

# Disclosure or not, When There are Three Bidders?

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## Abstract

This paper provides a more general sufficient condition than Hummel and McAfee (2015) for optimal information disclosure in auctions when there are three bidders. We show that the optimal disclosure policy is related to the skewness of the distribution of bidders' valuations. Specifically, if the distribution is skewed to the left (right), it is optimal for the seller to reveal full (no) information to the bidders. And if it is symmetric, then there's no difference between revealing information or not.

**JEL Classification:** D44, D83, M37

## 1 Introduction

Information disclosure and advertising are pervasive in the real world, and a natural question of interest is to investigate optimal disclosure policy in the presence of preference differentiation. When consumers' preferences are differentiated, revealing product information will drive up the valuations of some consumers, yet at the same time drive down those of others. In the context of monopoly pricing, Lewis and Sappington (1994) and Johnson and Myatt (2006) state that, the optimal disclosure policy is extreme, in the sense that it is optimal for a seller to reveal either full or no information of the product to the consumers. The intuition behind is that, in the presence of preference differentiation, information disclosure will induce more dispersed distribution of consumers' posterior valuations, and thus the clockwise rotation of the demand curve. As a result, the seller faces the trade-off between a mass marketing strategy, by revealing less information, charging a low price and serving all consumers, and a niche marketing strategy, by revealing more information, charging a high price yet just serving a portion of the consumers. They show that, when information is costless, the expected profit is quasi-convex in the informativeness of advertising, and therefore the optimal disclosure policy is extreme.

Similar results are also reported in the context of auctions (Ganuza, 2004; Board, 2009; Ganuza and Panelva, 2010; Hummel and McAfee, 2015). In auctions, it is found that the seller's incentive to reveal product information increases in the number of bidders, denoted by  $n$ . Specifically, when there are just 2 bidders, it is optimal for the seller not to reveal information (Board, 2009; Hummel and McAfee, 2015). And there exists a cutoff number of  $N_0$ , such that when  $n \geq N_0$ , it is optimal for the seller to reveal full product information to the bidders, if information is costless. The intuition is that, revealing information induces more dispersed distribution of bidders' valuations, which will increase both the winning bidder's valuation and

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his information rent, and the joint effect on auction revenue is then mixed. On the other hand, increasing competition commonly squeezes out the information rent of the winning bidder, and therefore, when the number of bidders increases, the seller will have more incentive to reveal product information.

Though Board (2009) and Ganuza and Panelva (2010) show the existence of a cutoff number  $N_0$ , they do not characterize it and specify what will happen when the number of bidders is inbetween, that is, when  $2 < n < N_0$ . Hummel and McAfee (2015) provide a nice result that, when bidders' valuation distribution satisfies increasing failure rate (IFR), the cutoff number  $N_0 = 4$ . They further provide a sufficient condition for optimal information disclosure when  $n = 3$ , but that condition imposes strict monotonicity requirement on the probability density function, which is clearly quite restrictive. In this paper, we provide a more general sufficient condition for that, and show that when  $n = 3$ , the optimal disclosure policy is related to the skewness of the distribution of bidders' valuations. Our results help to fill the last gap in the long exploration of this question.

## 2 The Model and Main Results

An auctioneer sells a product to  $n \geq 2$  risk-neutral bidders, indexed by  $i \in \{1, 2, \dots, n\}$ , using a standard auction. The bidders do not know their valuations in advance, which depend on the realizations of a random variable  $X \in [0, V]$ , where  $V$  can be infinity. The cumulative distribution function of  $X$  is  $F(x)$ , with a probability density function of  $f(x)$  and finite mean  $E[X]$ .

If there's no information disclosure, all the bidders will share the same expected valuation of  $E[X]$ . Assume revealing information is costless. If the seller reveals product information prior to the auction, then the bidders' valuations,  $X_i$ 's, are  $n$  independent draws from the distribution  $F$ , and the corresponding order statistics are

$$X_{1:n} \geq X_{2:n} \geq \dots \geq X_{n:n}$$

We denote the cumulative distribution function of  $X_{i:n}$  by  $F_{i:n}(x)$ .

In a standard auction without reserve price, the bidder offering the highest bid wins and the expected payment is equal to the expected value of the second highest valuation. Therefore, the expected auction revenue is  $E[X_{2:n}]$ , under information disclosure. On the other hand, if there's no information disclosure, the expected auction revenue is clearly  $E[X]$ . We denote the difference between the expected revenues by

$$\Delta_n = E[X_{2:n}] - E[X] \tag{1}$$

Then the question is, under what condition, revealing information generates higher expected revenues for the seller?

Board (2009) and Hummel and McAfee (2015) both show that when  $n = 2$ , it is optimal for the seller to withhold product information. The idea is that, when  $n = 2$ ,  $X_{2:2} = \min\{X_1, X_2\}$  is a concave function, and by Jensen's inequality, we have  $E[X_{2:2}] = E[\min\{X_1, X_2\}] \leq$

$\min\{E[X_1], E[X_2]\} = E[X]$ . And therefore  $\Delta_2 \leq 0$  and withholding information is optimal for the seller. It is worthy of attention that, when bidders' valuations are draws from different distributions, this result for  $n = 2$  still holds.

As mentioned above, Board (2009), Ganuza and Panelva (2010) and Hummel and McAfee (2015) all show that, there exists a cutoff number  $N_0$ , and when  $n \geq N_0$ , it is optimal for the seller to reveal full product information. Under the IFR assumption, Hummel and McAfee (2015) further show that  $N_0 = 4$ , and provide a sufficient condition for optimal information disclosure when there are 3 bidders. That sufficient condition is rather restrictive, and we quote their result as below.

**Proposition 1 (Hummel & McAfee, 2015)** *For  $n = 3$ , i) if  $f(x)$  is increasing in  $x$  throughout its support, then  $\Delta_3 > 0$ ; ii) if  $f(x)$  is decreasing in  $x$  throughout its support, then  $\Delta_3 < 0$ ; iii) if  $f(x)$  is constant in  $x$  throughout its support, then  $\Delta_3 = 0$ .*

This sufficient condition imposes strict monotonicity requirements on  $f(x)$ , which is quite restrictive for density functions. In this article, we provide a more general sufficient condition than that, and show that when  $n = 3$ , the optimal disclosure policy is related to the skewness of the distribution  $F$ . With our model setup, we have

$$\Delta_n = E[X_{2:n}] - E[X] = \int_0^V [F(x) - F_{2:n}(x)] dx$$

where the cumulative distribution function  $F_{2:n}(x) = F^n(x) + n(1 - F(x))F^{n-1}(x)$ . When  $n = 3$ , it follows that

$$\Delta_3 = \int_0^V F(x)(1 - F(x))[1 - 2F(x)] dx \quad (2)$$

It is interesting to observe that  $X_{2:3}$  is the sample median of the order statistics. So the comparison between  $E[X_{2:3}]$  and  $E[X]$  is just the comparison between the expectation of sample median and the mean of the distribution  $F$ . If  $F$  is symmetric, then by intuition they should be equal and thus  $\Delta_3 = 0$ . We then provide the first result as below.

**Lemma 2** *If  $F(x)$  is symmetric, then  $\Delta_3 = 0$ .*

**Proof.** By definition, if  $F(x)$  is symmetric, then  $F(x) = 1 - F(V - x)$ . From (2),

$$\Delta_3 = \int_0^{\frac{V}{2}} F(x)(1 - F(x))[1 - 2F(x)] dx + \int_{\frac{V}{2}}^V F(x)(1 - F(x))[1 - 2F(x)] dx$$

where the second integral

$$\begin{aligned} &= \int_{\frac{V}{2}}^V [1 - F(V - x)] F(V - x) [1 - 2[1 - F(V - x)]] dx \\ &= - \int_0^{\frac{V}{2}} F(y) [1 - F(y)] [1 - 2F(y)] dy \end{aligned}$$

And therefore  $\Delta_3 = 0$ . ■

The symmetric condition for  $F(x)$  is obviously weaker than the condition of constant  $f(x)$  in Hummel and McAfee (2015). We take this idea onwards and show that the value of  $\Delta_3$  depends on the asymmetry or skewness of the distribution  $F$ . For instance, for a unimodal distribution  $F$ , if it is skewed to the right, then  $\Delta_3 < 0$ ; and if it is skewed to the left, then  $\Delta_3 > 0$ . We first introduce the following function

$$\eta(v) = f[F^{-1}(0.5 + v)] - f[F^{-1}(0.5 - v)] \text{ for } v \in (0, 0.5) \quad (3)$$

Specifically, if  $\eta(v) < 0$ , we would say that  $F$  is right-skewed. The idea is that, for any two values  $x_1 = F^{-1}(0.5 - v)$  and  $x_2 = F^{-1}(0.5 + v)$  such that both have equal tail probabilities, we have  $x_1 < x_2$  and the density  $f(x_1) > f(x_2)$ , which implies that the distribution is skewed to the right. We next provide the main result of this paper.

**Proposition 3** For  $n = 3$ , if for all  $v \in (0, \frac{1}{2})$

- (i)  $\eta(v) > 0$ , then  $\Delta_3 > 0$  and revealing information is better;
- (ii)  $\eta(v) < 0$ , then  $\Delta_3 < 0$  and withholding information is better.
- (iii)  $\eta(v) = 0$ , then  $\Delta_3 = 0$  and there's no difference between revealing or not.

**Proof.** From (2), let  $u = F(x) \in [0, 1]$ , and we have

$$\begin{aligned} \Delta_3 &= \int_0^1 u(1-u)[1-2u] dF^{-1}(u) = \int_0^1 \frac{1}{f[F^{-1}(u)]} u(1-u)(1-2u) du \\ &= 2 \int_0^{\frac{1}{2}} \frac{1}{f[F^{-1}(\frac{1}{2}-v)]} \left(\frac{1}{2}-v\right) \left(\frac{1}{2}+v\right) v dv \\ &\quad - 2 \int_0^{\frac{1}{2}} \frac{1}{f[F^{-1}(\frac{1}{2}+v)]} \left(\frac{1}{2}+v\right) \left(\frac{1}{2}-v\right) v dv \\ &= 2 \int_0^{\frac{1}{2}} \left\{ \frac{1}{f[F^{-1}(\frac{1}{2}-v)]} - \frac{1}{f[F^{-1}(\frac{1}{2}+v)]} \right\} \left(\frac{1}{2}-v\right) \left(\frac{1}{2}+v\right) v dv \end{aligned}$$

If  $\eta(v) > 0$  for all  $v \in (0, \frac{1}{2})$ , then the integrand is positive and thus  $\Delta_3 > 0$ . ■

The sufficient condition on  $\eta(v)$  is apparently weaker than the monotonic density condition proposed in Hummel and McAfee (2015), as  $f'(x) > 0$  implies  $\eta(v) > 0$ , yet not vice versa. It is obvious that  $\eta(v)$  is introduced naturally in our proof of Proposition 3, yet an equivalent version of it is also introduced in Van Zwet (1979). Different from Van Zwet's study of a single distribution, here we focus on the comparison between the expectation of the sample median of order statistics and the mean of  $F$ . Furthermore, as shown in the following example, the median of  $F$  is not necessarily equal to the expectation of the sample median.

We next introduce a simple example of asymmetric triangular distribution, where the density function  $f(x)$  is not monotonic, it first increasing until the mode and then decreasing afterward. The example helps to illustrate the main result of this paper.

**Example 4** Consider an example of asymmetric triangular distribution on  $x \in [0, 1]$ , with mode

$\frac{1}{3}$ . The probability density and cumulative distribution function are respectively

$$f(x) = \begin{cases} 6x & \text{if } x \in [0, \frac{1}{3}] \\ 3(1-x) & \text{if } x \in (\frac{1}{3}, 1] \end{cases} \quad F(x) = \begin{cases} 3x^2 & \text{if } x \in [0, \frac{1}{3}] \\ 1 - \frac{3}{2}(1-x)^2 & \text{if } x \in (\frac{1}{3}, 1] \end{cases}$$

It is clear that the distribution of  $X$  is skewed to the right, and its median,  $1 - \sqrt{1/3}$ , is smaller than its mean of  $E[X] = \frac{4}{9}$ . The quantile function of  $X$  is

$$F^{-1}(u) = \begin{cases} \sqrt{\frac{1}{3}u} & \text{if } u \in [0, \frac{1}{3}] \\ 1 - \sqrt{\frac{2}{3}(1-u)} & \text{if } u \in (\frac{1}{3}, 1] \end{cases}$$

We first show that  $\eta(v) < 0$  for all  $v \in (0, \frac{1}{2})$ , as follows

$$\begin{aligned} \eta(v) &= f[F^{-1}(0.5+v)] - f[F^{-1}(0.5-v)] \\ &= \begin{cases} \sqrt{3(1-2v)} - \sqrt{3(1+2v)} & \text{if } v \in (0, \frac{1}{6}] \\ \sqrt{3(1-2v)} - \sqrt{6(1-2v)} & \text{if } v \in [\frac{1}{6}, \frac{1}{2}) \end{cases} < 0 \end{aligned}$$

Based on Proposition 3, if  $\eta(v) < 0$ , then  $\Delta_3 < 0$  and withholding information is better. And a simple calculation verifies this

$$\begin{aligned} \Delta_3 &= \int_0^{\frac{1}{3}} F(x)(1-F(x))[1-2F(x)] dx + \int_{\frac{1}{3}}^1 F(x)(1-F(x))[1-2F(x)] dx \\ &= \left[ \left(\frac{1}{3}\right)^3 - \frac{9}{5} \left(\frac{1}{3}\right)^4 + \frac{18}{7} \left(\frac{1}{3}\right)^6 \right] + \left[ -4 \left(\frac{1}{3}\right)^3 + \frac{72}{5} \left(\frac{1}{3}\right)^4 - \frac{288}{7} \left(\frac{1}{3}\right)^6 \right] \\ &= -\frac{8}{35} \left(\frac{1}{3}\right)^3 < 0 \end{aligned}$$

### 3 Conclusion

This paper provides a more general sufficient condition than Hummel and McAfee (2015) for optimal information disclosure when there are 3 bidders. We show that the optimal disclosure is related to the skewness of the distribution of bidders' valuations. Specifically, if the valuation distribution is skewed to the left (right), then it is optimal for the seller to reveal (withhold). If the distribution is symmetric, then there's no difference between revealing information or not.

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