

Partial Inflation Indexation and Long-run Inflation Targeting in a Growing Economy: A Comparison of Calvo and Rotemberg Pricing Models

Parantap Basu^{*,a}, Agnirup Sarkar^b

^a*Durham University Business School, Durham University, Mill Hill Lane, DH1 3LB, Durham, UK*

^b*Durham University Business School, Durham University, Mill Hill Lane, DH1 3LB, Durham, UK*

Abstract

The observed 2% long run inflation target in most developed industrial nations is in variance with the zero or negative optimal inflation rates predicted by prominent monetary theories. Using a calibrated simple New-Keynesian model with endogenous growth and nominal rigidity, we compare two price setting environments of Calvo (1983) and Rotemberg (1982). In our growth model, the steady state welfare maximizing inflation takes into account the growth effect as well as the price distortionary effects of inflation. The long-run welfare maximizing trend inflation could be positive in economies with nominal rigidity in the form of partial inflation indexation and price stickiness. A higher degree of inflation indexation lowers the steady state price distortion in the Calvo model and steady state price adjustment cost in Rotemberg model and raises the long run optimal inflation. Since the productive inefficiency caused by partial inflation indexation is higher in Calvo economy compared to Rotemberg, the long run optimal trend inflation is higher in Rotemberg than in Calvo. In both models, a two percent long run inflation target is attainable for a reasonable degree of inflation indexation.

Key words:

Inflation Target, Partial Inflation Indexation, Endogenous Growth, Calvo Pricing, Rotemberg Pricing.

*Corresponding author

Email address: parantap.basu@durham.ac.uk (Parantap Basu)

1. Introduction

In major industrial nations, Central Banks set a long run inflation target around 2%. The well known Friedman rule mandates a negative inflation rate for the nominal interest rate to be zero. Schmitt-Grohe and Uribe (2011) and Woodford (2003) depart from this Friedman rule and argue that the optimal inflation rate is zero in reasonable economic environments. This gives rise to an anomaly because in reality the long run inflation target (known as trend inflation) is positive and around 2% for major industrial and emerging market economies.

This paper aims to revisit this issue in a New Keynesian endogenous growth setting. The focus of this paper is purely on long run growth and welfare implications of trend inflation and that is why we abstract from any aggregate shocks to the economy. Even though there is a rapidly growing literature on trend inflation,¹ little effort is directed to understand the long run growth and welfare effects of a positive inflation targeting in the presence of nominal and real frictions in the economy. In our model, the link between trend inflation and growth is in the form of imperfect inflation indexation. If inflation is not fully indexed, a higher trend inflation by lowering the price-marginal cost markup could redistribute income from monopolistic profit to rental income earners. Through this income redistribution channel, trend inflation could positively impact long run growth. The quantitative effect of trend inflation on growth depends crucially on the nature of price setting which is the central theme of this study.²

As in Ascari and Rossi (2012), we analyze two types of price setting behaviour, namely (i) Calvo (1983) where firms randomly reset prices, and (ii) Rotemberg (1982) where all firms continuously set prices subject to some price adjustment costs. Growth effects of a trend inflation are very different in these two price setting environments. As in King and Wolman (1996), a higher trend inflation has an ambiguous

¹See Ascari and Sbordone (2014) for a comprehensive survey.

²The positive effect of trend inflation on growth via capital accumulation resembles the Tobin (1965) effect although the mechanism in our model is very different. In Tobin (1965), a higher inflation causes reallocation of portfolio from money to capital while in our setting, a higher trend inflation could redistribute income from profit to rental income due to long run nominal rigidity.

effect on the average price-marginal cost markup of monopolistic intermediate goods firms in a Calvo price setting world. This happens due to the tension between *price adjustment* and *marginal markup effects*. In the Calvo model, this ambiguity translates into a hump shaped relation between long run growth and trend inflation. On the other hand, in a Rotemberg price setting environment, higher trend inflation unambiguously erodes the average markup of the intermediate goods firms along the same transmission channel described in Ascari and Rossi (2012) and promotes long run growth via boosting the rental price of capital.³

The effect of trend inflation on the steady state welfare depends not only on how the trend inflation impacts long run growth but also on how the trend inflation influences the price distortion. In the Calvo model, the price distortion arises due to the dispersion of prices among sticky and flexible price firms. In Rotemberg model, the price distortion arises solely due to the price adjustment costs. In both models, price distortion by causing productive inefficiency and loss of resources engenders adverse wealth effect on the representative household and depresses steady state welfare. After factoring the growth and price distortionary effects, our calibrated growth model still yields a positive welfare maximizing inflation in both models. As in Lombardo and Vestin (2008) the inefficiency due to nominal rigidity is higher in the Calvo model than in Rotemberg. The relative output loss in Calvo model progressively rises when the trend inflation is higher. This makes the welfare maximizing trend inflation lower in the Calvo model compared to the Rotemberg model. The immediate implication is that the optimal long run inflation is higher in the Rotemberg model than in the Calvo model. Both models yield a positive optimal long run annual inflation rate. A sensitivity analysis of welfare maximizing inflation shows that the optimal inflation is very sensitive to the degree of inflation indexation. Higher inflation indexation raises the optimal inflation rate in both models because more indexation of inflation dampens the price distortionary effects. In addition, the de-

³Ascari and Rossi (2012) provide useful insights about the differential effects of Calvo and Rotemberg models of price fixing but they do not address the issue of welfare maximizing trend inflation in an endogenous growth setting as we do.

gree of competition also plays an important role in determining the optimal trend inflation.

In our growth model, the optimality of positive trend inflation rests on a comparison across inefficient steady states. Two types of inefficiencies are present in the steady state of our model economy. The first inefficiency is due to market imperfection while the second inefficiency results from nominal rigidity. A zero trend inflation eliminates the second inefficiency but could not overcome the first inefficiency. We demonstrate that in such a second best environment, a small amount of trend inflation could be welfare improving.

The rest of the paper is organized as follows. In the following section we review the relevant literature on trend inflation. In section 3, we lay out the basic setup and compare the balanced growth and long run welfare properties of Calvo and Rotemberg models. Section 4 reports the calibration. Section 5 concludes.

2. Literature Review

There is a substantial literature that rationalizes the benefit of a positive trend inflation. Svensson (1997) and Mishkin (2004) argue that a higher trend inflation could reduce inflation volatility and the impact of shocks. Blanchard et al. (2010) and Williams (2009) argue that in the presence of a zero lower bound (ZLB) for the nominal interest rate, a higher long run inflation target gives the Central Bank greater latitude to lower the nominal interest rate. Billi (2011) develops a small open economy new Keynesian model with ZLB for the nominal interest rate and argues that the long run inflation target is very low if the government commits in advance to a future policy plan. On the other hand, if the government follows discretion, the long run inflation target could be inordinately high. Ascari and Sbordone (2014) undertake a comprehensive analysis of the adverse effect of a higher trend inflation on the stability of the aggregate economy. A higher trend inflation shrinks the region of determinacy of the model. In view of this result, they caution about the pitfall of a positive inflation target to mitigate the ZLB problem.

Although this literature provides useful insights about the rationale for a positive trend inflation, it does not factor into account the long run growth consequences of a

positive inflation target because of not including capital stock as a reproducible input. Ascari and Sbordone (2014) note that little work has been done on the effects of trend inflation on the aggregate economy with capital. Our novelty is to understand the welfare consequence of trend inflation via the growth channel which necessitates the use of an endogenous growth model. Since the thrust of this paper is on the effect of trend inflation on long run growth and welfare, unlike Ascari and Sbordone (2014), we abstract from the effect of trend inflation on the volatility and stability of the economy and only focus on the balanced growth path using a deterministic model of endogenous growth with a simple "AK" technology as in Rebelo (1991). Our model connects to a growing literature that highlights the difference between Calvo and Rotemberg price settings. Ascari and Rossi (2012) focus on the long run new Keynesian Phillips curve. Damjanovic and Nolan (2010), and Leith and Liu (2014) focus on optimal inflation. However, these models do not look into growth effects of monetary policy.

Although the primary thrust of our paper is on optimal trend inflation in an endogenous growth model, our model has indirect implications for the relationship between long run growth and inflation. There is an old thread of literature which studies the cross country relation between growth and inflation. The nexus between growth and inflation is still an unsettled question. Kormendi and Meguire (1985) use cross country data to establish that the long run average growth and long run inflation rate are negatively correlated. Sala-i-Martin (1997) finds rather negligible effect of inflation on growth for their cross country growth regression. In our context, the effect of trend inflation on growth depends on the price setting environment.

There is also a sparse literature exploring the growth and welfare effects of inflation in a new Keynesian endogenous growth setting. Amano, Carter and Moran (2012) have a rich endogenous growth model with nominal price and wage rigidities and find that the negative growth and welfare effects of inflation primarily arise from the distortionary effects of inflation on labour supply. Their model basically highlights the welfare loss of inflation and thus unable to rationalize the positive long run inflation target in a growing economy. Arato (2009) develops an endogenous growth model with endogenous contract duration and explores the growth effect of inflation

but does not specifically address the issue of welfare maximizing inflation target. On the other hand, the novelty of our paper is that we use a simple New Keynesian endogenous growth model and after factoring into account the positive and negative effects of trend inflation on steady state welfare, we find that the welfare maximizing trend inflation is still positive and for a plausible price setting environment, it comes close to what we observe in the real world.

3. Basic Setup

We lay out a simple New-Keynesian model with three players: firms, households and the Central Bank. There is a continuum of intermediate goods firms in the economy in the unit interval. Each variety (j) of such goods is produced with a linear technology as follows:

$$x_{jt} = Ak_{jt} \quad (1)$$

where x_{jt} is the amount produced of such good, k_{jt} the capital used in the production and A is a productivity parameter. The linear technology (AK type as in Rebelo, 1991) is the vehicle of endogenous growth. Each variety is produced by a firm with a patent right which disallows the entry of new firms to replicate this variety. Final goods firms transform these intermediate goods into the production of final goods (y_t) using the CES aggregator:

$$y_t = \left[\int_0^1 x_{jt}^{(\sigma-1)/\sigma} dj \right]^{\frac{\sigma}{\sigma-1}}. \quad (2)$$

where σ is the elasticity of substitution between intermediate goods.

The household owns the capital and all the firms. It rents capital to intermediate goods firms for production. There is no aggregate risk in this environment. The representative household's maximization problem is given by:

$$Max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

s.t.

$$\begin{aligned} P_t c_t + P_t(k_{t+1} - (1 - \delta)k_t) + B_{t+1} \\ = (1 + i_t)B_t + R_t k_t + D_t \end{aligned} \quad (3)$$

and the usual solvency condition, $\lim_{T \rightarrow \infty} \frac{\beta^T u'(c_{t+T})}{u'(c_t)} \frac{B_{t+T}}{P_{t+T}} \geq 0$ for all t . Notations are: t =time, c_t =per capita real consumption, P_t =nominal price, k_t =average capital stock, B_t =stock of nominal one period discount bonds in zero net supply, i_t =nominal interest rate, R_t =nominal rental price of capital, D_t =nominal profit from the intermediate goods firms, δ is the fractional rate of depreciation and β is the subjective discount factor in the unit interval.⁴

The Euler equations of the household are given by:

$$k_{t+1}: u'(c_t) = \beta u'(c_{t+1})(1 + r_{t+1} - \delta) \quad (4)$$

$$B_{t+1}: u'(c_t) = \beta u'(c_{t+1})(1 + i_{t+1})(P_t/P_{t+1}) \quad (5)$$

where r_t is the real rental price of capital equal to R_t/P_t .

Using (4) and (5) one obtains:

$$(1 + i_{t+1}) = (1 + r_{t+1} - \delta)(P_{t+1}/P_t) \quad (6)$$

The Central Bank (CB) sets a long run target Π (i.e., trend inflation) that satisfies the arbitrage condition (6). This means that along a balanced growth path the CB sets an interest rate as follows:

$$1 + i = (1 + r - \delta)\Pi$$

⁴The long run property of the model is invariant to external habit formation and investment adjustment cost, which we ignore for simplicity.

3.1. Equilibrium and Balanced Growth

In equilibrium, agents optimize which means that the first order conditions (4) and (5) hold. All markets clear meaning, $c_t + k_{t+1} - (1 - \delta)k_t = y_t$ and $B_t = 0$. Using the household's Euler equation (4) and assuming a logarithmic utility function, $u(c_t) = \ln c_t$, one obtains the balanced growth rate, G :

$$G = \beta(1 + r - \delta) \quad (7)$$

Because of the linearity of intermediate goods technology (1), the final output production function (2) takes a Rebelo (1991) type "Ak" form with k as the average capital. Since there is no diminishing return to reproducible input, it means that the growth is self-sustained. The balanced growth, G thus depends on the steady state rental price of capital, r . The exact relationship between the trend inflation and the steady state rental price depends on the nature of the price formation which we discuss later.

Finally, to accommodate this balanced growth (G) and the inflation target (Π), the CB lets the money supply grow at a rate ΠG .

3.2. Calvo Model

We now turn to the price setting scenario of Calvo (1983). In this model, all intermediate goods firms facing the same technology are ex ante identical. Each period a firm receives a random "price change" signal with a probability $1 - \theta$. In the spirit of Yun (1996), if the intermediate goods firm does not receive a price signal, its price is increased at the steady state rate of inflation (Π) subject to an inflation indexation parameterized by $\gamma \in (0, 1)$. Lower γ means less indexation. This partial inflation indexation formulation is borrowed from Smets and Wouters (2003).

The cost minimization from the final goods sector yields the conditional input demand functions:

$$x_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\sigma} y_t \quad (8)$$

where $P_{jt} = \Pi^\gamma P_{jt-1}$ if $j \in (0, \theta)$ and $P_{jt} = P_t^*$ otherwise. P_t is the general price level at date t .

Firms solve the optimal price setting problem:

$$\text{Max} \sum_{k=0}^{\infty} \theta^k M_{t,t+k} (\Pi^{\gamma k} P_t^* x_{t+k|t} - TC_{t+k|t}(x_{t+k|t}))$$

subject to the demand functions,

$$x_{t+k|t} = \left(\frac{\Pi^{\gamma k} P_t^*}{P_{t+k}} \right)^{-\sigma} y_{t+k}$$

where $M_{t,t+k}$ is the firm's nominal discount factor and $TC_{t+k|t}$ is the price setter's forecast of the nominal total cost at time $t+k$ conditional on the information at date t .

The optimal price (P_t^*) is nonstationary and thus it is normalized by the general price level P_t . It is straightforward to verify that:

$$\frac{P_t^*}{P_t} = \left(\frac{\sigma}{\sigma - 1} \right) \frac{\sum_{k=0}^{\infty} (\theta \Pi^{-\sigma \gamma})^k M_{t,t+k} \Pi_{t,t+k}^{\sigma+1} mc_{t,t+k} \left(\frac{y_{t+k}}{y_t} \right)}{\sum_{k=0}^{\infty} (\theta \Pi^{\gamma(1-\sigma)})^k M_{t,t+k} \Pi_{t,t+k}^{\sigma} \left(\frac{y_{t+k}}{y_t} \right)} \quad (9)$$

where $mc_{t,t+k}$ is the k -period ahead forecast of the real marginal cost. Given the linear production function (1), $mc_{t,t+k} = r_{t,t+k}/A$ where $r_{t,t+k}$ is the k -period ahead forecast of the real rental price of capital, where $mc_{t,t+k}$ is the k -period ahead forecast of the real marginal cost. Given the linear production function (1), $mc_{t,t+k} = r_{t,t+k}/A$ where $r_{t,t+k}$ is the k -period ahead forecast of the real rental price of capital.

The law of motion of the general price level is given by:

$$P_t = [\theta(\Pi^\gamma P_{t-1})^{1-\sigma} + (1-\theta)P_t^{*1-\sigma}]^{\frac{1}{1-\sigma}} \quad (10)$$

Based on the same principle as in King and Wolman (1996), one gets the following

expression for the average markup along the balanced growth path (BGP).⁵

$$\begin{aligned}\frac{P_t}{MC_t} &= \left(\frac{P_t^*}{MC_t}\right) \left(\frac{P_t}{P_t^*}\right) \\ &= \mu_n \left(\frac{P_t}{P_t^*}\right)\end{aligned}\tag{11}$$

where MC_t is the nominal marginal cost. Using (9), μ_n is the marginal markup defined as

$$\mu_n = \frac{\sigma}{\sigma - 1} \left[\frac{1 - \theta\beta\Pi^{(1-\gamma)(\sigma-1)}}{1 - \theta\beta\Pi^{(1-\gamma)\sigma}} \right]\tag{12}$$

and based on (10), the price adjustment gap, P_t/P_t^* is given by:

$$\frac{P_t}{P_t^*} = \left[\frac{1 - \theta\Pi^{(1-\gamma)(\sigma-1)}}{1 - \theta} \right]^{1/(\sigma-1)}\tag{13}$$

Higher trend inflation erodes the markup in a world with imperfect inflation indexation as seen from the price adjustment gap term (13). To combat this, price setting firm raises the marginal markup.⁶ As in King and Wolman (1996), the effect of a higher trend inflation on the average markup is thus ambiguous due to the conflicting effects on the marginal markup and the price adjustment gap.

The ambiguous effect of trend inflation on average markup could potentially give rise to a hump shaped relationship between the long run growth and trend inflation. This happens because the balanced growth (7) is driven by the real rental price of capital (r) which equals $A.mc$. Since the real marginal cost, mc is the reciprocal of the average markup P_t/MC_t , a higher trend inflation has an ambiguous effect on the balanced growth rate.⁷

⁵The appendix provides a derivation of (11). For the steady state average markup to exist one needs the convergence condition that $\Pi < \theta^{1/(\gamma-1)\sigma}$ which is 5.8% for the baseline parameter values. This upper limit accords well with Bakshi et al. (2007) although their model is very different from ours.

⁶The appendix shows that $\partial\mu_n/\partial\Pi > 0$ as long as $\Pi > 1$.

⁷While King and Wolman (1996) focus on the conflicting output effects of trend inflation, in our

3.3. Rotemberg Model

In Rotemberg (1982) all firms continuously adjust prices but all of them are subject to a quadratic price adjustment cost measured in terms of final goods. We follow Ascari and Rossi (2012) and Ireland (2007) in specifying the price adjustment cost function subject to imperfect inflation indexation as follows:

$$\frac{\varphi}{2} \left(\frac{P_{jt}}{\Pi^\gamma P_{jt-1}} - 1 \right)^2 y_t \quad (14)$$

where $\varphi > 0$ is the degree of nominal rigidity, Π^γ represents the indexation of the last period price level based on the trend inflation, Π , and γ is the degree of price indexation as before.

The optimal price fixing problem of each intermediate goods firm is, therefore,

$$Max \sum_{k=0}^{\infty} M_{t,t+k} \left[P_{jt+k} x_{j+k} - MC_{t+k} x_{j+k} - \frac{\varphi}{2} \left(\frac{P_{jt}}{\Pi^\gamma P_{jt-1}} - 1 \right)^2 P_{t+k} y_{t+k} \right] \quad (15)$$

subject to the same sequence of demand functions as in (8). Since all firms face the same technology and the same price adjustment costs, there is no heterogeneity in price fixing behaviour as in Calvo. Thus in a symmetric equilibrium $P_{jt} = P_t$ for all j . Unlike Calvo (1983), there is no difference between average and marginal markup because all firms charge the same price. Along the BGP, the average markup is given by (the proof is relegated to the appendix):

$$\frac{P_t}{MC_t} = \left[\frac{\varphi(1-\beta)\Pi^{1-\gamma}(\Pi^{1-\gamma}-1)}{\sigma} + \frac{\sigma-1}{\sigma} \right]^{-1} \quad (16)$$

Higher trend inflation (Π) lowers the markup due to imperfect inflation indexation. Since the steady state rental price (r) is $A.mc$, this means that a higher trend inflation unambiguously raises the balanced growth rate in the Rotemberg price setting

model the trend inflation has similar ambiguous effect in determining the long run growth. The positive effect is likely to prevail at a low trend inflation.

scenario.⁸

3.4. Long run Welfare in Calvo and Rotemberg Models

For any arbitrary balanced growth (G), the steady state welfare (W) function is given by:

$$W = \frac{\ln c_0}{1 - \beta} + \frac{\beta \ln G}{(1 - \beta)^2} \quad (17)$$

where c_0 is the initial consumption.

The initial consumption (c_0) and the balanced growth rate (G) differ between Calvo and Rotemberg price setting regimes. Hereafter, we distinguish between these two regimes with superscripts C and R respectively. Using (11) and noting that $r = A.mc$, the balanced growth rate (7) in the Calvo scenario is given by:⁹

$$G^C = \beta \left[A \left\{ \mu_n \frac{P_t}{P_t^*} \right\}^{-1} + 1 - \delta \right] \quad (18)$$

and the initial consumption (the proof is relegated to the appendix) is given by

$$c_0^C = [(A\bar{s}^{-1} + 1 - \delta) - G^C] \quad (19)$$

where \bar{s} is the steady state price dispersion given by,¹⁰

$$\bar{s} = \frac{(1 - \theta\Pi^{(1-\gamma)(\sigma-1)})^{\sigma/(\sigma-1)}}{(1 - \theta\Pi^{\sigma(1-\gamma)})(1 - \theta)^{1/(\sigma-1)}} \quad (20)$$

The steady state welfare depends on the long run inflation rate (Π) through two channels: (i) the balanced growth (G), and (ii) relative price dispersion channel (\bar{s}). The trend inflation has ambiguous effect on growth due to conflict between marginal

⁸Note that growth cannot indefinitely rise because a lower markup depresses the profits of the intermediate goods firms which adversely affects household's consumption.

⁹Further details of the derivation of the steady state welfare functions are relegated to the appendix.

¹⁰See the appendix for the derivation of the steady state price dispersion.

markup and price adjustment effects. On the other hand, a higher trend inflation elevates the steady state price dispersion along the same channel described in Ascari and Sbordone (2014).¹¹ Although this depresses the initial consumption via the first term of (19), the ambiguous inflation effects on growth makes the sign of the comparative statics $\partial c_0^C / \partial \Pi$ ambiguous. The overall effect of trend inflation on long run welfare is thus ambiguous in the Calvo model.

For the Rotemberg model, the balanced growth rate (7) is obtained by plugging (16) into (7):

$$G^R = \beta \left[A \left\{ \frac{\varphi(1-\beta)\Pi^{1-\gamma}(\Pi^{1-\gamma}-1)}{\sigma} + \frac{\sigma-1}{\sigma} \right\} + 1 - \delta \right] \quad (21)$$

and the initial consumption (the derivation is relegated to the appendix) is given by:

$$c_0^R = [(A(1-\tau) + 1 - \delta) - G^R]k_0 \quad (22)$$

where

$$\tau = \frac{\varphi}{2} (\Pi^{1-\gamma} - 1)^2 \quad (23)$$

Since all firms are homogeneous, there is no price dispersion in the Rotemberg model. Instead of price dispersion, the price adjustment cost acts as an implicit tax on TFP. Higher trend inflation has a hump shaped effect on the steady state welfare in the Rotemberg model due to the conflicting effect of trend inflation on the steady state welfare. First, it unambiguously promotes growth, G^R which has a direct positive effect on the steady state welfare via the second term in (17). Second, it has a negative effect on the welfare via the initial consumption term, c_0^R term (22) which falls because of a higher implicit tax (τ) on output as well as a higher growth, G^R .

¹¹See the appendix for a proof.

4. Calibration

To assess the relationship between W and Π quantitatively, we fix $\beta = 0.99$ and $\delta = 0.025$ at the conventional levels. The demand elasticity parameter σ is fixed at 6.00 as in Kollmann (2002). The productivity parameter A is fixed to target the long run per capita quarterly real GDP growth rate at 0.49% which means an annualized growth rate of 1.97% for the sample period 1947-2014.¹² There is considerable disagreement in the literature about the range of values for the price stickiness parameter, θ . Kollmann (2002) uses 0.75 as the baseline value while Smets and Wouters (2003) estimate a higher value of θ around 0.91. These values basically mean that the average duration of prices to remain sticky is 4 to 10 quarters. We take an average of these extreme values and set θ equal to 0.85 as a baseline.

A similar ambiguity arises about the size of the inflation indexation parameter γ . For Euro regions, Smets and Wouters (2003) estimate γ around 0.52. Using GMM approach Sahuc (2004) comes up with an estimate of γ around 0.41 for Euro regions and 0.64 for the US. As a baseline we start off with a conservative estimate, $\gamma = 0.52$ which is close to Smets and Wouters (2003) estimate or an average of the Euro and US estimates of Sahuc (2004). We then carry out a sensitivity analysis to check how the optimal long run inflation depends on the size of γ .

It is difficult to find an estimate of the price adjustment cost parameter, φ , that is consistent with our growth model. Keen and Wang (2005) calibrate this by matching the slopes of the New Keynesian Phillips curves from Calvo and Rotemberg models. In our context, balanced growth rate is a crucial link between Calvo and Rotemberg models. In a similar vein, we calibrate φ by matching the balanced growth rates, G^C and G^R which yields an analytical expression for φ as follows:

$$\varphi = \left[\left\{ \frac{\sigma}{\sigma - 1} \mu_n \frac{P_t}{P_t^*} \right\} - \frac{\sigma - 1}{\sigma} \right] \left[\frac{\sigma}{(1 - \beta)\Pi^{1-\gamma}(\Pi^{1-\gamma} - 1)} \right] \quad (24)$$

The price adjustment cost parameter depends nonlinearly on the trend inflation Π .

¹²Data for annual per capita real GDP in chained 2009 US dollars came from the Bureau of Economic Analysis.

As a baseline we evaluate φ at a zero inflation level which yields¹³:

$$\varphi = \left(\frac{\sigma - 1}{1 - \beta} \right) \left(\frac{\theta}{1 - \theta} - \frac{\theta\beta}{1 - \theta\beta} \right) \quad (25)$$

Note that φ is increasing in θ and not surprisingly at zero inflation steady state, inflation indexation parameter, γ plays no role in determining φ . For our calibrated values of σ, β and θ , the price adjustment cost parameter, φ equals 178.76. This value is close to Keen and Wang (2005) for a price markup in the range 10 to 20% and Calvo nominal rigidity parameter θ around 0.8.¹⁴

Figure 1 plots the growth effect of trend inflation for the Calvo model where both growth and inflation rates are annualized. The hump shaped relationship arises which bears out the intuition described earlier about the conflict between marginal markup effect and price adjustment effect caused by a higher trend inflation. The positive price adjustment effect dominates first and then the marginal markup effect picks up. The growth maximizing inflation is around 1.49%.

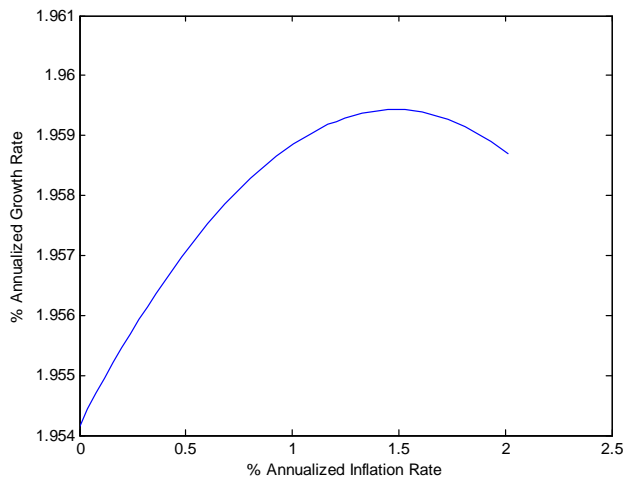


Figure 1: Growth Effects of Inflation in the Calvo Model

¹³The expression for φ in eq (25) comes from an application of L'Hospital's rule to (24).

¹⁴The balanced growth rate in our calibrated Rotemberg model is not very sensitive to the value of φ .

Figure 2 plots the welfare effect of annualized trend inflation in the Calvo model. A welfare maximizing inflation results around 0.52% after balancing the negative price distortion effects and the hump-shaped effects of trend inflation on growth. Not surprisingly the welfare maximizing inflation is lower than the growth maximizing inflation because of an additional negative price distortionary effect on the steady state welfare.

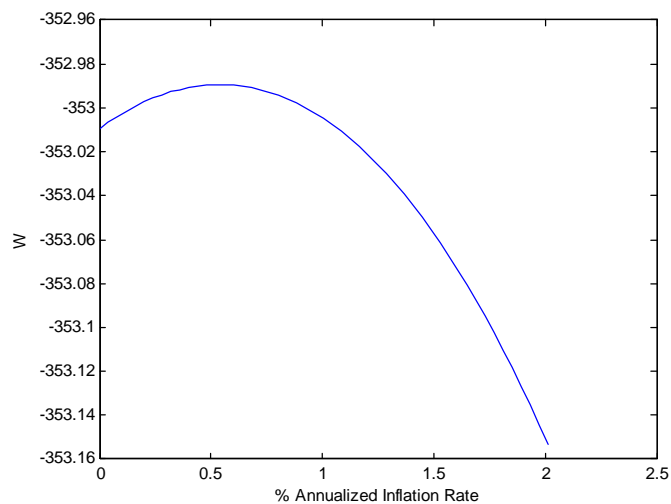


Figure 2: Welfare maximizing inflation in the Calvo model

Figure 3 plots the effect of trend inflation on the balanced growth in the Rotemberg model. Since the average price markup unambiguously decreases due to higher inflation, the steady state rental price rises which promotes growth unambiguously.

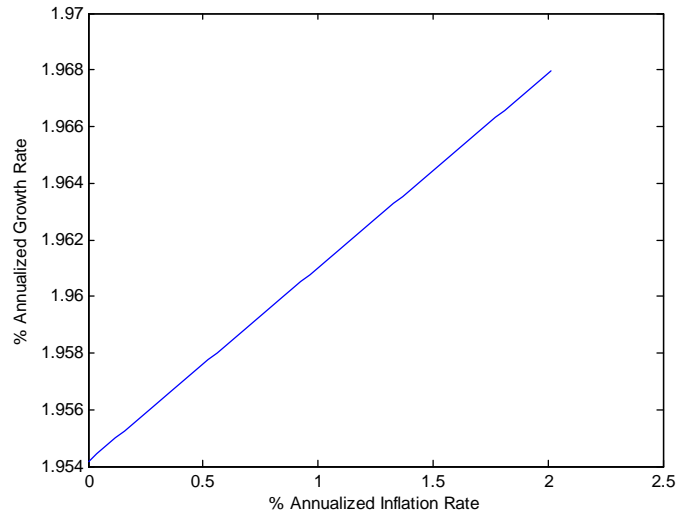


Figure 3: Growth Effects of Inflation in the Rotemberg Model

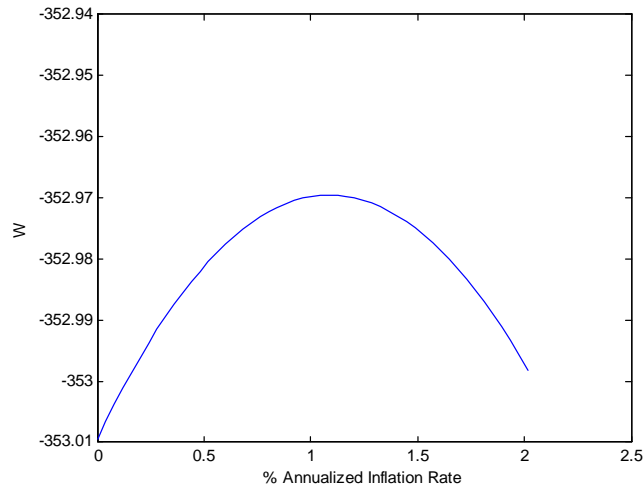


Figure 4: Welfare effects of trend inflation in the Rotemberg model

Figure 4 plots the effect of trend inflation on the steady state welfare in the Rotemberg model. A hump shaped relationship again emerges which picks at 1.08%. The welfare maximizing inflation in Rotemberg model is higher than the Calvo model. In both models, the growth at the welfare maximizing inflation is 1.96% on par with the annual growth rate in the US.

To understand the reasons for the higher optimal long run inflation in Rotemberg model compared to Calvo, we compute the inefficiency in both models due to nominal rigidity. In the Calvo model the efficiency loss arises due to the price dispersion, \bar{s} as shown in (20). In the appendix, we demonstrate that the steady state implicit tax on TFP due to price dispersion is given by $(\bar{s} - 1)/\bar{s}$. On the other hand, in the Rotemberg model the efficiency loss arises due to the price adjustment cost which results in an implicit steady state tax on TFP equal to $\frac{\rho}{2} (\Pi^{1-\gamma} - 1)^2$ as seen from (22) and (23). Figure 5 plots the ratio of these two implicit taxes in Calvo and Rotemberg for a range of annualized trend inflation rates. Higher trend inflation unambiguously raises the inefficiency in the Calvo model compared to Rotemberg. This progressive relative output loss in the Calvo model vis-a-vis Rotemberg is at the very foundation of a higher optimal long run inflation in the Rotemberg model.

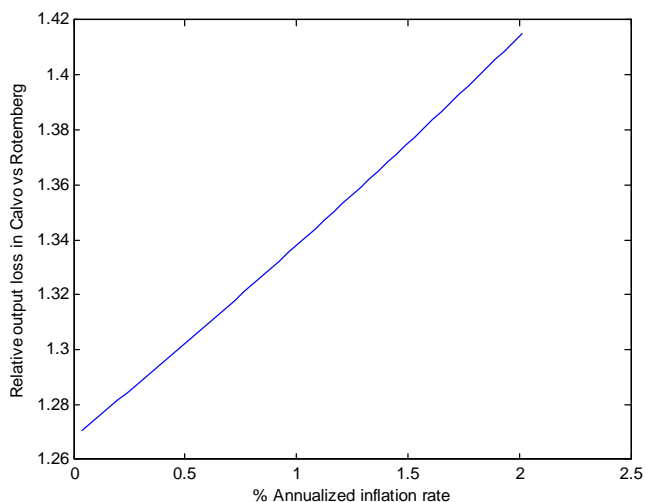


Figure 5: Relative output loss in Calvo vs Rotemberg due to nominal rigidity

Table 1 demonstrates the sensitivity of the welfare maximizing inflation with respect to θ and γ for these two models, C and R . The welfare maximizing inflation is sensitive to the inflation indexation parameter, γ . A higher indexation raises the optimal inflation because it dampens the negative distortionary effect of trend inflation on welfare in both Calvo and Rotemberg models (see eqs (20) and (23)).

For $\gamma = 0.87$, the Calvo model reproduces an optimal trend inflation around 2%. For the Rotemberg model, the 2% target inflation is obtained for a smaller value of γ around 0.74. The optimal inflation is not very sensitive to change in the value of the price rigidity parameter θ except for high values exceeding 0.9. The less sensitivity of the trend inflation to θ reflects the fact that the Calvo price rigidity parameter generates primarily short run effects of monetary policy.

A 2% long run inflation target is, therefore, obtained for a value of the inflation indexation parameter γ higher than standard estimates in the literature. Is such a high value of γ empirically plausible? As discussed earlier, in the extant literature the rule of indexation varies a lot and it is often based on ad hoc estimates. Del Negro et al. (2012) uses a hybrid indexation rule that makes indexation a weighted average of last period inflation and the trend inflation. Carrillo et al. (2015) find that the size of the indexation rule depends on the predominant source of the shock in a structural model. During the great inflation periods, the indexation was close to 0.89 while during the great moderation period, it is close to zero. Although these numbers are not directly comparable to our long run framework, it at least indicates that there is no clear conventional wisdom of the exact degree of inflation indexation.

The most relevant study in our context is by Ascari and Branzoli (2010) who make a persuasive case that the optimal inflation indexation in the presence of a positive trend inflation could be quite high. Their theoretical estimate of γ is 0.87 that is remarkably close to the estimate of γ for which the Calvo model reproduces a 2 percent inflation. It is also noteworthy that in our model, a high value of γ is associated with a relatively high value of the degree of competition parameter, σ . We next show that a 2 percent optimal inflation can be obtained for a lower value of γ if σ is calibrated at a lower level.

<Table 1 comes here>

4.1. Degree of Competition and the Optimal Inflation

How does the optimal trend inflation depend on the degree of competition parameter σ ? A lower σ raises the flexible price markup. In the Calvo model, a lower σ raises the marginal markup (12) but it lowers the price adjustment gap (13) making

the growth effect ambiguous. A similar ambiguity arises in the Rotemberg growth rate (21) because a change in σ has conflicting effects on the first and second terms of the average markup (16). After factoring these conflicting growth effects into account, the optimal trend inflation in both models, however, still rises as σ falls. Table 2 summarizes the result of this sensitivity analysis. In the Calvo model, the optimal trend inflation of 2 percent is achieved for σ equal to 2.25 while in the Rotemberg model, it is reproduced when σ is 4.44.

<Table 2 comes here>

The upshot of all this sensitivity analysis in both Tables 1 and 2 is that the Calvo model always falls short in reproducing a 2 percent trend inflation target vis-a-vis Rotemberg for reasonable parameter values. The low optimal trend inflation in Calvo basically reflects the greater inefficiency due to price distortion explained earlier in terms of Figure 5.

4.2. Why do nominal frictions inefficiency persist in the long run?

The conventional wisdom is that all prices and quantities flexibly adjust in the long run and thus no inefficiency due to nominal frictions persist in the long run. In our model, the principal driver of nominal friction is partial inflation indexation in the long run. If γ equals one, both Calvo and Rotemberg pricing models revert to a flexible price model as seen from (11) and (16). The question is: what is the rationale for assuming partial inflation indexation in the long run? We follow the reasoning of Ascari and Branzoli (2010) that neither zero nor full inflation indexation is optimal in the long run in a staggered price setting environment with a positive trend inflation. Ascari and Branzoli (2010) focus on the Calvo price setting without growth and argue that a lower γ has conflicting effects on the price adjustment gap (P_t/P_t^*) and marginal markup (μ_n). A similar reasoning applies to our growth model. A higher indexation has conflicting effects on long run growth via tensions on the steady state rental price of capital. In addition, in our model, the steady state welfare effect of indexation is further complicated by the fact that a lower indexation elevates the

price distortion in the Calvo setting which depresses initial consumption (C_0^C).¹⁵

Ascari and Branzoli do not analyze the case of Rotemberg price setting model. It is straightforward to verify that a lower γ also raises the price adjustment cost term (14). Thus by lowering average markup (16), it unambiguously raises the balanced growth rate. On the other other hand, the initial consumption (C_0^R) falls because a lower γ raises τ via a higher price adjustment cost. These conflicting effects on growth and initial consumption make the effect of a higher indexation on long run welfare ambiguous in the Rotemberg model. Thus neither zero nor full inflation indexation is optimal also in the Rotemberg model.

4.3. *Why is positive trend inflation welfare improving?*

In both Calvo and Rotemberg models price distortions either in the form of price dispersion or price adjustment cost give rise to misallocation of resources. The question arises: why does not the Central Bank eliminate this distortion right at the outset by setting the trend inflation equal to zero? Setting trend inflation to zero would eliminate the nominal friction and lead the economy to a flexible price steady state which is still subject to real friction arising from market imperfection. Such a steady state is, therefore, an inefficient steady state due to the existence of market imperfection. If one compares a zero inflation inefficient steady state with an inefficient steady state with a small dose of trend inflation, the latter could be welfare improving. To see this more clearly first note that the real rental price in a flexible price inefficient steady state is given by $A(\sigma - 1)/\sigma$. The reciprocal of the flexible price markup imposes a tax on the TFP. The immediate consequence is that the balanced growth is given by:

$$G = \beta(A(\sigma - 1)/\sigma + 1 - \delta) \tag{26}$$

As seen from Figures 1 and 3, a positive trend inflation up to a threshold could improve the long run growth rate in both Calvo and Rotemberg models compared

¹⁵A full blown analysis of an optimal inflation indexation in our present endogenous growth setting requires endogenizing γ which is beyond the scope of this paper.

to a flexible price inefficient balanced growth rate as in (26).

The welfare comparison is less straightforward. To compare the steady state welfare between zero and positive inflation inefficient steady states, we need to first derive the steady state welfare function for the flexible price steady state. Given our log-utility function and the linear production function, the optimal consumption and investment rules in a flexible price steady state are given by the Solow saving rules as follows:

$$k_{t+1} = \beta(A(\sigma - 1)/\sigma + 1 - \delta)k_t \quad (27)$$

and

$$c_t = \left[A\left(1 - \beta\frac{\sigma - 1}{\sigma}\right) + (1 - \delta)(1 - \beta) \right] k_t \quad (28)$$

The appendix outlines the derivations of (27) and (28). Plugging the consumption decision rule (28) and the balanced growth (26) into the welfare function (17) and normalizing the initial capital stock (k_0) to unity one gets

$$W^{flex} = \frac{\ln \left[A\left(1 - \beta\frac{\sigma - 1}{\sigma}\right) + (1 - \delta)(1 - \beta) \right]}{1 - \beta} + \frac{\beta \ln G}{(1 - \beta)^2}$$

where W^{flex} represents the steady state welfare in an inefficient flexible price economy. For the baseline values of the parameters, we find that $W^{flex} = -353.0095$. In the Calvo model, as seen from Figure 2 the steady state welfare at the optimal inflation is -352.9894 and for the Rotemberg model, the same is -352.9696 as seen from Figure 4. Thus a flexible price zero inflation steady state welfare is lower than a positive inflation steady state welfare for our calibrated economy. The policy authority can do better in inflating the economy a bit compared to a zero inflation flexible price steady state.¹⁶

¹⁶In our present setting the real imperfection due to the existence of monopolistic market power is mitigated by a small dose of inflation. Alternatively one can think of a production subsidy financed by lump sum taxation as in Galí (2015) to eliminate this inefficiency. Although this remains a theoretical possibility there may be practical issues of implementability of such production subsidy. In this paper, we abstract from such fiscal subsidy.

5. Conclusion

The inflation targets of the major industrial nations are consistently above zero which goes against the conventional wisdom of zero or negative inflation based on welfare theoretic considerations. We set up a new-Keynesian endogenous growth model to address this apparent anomaly. Due to partial inflation indexation, a higher long run inflation gives rise to opposing effects on welfare via its conflicting effects on growth and price distortion. An optimal inflation rate exists which maximizes the long run aggregate welfare. For plausible parameter values we find that this optimal inflation rate is positive. The level of this inflation depends on the nature of price setting in the model. The welfare maximizing inflation is consistently higher in the Rotemberg model compared to Calvo due to greater inefficiency in the latter caused by price dispersion. A future extension of this paper would be to examine the short run implications of inflation targets in the presence of aggregate shocks. In this paper, we have focused only on Calvo and Rotemberg price setting scenarios. Another possible avenue of extension would be to investigate the implications of a more generalized pricing rule such as state dependent pricing as in Sheedy (2010).

Acknowledgement

We acknowledge Simon Vicary for inspiring us to undertake this project. Leslie Reinhorn, Martin Ellison and Elisa Keller are acknowledged for useful comments. Participants at the workshops in Newcastle University, Keele University, Bank of England and Bank of Japan are also gratefully acknowledged for comments. The usual disclaimer applies.

References

- Amano, R., Carter, T., Moran, K. 2012. Inflation and Growth: A New Keynesian Perspective. Bank of Canada Working Paper No. 23.
- Arato, H., 2009. Long-run relationship between inflation and growth in a New Keynesian Framework. *Economics Bulletin* 29,3.
- Ascari, G., Branzoli, N. 2007. The long-run optimal degree of indexation in new Keynesian models with price staggering a'la Calvo. *Economics Bulletin* 30(1).
- Ascari, G, Rossi, L. 2012. Trend Inflation and Firms Price-Setting: Rotemberg Versus Calvo. *The Economic Journal* 122,563, 1115–1141.
- Ascari, G and A.M. Sbordone. 2014. The Macroeconomics of Trend Inflation. *Journal of Economic Literature* 52,3, 679-739.
- Bakshi, H., Khan, H., Burriel-Llombart, P, Rudolf, B. 2007. The New Keynesian Phillips Curve under trend inflation and strategic complementarity. *Journal of Macroeconomics* 29, 37-59.
- Billi, R.M. 2011. Optimal Inflation for the US Economy. *American Economic Journal: Macroeconomics* 3,3, 29-52.
- Blanchard, O, Del Araccia, G, Mauro, P. 2010. Rethinking Macroeconomic Policy. *International Monetary Fund* 42,1, 199-215.
- Calvo, G. A. 1983. Staggered Contracts in a Utility-Maximizing Framework. *Journal of Monetary Economics* 12, 383-98.

- Carrillo, Julio A., Peersman, G., Wauters, J. 2014. Endogenous Wage Indexation and Aggregate Shocks. University of Ghent Working Paper No 881.
- Damjanovic, T, Nolan. C. 2010. Seigniorage-Maximizing Inflation under Sticky Prices. *Journal of Money, Credit and Banking* 42, 503-519.
- Del Negro, F., Schorfheide, F., Smets, F., and Wouters, R. 2012, On the Fit of New Keynesian Models. *Journal of Business & Economic Statistics* 25, 2, 123-143.
- Gali, J, *Monetary Policy, Inflation and the Business Cycle: An Introduction to New Keynesian Framework and Its Application. Second Edition*, Princeton University Press, 2015.
- Keen, B., Wang, Y. 2005. What is a Realistic Value for Price Adjustment Costs in new Keynesian Models? *Applied Economics Letters*, 14,11, 789-793.
- King, R., Woolman,A. 1996. Inflation Targeting in a St. Louis Model of the 21st Century. *Federal Reserve Bank of St. Louis Bulletin* 78,3, 83-107.
- Kollmann, R. 2002. Monetary policy rules in the open economy: effects on welfare and business cycles. *Journal of Monetary Economics* 49, 989-1015.
- Kormendi, R.C., Meguire, P.G. 1985. Macroeconomic determinants of growth: Cross-Country Evidence. *Journal of Monetary Economics*, 16, 141-163.
- Leith, C, D. Liu. 2014. The inflation bias under Calvo and Rotember Pricing. Glasgow University working paper.
- Lombardo, P., Vestin, G. 2008. Welfare implications of Calvo vs. Rotemberg-pricing assumptions. *Economics Letters* 100, 2, 275-279
- Mishkin, F.S. 2004. Can Central Bank Transparency Go Too Far? NBER Working Paper No 10829.
- Prescott, E.C. 1986. Theory Ahead of Business Cycle Measurement. *Quarterly Review*, 10, 4, 9-22.

- Rebelo, S. 1991. Long -run Policy Analysis and Long-run Growth. *Journal of Political Economy* 99, 500-521.
- Sahuc, J.-G. 2004. Partial indexation, trend inflation, and the hybrid Phillips curve. Working Paper no. 04/05, University of Evry Val d'Essonne.
- Sahuc, J.-G. 2004. Partial indexation and inflation dynamics: what do the data say? *Applied Economics Letters*, 11, 827–832.
- Sala-I-Martin, Xavier, X. 1997. I Just Ran Two Million Regressions. *American Economic Review* 87 (2), 178-183.
- Schmitt-Grohe, S.,M. Uribe. 2011. The Optimal Rate of Inflation. *Handbook of Monetary Economics* edited by Benjamin M. Friedman and Michael Woodford 3B, Elsevier, San Diego CA, 653-722.
- Sheedy, K.D. 2010. Intrinsic Inflation Persistence. *Journal of Monetary Economics* 57, 1049-1061.
- Smets, F., R. Wouters. 2003. An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association* 1, 1123-1175.
- Svensson, L.E. 1997. Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets. *European Economic Review* 41, 111–1146.
- Tobin, J. 1965. Money and Economic Growth, *Econometrica* 33, 4, 671-684.
- Williams, J.C. 2009. Heeding Daedalus: Optimal Inflation and the Zero Lower Bound. *Brookings Papers on Economic Activity*,1-37.
- Woodford, Michael. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton, NJ: Princeton University Press.
- Yun, T. 1996. Nominal price rigidity, money supply endogeneity and business cycles. *Journal of Monetary Economics* 37, 345-370.

A. Appendix

Derivation of equation (11)

Assuming a logarithmic utility function of the form $u(c_{t+k}) = \ln(c_{t+k})$, the nominal discount factor (9) can be written as:

$$M_{t,t+k} = \beta^k \left(\frac{c_t}{c_{t+k}} \right) \left(\frac{P_t}{P_{t+k}} \right) \quad (\text{A.1})$$

which can be written along the BGP as:

$$M = \left(\frac{\beta}{\Pi G} \right)^k \quad (\text{A.2})$$

Along the BGP, the numerator of (9), $mc \sum_{k=0}^{\infty} (\theta \beta \Pi^{\sigma(1-\gamma)})^k = \frac{mc}{1 - \theta \beta \Pi^{\sigma(1-\gamma)}}$ and the denominator is $\sum_{k=0}^{\infty} (\theta \beta \Pi^{(\sigma-1)(1-\gamma)})^k = \frac{1}{1 - \theta \beta \Pi^{(\sigma-1)(1-\gamma)}}$.

Thus along the BGP, the optimal price setting equation reduces to:

$$\frac{P_t^*}{P_t} = \left(\frac{\sigma}{\sigma - 1} \right) \left(\frac{1 - \theta \beta \Pi^{(\sigma-1)(1-\gamma)}}{1 - \theta \beta \Pi^{\sigma(1-\gamma)}} \right) mc$$

which implies

$$\frac{P_t}{MC_t} = \mu_n \frac{P_t}{P_t^*}$$

same as (11).

Derivation of $\frac{\partial \ln \mu_n}{\partial \Pi} > 0$

We have

$$\mu_n = \frac{\sigma}{\sigma - 1} \left[\frac{1 - \theta \beta \Pi^{(1-\gamma)(\sigma-1)}}{1 - \theta \beta \Pi^{(1-\gamma)\sigma}} \right]$$

Therefore,

$$\ln \mu_n = \ln \left(\frac{\sigma}{\sigma - 1} \right) + \ln (1 - \theta\beta\Pi^{(1-\gamma)(\sigma-1)}) - \ln (1 - \theta\beta\Pi^{(1-\gamma)\sigma})$$

Next note that for $\Pi > 1$,

$$\frac{\partial \ln \mu_n}{\partial \Pi} = \frac{\theta\beta(1-\gamma)\sigma\Pi^{(1-\gamma)\sigma-1}}{1 - \theta\beta\Pi^{(1-\gamma)\sigma}} - \frac{\theta\beta(1-\gamma)(\sigma-1)\Pi^{(1-\gamma)(\sigma-1)-1}}{1 - \theta\beta\Pi^{(1-\gamma)(\sigma-1)}} > 1$$

Derivation of (16)

The first order condition for this price setting problem (15) of the j th intermediate goods firm yields:

$$(1-\sigma) \left(\frac{P_{jt}}{P_t} \right)^{-\sigma} y_t + \frac{r_t}{A} \sigma \left(\frac{P_{jt}}{P_t} \right)^{-\sigma-1} y_t - \varphi P_t \left(\frac{P_{jt}}{\Pi^\gamma P_{jt-1}} - 1 \right) \frac{y_t}{\Pi^\gamma P_{jt-1}} + \beta \frac{u'(c_{t+1})}{u'(c_t)} \varphi P_t \left(\Pi^{-\gamma} \frac{P_{jt+1}}{P_{jt}} - 1 \right) \Pi^{-\gamma} P_{jt+1} P_{jt}^{-2} y_{t+1} = 0 \quad (\text{A.3})$$

Since all firms are homogeneous, they charge the same price in a symmetric equilibrium which means $P_{jt} = P_t$. Eq (A.3) thus reduces to

$$(1-\sigma)y_t + \frac{r_t}{A} \sigma y_t - \frac{\varphi \Pi_t}{\Pi^\gamma} \left(\frac{\Pi_t}{\Pi^\gamma} - 1 \right) y_t + \beta \frac{u'(c_{t+1})}{u'(c_t)} \varphi \Pi^{-\gamma} \Pi_t (\Pi^{-\gamma} \Pi_t - 1) y_{t+1} = 0 \quad (\text{A.4})$$

Along the BGP, $\Pi_t = \Pi$, $r_t = r$ and $\frac{y_{t+1}}{y_t} = \frac{c_{t+1}}{c_t} = G$. With log utility, eq (A.4) reduces to

$$\frac{r}{A} = \left[\frac{\varphi(1-\beta)\Pi^{1-\gamma}(\Pi^{1-\gamma} - 1)}{\sigma} + \frac{\sigma - 1}{\sigma} \right]$$

Since $\frac{r}{A} = mc_t = \frac{MC_t}{P_t}$, from the above equation the steady state average mark

up can be expressed as

$$\frac{P_t}{MC_t} = \left[\frac{\varphi(1-\beta)\Pi^{1-\gamma}(\Pi^{1-\gamma}-1)}{\sigma} + \frac{\sigma-1}{\sigma} \right]^{-1}$$

which is equation (16).

Derivation of the steady state welfare function for Calvo and Rotemberg models

Along the BGP the steady state welfare can be written as:

$$\begin{aligned} W &= \sum_{t=0}^{\infty} \beta^t \ln c_0 G^t \\ &= \frac{\ln c_0}{1-\beta} + \frac{\beta \ln G}{(1-\beta)^2} \end{aligned} \tag{A.5}$$

The equilibrium resource constraint (3) facing the household is given by:

$$c_t + k_{t+1} - (1-\delta)k_t = r_t k_t + d_t \tag{A.6}$$

Dividing through by k_t and using the balanced growth condition:

$$\frac{c_t}{k_t} + G = (r + 1 - \delta) + \frac{d_t}{k_t} \tag{A.7}$$

To derive the expression for (19), first aggregate the capital of all firms as:

$$k_t = \int_0^1 k_{jt} d_j$$

Using (1) and (8):

$$\begin{aligned} k_t &= y_t A^{-1} \int_0^1 \left(\frac{P_{jt}}{P_t} \right)^{-\sigma} d_j \\ &= y_t A^{-1} s_t \end{aligned}$$

The final goods production function thus reduces to:

$$y_t = As_t^{-1}k_t$$

which means that the implicit tax on output is $(s_t - 1)/s_t$. Next note that the dividend in the Calvo model is given by:

$$\begin{aligned} d_t &= y_t - r_t k_t \\ &= As_t^{-1}k_t - r_t k_t \\ &= (As_t^{-1} - r_t)k_t \end{aligned}$$

which implies

$$\frac{d_t}{k_t} = (As_t^{-1} - r_t)$$

In the steady state $s_t = \bar{s}$ which means that

$$\frac{c_t}{k_t} + G^C = (r + 1 - \delta) + \frac{d_t}{k_t} \Rightarrow \frac{c_t}{k_t} = (r + 1 - \delta) - G^C + (A\bar{s}^{-1} - r) = \left(\frac{1-\beta}{\beta}\right)G^C + (A\bar{s}^{-1} - r).$$

From (A.7) this implies (given the normalization $k_0 = 1$):

$$c_0^C = [(A\bar{s}^{-1} + 1 - \delta) - G^C] \tag{A.8}$$

An expression for \bar{s} is derived the following section.

For the Rotemberg model, along the BGP the dividend is given by

$$\frac{d_t}{k_t} = (A(1 - \tau) - r)$$

where τ is given by (23). From (A.7) this implies (given the normalization $k_0 = 1$)

$$c_0^R = [(A(1 - \tau) + 1 - \delta) - G^R]$$

Derivation of (20)

We follow Schmitt-Grohe and Uribe (2011). Define the price dispersion as:

$$s_t = \int_0^1 \left(\frac{P_{jt}}{P_t} \right)^{-\sigma} dj \quad (\text{A.9})$$

(A.9) can be rewritten as a recursion as:

$$s_t = \Pi^{-\gamma\sigma} \Pi_t^\sigma \theta s_{t-1} + (1 - \theta) \left(\frac{P_t^*}{P_t} \right)^{-\sigma} \quad (\text{A.10})$$

Plugging $\Pi_t = \Pi$ and (13) into (A.10), we get the following steady state price dispersion (subject to the same convergence condition in footnote 4):

$$\bar{s} = \frac{(1 - \theta \Pi^{(1-\gamma)(\sigma-1)})^{\sigma/(\sigma-1)}}{(1 - \theta \Pi^{\sigma(1-\gamma)})(1 - \theta)^{1/(\sigma-1)}} \quad (\text{A.11})$$

Check that

$$\frac{\partial \ln \bar{s}}{\partial \Pi} = \frac{\theta \sigma (1 - \gamma) \Pi^{\sigma(1-\gamma)} (1 - \Pi^{-1})}{\Pi (1 - \theta \Pi^{\sigma(1-\gamma)}) (1 - \theta \Pi^{(1-\gamma)(\sigma-1)})} > 0 \text{ if } \Pi > 1 \quad (\text{A.12})$$

The denominator is positive given that the convergence condition holds.

Derivation of (27) and (28)

In a flexible price steady state with log utility, the rental Euler equation (4) can be written as:

$$\frac{1}{c_t} = \beta \left[\frac{1 + r - \delta}{c_{t+1}} \right] \quad (\text{A.13})$$

where

$$r = \left(\frac{\sigma - 1}{\sigma} \right) A$$

We make the following conjectures for the optimal consumption and investment policy rules:

$$c_t = \lambda(A + 1 - \delta)k_t$$

and

$$k_{t+1} = (1 - \lambda)(A + 1 - \delta)k_t$$

Plugging these conjectures in the Euler equation in (A.13), we can solve λ as

$$\lambda = \frac{A(1 - \beta^{\frac{\sigma-1}{\sigma}}) + (1 - \delta)(1 - \beta)}{A + 1 - \delta}$$

which yields (27) and (28).

Table 1: Sensitivity Analysis of the Optimal Inflation Rate in Calvo and Rotemberg Models (in percent)

| | $\theta = 0.75$ | $\theta = 0.80$ | $\theta = 0.85$ | $\theta = 0.90$ | $\theta = 0.95$ |
|-----------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| $\gamma = 0.00$ | C = 0.26 R = 0.52 | C = 0.26 R = 0.52 | C = 0.26 R = 0.52 | C = 0.26 R = 0.52 | C = 0.25 R = 0.51 |
| $\gamma = 0.20$ | C = 0.33 R = 0.65 | C = 0.33 R = 0.65 | C = 0.32 R = 0.65 | C = 0.32 R = 0.65 | C = 0.31 R = 0.64 |
| $\gamma = 0.40$ | C = 0.44 R = 0.87 | C = 0.44 R = 0.87 | C = 0.43 R = 0.87 | C = 0.43 R = 0.87 | C = 0.41 R = 0.85 |
| $\gamma = 0.50$ | C = 0.52 R = 1.05 | C = 0.52 R = 1.05 | C = 0.52 R = 1.01 | C = 0.51 R = 1.01 | C = 0.49 R = 1.01 |
| $\gamma = 0.60$ | C = 0.66 R = 1.31 | C = 0.65 R = 1.31 | C = 0.65 R = 1.31 | C = 0.64 R = 1.30 | C = 0.61 R = 1.27 |
| $\gamma = 0.70$ | C = 0.87 R = 1.73 | C = 0.88 R = 1.73 | C = 0.87 R = 1.73 | C = 0.86 R = 1.73 | C = 0.82 R = 1.27 |
| $\gamma = 0.74$ | C = 1.01 R = 2.02 | C = 1.00 R = 2.02 | C = 1.00 R = 2.02 | C = 0.99 R = 2.02 | C = 0.95 R = 1.97 |
| $\gamma = 0.80$ | C = 1.31 R = 2.63 | C = 1.31 R = 2.63 | C = 1.30 R = 2.63 | C = 1.29 R = 2.63 | C = 1.23 R = 2.57 |
| $\gamma = 0.87$ | C = 2.03 R = 4.06 | C = 2.02 R = 4.06 | C = 2.01 R = 4.06 | C = 1.99 R = 4.06 | C = 1.90 R = 3.97 |

Table 2: Sensitivity Analysis of the Optimal Inflation Rate in Calvo and Rotemberg Models (in percent) with respect to markup

| | |
|-----------------|-----------------------------|
| $\sigma = 2$ | C = 2.27 R = 4.06 |
| $\sigma = 2.25$ | C = 2.00 R = 4.06 |
| $\sigma = 3$ | C = 1.42 R = 4.06 |
| $\sigma = 4$ | C = 0.97 R = 2.48 |
| $\sigma = 4.44$ | C = 0.84 R = 2.00 |
| $\sigma = 5$ | C = 0.71 R = 1.57 |
| $\sigma = 6$ | C = 0.54 R = 1.09 |