010 \, \text{ps}^{-1} \,. \tag{3.2}$$

Now the theory error is very close to the experimental one and it would be desirable to have more precise values in theory and in experiment. In that case we also get an indication of the short-term perspectives for duality violating bounds. The above numbers indicate an uncertainty of ± 0.145 for the ratio $\Delta \Gamma_s / \Delta M_s$, which corresponds - in the case of a perfect agreement of experiment and theory - to a bound on δ of ± 0.037. This would already be a considerable improvement compared to the current situation.

4. Finally the slight deviation might be a first hint for NP effects.

   (a) To explain the deviation in the decay rate difference one needs new physics effects in tree level decays, while deviation in M_{12} might be solved by new physics effects in loop contributions.

   (b) In principle one can think of the possibility of new tree-level effects that modify both $\Delta \Gamma_s$ and ΔM_s, but which cancels in the ratio. ΔM_s is affected by a double insertion of the new tree-level operators. Following the strategy described in e.g. [61], we found, however, that the possible effects on the mass difference are much too small.

   (c) Finally there is also the possibility of having a duality violation of about 20% in $\Delta \Gamma_s$, while the effect in ΔM_s is due to new physics in loops. This possibility can be tested in future by more precise investigations of the lifetime ratio $\tau(B_s^0)/\tau(B_d^0)$.

In order to draw any definite conclusions about these interesting possibilities, we need improvements in several sectors: from experiment we need more precise values for $\Delta \Gamma_s$ and $\tau(B_s^0)/\tau(B_d^0)$. A first measurement of $\Delta \Gamma_d$ will also be very helpful. A measurement of the semileptonic asymmetries outside the duality-allowed regions would already be a clear manifestation of new physics in the mixing system. From the theory side we need an improvement in the three dominant sources of theoretical errors in Γ_{12}. These are in ranked order:

1. Dimension 7 operators $\Rightarrow$ a first principle determination of the dimension 7 operators $B_{R_{2,3}}$ and the corresponding colour-rearranged ones.

2. Dimension 6 operators $\Rightarrow$ Independent non-perturbative determinations (lattice, sum rules) of the matrix elements of Q, Q_S, $\tilde{Q}_S$, R_0, R_1 and $\tilde{R}_1$.

3. Renormalisation scale dependence $\Rightarrow$ NNLO QCD calculations for the perturbative part of Γ_{12}, i.e. the coefficients $\Gamma_3^{(2)}$ and $\Gamma_4^{(1)}$ in Eq.(2.7).

These improvements seem possible in the next few years and they might lead to a reduction of the theory error as low as 5% and thus might be the path to a detection of new physics effects in meson mixing.

4 D-mixing

D-mixing is by now experimentally well established and the values of the mixing parameters are quite well measured [40]:

$$x = (0.37 \pm 0.16) \cdot 10^{-2} \,, \quad y = (0.66^{+0.07}_{-0.10}) \cdot 10^{-2} \,. \tag{4.1}$$

Using $\tau(D^0) = 0.4101$ ps [70], this can be translated into

$$\Delta M_D = \frac{x}{\tau(D^0)} \approx 0.009 \,\mathrm{ps}^{-1} \,, \quad \Delta\Gamma_D = 2\frac{y}{\tau(D^0)} \approx 0.03 \,\mathrm{ps}^{-1} \,. \tag{4.2}$$

When trying to compare these numbers with theory predictions, we face the problem that it is not obvious if our theory tools are also working in the D system. Till now the mixing quantities have been estimated via exclusive and inclusive approaches. The exclusive approach is mostly based on phase space and $SU(3)_F$-symmetry arguments, see e.g. [71, 72]. Within this approach values for x and y of the order of 1% can be obtained. Thus, even if it is not a real first principle approach, this method seems to be our best currently available tool to describe D mixing. Inclusive HQE calculations worked very well in the B system, but their naive application to the D system gives results that are several orders of magnitude lower than the experimental result [73, 74]. Hence it seems we are left with some of the following options:

- The HQE is not valid in the charm system. This obvious solution might however, be challenged by the fact that the tiny theoretical D mixing result is solely triggered by an extremely effective GIM cancellation [52], see e.g. the discussion in [75], and not by the smallness of the first terms of the HQE expansion. A breakdown of the HQE in the charm system could best be tested by investigating the lifetime ratio of D mesons. From the theory side, the NLO QCD corrections have been determined for the lifetime ratio in [76] and it seems that the experimental measured values can be reproduced. To draw a definite conclusion about the agreement of experiment and theory for lifetimes and thus about the convergence of the HQE in the charm system, lattice evaluations of the unknown charm lifetime matrix elements are urgently needed. So this issue is currently unsettled.

- Bigi and Uraltsev pointed out in 2000 [77] that the extreme GIM cancellation in D mixing might be lifted by higher terms in HQE, i.e. the $1/m_c$-suppression of higher terms in the HQE is overcompensated by a lifting of the GIM cancellation in higher order terms. There are indications for such an effect, see [75, 78], but it is not clear whether the effect is large enough to explain the experimental mixing values. To make further progress in that direction we need the perturbative calculation of the dimension 9 and 12 terms of the OPE and an idea of how to estimate the matrix elements of the arising D=9 and D=12 operators. Hence this possibility is not ruled out yet.

- The deviation of theory and experiment could of course also be due to new physics effects. Bounds on new physics models from determining their contributions to D mixing, while more or less neglecting the standard model contributions were studied e.g. in [79].

In this work we will investigate the related question, whether relatively small duality violating effects in inclusive charm decays could explain the deviation between experiment and the inclusive approach. We consider the decay rate difference $\Delta\Gamma_D$ for this task. According to the relation (see [80])

$$\Delta\Gamma_D \leq 2|\Gamma_{12}|\,, \tag{4.3}$$

we will only study $|\Gamma_{12}|$ and test whether it can be enhanced close to the experimental value of the decay rate difference. This is of course only a necessary, but not a sufficient condition for an agreement of experiment and theory. A complete answer would also require a calculation of $|M_{12}|$, which is beyond the scope of this work. Γ_{12} consists again of three CKM contributions

$$\Gamma_{12} = -\left(\lambda_s^2\Gamma_{12}^{ss} + 2\lambda_s\lambda_d\Gamma_{12}^{sd} + \lambda_d^2\Gamma_{12}^{dd}\right)\,, \tag{4.4}$$

with the CKM elements $\lambda_d = V_{cd}V_{ud}^*$ and $\lambda_s = V_{cs}V_{us}^*$. Using again the unitarity of the CKM matrix ($\lambda_d + \lambda_s + \lambda_b = 0$) we get

$$\Gamma_{12} = -\lambda_s^2\left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd}\right) + 2\lambda_s\lambda_b\left(\Gamma_{12}^{sd} - \Gamma_{12}^{dd}\right) - \lambda_b^2\Gamma_{12}^{dd}\,. \tag{4.5}$$

The CKM-factor have now a very pronounced hierarchy, $|\lambda_b| \ll |\lambda_s|$. We find in Eq.(4.5) an extreme GIM cancellation in the CKM-leading term, while the last term without any GIM cancellation is strongly CKM suppressed. We get

$$\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd} = 1.17\bar{z}^2 - 59.5\bar{z}^3 + ...\,, \tag{4.6}$$

$$\Gamma_{12}^{sd} - \Gamma_{12}^{dd} = -2.76\bar{z} + ...\,. \tag{4.7}$$

Using our simplest duality violating model

$$\Gamma_{12}^{ss} \to \Gamma_{12}^{ss}(1 + 4\delta)\,, \tag{4.8}$$

$$\Gamma_{12}^{sd} \to \Gamma_{12}^{sd}(1 + \delta)\,, \tag{4.9}$$

$$\Gamma_{12}^{dd} \to \Gamma_{12}^{dd}(1 + 0\delta)\,, \tag{4.10}$$

we find

$$\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd} = 1.17\bar{z}^2 - 59.5\bar{z}^3 + ... + \delta\left(3.7392 - 16.5692z - 40.276z^2 + ...\right)\,,$$

$$\Gamma_{12}^{sd} - \Gamma_{12}^{dd} = -2.76\bar{z} + ... + \delta\left(1.8696 - 2.7616z - 7.4906 + ...\right)\,. \tag{4.11}$$

Eq.(4.11) shows that our duality violating model completely lifts the GIM cancellation and that even tiny values of δ will lead to an overall result that is much bigger than the usual standard model predictions within the inclusive approach. For our final conclusions we will
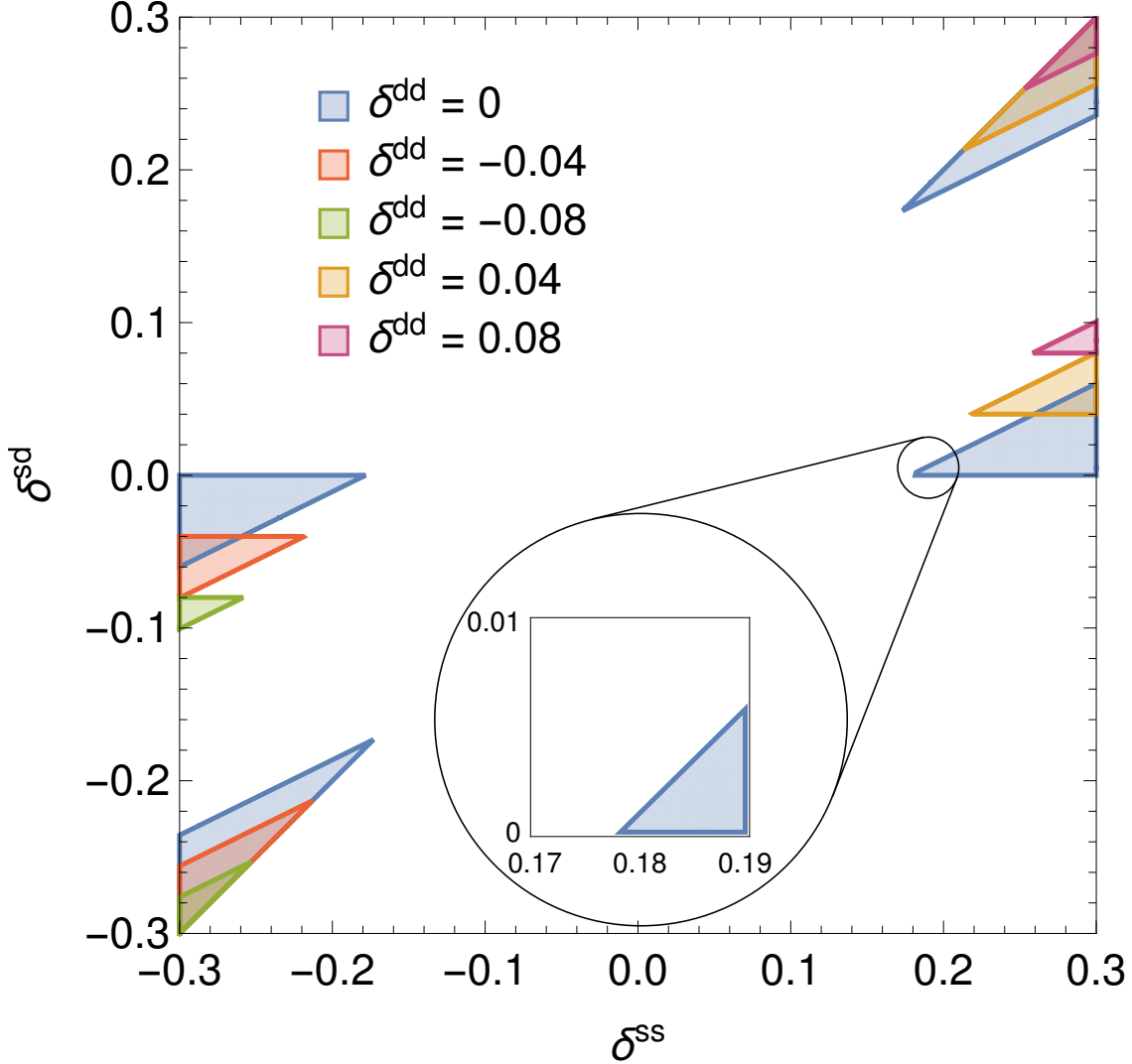

Figure 6: 95% confidence limits on δ^{ss}, δ^{sd} and δ^{dd} for the D system from a comparison of the experimentally allowed region of $\Delta\Gamma_D$ with the theory prediction based on the HQE. The allowed regions for the δs are shaded. Depending on the values of δ^{dd}, different colours are used. As expected for small values of δ the experimental value of $\Delta\Gamma_D$ can not be reproduced. Thus the area in the centre is free. Starting from values of about 20% on duality violation can explain the difference between experiment and HQE. To see more precisely, where the smallest possible value of δ lies, we have zoomed into the overlap region.

use the generalised duality violating model

$$\Gamma_{12}^{ss} \to \Gamma_{12}^{ss}(1 + \delta^{ss}) \,, \qquad (4.12)$$

$$\Gamma_{12}^{sd} \to \Gamma_{12}^{sd}(1 + \delta^{sd}) \,, \qquad (4.13)$$

$$\Gamma_{12}^{dd} \to \Gamma_{12}^{dd}(1 + \delta^{dd}) \,, \qquad (4.14)$$

20

with $\delta^{ss} \geq \delta^{sd} \geq \delta^{dd}$. Next we test for what values of δ the inclusive approach can reproduce the experimental results for $\Delta\Gamma_D$. The corresponding allowed regions for $\delta^{ss,sd,dd}$ are given as shaded areas in Fig. 6. As expected, very small values of δ cannot give an agreement between HQE and experiment, surprisingly, however, values as low as $\delta^{ss} \approx 0.18$ can explain the current difference. So a duality violation of the order of 20% in the HQE for the charm system is sufficient to explain the huge discrepancy between a naive application of the HQE and the measured value for $\Delta\Gamma_D$.

5   Summary and Conclusions

In this paper we have explored the possibility of duality violations in heavy meson decays. The study of such effects has a long tradition in flavour physics. Since the direct measurement of $\Delta\Gamma_s$ in 2012 by the LHCb collaboration huge duality violating effects are excluded [20] by experiment. But there is still space for duality violating effects of the order of 20%. Because of the constantly improving experimental precision in flavour physics it is crucial to consider corrections of the order of 20% and to investigate whether, and how, such a bound can be improved.

To do so, we introduced a simple parameterisation of duality violating effects, see Eq. (2.20) - (2.22), that relies solely on phase space arguments: the smaller the remaining phase space is in a heavy hadron decay, the larger duality violations might be. In such a model, decay rate differences depend moderately on the duality violating parameter, δ, whereas semi-leptonic asymmetries have a strong δ dependence, see Eq.(2.24). Currently we get the strongest bound on δ from Eq.(2.9)

$$\frac{\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\text{Exp}}}{\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\text{SM}}} = 0.96 \pm 0.22\,, \quad |\delta| \lesssim 0.1. \tag{5.1}$$

If the semileptonic asymmetries would agree with a similar precision between experiment and theory then the bound on δ would go down to ± 0.01. Unfortunately, the semileptonic asymmetries are not observed yet, and we have only experimental bounds. The same is true for the decay rate difference $\Delta\Gamma_d$. Thus we use our bounds on δ from $\Delta\Gamma_s$ to determine the maximal possible size of a_{sl}^q and $\Delta\Gamma_d$, if duality is violated. These regions are compared with current experimental ranges in Fig.3. Any measurement outside the region allowed by duality violation is a clear signal for new physics. We also show a future scenario in which the duality violation is further constrained by more precise values of $\Delta\Gamma_s$ both in experiment and theory.

Duality violations would also affect the still unsolved problem of the dimuon asymmetry measured by the D0 collaboration, since it depends on a_{sl}^d, a_{sl}^s and $\Delta\Gamma_d$. We found, however, that an agreement between experiment and theory for the dimuon asymmetry would require values of δ in the region of -0.2 to -2.0, which is considerably outside the allowed region found above. Taking only allowed values of δ we find that the theory prediction including duality violation is still an order of magnitude smaller than experiment. Hence duality violation cannot explain the value of the dimuon asymmetry.

We have shown that the duality violating parameter δ will also affect the lifetime ratio $\tau(B_s^0)/\tau(B_d^0)$, where we currently have a deviation of about 2.5 standard deviations between experiment and theory. Looking at the historical development of this ratio depicted in Fig. 1 one might, however, be tempted to assume a statistical fluctuation in the data. Taking

the current deviation seriously, it is either a hint for new physics or for a sizeable duality violations of the order of $\delta \sim 0.5$, which is inconsistent with our bounds on δ derived from $\Delta\Gamma_s$. Here a future reduction of the experimental error of $\tau(B_s^0)/\tau(B_d^0)$ will give us valuable insight. We have studied two future scenarios in Fig. 5, which would either point towards new physics and duality violations or stronger bounds on duality violation. It is very important to note here that the theory prediction has a very strong dependence on almost unknown lattice parameters. Any new calculation of the bag parameters $\epsilon_{1,2}$ would bring large improvements in the theory prediction for $\tau(B_s^0)/\tau(B_d^0)$.

By now we already mentioned several times necessary improvements in both experiment and theory for mixing observables and in particular for $\Delta\Gamma_s$. Therefore we presented an update of the SM predictions for the observables $\Delta\Gamma, \Delta M$, and a_{sl} in both the B_s^0 and B_d^0 systems, based on the recent Fermilab-MILC lattice results [67] for non-perturbative matrix elements, the latest CKM parameters from CKMfitter [50], and an aggressive error estimate on the unknown bag parameters of dimension seven operators. With this input the current theory error in the mixing observables could be reduced by a factor of two for ΔM_s or $1/3$ for $\Delta M_d, \Delta\Gamma_s$, and $\Delta M_s/\Delta\Gamma_s$. Thus we get for our fundamental relation to establish the possible size of duality violation

$$\frac{\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\text{Exp}}}{\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\text{SM agr.}}} = 0.95 \pm 0.15 \,. \tag{5.2}$$

As expected, the overall uncertainty drops considerably, with a theory uncertainty almost compatible with the experimental one - thus demanding more precise experimental values of $\Delta\Gamma_s$. On the other hand, we found in this new analysis that the central values of the mass differences and decay rate differences are enhanced to values of about 20% above the measurements with a significance of around 2 standard deviations. To find out whether this enhancement is real, we need several ingredients: 1) an independent confirmation of the larger values of the matrix element $\langle Q \rangle$ found by [67]. 2) a first principle calculation of $\langle R_{2,3} \rangle$ - triggered by the results of [67] we simply assumed small deviations from vacuum insertion approximation. If the new central values turn out to be correct, we will get profound implications for new physics effects and duality violation in the B-system. For a further improvement of the theory uncertainties NNLO-QCD corrections for mixing have to be calculated.

We finally focus on the charm system, where a naive application of the HQE gives results that are several orders of magnitude below the experimental values. We found the unexpected result that duality violating effects as low as 20% could solve this discrepancy. Such a result might have profound consequences on the applicability of the HQE. As a decisive test we suggest a lattice calculation of the matrix elements arising in the ratio of charm lifetimes. This ratio is free of any GIM cancellation, which are very severe in mixing.

Acknowledgements

References

[1] E. C. Poggio, H. R. Quinn and S. Weinberg, Phys. Rev. D **13** (1976) 1958. doi:10.1103/PhysRevD.13.1958

[2] E. D. Bloom and F. J. Gilman, Phys. Rev. Lett. **25** (1970) 1140. doi:10.1103/PhysRevLett.25.1140

[3] E. D. Bloom and F. J. Gilman, Phys. Rev. D **4** (1971) 2901. doi:10.1103/PhysRevD.4.2901

[4] V. A. Khoze and M. A. Shifman, Sov. Phys. Usp. **26** (1983) 387. doi:10.1070/PU1983v026n05ABEH004398

[5] M. A. Shifman and M. B. Voloshin, Sov. J. Nucl. Phys. **41** (1985) 120 [Yad. Fiz. **41** (1985) 187].

[6] I. I. Y. Bigi and N. G. Uraltsev, Phys. Lett. B **280** (1992) 271. doi:10.1016/0370-2693(92)90066-D

[7] I. I. Y. Bigi, N. G. Uraltsev and A. I. Vainshtein, Phys. Lett. B **293** (1992) 430 [Phys. Lett. B **297** (1993) 477] doi:10.1016/0370-2693(92)90908-M [hep-ph/9207214].

[8] B. Blok and M. A. Shifman, Nucl. Phys. B **399** (1993) 441 doi:10.1016/0550-3213(93)90504-I [hep-ph/9207236].

[9] B. Blok and M. A. Shifman, Nucl. Phys. B **399** (1993) 459 doi:10.1016/0550-3213(93)90505-J [hep-ph/9209289].

[10] J. Chay, H. Georgi and B. Grinstein, Phys. Lett. B **247** (1990) 399. doi:10.1016/0370-2693(90)90916-T

[11] M. E. Luke, Phys. Lett. B **252** (1990) 447. doi:10.1016/0370-2693(90)90568-Q

[12] A. Lenz, arXiv:1405.3601 [hep-ph]. A. Lenz, Int. J. Mod. Phys. A **30** (2015) 10, 1543005. doi:10.1142/S0217751X15430058

[13] A. J. Lenz, Phys. Rev. D **84** (2011) 031501 doi:10.1103/PhysRevD.84.031501 [arXiv:1106.3200 [hep-ph]].

[14] M. A. Shifman, In *Shifman, M. (ed.): At the frontier of particle physics, vol. 3* 1447-1494 [hep-ph/0009131].

[15] I. I. Y. Bigi and N. Uraltsev, Int. J. Mod. Phys. A **16** (2001) 5201 doi:10.1142/S0217751X01005535 [hep-ph/0106346].

[16] B. Grinstein, Phys. Rev. D **64** (2001) 094004 doi:10.1103/PhysRevD.64.094004 [hep-ph/0106205].

[17] B. Grinstein, PoS HEP **2001** (2001) 099.

[18] R. F. Lebed and N. G. Uraltsev, Phys. Rev. D **62** (2000) 094011 doi:10.1103/PhysRevD.62.094011 [hep-ph/0006346].

[19] I. I. Y. Bigi, M. A. Shifman, N. Uraltsev and A. I. Vainshtein, Phys. Rev. D **59** (1999) 054011 doi:10.1103/PhysRevD.59.054011 [hep-ph/9805241].

[20] A. Lenz, arXiv:1205.1444 [hep-ph].

[21] M. Artuso, G. Borissov and A. Lenz, arXiv:1511.09466 [hep-ph].

[22] J. R. Ellis, M. K. Gaillard, D. V. Nanopoulos and S. Rudaz, Nucl. Phys. B **131** (1977) 285 Erratum: [Nucl. Phys. B **132** (1978) 541]. doi:10.1016/0550-3213(77)90374-1

[23] J. S. Hagelin, Nucl. Phys. B **193** (1981) 123. doi:10.1016/0550-3213(81)90521-6

[24] E. Franco, M. Lusignoli and A. Pugliese, Nucl. Phys. B **194** (1982) 403. doi:10.1016/0550-3213(82)90018-9

[25] L. L. Chau, Phys. Rept. **95** (1983) 1. doi:10.1016/0370-1573(83)90043-1

[26] A. J. Buras, W. Slominski and H. Steger, Nucl. Phys. B **245** (1984) 369. doi:10.1016/0550-3213(84)90437-1

[27] V. A. Khoze, M. A. Shifman, N. G. Uraltsev and M. B. Voloshin, Sov. J. Nucl. Phys. **46** (1987) 112 [Yad. Fiz. **46** (1987) 181].

[28] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Phys. Lett. B **459** (1999) 631 doi:10.1016/S0370-2693(99)00684-X [hep-ph/9808385].

[29] M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C. Tarantino, JHEP **0308** (2003) 031 doi:10.1088/1126-6708/2003/08/031 [hep-ph/0308029].

[30] M. Beneke, G. Buchalla, A. Lenz and U. Nierste, Phys. Lett. B **576** (2003) 173 doi:10.1016/j.physletb.2003.09.089 [hep-ph/0307344].

[31] M. Beneke, G. Buchalla and I. Dunietz, Phys. Rev. D **54** (1996) 4419 [Phys. Rev. D **83** (2011) 119902] doi:10.1103/PhysRevD.54.4419, 10.1103/PhysRevD.83.119902 [hep-ph/9605259].

[32] A. S. Dighe, T. Hurth, C. S. Kim and T. Yoshikawa, Nucl. Phys. B **624** (2002) 377 doi:10.1016/S0550-3213(01)00655-1 [hep-ph/0109088].

[33] A. Badin, F. Gabbiani and A. A. Petrov, Phys. Lett. B **653** (2007) 230 doi:10.1016/j.physletb.2007.07.049 [arXiv:0707.0294 [hep-ph]].

[34] D. Becirevic, V. Gimenez, G. Martinelli, M. Papinutto and J. Reyes, JHEP **0204** (2002) 025 doi:10.1088/1126-6708/2002/04/025 [hep-lat/0110091].

[35] C. M. Bouchard, E. D. Freeland, C. Bernard, A. X. El-Khadra, E. Gamiz, A. S. Kronfeld, J. Laiho and R. S. Van de Water, PoS LATTICE **2011** (2011) 274 [arXiv:1112.5642 [hep-lat]].

[36] N. Carrasco *et al.* [ETM Collaboration], JHEP **1403** (2014) 016 doi:10.1007/JHEP03(2014)016 [arXiv:1308.1851 [hep-lat]].

[37] R. J. Dowdall, C. T. H. Davies, R. R. Horgan, G. P. Lepage, C. J. Monahan and J. Shigemitsu, arXiv:1411.6989 [hep-lat].

[38] A. Lenz and U. Nierste, JHEP **0706** (2007) 072 doi:10.1088/1126-6708/2007/06/072 [hep-ph/0612167].

[39] A. Lenz and U. Nierste, arXiv:1102.4274 [hep-ph].
 [40]

[40] Y. Amhis *et al.* [Heavy Flavor Averaging Group (HFAG) Collaboration], arXiv:1412.7515 [hep-ex].

[41] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **117** (2016) no.6, 061803 doi:10.1103/PhysRevLett.117.061803 [arXiv:1605.09768 [hep-ex]].

[42] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. **83** (1999) 1914 doi:10.1103/PhysRevLett.83.1914 [hep-ph/9905312].

[43] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **591** (2000) 313 doi:10.1016/S0550-3213(00)00559-9 [hep-ph/0006124].

[44] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **606** (2001) 245 doi:10.1016/S0550-3213(01)00251-6 [hep-ph/0104110].

[45] R. Aleksan, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Lett. B **316** (1993) 567. doi:10.1016/0370-2693(93)91045-O

[46] C. K. Chua, W. S. Hou and C. H. Shen, Phys. Rev. D **84** (2011) 074037 doi:10.1103/PhysRevD.84.074037 [arXiv:1107.4325 [hep-ph]].

[47] M. A. Shifman and M. B. Voloshin, Sov. J. Nucl. Phys. **47** (1988) 511 [Yad. Fiz. **47** (1988) 801].

[48] I. Dunietz, R. Fleischer and U. Nierste, Phys. Rev. D **63** (2001) 114015 doi:10.1103/PhysRevD.63.114015 [hep-ph/0012219].

[49] C. Patrignani *et al.* [Particle Data Group Collaboration], Chin. Phys. C **40** (2016) no.10, 100001. doi:10.1088/1674-1137/40/10/100001

[50] J. Charles *et al.* [CKMfitter Group Collaboration], Eur. Phys. J. C **41** (2005) 1 doi:10.1140/epjc/s2005-02169-1 [hep-ph/0406184].

[51] M. Ciuchini, G. D'Agostini, E. Franco, V. Lubicz, G. Martinelli, F. Parodi, P. Roudeau and A. Stocchi, JHEP **0107** (2001) 013 doi:10.1088/1126-6708/2001/07/013 [hep-ph/0012308].

[52] S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D **2** (1970) 1285. doi:10.1103/PhysRevD.2.1285

[53] W. F. Palmer and B. Stech, Phys. Rev. D **48** (1993) 4174. doi:10.1103/PhysRevD.48.4174

[54] I. Dunietz, J. Incandela, F. D. Snider and H. Yamamoto, Eur. Phys. J. C **1** (1998) 211 doi:10.1007/BF01245810 [hep-ph/9612421].

[55] A. Lenz, U. Nierste and G. Ostermaier, Phys. Rev. D **56** (1997) 7228 doi:10.1103/PhysRevD.56.7228 [hep-ph/9706501].

[56] G. Borissov and B. Hoeneisen, Phys. Rev. D **87** (2013) 7, 074020 doi:10.1103/PhysRevD.87.074020 [arXiv:1303.0175 [hep-ex]].

[57] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **82** (2010) 032001 doi:10.1103/PhysRevD.82.032001 [arXiv:1005.2757 [hep-ex]].

[58] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **105** (2010) 081801 doi:10.1103/PhysRevLett.105.081801 [arXiv:1007.0395 [hep-ex]].

[59] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **84** (2011) 052007 doi:10.1103/PhysRevD.84.052007 [arXiv:1106.6308 [hep-ex]].

[60] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **89** (2014) 1, 012002 doi:10.1103/PhysRevD.89.012002 [arXiv:1310.0447 [hep-ex]].

[61] C. Bobeth, U. Haisch, A. Lenz, B. Pecjak and G. Tetlalmatzi-Xolocotzi, JHEP **1406** (2014) 040 doi:10.1007/JHEP06(2014)040 [arXiv:1404.2531 [hep-ph]].

[62] U. Nierste, talk at CKM2014 in Vienna.

[63] E. Franco, V. Lubicz, F. Mescia and C. Tarantino, Nucl. Phys. B **633** (2002) 212 doi:10.1016/S0550-3213(02)00262-6 [hep-ph/0203089].

[64] N. G. Uraltsev, Phys. Lett. B **376** (1996) 303 doi:10.1016/0370-2693(96)00305-X [hep-ph/9602324].

[65] M. Neubert and C. T. Sachrajda, Nucl. Phys. B **483** (1997) 339 doi:10.1016/S0550-3213(96)00559-7 [hep-ph/9603202].

[66] D. Becirevic, PoS HEP **2001** (2001) 098 [hep-ph/0110124].

[67] A. Bazavov *et al.* [Fermilab Lattice and MILC Collaborations], arXiv:1602.03560 [hep-lat].

[68] M. Blanke and A. J. Buras, arXiv:1602.04020 [hep-ph].

[69] S. Aoki *et al.*, Eur. Phys. J. C **74** (2014) 2890 doi:10.1140/epjc/s10052-014-2890-7 [arXiv:1310.8555 [hep-lat]].

[70] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38** (2014) 090001. doi:10.1088/1674-1137/38/9/090001

[71] A. F. Falk, Y. Grossman, Z. Ligeti and A. A. Petrov, Phys. Rev. D **65** (2002) 054034 doi:10.1103/PhysRevD.65.054034 [hep-ph/0110317].

[72] A. F. Falk, Y. Grossman, Z. Ligeti, Y. Nir and A. A. Petrov, Phys. Rev. D **69** (2004) 114021 doi:10.1103/PhysRevD.69.114021 [hep-ph/0402204].

[73] H. Georgi, Phys. Lett. B **297** (1992) 353 doi:10.1016/0370-2693(92)91274-D [hep-ph/9209291].

[74] T. Ohl, G. Ricciardi and E. H. Simmons, Nucl. Phys. B **403** (1993) 605 doi:10.1016/0550-3213(93)90364-U [hep-ph/9301212].

[75] M. Bobrowski, A. Lenz, J. Riedl and J. Rohrwild, JHEP **1003** (2010) 009 doi:10.1007/JHEP03(2010)009 [arXiv:1002.4794 [hep-ph]]. M. Bobrowski, A. Lenz, J. Riedl and J. Rohrwild, arXiv:0904.3971 [hep-ph].

[76] A. Lenz and T. Rauh, Phys. Rev. D **88** (2013) 034004 doi:10.1103/PhysRevD.88.034004 [arXiv:1305.3588 [hep-ph]].

[77] I. I. Y. Bigi and N. G. Uraltsev, Nucl. Phys. B **592** (2001) 92 doi:10.1016/S0550-3213(00)00604-0 [hep-ph/0005089].

[78] M. Bobrowski, A. Lenz and T. Rauh, arXiv:1208.6438 [hep-ph].

[79] E. Golowich, S. Pakvasa and A. A. Petrov, Phys. Rev. Lett. **98** (2007) 181801 doi:10.1103/PhysRevLett.98.181801 [hep-ph/0610039].

[80] U. Nierste, arXiv:0904.1869 [hep-ph].