

What if LIGO’s gravitational wave detections are strongly lensed by massive galaxy clusters?

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ABSTRACT

Motivated by the preponderance of so-called ‘heavy black holes’ in the binary black hole (BBH) gravitational wave (GW) detections to date, and the role that gravitational lensing continues to play in discovering new galaxy populations, we explore the possibility that the GWs are strongly lensed by massive galaxy clusters. For example, if one of the GW sources were actually located at $z = 1$, then the rest-frame mass of the associated BHs would be reduced by a factor of ~ 2 . Based on the known populations of BBH GW sources and strong-lensing clusters, we estimate a conservative lower limit on the number of BBH mergers detected per detector year at LIGO/Virgo’s current sensitivity that are multiply-imaged, of $R_{\text{detect}} \simeq 10^{-5} \text{ yr}^{-1}$. This is equivalent to rejecting the hypothesis that one of the BBH GWs detected to date was multiply-imaged at $\lesssim 4\sigma$. It is therefore unlikely, but not impossible, that one of the GWs is multiply-imaged. We identify three spectroscopically confirmed strong-lensing clusters with well-constrained mass models within the 90 per cent credible sky localizations of the BBH GWs from LIGO’s first observing run. In the event that one of these clusters multiply-imaged one of the BBH GWs, we predict that 20–60 per cent of the putative next appearances of the GWs would be detectable by LIGO, and that they would arrive at Earth within 3yr of first detection.

Key words: gravitational lensing; strong – gravitational waves – galaxies: clusters: individual: 1E0657–558, MACS J0140.0–0555, MACS J1311.0–0311, RCS0224–0002.

1 INTRODUCTION

Strong gravitational lensing – i.e. multiple-imaging of a single galaxy – by massive galaxy clusters plays an invaluable role in discovering and studying new populations of objects at high redshift (e.g. Mellier et al. 1991; Franx et al. 1997; Ellis et al. 2001; Smith et al. 2002; Kneib et al. 2004; Stark et al. 2007; Willis et al. 2008; Wardlow et al. 2013; Bouwens et al. 2014; Zheng et al. 2014; Atek et al. 2015; McLeod et al. 2015). Indeed, gravitational magnification by massive clusters – albeit not multiple-imaging – was instrumental in the first detections of sub-mm galaxies (Smail, Ivison & Blain 1997; Ivison et al. 1998). More recent work has also

shown that an efficient hunting ground for strongly lensed sub-mm galaxies is the population with the brightest apparent fluxes (Wardlow et al. 2013). Theoretical considerations also underline the important role of strong lensing by galaxy clusters in discovering new high-redshift populations, because clusters dominate the lensing cross-section at the large gravitational magnifications associated with multiple-imaging ($|\mu| > 10$; see fig. 5 of Hilbert et al. 2008). As the LIGO/Virgo interferometers have begun to detect a new population of objects – mergers of binary compact objects (Abbott et al. 2016b; Abbott et al. 2017a,b), it is therefore natural to speculate on whether gravitational lensing played a role in any of these detections.

Strong lensing of gravitational waves (GWs) had been considered by numerous authors in advance of the advent of direct GW detections (Wang, Stebbins & Turner 1996; Takahashi & Nakamura

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2003; Seto 2004; Takahashi 2004; Varvella, Angonin & Tournenc 2004; Sereno et al. 2010, 2011; Piórkowska, Biesiada & Zhu 2013; Biesiada et al. 2014). In particular, the degeneracy between the luminosity distance to and thus source-frame mass of a GW source, and any gravitational magnification suffered by the source noted by Wang et al. (1996) is interesting in light of the reported BH masses thus far (see also Dai, Venumadhav & Sigurdson 2017). It is intriguing that six of the 10 BHs reported to date by LIGO/Virgo have rest-frame masses of $\gtrsim 20 M_{\odot}$ (Abbott et al. 2016b; Abbott et al. 2017a,b), and thus are more massive than the most massive stellar mass BHs observed in the local Universe (Farr et al. 2011). Whilst plausible astrophysical interpretations of these ‘heavy’ BHs do exist (Abbott et al. 2016a; Stevenson et al. 2017), it is also possible that the large detector-frame masses arise from lower mass sources at higher redshift that have been gravitationally magnified. Ignoring this gravitational magnification would cause the redshift of the GW sources to be underestimated, the BH masses to be overestimated, and raise the possibility of detecting the same object again in the future.

The GW detections have stimulated a flurry of articles on strong lensing of GWs, discussing the effect of lens magnification on the detectability of GWs (Dai et al. 2017), forecast event rates including the effects of strong lensing by galaxies (Ng et al. 2017), relative arrival times of GW and electromagnetic (EM) signals (Takahashi 2017), prospects for measuring the speed of GWs (Collett & Bacon 2017; Fan et al. 2017), and the impact of strong lensing on cosmography (Baker & Trodden 2017; Liao et al. 2017). In this article, we investigate the probability that one or more of the GW sources detected to date was strongly lensed by a massive galaxy cluster. We take an empirical/observational approach, in that after many years of investment by the cluster strong lensing and LIGO/Virgo communities, we are finally able to consider populations of observed lenses and GW sources. In essence, we clarify whether the perspective of an extragalactic observer that ‘surely the first detections have benefitted from lensing’ is valid. We also compare the sky localizations of GW sources to the celestial coordinates of known and spectroscopically confirmed cluster strong lenses, to identify candidate lensing clusters that might have magnified our view of the GWs. For these candidate clusters, we use detailed and well-constrained models of the cluster cores to answer the question: ‘what if LIGO’s GW detections are strongly lensed by massive galaxy clusters?’ – i.e. when would we see the same sources again, and is detection possible?

In Section 2, we review how strong gravitational lensing modifies GW signals and estimate the probability of strong lensing. We then identify the candidate cluster lenses in Section 3, and describe our lensing calculations in Section 4. In Section 5, we summarize and discuss our main results. We assume $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_M = 0.3$, and $\Omega_{\Lambda} = 0.7$.

2 STRONG LENSING OF GRAVITATIONAL WAVES

We consider merging compact objects as point sources of gravitational and EM radiation. The effect of strong gravitational lensing on short-wavelength radiation from point sources is to modify their flux (magnification) and arrival time, yet leave their frequency unaltered (e.g. Schneider, Ehlers & Falco 1992). In the case of GW sources, the ‘flux’ manifests itself as the amplitude of the strain signal detected by the interferometer. In fact, the interpretation of the strain amplitude, A , is degenerate to the gravitational magnification, μ , and the luminosity distance to the source, D_L , as follows:

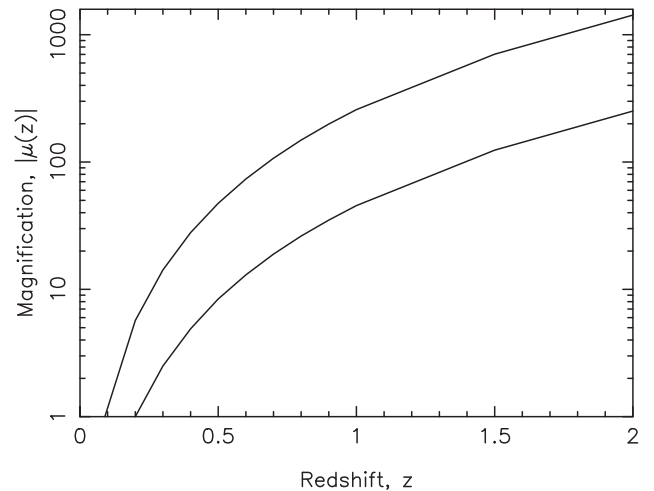


Figure 1. Gravitational magnification, $\mu_{\text{GW}}(z)$, required to modify the inferred luminosity distance to a GW source, based on GW150914, GW151226, GW170814 (upper curve) and LVT151012, GW170104 (lower curve) as a function of redshift.

$A \propto \sqrt{|\mu|}/D_L$ (Wang et al. 1996). The gravitational magnification required to reinterpret a GW source as being strongly lensed and thus at a higher redshift, z , than originally inferred is therefore given by $\mu_{\text{GW}} = [D_L(z)/D_{L,\mu=1}]^2$, where $D_{L,\mu=1}$ is the luminosity distance inferred assuming $\mu = 1$ (Fig. 1).

The binary black hole (BBH) GW sources detected to date have been interpreted, assuming no gravitational lensing, as lying at redshifts of $z \sim 0.1$ – 0.2 (Abbott et al. 2016b; Abbott et al. 2017a,b). To reinterpret a source initially identified at $z \simeq 0.1$ (GW150914, GW151226, GW170814) as actually being at $z = 1$ requires $\mu_{\text{GW}} \simeq 200$, and to reinterpret sources initially identified at $z \simeq 0.2$ (LVT151012, GW170104) as actually being at $z = 1$ requires $\mu_{\text{GW}} \simeq 45$. Increasing the redshift of the sources in this way would also lead to a reduction in the inferred rest-frame masses by a factor of $(1+z)$. The masses of sources identified at $z \simeq 0.1$ would reduce by a factor of ~ 1.8 and at $z \simeq 0.2$ by a factor of ~ 1.7 if they are actually located at $z = 1$. Typical strongly lensed galaxies suffer gravitational magnifications of $\mu \sim 10$ – 50 (e.g. Richard et al. 2010), i.e. generally less than those discussed here. Nevertheless, very high magnifications are physically possible because the physical region from which GWs emerge is $\sim 100 \text{ km}$ in size. It is therefore possible for a GW source to be very closely aligned with the caustic of a gravitational lens, and thus achieve a high magnification value (Ng et al. 2017). This is not the case for a galaxy with a typical size of ~ 1 – 10 kpc . In addition to revised redshifts and rest-frame masses, a strongly lensed (hereafter, multiply-imaged) GW source will arrive at Earth on multiple occasions due to the existence of several stationary points on the Fermat surface that describes the arrival time at Earth. The time delay between multiple images created by strong lensing by galaxy clusters can be as short as a few days and as long as $\sim 10 \text{ yr}$ (e.g. Jauzac et al. 2016).

We now consider how likely it is that a GW source is multiply-imaged, and write the number of multiply-imaged GW sources in the Universe per detector year as

$$R = \int_0^{z_{\text{max}}} dz_L \int_{z_L}^{\infty} \frac{dz}{1+z} \frac{dV_{\mu}}{dz dz_L} \frac{dn}{dV dt}, \quad (1)$$

where z_L is the redshift of the lens, z is the actual redshift of the GW source, $dn/dV/dt$ is the number of sources per unit comoving volume per source-frame year, and V_{μ} is the comoving volume that

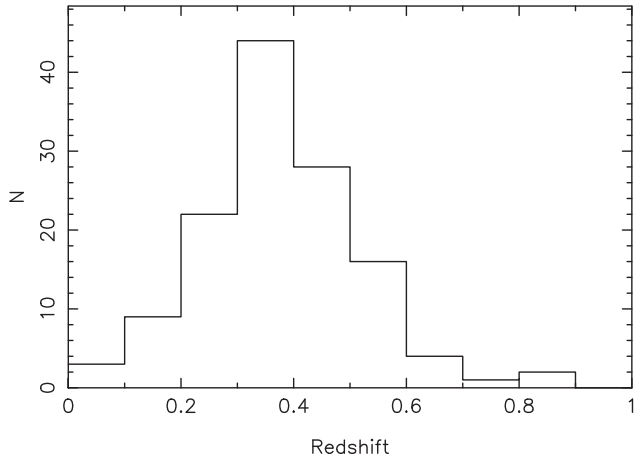


Figure 2. The redshift distribution of the 130 spectroscopically confirmed strong-lensing clusters discussed in Section 3.

is magnified by $\mu = \mu_{\text{GW}}$. Note that the sensitivity of the GW detectors is incorporated within V_μ , via the requirement for a given level of magnification to render a distant source detectable at a given detector sensitivity. In this article, we focus on multiple-imaging of BBH mergers by known strong-lensing clusters. This is motivated by the absence of EM counterparts and thus absence of precise sky localizations for BBH mergers to date, their ‘unlensed’ luminosity distances being sufficiently large as to not require very extreme values of μ_{GW} , that clusters dominate the lensing cross-sections at high magnification (Hilbert et al. 2008), and the availability of detailed models of the known cluster lenses (Sections 3 and 4). We therefore adapt equation (1) to estimate R_{detect} , the number of BBH mergers detected per detector year that are multiply-imaged by a known and spectroscopically confirmed strong-lensing cluster:

$$R_{\text{detect}} \simeq N_{\text{SL}} \int_{z_L}^{\infty} \frac{dz}{1+z} \left[\frac{dV_\mu}{dz} \right]_{\text{CL}} \frac{dn_{\text{BBH}}}{dV dt}, \quad (2)$$

where N_{SL} is the number of known and spectroscopically confirmed strong-lensing clusters, the dV_μ/dz term denotes the volume per unit redshift behind an example galaxy cluster that is magnified by $\mu = \mu_{\text{GW}}$, and the last term on the right-hand side is now specific to BBH mergers.

There are 130 spectroscopically confirmed strong-lensing clusters known at the present time (Section 3; i.e. $N_{\text{SL}} = 130$), with a median redshift of $z = 0.3$ (Fig. 2). We therefore adopt the cluster 1E 0657–558 (also known as the ‘Bullet Cluster’) at $z = 0.296$ as the example cluster upon which our calculations are based. We compute dV_μ/dz for this cluster using the detailed parametric mass model of this cluster (Paraficz et al. 2016), and also confirm that our calculation is numerically converged – i.e. the volume calculation is insensitive to the width of the redshift bins that we adopt.

In principle, the choice of $dn_{\text{BBH}}/dV/dt$ involves a circular argument, given that (1) the current estimate in the local Universe is $12\text{--}213 \text{ Gpc}^{-3} \text{ yr}^{-1}$, assuming $\mu = 1$ for all BBH GW sources to date (Abbott et al. 2017a), and (2) our goal is to explore the possibility that one or more of the sources were multiply-imaged and thus at higher redshift. To break the circularity implied by adopting the published BBH merger rate for our calculations, we compute the comoving volume at $0.35 < z < 2$ that is magnified by $\mu = \mu_{\text{GW}}(z)$ by the Bullet cluster, and express this as a fraction of the total comoving volume in this redshift range: $f_\mu \simeq 10^{-10}$. The probability of the GW sources detected to date being multiply-imaged is therefore very small, and we can safely adopt the published LIGO BBH

merger rate as the local rate, and thus effectively explore the possibility that one of the detections is multiply-imaged. Note that $f_\mu \simeq 10^{-10}$ is qualitatively consistent with Hilbert et al. (2008)’s predictions of the source frame optical depth to high magnifications.

We now estimate R_{detect} , assuming that the BBH merger rate does not evolve with redshift, and performing the integral in equation (2) over the redshift range $0.35 < z < 2$. The results of the calculation are insensitive to this choice. In particular, we note that $z = 0.35$ is $15\,000 \text{ km s}^{-1}$ beyond the cluster redshift and that the cross-section to strong lensing so close to the cluster is negligible. Also, varying the upper limit between $z = 1$ and $z = 3$ changes the result negligibly due to the tiny volume magnified by the very large factors required to push the GW source redshifts back to $z > 1$. We obtain a rate of detections of BBHs multiply-imaged by a known and spectroscopically confirmed strong-lensing cluster per detector year of $R_{\text{detect}} \simeq 7 \times 10^{-7} \text{--} 10^{-5} \text{ yr}^{-1}$, based on $dn_{\text{BBH}}/dV/dt = 12\text{--}213 \text{ Gpc}^{-3} \text{ yr}^{-1}$. Adopting a single and constant value of $dn_{\text{BBH}}/dV/dt = 50 \text{ Gpc}^{-3} \text{ yr}^{-1}$ yields $R_{\text{detect}} \simeq 3 \times 10^{-6} \text{ yr}^{-1}$. We also consider a BBH merger rate that tracks the star formation history of the Universe, as described by the fitting function in equation 15 of Madau & Dickinson (2014). This evolving BBH merger rate yields a $R_{\text{detect}} \simeq 4 \times 10^{-6} \text{--} 6 \times 10^{-5} \text{ yr}^{-1}$, with a rate of $R_{\text{detect}} \simeq 10^{-5} \text{ yr}^{-1}$ based on a local rate of $dn_{\text{BBH}}/dV/dt = 50 \text{ Gpc}^{-3} \text{ yr}^{-1}$ that evolves as discussed above.

In summary, based on the calculations detailed above, a reasonable estimate of the number of BBH mergers detected in LIGO’s first two observing runs and multiply-imaged by a known and spectroscopically confirmed strong-lensing cluster per detector year is $R_{\text{detect}} \simeq 10^{-5} \text{ yr}^{-1}$. This implies a small and non-zero probability, which is equivalent to saying that if one of the BBH detections to date has been multiply-imaged, then this implies getting lucky at the level of an $\sim 4\sigma$ outlier per detector year. This is a lower limit on the rate and an upper limit on the significance at which the hypothesis of strong lensing by a cluster can be ruled out, because our calculations are based on known strong-lensing clusters, and not the full population of clusters that have sufficiently dense cores to be able to produce strong-lensing effects. The low probability of strong lensing is due to a combination of (1) the large magnification factors required to reinterpret the strain signal as coming from a redshift beyond the population of known lenses, (2) constraints on the local BBH merger rate are already stringent enough to ensure that $R_{\text{detect}} \ll 1$ at the present time, and (3) the physically plausible redshift evolution is not strong enough and the luminosity function is not steep enough to allow lensed sources to dominate the early detections (see also Dai et al. 2017; Ng et al. 2017). Nevertheless, the rate is non-zero, and thus the possibility of parameter mis-estimation for apparently heavy BHs remains. In the next section, we therefore investigate whether any known cluster strong lenses are consistent with the GW sky localizations.

3 THE CLUSTER LENSES

We have assembled a list of 130 spectroscopically confirmed strong cluster lenses from the literature, drawing mainly on *HST* studies of X-ray selected clusters, and strong-lensing clusters from the Sloan Digital Sky Survey (Smith et al. 2005, 2009; Limousin et al. 2007, 2012; Richard et al. 2010; Christensen et al. 2012; Oguri et al. 2012; Jauzac et al. 2015; Paraficz et al. 2016; Umetsu et al. 2016). We compare the celestial coordinates of these clusters with the sky localization maps of the BBH merger detections from 2015, finding that none of the known strong-lensing clusters are located

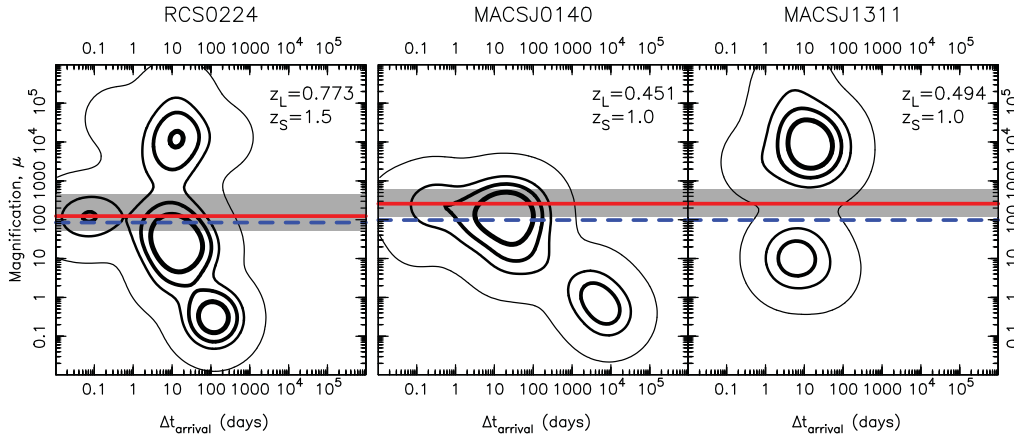


Figure 3. Magnification and time delay suffered by the putative next appearance of LVT151012 (left) and GW151226 (centre and right) in the scenario each has been multiply-imaged by a strong-lensing cluster located in their respective 90 per cent credible sky localizations. The solid red line shows μ_{GW} , and the grey band shows the range $\mu_{\text{GW}}^- < \mu < \mu_{\text{GW}}^+$. The blue dashed lines show μ_{GW}/α^2 , the magnification threshold above which the next appearance would be detectable by LIGO at $\text{SNR} \gtrsim 8$. Contours enclose 25 per cent, 50 per cent, 75 per cent, and 99 per cent of the probability density. Further details are discussed in Section 4.

within the 90 per cent credible sky localization of GW150914, two are located within the 90 per cent credible region of GW151226 (MACSJ0140.0–0555 at $z = 0.451$, and MACSJ1311.0–0310 at $z = 0.398$), and one is within the 90 per cent credible region of LVT151012 (RCS0224–0002 at $z = 0.773$). We note that there are no clusters in common between the sky localizations of GW151226 and LVT151012, and none of them lie in or near the intersection of the sky localizations of these two events (see for example, fig. 6 of Abbott et al. 2016b). Detailed mass models are available for all three clusters (Christensen et al. 2012; Ho, Ebeling & Richard 2012; Smit et al. 2017).

The sky localizations of all three GW events intersect the disc of the Milky Way. Unfortunately, severe dust extinction and stellar obscuration make it very difficult to find clusters at low galactic latitude, let alone identify whether any of them are strong-lenses. Despite some clusters being identified at low galactic latitude (e.g. Ebeling, Mullis & Tully 2002; Kocevski et al. 2007), searches for strong-lensing clusters have concentrated on high latitudes ($|b| > 20^\circ$). It is therefore possible that an unknown massive galaxy cluster at low galactic latitude strongly lensed one or more of the GW events. This underlines the fact that the rate of $R_{\text{detect}} \simeq 10^{-5} \text{ yr}^{-1}$ (Section 2), and the numbers of strong-lensing clusters that we find within the 90 per cent credible sky localizations are all lower limits on the incidence of strong lensing.

The detections of GW170104 and GW170814 were announced during the later stages of preparing this article, and while responding to the referee report, respectively (Abbott et al. 2017a,b). We identified two cluster lenses within the 90 per cent credible region of both of these detections. These findings do not alter any of the conclusions and discussion presented here. We will present our follow-up observations of clusters related to GW170814 in a future article.

4 TIME DELAY AND MAGNIFICATION CALCULATIONS

We estimate the arrival times and magnifications of putative future appearances of GW151226 and LVT151012 due to the three cluster lenses discussed in Section 3. The detailed mass models referred to above are all constrained by spectroscopically confirmed multiply-

imaged galaxies, thus breaking the redshift space degeneracies. The mass distribution of each cluster core was modelled as a superposition of mass components that represent the large-scale cluster mass distribution, and the cluster galaxies, and optimized using the publicly available LENSTOOL software (Jullo et al. 2007), following methods initially developed by Kneib et al. (1996) and Smith et al. (2005). Full details of the models are presented in Christensen et al. (2012), Ho et al. (2012), and Smit et al. (2017).

Starting from these models, we identify the sky locations in the $z_S = 1$ and $z_S = 1.5$ source planes of each cluster that are magnified by $\mu_{\text{GW}}^- < \mu < \mu_{\text{GW}}^+$, where μ_{GW}^- and μ_{GW}^+ are the values of μ_{GW} implied by the lower and upper 90 per cent confidence intervals, respectively, on the unlensed luminosity distance to the sources. Then, we ray traced these sky locations through the relevant lens models to obtain the respective image positions, $\vec{\theta}$. Given the large magnification values, all of these sky locations are multiply-imaged. We then measured the gravitational potential at the image positions, $\phi(\vec{\theta})$. The arrival-time surface for a light ray emitted by a lensed source, at the source-plane position $\vec{\beta}$, traversing the cluster lens at the image-plane position $\vec{\theta}$, is given by

$$\tau(\vec{\theta}, \vec{\beta}) = \frac{1 + z_L}{c} \frac{D_{\text{OL}} D_{\text{OS}}}{D_{\text{LS}}} \left[\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \phi(\vec{\theta}) \right], \quad (3)$$

where c is the speed of light in vacuum, z_L is the redshift of the cluster lens, D_{OL} , D_{OS} , and D_{LS} are the observer-lens, observer-source, and lens-source angular diameter distances, respectively, and $\phi(\vec{\theta})$ represents the projected cluster gravitational potential (Schneider 1985). These calculations were performed following the analytic procedure described by Jauzac et al. (2016).

The distribution of time delay between the first arrival of an image that suffers $\mu_{\text{GW}}^- < \mu < \mu_{\text{GW}}^+$ and the next arrival of an image from the same source location that is detectable by LIGO, $\Delta t_{\text{arrival}}$, spans a fraction of a day to a few years (Fig. 3). We classify these ‘next images’ as being detectable if they are magnified by $\mu \geq \mu_{\text{GW}}/\alpha^2$ where $\alpha = 13/8$ and $9.7/8$ for GW151226 and LVT151012, respectively, i.e. the ratio of the signal-to-noise ratio (SNR) at which each was detected in 2015 and the minimum SNR required of a detection by LIGO (Abbott et al. 2016c). Based on these calculations, we estimate the fraction of the next images that would be detectable by LIGO is ~ 20 –60 per cent. Note that in Fig. 3 we show results for $z_S = 1$ for the clusters at $z_L < 0.5$, in order to restrict our attention to

values of μ_{GW} that are not too extreme. However, we show results for $z_S = 1.5$ for RCS0224–0002, because the cross-section of this high-redshift cluster to a source at $z_S = 1$ is tiny. Our results are insensitive to these choices of z_S .

5 SUMMARY AND DISCUSSION

The degeneracy between gravitational magnification (μ) and luminosity distance (Wang et al. 1996) causes the luminosity distance to a GW source to be revised upwards by a factor of $\sqrt{\mu}$ if it is gravitationally magnified, and the inferred source frame masses of the compact objects to be revised down by a factor of $(1+z)$. This is interesting because some of the early GW detections appear to come from heavy BHs (Abbott et al. 2016a; Stevenson et al. 2017), and gravitational lensing by massive galaxy clusters has been central to the first detection of distant galaxy populations, most notably at sub-mm wavelengths (Smail et al. 1997; Ivison et al. 1998).

We estimate, based on the known populations of BBH GW sources and strong-lensing clusters, that the number of BBH mergers detected per detector year at LIGO/Virgo's current sensitivity that are multiply-imaged by known and spectroscopically confirmed strong-lensing clusters is $R_{\text{detect}} \simeq 10^{-5} \text{ yr}^{-1}$. This calculation takes into account the gravitational optics of clusters with sky positions consistent with the GW detections, the local BBH merger rate, and the star formation history of the Universe. It is a conservative lower limit on the true rate of multiply-imaged GWs to date, because it is based only on the known lenses, and in particular ignores any strong lenses obscured by the Galactic plane. The rate that we have computed is equivalent to saying that the hypothesis that one of the BBH GWs detected to date was multiply-imaged can be rejected at $\lesssim 4\sigma$. It is therefore unlikely, but not impossible, that one of the GWs detected to date was multiply-imaged.

Our search for candidate strong-lensing clusters that might have multiply-imaged GW sources concentrates on the BBH detections in LIGO's first observing run. Based on a comparison of the celestial coordinates of 130 spectroscopically confirmed strong-lensing clusters with the GW sky localization maps, we have identified no candidate lenses within the 90 per cent credible sky localization of GW150914, two within the 90 per cent credible sky localization of GW151226 (MACS J0410.0–0555 and MACS J1311.0–0311), and one within the 90 per cent credible sky localization of LVT151012 (RCS0224–0002). We used detailed mass models of these three clusters to calculate the magnifications and time delays suffered by the putative next appearance of GW151226 and LVT151012, in the scenario that they have indeed been multiply-imaged. We find that 20–60 per cent of the next appearances would be detectable by LIGO/Virgo at the sensitivity achieved in the first and second observing runs, and that they would arrive at Earth within 3 yr of the original detections.

Finally, we consider what it would take to identify unambiguously a multiply-imaged GW. Identifying a temporally coincident optical transient in the strong-lensing region of a massive galaxy cluster located within the GW sky localization would be an ideal scenario. This would allow the previous and subsequent appearances of the optical transient and presumed associated GW source to be computed, based on a detailed model of the cluster mass distribution. Predictions of previous appearances could then be compared with earlier GW detections and archival optical observations, and predictions of future appearances would inform future observations. Based on this outline, the gold standard would therefore be temporal and celestial sphere coincidence of the sky localizations of two GW detections and two optical transients with the strong-lensing

region of a single-cluster lens. Moreover, consistency between the strain signals detected by LIGO/Virgo would be required; this is efficiently phrased as requiring consistency between the detector frame chirp masses of the two GW detections. Chirp masses are currently measured to few per cent precision (e.g. Abbott et al. 2017b).

Given the sensitivity and thus reach of LIGO/Virgo, it is more realistic to contemplate searching for multiply-imaged GW sources that include one or, preferably, two BHs. Whilst it is an open question as to whether BBH mergers emit any EM radiation, it is reasonable to expect that any optical emission will be faint, notwithstanding any boost to the flux level thanks to gravitational magnification. It is therefore appropriate to consider deep follow-up optical observations of candidate strong-lensing clusters located in the sky localizations of BBH and BHNS GW sources, and also to consider whether and how a multiply-imaged GW source might be identified without an EM counterpart. On the first point, we commenced deep follow-up observations of candidate strong-lensing clusters in the latter stages of LIGO's second observing run with MUSE on VLT and GMOS on Gemini-South. These observations aim to reach a depth of AB $\simeq 25$ per epoch, which is considerably deeper than typical observations of GW sky localizations with wide-field instruments. We will report on these follow-up observations in a future article. On the later point, we intend to explore the feasibility of basing the discovery of multiply-imaged GW sources on solely the sky localizations, chirp masses, and available strong-lensing clusters for a given pair of GW detections.

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