Modeling fundamental analysis into portfolio selection

Huazhu Zhang †, Cheng Yan *‡

† *Nomura International Plc, 1 Angel Lane, London, EC4R 3AB, UK.*
‡ *Durham University Business School, Millhill Lane, Durham, DH1 3LB, UK.*

(Received 11 May 2016; Revised 14 March 2017; 5 August 2017)

Accepted in **Quantitative Finance**

Abstract: We derive a closed-form appraisal/information ratio of the investors who are able to observe some information about security fundamentals, by solving a simple instantaneous mean-variance portfolio choice problem in a continuous-time framework. Both analytical and numerical results suggest that investors should choose securities with a more volatile mispricing, a less volatile fundamental, a higher mean-reverting speed and a larger dividend. Our model calibrated with realistic parameters easily outperforms top-percentile portfolio managers in reality, which suggests that the implementation of fundamental analysis may be impeded in practice due to limits of arbitrage. Our paper is a first, necessarily simple, step toward filling the gap of modeling fundamental analysis into portfolio selection.

Keywords: Fundamental Analysis; Mean-reversion; Appraisal/Information Ratio; Portfolio Selection.
JEL classification: C61; G11
Word count: 8000+.

---

* ‡Corresponding author: Cheng Yan, Tel: +44 (0) 191 3345 197. Email: Cheng.Yan@dur.ac.uk (Cheng Yan). †Huazhu Zhang, Tel: +44 (0) 741 0636 810. Email: huazhu.zhang@hotmail.com (Huazhu Zhang). The authors have contributed equally to this work. We are grateful to the editors (Michael Dempster and Jim Gatheral), the Publishing Editor (Sadie Thrift), two anonymous referees, Anurag Banerjee, Sohnke Bartram, Travis Box (discussant), Xiaoshan Chen, Vladislav Damjanovic, Stewart Hodges, Hening Liu, Anthony Neuberger, Qiusha Peng, Dennis Philip, Thomas Renstrom, Jing Xu and participants in the 2011 Warwick, Oxford and Cambridge (Woxbridge) Spring Doctoral Conference in University of Warwick, the II Annual PhD conference of University of Leicester, the Micro Workshop in Durham University, and the 2016 FMA annual conference in Las Vegas for helpful comments and discussions. We are also grateful to the fund from National Natural Science Foundation of China (No. 71703045) and the durham university business school seedcorn fund. The views expressed herein are those of the authors and do not necessarily reflect or represent those of Nomura. All errors are our own.

Modeling fundamental analysis into portfolio selection

Abstract: We derive a closed-form appraisal/information ratio of the investors who are able to observe some information about security fundamentals, by solving a simple instantaneous mean-variance portfolio choice problem in a continuous-time framework. Both analytical and numerical results suggest that investors should choose securities with a more volatile mispricing, a less volatile fundamental, a higher mean-reverting speed and a larger dividend. Our model calibrated with realistic parameters easily outperforms top-percentile portfolio managers in reality, which suggests that the implementation of fundamental analysis may be impeded in practice due to limits of arbitrage. Our paper is a first, necessarily simple, step toward filling the gap of modeling fundamental analysis into portfolio selection.
Keywords: Fundamental Analysis; Mean-reversion; Appraisal/Information Ratio; Portfolio Selection.
JEL classification: C61; G11
Word count: 8000+

1 Introduction

Fundamental analysis is popular in practitioners (e.g., Wermers et al., 2012), while one cornerstone of the Market Efficiency Hypothesis (EMH) is that fundamental analysis should "not work" (Bartram and Grinblatt, 2017). Two specific questions require answers before concluding whether fundamental analysis works. The first one is how to estimate the fundamental (fundamental value, fair value, or intrinsic value). The second one, perhaps of more interest to the practitioners, is how to make the best use of the information revealed by fundamental analysis. The extant Accounting and Finance literature on the first one is almost purely empirical (e.g., Lewellen, 2010), '…*however, empirical research is (or should be) informed by theory, because the interpretation of empirical analysis is impossible without theoretical guidance* (Richardson et al., 2010, page 411)'. Perhaps because the fundamental research efforts are unobservable, the second question has rarely been scrutinized by the literature (e.g., Wermers et al., 2012; Bartram and Grinblatt, 2017; Yan and Zheng, 2017). Our paper is a first, necessarily simple, step toward filling the gap of modeling fundamental analysis into portfolio selection and endorsing fundamental analysis from a theoretical perspective.

Our question is not trivial, but seriously under-research. Most academic studies gauge the fundamental and simply compare the performance of the portfolios sorted by the fundamental signal (see, e.g. Bartram and Grinblatt, 2017). Although this approach suits their academic purpose, it probably is less desired in industry. The *fundamental investor (e.g.,* active managers in Wermers et al., 2012, or more generally the arbitrageurs in Grossman and Stiglitz, 1980) who have information about the fundamentals, are prone to maximize their potential reward from fundamental analysis results. However, the mainstream literature neglects this need, and the optimal trading strategy is unknown. This insight stems from Treynor and Black (1973) and Black and Litterman (1992). Departing from their old-school discrete settings, our analysis is tamed in a modern continuous-time framework with the new perspective of fundamental analysis.

We resist the temptation to add another paper on designing a 'superior mousetrap' to *empirically* capture the fundamental for three reasons. Firstly, even if we ignore the critique of data-snooping and make room for it, our contribution in this regard will be marginal, given the large number of existing papers. The 'anomaly' literature has identified more than 330 predictors of future returns and a large portion of them are accounting variables, although most of them are sample/model sensitive (Green et al., 2013; Harvey et al., 2016; McLean and Pontiff, 2016).

Secondly, despite numerous papers on the first moment, there is scarce discussion on the second moment, which is much more attainable in practice (e.g. Campbell and Shiller, 1987, 1988a, b; Campbell, 1991). We focus on the second moment of the fundamental and sidestep for now the thorny issues of forecasting the first moment, as the forecast of the first moment is likely to be inaccurate and time-varying, which is clear in an Arrow-Debreu world.

Finally, even if some investors accurately observe the first moment of the fundamental, they can at most predict one future moving direction of security prices by using the captured fundamental values only, but not when the security prices start to revert to their fundamentals and how to maximize the profit. There are various reasons why the 'asset bubble' may persist (e.g. Shleifer and Vishny, 1997). Fundamental investors can reap their profit from fundamental analysis from either the mean-reverting market price or the distributions of dividends, takeovers, private buyouts or asset liquidation, which means that at least both the reversion speed and dividend should also have been taken into account. We are the first to explicitly tackle this issue, which makes our paper complementary to Bartram and Grinblatt (2017), Yan and Zheng (2017).

We demonstrate our idea in the most parsimonious model possible under standard assumptions. We employ the framework from Treynor and Black (1973) and Sharpe's Diagonal Model (1963): the investor actively allocates her wealth into *n almost independent* risky securities and one risk-free asset by minimizing the instantaneous risk of her portfolio while holding the instantaneous expected excess return fixed. We assume that the risky security has a market price and a fundamental, and model the market price as its fundamental scaled by a mispricing multiplier (e.g. Arnott et al., 2014). Whereas the fundamental follows a geometric Brownian motion and satisfies the ICAPM (see, e.g. Wang, 1993), the mispricing is governed by a geometric mean-reversion process (see, e.g. Pindyck, 1991).

Our mean-reversion setting is backed up by a large literature arguing that security prices mean-revert to their fundamental values in the long term (e.g. Shiller, 1981; Summers, 1986; Fama and French, 1988). Guasoni (2006) extends Shiller (1981) and Summers (1986) models to the purely continuous random setting and studies a continuous-time version of these models from an informed investor, who observe both fundamental and market values. Buckley et al. (2012) extend Guasoni's model for stocks following geometric Brownian motion to constant relative risk-averse investors when mispricing follows a continuous mean-reverting process, and obtain more general results which nests those of Guasoni (2006) as a special case of the relative risk aversion being one. With the notions of asymmetric information and fads analogous to Wang (1993), Guasoni (2006), Buckley et al. (2012), Buckley and Long (2015) model the asset dynamics using a Levy process that can be applied to any asset that has a fundamental value and the mispricing is simply the difference between the fundamental value and the observed value of the asset. Analogically, we model the price as a multiplicative rather than additive process (e.g., Arnott et al., 2014), where the mispricing follows a geometric mean-reversion (e.g. Pindyck, 1991; Dixit and Pindyck, 1994; Metcalf and Hassett, 1995; Epstein et al., 1998; Ewald and Zhang, 2006; Ewald and Yang, 2007; Yang and Ewald, 2010). Our model complements the recent additive price settings such as the two Brownian motions in Cvitanić et al. (2006), and one geometric Brownian motion and one OU process in Buckley et al. (2012, 2014), Buckley and Long (2015). The tractability of our model allows us to analytically derive the *appraisal ratio* and *information ratio* of the optimal portfolio in the Treynor-Black framework.

We solve the instantaneous mean-variance portfolio choice problem based on quadratic utility in the presence of mispricing. Analytical results suggest that investors use fundamental analysis to pick up securities with a more volatile mispricing, a less volatile fundamental, a higher mean-reverting speed and a larger dividend. The intuition is that a volatile mispricing implies many investment opportunities, while a volatile fundamental can only increase the active risk and thus has a negative effect on the expected appraisal ratio. The contribution from the mean-reverting speed and dividends to the expected appraisal ratio is positive.

Our numerical experiments reveal that a large realistic domain for parameter values, in which the information ratios are higher than the ones of top-percentile portfolio managers. Since our model simply maps the features of the fundamental and the mispricing processes into the information ratio, the discrepancy between our calibration and those achieved in reality may be due to various realistic limits of arbitrage (e.g. Shleifer and Vishny, 1997) such as trading costs, costs to produce fundamental information, market participants' competitions.

The paper unfolds as follows. In Section 2, we introduce our basic model and derive the dynamics of the security price, the appraisal ratio and the expected appraisal ratio. Section 3 presents the comparative statics analysis of the effects of model parameters on the expected appraisal ratio and the results from our numerical experiments. Section 4 concludes.

2 Theoretical Modeling

In this section, we firstly introduce our model for security prices and the details of the two stochastic processes that govern the mispricing and the fundamental, respectively. After that, we derive the dynamics of the security price, the appraisal ratio and the expected appraisal ratio.

2.1 The multiplicative security price

Let $P_i(t)$ and $F_i(t)$ denote the market price and the fundamental value of the ith security, respectively. We assume

$$P_i(t) = \alpha_i(t) F_i(t) \tag{1}$$

where $\alpha_i(t)$ is a non-negative multiplier, which is assumed to be independent of $F_i(t)$ in the spirit of Cvitanić et al. (2006), Buckley et al. (2012, 2014), and Buckley and Long (2015). A zero $\alpha_i(t)$ can be explained as the security issuer being in severe financial distress or bankruptcy. Intuitively, the mispricing $\alpha_i(t)$ arises from the asymmetric information between the fundamental investors, who observe both the market price and some fundamental information, and the uninformed investors who lack the aid of fundamental analysis and observe market prices only. We explore how good the performance relative to the benchmark the fundamental investors can achieve and what affects it, i.e. the magnitude of the information/appraisal ratios and the underlying factors.

This specification for the security price is straightforward. The multiplier $\alpha_i(t)$ measures to what extent the security price $P_i(t)$ deviates from its fundamental value $F_i(t)$ at time t. For this reason, we refer to it as the mispricing in the rest of this paper. The security is underpriced or traded at a discount relative to its fundamental value at time t if $\alpha_i(t) < 1$, overpriced or traded at a premium if $\alpha_i(t) > 1$, and just priced or traded at par if $\alpha_i(t) = 1$. This multiplicative price form is also used in other papers, see, e.g. Arnott et al. (2014) and the references therein.

2.2 The geometric mean-reversion process for the mispricing

The mispricing $\alpha_i(t)$ in equation (1) has to be non-negative for the price $P_i(t)$ to be meaningful, and the price converges to its fundamental value $F_i(t)$ in the long term. One suitable choice for $\alpha_i(t)$ is the geometric mean-reversion process, which satisfies both requirements. Another advantage of this process is that it has a stationary distribution under certain restrictions which has been derived in Ewald and Yang (2007) and we will discuss later.

The geometric mean-reversion process for $\alpha_i(t)$ is

$$d\alpha_i(t) = \theta_i\big(\eta_i - \alpha_i(t)\big)\alpha_i(t)dt + \varsigma_i\alpha_i(t)dZ_i(t), i = 1, \dots, n \tag{2}$$

where $Z_1(t), \dots, Z_n(t)$ are n independent Wiener processes and the source of uncertainty in the mispricing, θ_i is the mean-reverting speed, η_i is the mean-reversion level for $\alpha_i(t)$, and ς_i is the idiosyncratic volatility due to mispricing. This geometric mean-revision process has been used to model commodity prices, irreversible investment, and corporate earnings, etc, in the previous literature (e.g. Pindyck, 1991; Dixit and Pindyck, 1994; Metcalf and Hassett, 1995; Epstein et al., 1998; Ewald and Zhang, 2006; Ewald and Yang, 2007; Yang and Ewald, 2010), and its application to describe the mispricing in this study is novel.

$\alpha_i(t)$ has several nice properties. First, unlike the OU process, the geometric mean-reversion process is strictly positive and 'the boundary values 0 and ∞ are inaccessible in the language of Merton (Ewald and Yang, 2007, page 11)', when its stationary distribution exists under the assumption $2\eta_i\theta_i > \varsigma_i^2$. Second, like the OU process, the geometric mean-reversion process automatically adjusts upward if $\alpha_i(t) < \eta_i$ and downward if $\alpha_i(t) > \eta_i$. As long as the mispricing is not a constant (otherwise degenerate to a special case of the OU process), its drift term $\theta_i\big(\eta_i - \alpha_i(t)\big)\alpha_i(t)$ is quadratic in $\alpha_i(t)$ and its diffusion $\varsigma_i\alpha_i(t)$ is dependent on the level of $\alpha_i(t)$, , which offers it an advantage over the OU process in terms of capturing the non-linearity and state-dependent diffusion, respectively. Third, while $\alpha_i(t)$ always adjusts toward the mean-reversion level η_i, the mean of its stationary distribution is not equal to η_i in all but trivial cases, which makes our model applicable to more general cases. Finally, the mean of the stationary distribution is affected by its higher order moments.

2.3 The geometric Brownian motion for the fundamentals

Let r_f, $P_m(t)$ and β_i denote the constant risk-free interest rate, the value of the market index and the beta coefficient of the ith security, respectively. We assume that dividends are paid continuously at a constant fraction δ_i of the fundamental value $F_i(t)$, so that dividends offer an alternative way for investors to reap abnormal profits (see, e.g. Bartram and Grinblatt, 2017).

The constant fraction δ_i is not limited to stock dividend, as it can also be viewed as takeovers, private buyouts or asset liquidation if it is assumed to be one.

For illustrating reasons, we simply specify the stochastic process for the fundamental $F_i(t)$ so that it satisfies the intertemporal CAPM of Merton (1973)

$$\frac{dF_i(t)}{F_i(t)} = r_f dt + \beta_i \left(\frac{dP_m(t)}{P_m(t)} - r_f dt\right) - \delta_i dt + v_i dB_i(t), i = 1, \dots, n \tag{3}$$

$$\frac{dP_m(t)}{P_m(t)} = (\mu_{mt} + r_f)dt + \sigma_m dB_m(t) \tag{4}$$

Where v_i measures the idiosyncratic risk of fundamental returns, μ_{mt} is the instantaneous excess return on the market index, σ_m is the volatility of the market return, $B_i(t), i = 1, \dots, n$ and $B_m(t)$ are independent standard Brownian motions and the source of uncertainty in the fundamental, uncorrelated with $Z_m(t)$, the standard Brownian motion that drives the mispricing $\alpha_i(t)$.

Substituting equation (4) into equation (3) and rearranging equation (3), we have the total return on the ith security over an infinitesimally small interval dt

$$\frac{dF_i(t)}{F_i(t)} + \delta_i dt = r_f dt + \beta_i \mu_{mt} dt + v_i dB_i(t) + \beta_i \sigma_m dB_m(t) \tag{5}$$

We can see that the volatility of fundamental returns can be attributed to two sources of risk: the idiosyncratic risk v_i and the systematic risk $\beta_i \sigma_m$.

Moving $r_f dt$ to the left of equation (5) and taking expectation on both sides, we find that $F_i(t)$ satisfies the intertemporal CAPM of Merton (1973)

$$E\left[\frac{dF_i(t)}{F_i(t)} + \delta_i dt\right] - r_f dt = \beta_i \mu_{mt} dt \tag{6}$$

which means that the expected excess return on the fundamental of the security is determined by the market risk premium and its exposure to the market risk. The only difference is that, Merton (1973) assumes that all dividend payments are in the form of share repurchase and thus the expected rate of return on the security is simply $E\left[\frac{dF_i(\mathrm{t})}{F_i(\mathrm{t})}\right]$.

2.4 Asset price dynamics

This sub-section solves the stochastic differential equation for the market price.

Given the multiplicative function for $P_i(t)$ in equation (1) and the two stochastic processes for $\alpha_i(t)$ and $F_i(t)$ in equation (2) and equation (5), we can obtain the stochastic differential equation for the price $P_i(t)$:

$$\frac{dP_i(t)}{P_i(t)} = \left(\theta_i\left(\eta_i - \alpha_i(t)\right) + \beta_i\mu_{mt} - \delta_i + r_f\right)dt + \varsigma_i dZ_i(t)$$
$$+v_i dB_i(t) + \beta_i\sigma_m dB_m(t) \tag{7}$$

For the proof of the above equation, please refer to Appendix A.

The expected instantaneous return on the security conditional on $\alpha_i(t)$ is

$$E[\frac{dP_i(t)}{P_i(t)}] = \left(\theta_i\left(\eta_i - \alpha_i(t)\right) + \beta_i\mu_{mt} - \delta_i + r_f\right)dt \tag{8}$$

We see that the expected capital gains on the security are due to two sources: the mispricing and the change in the fundamental value. The intertemporal CAPM holds when $\alpha_i(t) = \eta_i$. In other words, if the mispricing stays at its mean-reversion level, no abnormal returns can be obtained from investing in this security.

2.5 The appraisal ratio

This sub-section derives the appraisal ratio of the representative investor.

Without loss of generality, we use appraisal ratio and the root of appraisal ratio, information ratio to measure the performance of portfolio selection. Following Treynor and Black (1973) and Sharpe's Diagonal Model (1963), we consider the portfolio choice problem facing an investor who allocates her wealth into *n* almost independent risky securities and one risk-free asset by solving the instantaneous mean-variance optimization problem based on quadratic utility.

Let x_{it} denote the fraction of the investor's wealth V_t allocated to the ith security at time t. The excess return on her portfolio at time t, denoted by r_{pt}, is

$$r_{pt} = \frac{dV_t}{V_t} - r_f dt = \sum_{i=1}^n x_{it}\left(\frac{dP_i(t)}{P_i(t)} + \frac{\delta_i F_i(t)dt}{P_i(t)} - r_f dt\right) \tag{9}$$

In the above expression, the total return on the ith security consists of two parts: the capital gains $dP_i(t)/P_i(t)$ and the dividend yield $\delta_i F_i(t)dt/P_i(t)$. We assume the dividends are paid continuously as a constant fraction of the fundamental $F_i(t)$ and that is why the dividend income over the infinitesimally small interval dt is calculated as $\delta_i F_i(t)dt$.

Using equation (1) and (7), we partition r_{pt} into two parts as follows:

$$r_{pt} = \sum_{i=1}^n x_{it}\left(\frac{dP_i(t)}{P_i(t)} + \frac{F_i(t)}{P_i(t)}\delta_i dt - r_f dt\right)$$

$$= \sum_{i=1}^{n} x_{it} \Big(\big(\theta_i\big(\eta_i - \alpha_i(t)\big) + \beta_i\mu_m - \delta_i\big)dt + \varsigma_i dZ_i(t) + v_i dB_i(t)$$
$$+ \beta_i\sigma_m dB_m(t) + \alpha_i^{-1}(t)\delta_i dt \Big)$$
$$= \sum_{i=1}^{n} x_{it} \left(\mu_i dt + \sigma_i dW_i(t)\right) + \left(\sum_{i=1}^{n} x_{it}\beta_i\right)\left(\mu_m dt + \sigma_m dB_m(t)\right)$$
$$= \sum_{i=1}^{n} x_{it}\, r_{it} + \left(\sum_{i=1}^{n} x_{it}\beta_i\right) r_{mt} \tag{10}$$

where

$$\mu_{it} = \theta_i\big(\eta_i - \alpha_i(t)\big) + (\alpha_i^{-1}(t) - 1)\delta_i \tag{11}$$

$$\sigma_i = \sqrt{\varsigma_i^2 + v_i^2} \tag{12}$$

$$dW_i(t) = \frac{\varsigma_i}{\sigma_i} dZ_i(t) + \frac{v_i}{\sigma_i} dB_i(t) \tag{13}$$

$$r_{it} = \mu_{it} dt + \sigma_i dW_i(t) \tag{14}$$

$$r_{mt} = \mu_{mt} dt + \sigma_m dB_m(t) \tag{15}$$

r_{mt} is the instantaneous excess return on the market index, r_{it} is the active return on the ith security, μ_{it} is the expected active return conditional on $\alpha_i(t)$, and σ_i is the active risk.

If we take an explicit position x_{mt} in the market index, the equation (10) needs to be modified to

$$r_{pt} = \sum_{i=1}^{n} x_{it} r_{it} + \left(x_{mt} + \sum_{i=1}^{n} x_{it}\beta_i\right) r_{mt} = \sum_{i=1}^{n+1} x_{it} r_{it} \tag{16}$$

where $r_{(n+1)t} = r_{mt}$ and $x_{(n+1)t} = x_{mt} + \sum_{i=1}^{n} x_{it}\beta_i$. Note that the investment in the market index is due to two sources: the explicit investment in the market index x_{mt} and $\sum_{i=1}^{n} x_{it}\beta_i$, the exposure to the market risk of individual securities.

Since r_{mt} and $r_{it}, i = 1, \dots, n$ are mutually independent in the standard Treynor-Black framework (1973) and Sharpe's Diagonal Model (1963), the portfolio choice problem amounts to making optimal bets on $n + 1$ independent assets, the market index and the n active returns $r_{it}, i = 1, \dots, n$. Let $\mu_{(n+1)t} = \mu_{mt}$ and $\sigma_{n+1}^2 = \sigma_m^2$. Then the conditional instantaneous mean and variance of the excess returns on the portfolio are $\mu_{pt} = \sum_{i=1}^{n+1} x_{it}\mu_{it}$ and $\sigma_{pt}^2 = \sum_{i=1}^{n+1} x_{it}^2\sigma_i^2$, respectively.

Consistent with mean-variance framework based on quadratic utility (e.g. Sharpe, 1963; Treynor and Black, 1973), we assume that the investor's problem is to minimize the instantaneous variance of her portfolio σ_{pt}^2 while holding the instantaneous expected excess return μ_{pt} fixed

$$min\frac{1}{2}\sum_{i=1}^{n+1} x_{it}^2\,\sigma_i^2,\ \ s.t.\ \ \sum_{i=1}^{n+1} x_{it}\,\mu_{it} = \mu_{pt} \tag{17}$$

Now our portfolio choice problem is the same as Treynor and Black (1973, equation 7 on page 71). We follow their procedures to solve the problem. We have to emphasize that the portfolio in our model is rebalanced continuously while Treynor and Black's model is static.

The optimal portfolio and its squared Sharpe Ratio is

$$x_i^* = \frac{\mu_{it}}{\mu_{pt}}\frac{\sigma_p^2}{\sigma_i^2}, i = 1, \dots, n \tag{18}$$

$$\left(\frac{\mu_{pt}}{\sigma_{pt}}\right)^2 = \left(\frac{\mu_{mt}}{\sigma_m}\right)^2 + \sum_{i=1}^{n}\frac{\mu_{it}^2}{\sigma_i^2} \tag{19}$$

The first term on the right-hand side of the above equation measures the influence of market wide deviations from fundamentals on investment performance (known as 'market premium'). The second term is the so-called 'appraisal ratio', which represents the total contribution of security selection and can be viewed as the better/worse performance relative to the benchmark buy-and-hold strategy of the market portfolio. Following Treynor and Black (1973), we focus on the appraisal ratio rather than the Squared Sharpe Ratio from here onwards and denote it by AR_p. Substituting the equation (11) for μ_{it} into equation (19), we obtain

$$AR_p = \sum_{i=1}^{n}\frac{\left(\theta_i(\eta_i - \alpha_i(t)) + (\alpha_i^{-1}(t) - 1)\delta_i\right)^2}{\sigma_i^2} \tag{20}$$

2.6 The expected appraisal ratio

This sub-section derives the expected appraisal ratio of the representative investor.

We evaluate the appraisal ratio in equation (20) with respect to the stationary probability distribution of $\alpha_i(t)$ to eliminate its dependence on $\alpha_i(t)$. The resulting expected appraisal ratio reflects the *ex-ante* investment opportunity when we have no prior knowledge of the level of $\alpha_i(t)$. In case that it is appropriate to approximate the cross-sectional distribution of the returns on the securities using this stationary distribution (Yang and Ewald, 2010), it reveals the *ex-ante* investment opportunity among these securities.

As we show in Appendix B, to obtain the expected appraisal ratio, we have to evaluate $E\left[\left(\alpha_i(\infty)\right)^{-2}\right]$, which exists only if $3\varsigma_i^2/2\theta_i < \eta_i$. For this reason, we assume $3\varsigma_i^2/2\theta_i < \eta_i$ in the rest of this paper, and the expected appraisal ratio is

$$E\left[AR_p\right] = \sum_{i=1}^{n}\left(\frac{\theta_i\varsigma_i^2}{2\left(\varsigma_i^2+v_i^2\right)} + \frac{\left(2(\eta_i-1)^2\theta_i^2+3\varsigma_i^4+\left(6-5\eta_i\right)\theta_i\varsigma_i^2\right)\delta_i^2}{(2\eta_i\theta_i-3\varsigma_i^2)(\eta_i\theta_i-\varsigma_i^2)\left(\varsigma_i^2+v_i^2\right)} + \frac{\left(\varsigma_i^2+(2-\eta_i)\theta_i\right)\delta_i\varsigma_i^2}{(\eta_i\theta_i-\varsigma_i^2)\left(\varsigma_i^2+v_i^2\right)}\right) \quad (21)$$

For $\eta_i = 1$, the restriction $3\varsigma_i^2/2\theta_i < \eta_i$ becomes $3\varsigma_i^2/2 < \theta_i$ and the above expression can be simplified to

$$E\left[AR_p\right] = \sum_{i=1}^{n}\left(\frac{\theta_i\varsigma_i^2}{2\left(\varsigma_i^2+v_i^2\right)} + \frac{(\theta_i+3\varsigma_i^2)\delta_i^2\varsigma_i^2}{(2\theta_i-3\varsigma_i^2)(\theta_i-\varsigma_i^2)\left(\varsigma_i^2+v_i^2\right)} + \frac{(\theta_i+\varsigma_i^2)\delta_i\varsigma_i^2}{(\theta_i-\varsigma_i^2)\left(\varsigma_i^2+v_i^2\right)}\right) \quad (22)$$

In the following analysis, we focus on the case $\eta_i = 1$ and the mispricing $\alpha_i(t)$ mean-reverts to one.

3 Comparative statics analysis and numerical experiments

In this section, we examine how the pivotal parameters listed in Table 1 affect the expected appraisal ratio of the optimal portfolio when the mean-reversion level $\eta_i = 1$, according to the large literature arguing that security prices mean-revert to their fundamental values in the long term (e.g. Shiller, 1981; Summers, 1986; Fama and French, 1988). Given the difficulty in quantifying the (probably time-varying) magnitude of the fundamental and the mispricing empirically, we take a conservative approach to calibrate our model to a large domain of reasonable parameter values, rather than to any one set of parameter values only.

[Insert Table 1 around here]

To be specific, we deliberately set the range of the mean-reversion speed of the mispricing (θ_i) from 9.9% to 64.9%, based on the empirical results documented over a sample of 12024 stocks during the period 2004-2014 in Giannetti and Kahraman (2015, page 16) with the measure from Della Vigna and Pollet (2009). It is perhaps a little bit difficult to quantify the average number of stocks held by investors. We adopt 90 from Saap and Yan (2008, page 33), who find that the average number of holdings is approximately 90 over a large sample of 2278 funds during 1984–2002. We start with the case of zero dividends and proceed to the cases of non-zero dividends. In the non-zero dividend case, we adopt an average dividend yield of 1.5%

from Hartzmark and Solomon (2013, Table 1, page 645) over the sample of all common shares of US companies listed on the New York Stock Exchange (NYSE), American Stock Exchange (Amex) and Nasdaq Exchange from January 1927 until December 2011, which is also confirmed in Harris et al. (2015) using fund data. In order to make sure that we are on the safe side, we conservatively set the active risk (σ_i) at 14% with all possible combinations of the volatilities of fundamental (v_i) and mispricing (ς_i), as Cremers and Petajisto (2009, Table 1, page 13) find that about 97%(= 1– 48/1678) of the US all-equity mutual funds have an active risk no more than 14% in 2002. The information ratios will be higher if we assign a smaller value to active risk.

We confirm the robustness of our results using many alternative domains and the results are available upon request. We do not consider how the trivial parameters affect the expected appraisal ratio, as the mean-reversion level (η_i) of the mispricing should be one in the long-run; clearly the effect of the number of investable securities (n) is positive; and the effect of the active risk (σ_i) depends on the relative variation between the volatility of the mispricing (ς_i) and the fundamental (v_i). We have also considered if and only if a fraction of the securities pay dividends, and the results are similar and available upon request.

3.1 Expected appraisal ratios when zero dividends

This sub-section explores the expected appraisal ratio when no security pays dividends.

3.1.1 Comparative statics analysis. When no security pays dividends, we have $\delta_i = 0$ for all i. For $\eta_i = 1$, the expected active return μ_i in equation (11) is completely due to the mispricing, and the expected appraisal ratio in equation (22) is reduced to

$$E\left[AR_p\right] = \sum_{i=1}^{n} \frac{\theta_i \varsigma_i^2}{2(\varsigma_i^2 + v_i^2)} \tag{23}$$

We see that the value added by security selection is determined by the mean-reverting speed θ_i, the volatility of mispricing ς_i and the volatility of the fundamental value v_i. *Ceteris paribus*, the expected appraisal ratio is increasing in θ_i and ς_i^2, but decreasing in v_i^2, which means that investors prefer securities with higher mean-reverting speeds, more volatile mispricing and less volatile fundamental value. Intuitively, a fast adjustment of mispricing allows a quick reaping of benefits from betting on the security. In addition, as ς_i^2 rises, the probability for the occurrence of more extreme mispricing will increase, which implies better investment opportunities. Finally, the rise in the volatility of fundamental v_i can only increase the active risk and thus has a negative effect on the expected appraisal ratio.

3.1.2 Numerical experiments. Given the effects of model parameters on the appraisal ratio from previous analysis, we are wondering the extent to which the fundamental analysis can be used for portfolio selection. We answer this question by conducting several numerical experiments under assumed values for parameters. For comparison reasons with existing studies, here we turn to information ratios, the square root of appraisal ratios, rather than appraisal ratios themselves. To our surprise, our numerical results show that under plausible parameter values, fundamental analysis can lead to information ratios well above those achieved by practitioners.

In the following analysis, we assume independent investable securities for all the numerical experiments after extracting the common market component in equation (19), which is in consistence with Treynor and Black (1973). We focus our attention on the relationship between information ratios, the mean-reverting speed θ_i, the volatility of mispricing ς_i.

Figure 1 illustrates how the information ratio varies according to the changes in the mean-reverting speed θ_i and the volatility of mispricing ς_i, when the active risk σ_i is fixed at 14% and none of the 90 independent and identical securities pays dividends. The two subplots are respectively the 3-D and 2-D contours, produced under the two restrictions $3\varsigma_i^2/2 < \theta_i$ and $\varsigma_i^2 \leq \sigma_i^2$. The labels on the plots indicate the levels of the information ratios.

[Insert Figure 1 around here]

From the top graph, we see that the information ratio is no greater than 3 and is increasing in both the mean-reverting speed θ_i and the volatility of mispricing ς_i. A larger volatility of mispricing ς_i implies not only potentially more extreme values of $\alpha(t)$ but also increased probabilities for their occurrences, and thus better investment opportunities. A high mean-reverting speed θ_i allows us to quickly reap the benefits of making bets on the mispricing. The 2-D contour is convex to the origin, implying that the marginal increase in the information ratio is decreasing in both the mean-reverting speed θ_i and the volatility of mispricing ς_i. As we move down the isoline, more increase in the volatility of mispricing ς_i has to be traded for the same decrease in the mean-reverting speed θ_i in order to obtain the same information ratio.

[Insert Table 2 around here]

Table 2 reports information ratios of the optimal portfolio for different combinations of the mean-reverting speed θ_i and the volatility of mispricing ς_i, when the active risk σ_i is fixed at 14% and none of the 90 independent and identical securities pays dividends. The information ratio is calculated as the square root of the expected appraisal ratio in equation (22) and the missing values are due to the restrictions of $3\varsigma_i^2/2 < \theta_i$ and $\varsigma_i^2 \leq \sigma_i^2$.

We see that given the volatility of mispricing ς_i, the information ratio is increasing in the mean-reverting speed θ_i. For $\varsigma_i = 1\%$ and $\theta_i = 9.9\%$, the information ratio is 0.151, which increases to 0.386 when $\varsigma_i = 1\%$ and $\theta_i = 64.9\%$. Given the level of θ_i, the information ratio is also increasing in ς_i. For example, at $\theta_i = 9.9\%$ and $\varsigma_i = 14\%$, the information ratio increases to 2.111. We find in 89.88% (76.79%) of our results, the information ratios is greater than one-half (one), which is the a good performance can only achieved by the top 25% (10%) of the active portfolio managers, according to Grinold and Kahn (2000) and Kahn and Rudd (2003). For reasonable magnitude of parameters such as $\theta_i \geq 14.9\%$ and $\varsigma_i \geq 3\%$, all the information ratios are greater than one-half. In other words, our results suggest an important role for fundamental analysis in term of improving portfolio selection.

3.2 Expected appraisal ratios when non-zero dividends

This sub-section explores the expected appraisal ratio when there are non-zero dividends.

3.2.1 Comparative Statics Analysis. When securities pay dividends at a constant fraction δ_i of the fundamental value, the expected appraisal ratio $E\left[AR_p\right]$ is shown in equation (22). The introduction of dividends not only increases $E\left[AR_p\right]$ but also complicates the way that other parameters, especially the mean-reverting speed θ_i as we show below, influence $E\left[AR_p\right]$.

First, we analyze the effect of dividends on the expected appraisal ratio.

The expected active return in equation (11) can be rewritten as

$$\mu_{it} = \left(1 - \alpha_i(t)\right)\left[\theta_i + \frac{\delta_i}{\alpha_i(t)}\right] \tag{24}$$

The above equation indicates that given the level of $\alpha_i(t)$ and the mean-reverting speed θ_i, the growth of dividends increases the contribution from dividends to the optimal portfolio. More importantly, note that the effect of dividends to the expected active return is similar to increasing the mean-reverting speed by the amount of $\delta_i/\alpha_i(t)$, and the way dividends contribute to the appraisal ratio is very similar to mean-reverting speed according to equation (22). It does not only analytically blur the effects of the mean-reverting speed θ_i on the appraisal ratio, but also weaken the function of the mean-reverting speed to let investors to capitalize abnormal returns, which has important influence in latter analysis.

Based on equation (22), clearly the partial derivative of the expected appraisal ratio $E[AR_p]$ with respect to the volatility of the dividend yield δ_i is positive:

$$\frac{\partial E[AR_p]}{\partial \delta_i} = \sum_{i=1}^{n}\left(\frac{2(\theta_i+3\varsigma_i^2)\varsigma_i^2\delta_i}{(2\theta_i-3\varsigma_i^2)(\theta_i-\varsigma_i^2)(\varsigma_i^2+v_i^2)} + \frac{(\theta_i+\varsigma_i^2)\varsigma_i^2}{(\theta_i-\varsigma_i^2)(\varsigma_i^2+v_i^2)}\right) > 0 \quad (25)$$

where $3\varsigma_i^2/2 < \theta_i$ and the expected appraisal ratio is increasing in δ_i when $\delta_i > 0$.

However, we find the marginal effects of the mean-reverting speed θ_i on the expected appraisal ratio $E[AR_p]$ is decreasing in dividend yield δ_i, and vice versa, as the partial derivative of the marginal effects of the mean-reverting speed is negative:

$$\frac{\partial^2 E[AR_p]}{\partial \theta_i \partial \delta_i} = \sum_{i=1}^{n}\left(-\frac{2\varsigma_i^2\left(2\delta_i(\theta_i+(3+3\sqrt{2})\varsigma_i^2)(\theta_i+(3-3\sqrt{2})\varsigma_i^2)+\left(3\varsigma_i^3-2\varsigma_i\theta_i\right)^2\right)}{(3\varsigma_i^4-5\varsigma_i^2\theta_i+2\theta_i^2)^2(\varsigma_i^2+v_i^2)}\right) < 0 \quad (26)$$

where it is clear as $(\theta_i + (3-3\sqrt{2})\varsigma_i^2) > 0$ conditional on $3\varsigma_i^2/2 < \theta_i$.

Next, we examine whether the expected appraisal ratio is still increasing in the volatility of mispricing ς_i and decreasing in the volatility of the fundamental value v_i, after introducing dividends. Although the analytic effects of the volatility of mispricing ς_i on the expected appraisal ratio $E[AR_p]$ is no longer clear, we find that the partial derivative of the expected appraisal ratio $E[AR_p]$ with respect to the volatility of the fundamental value v_i remain negative:

$$\frac{\partial E[AR_p]}{\partial v_i^2} = \sum_{i=1}^{n}\left(-\frac{(\theta_i+3v_i^2)\delta_i^2 v_i^2}{(2\theta_i-3\varsigma_i^2)(\theta_i-\varsigma_i^2)\left(\varsigma_i^2+v_i^2\right)^2} - \frac{(\theta_i+v_i^2)\delta_i v_i^2}{(\theta_i-\varsigma_i^2)\left(\varsigma_i^2+v_i^2\right)^2} - \frac{\theta_i v_i^2}{2\left(\varsigma_i^2+v_i^2\right)^2}\right) < 0 \quad (27)$$

where it is clear as $(2\theta_i - 3\varsigma_i^2) > 0$ and $(\theta_i - \varsigma_i^2) > 0$ conditional on $3\varsigma_i^2/2 < \theta_i$.

Since the appraisal ratio is defined as the expected active return divided by the tracking error, without loss of generality we can hold the tracking error fixed and verify that the expected appraisal ratio is still increasing in the volatility of mispricing ς_i, as the volatility of mispricing $\varsigma_i$is perfectly negative correlated with the volatility of the fundamental value v_i as $\sigma_i^2 = \varsigma_i^2 + v_i^2$.

Finally, after the introduction of dividends, we find little analytical guidance regarding the effects of increasing the mean-reverting speed θ_i on the expected appraisal ratio $E[AR_p]$, for which we resort to numerical experiments.

3.2.2 Numerical Experiments. Table 3 reports information ratios of the optimal portfolio for different combinations of the mean-reverting speed θ_i and the volatility of mispricing ς_i, when the active risk σ_i is fixed at 14% and each of the 90 independent and identical securities pay fixed dividends $\delta_i = 1.5\%$.

[Insert Table 3 around here]

A couple of interesting observations can be made. On the one hand, we see that similar to the no-dividend case, the information ratio remains increasing in the volatility of mispricing ς_i, and in the mean-reverting speed θ_i. For example, a information ratio of 0.174 occurs when $\varsigma_i = 1\%$ and $\theta_i = 9.9\%$, and increases to 0.395 when θ_i increases to 64.9% with the same $\varsigma_i = 1\%$, and to 2.600 when ς_i increases to 14% if we keep the $\theta_i = 9.9\%$ unchanged.

On the other hand, the information ratios become substantially larger for the same combinations of ς_i and θ_i than in the no-dividend case, corroborating with previous comparative statics analysis on the point that the existence of dividends strengthens the importance of fundamental analysis in term of improving portfolio selection. For instance, we see that when $\theta_i \geq 14.9\%$ and $\varsigma_i \geq 3\%$, the information ratio is 0.611, well above one-half, the corresponding information ratio when no security pays dividends. Actually, in this scenario we find in 91.07% and 79.17% of our results, the information ratios is greater than one-half and one, respectively. We can also see from Table 3 that there is a large region in which combinations of plausible values of ς_i and θ_i, give rise to information ratios well above one-half, one and even two. For a mild magnitude of $\varsigma_i \geq 3\%$, all the information ratios are above 0.50; for $\varsigma_i \geq 6\%$, all the information ratios are above one.

Our results are in contrast to empirical findings on practitioners' performance. According to Grinold and Kahn (2000) and Kahn and Rudd (2003), empirical researches show that overall a top-percentile manager has a before-fee information ratio of one, and a top-quartile manager has a before-fee information ratio of one-half. However, even top portfolio managers' performance appears lacklustre compared to what our model reveals. As mentioned above, we find that a large number of combinations of reasonable parameter values can achieve an information ratio of one or above, especially when securities pay dividends. Our numerical experiments reveal that a large realistic domain for parameter values, in which the information ratios are higher than the ones of top-percentile portfolio managers. Since our model simply maps the features of the fundamental and the mispricing processes into the information ratio, the discrepancy between our calibration and those achieved in reality may be due to various realistic limits of arbitrage (e.g. Shleifer and Vishny, 1997) such as trading costs, costs to produce fundamental information, market participants' competitions. Our paper is a first, necessarily simple, step toward filling the gap of modeling fundamental analysis into portfolio selection and endorsing fundamental analysis from a theoretical perspective.

4 Concluding remarks

We develop a continuous-time model to examine the implications of the mean-reverting security prices to their fundamentals for active portfolio management. We model the security price as its fundamental scaled by a mispricing, where the mispricing follows a geometric mean-reversion process and the fundamental a geometric Brownian motion satisfying the ICAPM. Our investor allocates her wealth into n almost independent risky securities and one risk-free asset by solving the classic mean-variance portfolio choice problem based on quadratic utility.

Analytically, we suggest investors choose securities with a more volatile mispricing, a less volatile fundamental, a higher mean-reverting speed and a larger dividend. A volatile mispricing implies many investment opportunities, while a volatile fundamental can only increase the active risk and thus has a negative effect on the expected appraisal ratio. The contribution from the mean-reverting speed and dividends to the expected appraisal ratio is positive.

Our numerical experiments reveal that a large realistic domain for parameter values, in which the information ratios are higher than the ones of top-percentile portfolio managers. Since our model simply maps the features of the fundamental and the mispricing processes into the information ratio, the discrepancy between our calibration and those achieved in reality may be due to various realistic limits of arbitrage (e.g. Shleifer and Vishny, 1997) such as trading costs, costs to produce fundamental information, market participants' competitions. Our paper is a first, necessarily simple, step toward filling the gap of modeling fundamental analysis into portfolio selection and endorsing fundamental analysis from a theoretical perspective.

To illustrate our idea, we simply avoid introducing complex structures among securities, assuming dependence between the mispricing and the fundamental processes, and modeling the mispricing and/or the fundamental using more complicated stochastic processes. We view our model as a workhorse which is flexible to be extended in various directions. We share the normative flavor with Treynor and Black (1973) by ignoring the transaction costs in the tradition of Markowitz, Sharpe, Treynor and Black, and others. Our model is a simple and parsimonious micro approach to model fundamental analysis into portfolio section analysis and we make no attempt to exhaust other potential approaches and utility functions, although we are aware that the dynamic programming approach popular in macro area may also be able to address our question. Of course, these issues should be considered when carried to empirical datasets and we leave them for future research. Another noteworthy direction, of course, is on the counterpart of our focus: technical analysis.

References

Arnott, R.D., J.C. Hsu, J. Liu, and H. Markowitz, Can noise create the size and value effects? *Management Science*, 2014, **61**(11), 2569-2579.

Bartram, S.M. and M. Grinblatt, Agnostic fundamental analysis works, *Journal of Financial Economics*, 2017, forthcoming.

Black, F. and R. Litterman, Global portfolio optimization. *Financial Analysts Journal*, 1992, **48**(5), 28-43.

Buckley, W., H. Long, and S. Perera, A jump model for fads in asset prices under asymmetric information. *European Journal of Operational Research*, 2014, **236**(1), 200-208.

Buckley, W.S., G.O. Brown, and M. Marshall, A mispricing model of stocks under asymmetric information. *European Journal of Operational Research*, 2012, **221**(3), 584-592.

Buckley, W.S. and H. Long, A discontinuous mispricing model under asymmetric information. *European Journal of Operational Research*, 2015, **243**(3), 944-955.

Campbell, J.Y., A variance decomposition for stock returns. *Economic Journal*, 1991, **101**(405), 157-179.

Campbell, J.Y. and R.J. Shiller, Cointegration and tests of present value models. *Journal of Political Economy*, 1987, **95**(5), 1062-1088.

Campbell, J.Y. and R.J. Shiller, The dividend-price ratio and expectations of future dividends and discount. *Review of Financial Studies*, 1988a, **1**(3), 195-228.

Campbell, J.Y. and R.J. Shiller, Stock prices, earnings, and expected dividends. *Journal of Finance*, 1988b, **43**(3), 661-676.

Cremers, K.M. and A. Petajisto, How active is your fund manager? A new measure that predicts performance. *Review of Financial Studies*, 2009, **22**(9), 3329-3365.

Cvitanić, J., A. Lazrak, L. Martellini, and F. Zapatero, Dynamic portfolio choice with parameter uncertainty and the economic value of analysts' recommendations. *Review of Financial Studies*, 2006, **19**(4), 1113-1156.

Della Vigna, S. and J.M. Pollet, Investor inattention and Friday earnings announcements. *Journal of Finance*, 2009, **64**(2), 709-749.

Dixit, A.K. and R.S. Pindyck, *Investment under uncertainty*1994: Princeton university press.

Epstein, D., N. Mayor, P. Schonbucher, A.E. Whalley, and P. Wilmott, The valuation of a firm advertising optimally. *Quarterly Review of Economics and Finance*, 1998, **38**(2), 149-166.

Ewald, C.-O. and Z. Yang, Geometric mean reversion: Formulas for the equilibrium density and analytic moment matching. *University of Glasgow working paper (http://ssrn.com/abstract=999561)*, 2007.

Ewald, C.-O. and A. Zhang, A new technique for calibrating stochastic volatility models: the Malliavin gradient method. *Quantitative Finance*, 2006, **6**(02), 147-158.

Fama, E.F. and K.R. French, Permanent and temporary components of stock prices. *Journal of Political Economy*, 1988, 246-273.

Giannetti, M. and B. Kahraman, Who Trades Against Mispricing?, *Swedish House of Finance Research Paper 14-09 (available at* https://ssrn.com/abstract=2766449*)*, 2015 November.

Green, J., J.R. Hand, and X.F. Zhang, The supraview of return predictive signals. *Review of Accounting Studies*, 2013, **18**(3), 692-730.

Grinold, R.C. and R.N. Kahn, *Active portfolio management*2000: Probus Publishing.

Grossman, S.J. and J.E. Stiglitz, On the impossibility of informationally efficient markets.

American Economic Review, 1980, **70**(3), 393-408.
Guasoni, P., Asymmetric information in fads models. *Finance and Stochastics*, 2006, **10**(2), 159-177.
Harris, L.E., S.M. Hartzmark, and D.H. Solomon, Juicing the dividend yield: Mutual funds and the demand for dividends. *Journal of Financial Economics*, 2015, **116**(3), 433-451.
Hartzmark, S.M. and D.H. Solomon, The dividend month premium. *Journal of Financial Economics*, 2013, **109**(3), 640-660.
Harvey, C.R., Y. Liu, and H. Zhu, … and the cross-section of expected returns. *Review of Financial Studies*, 2016, **29**(1), 5-68.
Kahn, R., and A. Rudd, The persistence of equity style performance: Evidence from mutual fund data, in *The Handbook of Equity Style Management*, edited by Coggin, T. and F. Fabozzi *John Wiley & Sons, Inc., New York*, 2003, 259-271.
Lewellen, J., Accounting anomalies and fundamental analysis: An alternative view. *Journal of Accounting and Economics*, 2010, **50**(2), 455-466.
McLean, R.D. and J. Pontiff, Does academic research destroy stock return predictability? *Journal of Finance*, 2016, **71**(1), 5-32.
Merton, R.C., An intertemporal capital asset pricing model. *Econometrica*, 1973, **41**(5), 867-887.
Metcalf, G.E. and K.A. Hassett, Investment under alternative return assumptions comparing random walks and mean reversion. *Journal of Economic Dynamics and Control*, 1995, **19**(8), 1471-1488.
Pindyck, R.S., Irreversibility, uncertainty, and investment. *Journal of Economic Literature*, 1991, **29**(3), 1110-1148.
Richardson, S., İ. Tuna, and P. Wysocki, Accounting anomalies and fundamental analysis: A review of recent research advances. *Journal of Accounting and Economics*, 2010, **50**(2), 410-454.
Sapp, T. and X.S. Yan, Security concentration and active fund management: do focused funds offer superior performance? *Financial Review*, 2008, **43**(1), 27-49.
Sharpe, W.F., A simplified model for portfolio analysis. *Management Science*, 1963, **9**(2), 277-293.
Shiller, R.J., The use of volatility measures in assessing market efficiency. *Journal of Finance*, 1981, **36**(2), 291-304.
Shleifer, A. and R.W. Vishny, The limits of arbitrage. *Journal of Finance*, 1997, **52**(1), 35-55.
Summers, L.H., Does the stock market rationally reflect fundamental values? *Journal of Finance*, 1986, **41**(3), 591-601.
Treynor, J.L. and F. Black, How to use security analysis to improve portfolio selection. *Journal of Business*, 1973, **46**(1), 66–86.
Wang, J., A model of intertemporal asset prices under asymmetric information. *Review of Economic Studies*, 1993, **60**(2), 249-282.
Wermers, R., T. Yao, and J. Zhao, Forecasting stock returns through an efficient aggregation of mutual fund holdings. *Review of Financial Studies*, 2012, **25**(12), 3490-3529.
Yan X., and L. Zheng; Fundamental analysis and the cross-section of stock returns: a data-mining approach. *Review of Financial Studies*, 2017, forthcoming. hhx001. doi: 10.1093/rfs/hhx001.
Yang, Z. and C.-O. Ewald, On the non-equilibrium density of geometric mean reversion. *Statistics and Probability Letters*, 2010, **80**(7), 608-611.

Appendix A. Proof of Equation (7)

To simplify notation, we write $P_i(\mathrm{t})$ and $\alpha_i(\mathrm{t})$ as P_i and α_i, respectively. From equation (1), we know that the first and second order partial derivatives of P_i with respect to α_i and F_i are

$$P_{\alpha_i} = F_i, P_{\alpha_i F_i} = 1, P_{F_i} = \alpha_i, P_{\alpha_i \alpha_i} = P_{F_i F_i} = 0 \tag{A.1}$$

Using equation (2) and (5), we obtain

$$d\alpha_i \cdot dF_i = (\theta_i(\eta_i - \alpha_i)\alpha_i dt + \varsigma_i \alpha_i dZ_i)\left(\left(\beta_i \mu_m + r_f - \delta_i\right)dt + v_i dB_i + \beta_i \sigma_m dB_m\right)F_i$$
$$= 0 \tag{A.2}$$

where we employ the rules ${dZ_i}^2 = dt$, $dt^2 = dt \cdot dZ_i = dt \cdot dB_m = dt \cdot dB_i = 0$ and the independence assumption $dZ_i \cdot dB_m = dB_i \cdot dB_m = 0$.

Using Ito's formula, equations (A.1) and (A.2), we obtain the stochastic differential equation for the security price P_i

$$dP_i = P_{\alpha_i} d\alpha_i + P_{F_i} dF_i + \frac{1}{2} P_{\alpha_i \alpha_i} (d\alpha_i)^2 + P_{\alpha_i F_i} d\alpha_i \cdot dF_i + \frac{1}{2} P_{F_i F_i} (dF_i)^2$$
$$= F_i d\alpha_i + \alpha_i dF_i$$
$$= \alpha_i F_i(\theta_i(\eta_i - \alpha_i)dt + \varsigma_i dZ_i) + \alpha_i F_i\left(\left(\beta_i \mu_m + r_f - \delta_i\right)dt + v_i dB_i + \beta_i \sigma_m dB_m\right)$$
$$= P_i\left(\left(\theta_i(\eta_i - \alpha_i) + \beta_i \mu_m - \delta_i + r_f\right)dt + \varsigma_i dZ_i + v_i dB_i + \beta_i \sigma_m dB_m\right)$$

Divide P_i from both sides of the above expression, we obtain **Equation (7)**:

$$\frac{dP_i}{P_i} = \left(\theta_i(\eta_i - \alpha_i) + \beta_i \mu_m - \delta_i + r_f\right)dt + \varsigma_i dZ_i + v_i dB_i + \beta_i \sigma_m dB_m \tag{A.3}$$

Appendix B. Proof of Equation (21)

Let $pdf_i(\alpha)$ denote the stationary distribution of $\alpha_i(t)$, that is, the distribution of $\alpha_i(\infty) \equiv \lim_{t\to\infty} \alpha_i(t)$. For convenience, we let $D_i = \varsigma_i^2/2\theta_i$. Obviously, $distance_i$ is increasing in ς_i and decreasing in θ_i. Since by assumption $\theta_i > 0$, D_i is always positive. As we will see, D_i actually represents the distance between the non-central mean of the stationary distribution and the mean-reversion level η_i. According to the Proposition 2 in Ewald and Yang (2007, page 11), under the restriction $D_i < \eta_i$, $pdf_i(\alpha)$ can be written as

$$pdf_i(\alpha) = \frac{\left(\frac{1}{D_i}\right)^{\left(\frac{\eta_i}{D_i}-1\right)}}{\Gamma\left(\frac{\eta_i}{D_i}-1\right)} \alpha^{\left(\frac{\eta_i}{D_i}-2\right)} e^{-\frac{\alpha}{D_i}} \tag{B.1}$$

where $\Gamma(\cdot)$ is the Gamma function $\Gamma(x) = \int_0^\infty t^{x-1}\, e^{-t} dt$.

$pdf_i(\alpha)$ is similar to a Gamma distribution with the shape and scale parameters equal to D_i and $\eta_i/D_i - 1$, respectively. However, the shape and scale parameters are independent in a standard Gamma distribution, but related to each other through D_i for $pdf_i(\alpha)$ in our case.

According to the Proposition 4 in Ewald and Yang (2007, page 13), the non-central moments of $pdf_i(\alpha)$, denoted by $e(n)$, satisfy the following relationship:

$$e(n+1) = d_i\left(\frac{\eta_i}{D_i} - 1 + n\right)e(n) \tag{B.2}$$

for all $n \geq 0$ and $e(0) = 1$. This recursive relationship implies that higher order moments have an influence on the mean and the variance. In particular, the first two moments are

$$e(1) = \eta_i - D_i \tag{B.3}$$

$$e(2) = \eta_i(\eta_i - D_i) \tag{B.4}$$

From the expression for $e(1)$, we can see that D_i represents the distance between the mean-reversion level η_i and the non-central mean $e(1)$. Since d_i is positive, $e(1)$ always stays below the mean-reversion level of $\alpha_i(t)$.

To simplify notation, we let $X = \alpha_i(t)$. From equations (B.3) and (B.4), we have

$$E[X] = \eta_i - D_i, E[X^2] = \eta_i(\eta_i - D_i) \tag{B.5}$$

Next we evaluate $E[X^{-2}]$ and $E[X^{-1}]$ using the stationary distribution in equation (B.1)

$$E[X^{-n}] = \int_0^\infty x^{-n}\, pdf_i(x)dt = \int_0^\infty x^{-n} \frac{\left(\frac{1}{D_i}\right)^{\left(\frac{\eta_i}{D_i}-1\right)}}{\Gamma\left(\frac{\eta_i}{D_i} - 1\right)} x^{\left(\frac{\eta_i}{D_i}-2\right)} e^{-\frac{x}{D_i}} dx$$

$$= \frac{\left(\frac{1}{D_i}\right)^n}{\Gamma(\frac{\eta_i}{D_i} - 1)} \int_0^\infty \frac{1}{D_i}\left(\frac{x}{D_i}\right)^{\left(\frac{\eta_i}{D_i}-1-n\right)-1} e^{-\frac{x}{D_i}} dx$$

$$= \frac{\left(\frac{1}{D_i}\right)^n}{\Gamma\left(\frac{\eta_i}{D_i} - 1\right)} \int_0^\infty z^{\left(\frac{\eta_i}{D_i}-1-n\right)-1} e^{-z} dz$$

$$= \frac{\left(\frac{1}{D_i}\right)^n \Gamma(\frac{\eta_i}{D_i} - 1 - n)}{\Gamma(\frac{\eta_i}{D_i} - 1)}$$

where the penultimate equality is due to the change of variables $z = \frac{x}{D_i}$ and $dx = D_i dz$, and for $\Gamma(\cdot)$ to be well defined, it is required that $\eta_i > (1+n)D_i$.

Therefore, we have

$$E[X^{-1}] = \frac{1}{D_i}\left(\frac{\eta_i}{D_i} - 2\right)^{-1} = \frac{1}{\eta_i - 2D_i} \tag{B.6}$$

$$E[X^{-2}] = \left(\frac{1}{D_i}\right)^2 \left(\frac{\eta_i}{D_i} - 2\right)^{-1} \left(\frac{\eta_i}{D_i} - 3\right)^{-1} = \frac{1}{(\eta_i - 2D_i)(\eta_i - 3D_i)} \tag{B.7}$$

where we employ the property of Gamma function $\Gamma(z+1) = z\Gamma(z)$ and the restriction $\eta_i > 3D_i = 3\varsigma_i^2/2\theta_i$ is required. Alternatively, Equations (B.6) and (B.7) can also be derived from the fact that if $X \sim Gamma(a, b^{-1})$, then $X^{-1} \sim Inv - Gamma(a, b)$ where $Gamma(a, b^{-1})$ denotes the gamma distribution with shape and scale parameters a and b^{-1}, respectively, and $Inv - Gamma(a, b)$ denotes the inverse gamma distribution with shape and scale parameters a and b, respectively. In our case, the stationary distribution of $\alpha_i(t)$ is gamma with $a = \eta_i/D_i - 1$ and $b^{-1} = D_i$.

By expanding out the nominator of the equation (20), we obtain

$$AR_p = \sum_{i=1}^{n} \frac{\theta_i^2(\eta_i - X)^2 + \delta_i^2\left(X^{-1} - 1\right)^2 + 2\delta_i\theta_i(\eta_i - X)\left(X^{-1} - 1\right)}{\sigma_i^2} \tag{B.8}$$

Using (B.5), (B.6) and (B.7), we have

$$E[\theta_i^2(\eta_i - X)^2 + \delta_i^2(X^{-1} - 1)^2 + 2\delta_i\theta_i(\eta_i - X)(X^{-1} - 1)] = \theta_i^2(\eta_i^2 - 2\eta_i E[X] + E[X^2])$$
$$+\delta_i^2(E[X^{-2}] - 2E[X^{-1}] + 1) + 2\theta_i\delta_i(\eta_i E[X^{-1}] + E[X] - 1 - \eta_i)$$
$$= \theta_i^2\eta_i D_i + \frac{((\eta_i - 1)^2 + 6D_i^2 + (6 - 5\eta_i)D_i)\delta_i^2}{(\eta_i - 2D_i)(\eta_i - 3D_i)} + \frac{(4d_i^2 + (4 - 2\eta_i)D_i)\theta_i\delta_i}{(\eta_i - 2D_i)}$$

Plugging the above result into (B.9), we have **Equation (21)** and the proof is complete.

$$E\left[AR_p\right]$$
$$= \sum_{i=1}^{n}\left(\frac{\theta_i^2\eta_i D_i}{\sigma_i^2} + \frac{\left((\eta_i - 1)^2 + 6D_i^2 + (6 - 5\eta_i)D_i\right)\delta_i^2}{(\eta_i - 2D_i)(\eta_i - 3D_i)\sigma_i^2} + \frac{\left(4D_i^2 + (4 - 2\eta_i)D_i\right)\theta_i\delta_i}{(\eta_i - 2D_i)\sigma_i^2}\right)$$
$$= \sum_{i=1}^{n}\left(\frac{\theta_i\varsigma_i^2}{2(\varsigma_i^2 + v_i^2)} + \frac{\left(2(\eta_i - 1)^2\theta_i^2 + 3\varsigma_i^4 + \left(6 - 5\eta_i\right)\theta_i\varsigma_i^2\right)\delta_i^2}{(2\eta_i\theta_i - 3\varsigma_i^2)(\eta_i\theta_i - \varsigma_i^2)(\varsigma_i^2 + v_i^2)} + \frac{(\varsigma_i^2 + (2 - \eta_i)\theta_i)\delta_i\varsigma_i^2}{(\eta_i\theta_i - \varsigma_i^2)(\varsigma_i^2 + v_i^2)}\right)$$

Table 1. An Overview of Model Parameters

This table reports the symbols, values and the source/justification of the values of all the parameters used in this paper.

Symbol	Parameter	Value	Source/Justification
θ_i	the mean-reversion speed of the mispricing $\alpha_i(t)$	[9.9%, 64.9%]	Giannetti and Kahraman (2015)
ς_i	the volatility of the mispricing $\alpha_i(t)$	[0, 14%]	Bounded by zero and σ_i
v_i	the volatility of the fundamental $F_i(t)$	[0, 14%]	Bounded by zero and σ_i
δ_i	dividend yield, which is a constant fraction of the fundamental value $F_i(t)$	[0, 1.5%]	Hartzmark and Solomon (2013)
η_i	the mean-reversion level of the mispricing $\alpha_i(t)$	1	The literature about mean-reverting prices
n	the number of investable securities	90	Sapp and Yan (2008)
σ_i	$\sqrt{\varsigma_i^2 + v_i^2}$, active risk or tracking error, *i.e.* standard deviation of active return	14%	Cremers and Petajisto (2009)

Table 2. Information Ratios When Active Risk $\sigma_i = 14\%$ and No Dividends

Table 2 reports information ratios of the optimal portfolio for different combinations of the mean-reverting speed θ_i and the volatility of mispricing ς_i, when the active risk σ_i is fixed at 14% and none of the 90 independent and identical securities pays dividends. The information ratio is calculated as the square root of the expected appraisal ratio in equation (22) and the missing values are due to the restrictions of $3\varsigma_i^2/2 < \theta_i$ and $\varsigma_i^2 \leq \sigma_i^2$.

ς \ θ	9.9%	14.9%	19.9%	24.9%	29.9%	34.9%	39.9%	44.9%	49.9%	54.9%	59.9%	64.9%
1%	0.151	0.185	0.214	0.239	0.262	0.283	0.303	0.321	0.338	0.355	0.371	0.386
2%	0.302	0.370	0.427	0.478	0.524	0.566	0.605	0.642	0.677	0.710	0.742	0.772
3%	0.452	0.555	0.641	0.717	0.786	0.849	0.908	0.963	1.015	1.065	1.113	1.158
4%	0.603	0.740	0.855	0.956	1.048	1.132	1.211	1.284	1.354	1.420	1.483	1.544
5%	0.754	0.925	1.069	1.195	1.310	1.415	1.513	1.605	1.692	1.775	1.854	1.930
6%	0.905	1.110	1.282	1.435	1.572	1.698	1.816	1.926	2.031	2.130	2.225	2.316
7%	1.055	1.295	1.496	1.674	1.834	1.981	2.119	2.247	2.369	2.485	2.596	2.702
8%	1.206	1.480	1.710	1.913	2.096	2.265	2.421	2.569	2.708	2.840	2.967	3.088
9%	1.357	1.665	1.924	2.152	2.358	2.548	2.724	2.890	3.046	3.195	3.338	3.474
10%	1.508	1.850	2.137	2.391	2.620	2.831	3.027	3.211	3.385	3.550	3.708	3.860
11%	1.658	2.035	2.351	2.630	2.882	3.114	3.329	3.532	3.723	3.905	4.079	4.246
12%	1.809	2.219	2.565	2.869	3.144	3.397	3.632	3.853	4.062	4.260	4.450	4.632
13%	1.960	2.404	2.779	3.108	3.406	3.680	3.935	4.174	4.400	4.615	4.821	5.018
14%	2.111	2.589	2.992	3.347	3.668	3.963	4.237	4.495	4.739	4.970	5.192	5.404

Table 3. Information Ratios when Active Risk $\sigma_i = 14\%$ and Dividends $\delta_i = 1.5\%$

Table 3 reports information ratios of the optimal portfolio for different combinations of the mean-reverting speed θ_i and the volatility of mispricing ς_i, when the active risk σ_i is fixed at 14% and each of the 90 independent and identical securities pay fixed dividends $\delta_i = 1.5\%$.

ς \ θ	9.9%	14.9%	19.9%	24.9%	29.9%	34.9%	39.9%	44.9%	49.9%	54.9%	59.9%	64.9%
1%	0.174	0.204	0.230	0.254	0.275	0.295	0.314	0.332	0.349	0.365	0.380	0.395
2%	0.348	0.407	0.460	0.507	0.550	0.591	0.628	0.664	0.697	0.729	0.760	0.790
3%	0.522	0.611	0.690	0.761	0.826	0.886	0.942	0.996	1.046	1.094	1.140	1.185
4%	0.698	0.816	0.921	1.015	1.101	1.181	1.257	1.328	1.395	1.459	1.521	1.580
5%	0.874	1.021	1.151	1.269	1.377	1.477	1.571	1.660	1.744	1.824	1.901	1.975
6%	1.053	1.227	1.383	1.524	1.653	1.773	1.886	1.992	2.093	2.189	2.281	2.370
7%	1.233	1.434	1.615	1.779	1.929	2.069	2.200	2.324	2.442	2.554	2.662	2.765
8%	1.416	1.642	1.848	2.034	2.206	2.366	2.515	2.657	2.791	2.920	3.043	3.161
9%	1.601	1.852	2.081	2.290	2.483	2.662	2.831	2.990	3.141	3.285	3.423	3.556
10%	1.790	2.064	2.316	2.547	2.761	2.960	3.146	3.323	3.491	3.651	3.804	3.952
11%	1.984	2.277	2.552	2.805	3.039	3.257	3.462	3.656	3.841	4.017	4.186	4.348
12%	2.183	2.493	2.789	3.064	3.318	3.556	3.779	3.990	4.191	4.383	4.567	4.744
13%	2.388	2.711	3.028	3.323	3.598	3.854	4.096	4.324	4.542	4.750	4.949	5.140
14%	2.600	2.933	3.269	3.584	3.878	4.154	4.413	4.659	4.893	5.116	5.331	5.537

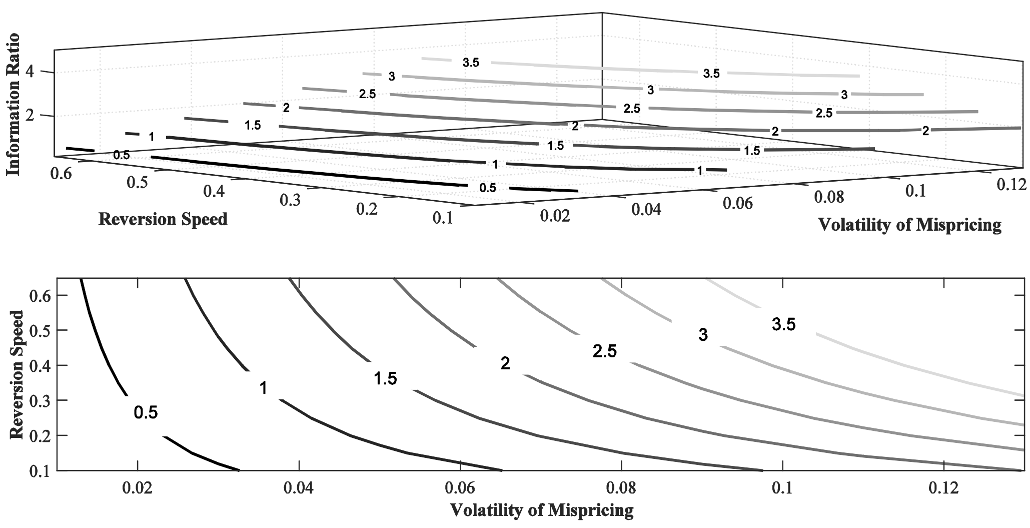

Figure 1. Information Ratios when Active Risk $\sigma_i = 14\%$ and No Dividends

Figure 1 illustrates how the information ratio varies according to the changes in the mean-reverting speed θ_i and the volatility of mispricing ς_i, when the active risk σ_i is fixed at 14% and none of the 90 independent and identical securities pays dividends. The two subplots are respectively the 3-D and 2-D contours, produced under the two restrictions $3\varsigma_i^2/2 < \theta_i$ and $\varsigma_i^2 \leq \sigma_i^2$. The labels on the plots indicate the levels of the information ratios.