Comparability of cirque size and shape measures between regions and between researchers

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With 11 Figures and 5 Tables

Summary –When comparably defined, cirque size and shape vary moderately but significantly between regions. For nine spatial divisions in three countries, differences in vertical dimensions (height range, amplitude, wall height) are greater than those in horizontal dimensions. Problems of data quality, especially contour interval and reliability, affect mainly comparisons of slope gradients between countries. Problems are more apparent from graphical displays of complete distributions than from means and extremes. A broader set of data from several authors shows greater variability, especially in mean values, for which there are several possible explanations.

"As maps and air photos of glaciated areas become increasingly available it should be possible to develop glacial morphometry to provide many valuable data in studying the processes that created the forms of both glacial erosion and glacial deposition." C.A.M. KING 1974, p. 162.

1. *Introduction*

Variation in results between data sources and between researchers is a general problem in environmental sciences. It is certainly important when geomorphologists define, map and measure landforms. Landforms are mental constructs (SMITH & MARK 2003, MARK & SMITH 2004): researchers map their models onto the continuous land surface and separate segments that satisfy their concept of each landform type (EVANS 2012). We aspire to achieve comparability of results by reducing the subjectivity of the definition process (EVANS & COX 1974, 2015).

Studies of the morphometry of landforms normally deal with single regions. This permits assessment of the effects of altitude, aspect and limited ranges of rock types, but not of differences between climatic regions, tectonic provinces and contrasts between regions covered by ice sheets at glacial maxima, and those not covered. All these contrasts and effects are probably important in the development of glacial cirques. When regions are compared, differences can be attributed to subjective differences between authors in their understanding of definitions and to differences in source material and methods, as well as to real differences. Progress in specific geomorphometry has thus been held back by the vagueness of definitions, so that data sets produced by different authors are rarely comparable.

Morphometric analysis of cirques is needed for determining their ranges of sizes and shapes and finding any characteristic relations between size and shape. Often morphometry provides the *only* evidence we have for past processes or environments of cirque glaciers, as the glaciers have melted, available sediments may be uninformative, or detailed fieldwork may be impracticable. Even if morphometric evidence is indirect, it is relatively easy to obtain systematically with moderate effort, and indirect evidence has to be preferred to no evidence at all. The question is, how to define variables in the most relevant way and to maximise information with a detailed and careful statistical/graphical analysis?

Geomorphometry requires precise, repeatable operational definitions permitting replicable closed outlines to be drawn around each landform. ARDELEAN et al. (2013) showed that the considerable differences between different authors in defining glacial cirques can be reduced if a common precise definition is applied. Otherwise the numbers reported in the Ţarcu mountain range (in Romania) varied between 23 and 60, and total area of cirques between 12.6 and 27.7 km^2. The definition agreed by a British Geomorphological Research Group meeting (reported by EVANS & COX 1974:151) is that a cirque is "a hollow, open downstream but bounded upstream by the crest of a steep slope ('headwall') which is arcuate in plan around a more gently sloping floor. It is glacial if the floor has been affected by glacial erosion while part of the headwall has developed subaerially, and a drainage divide was located sufficiently close to the top of the headwall (the cirque crest) for little or none of the ice that fashioned the cirque to have flowed in from outside". An attempt has been made to produce a series of data sets based on the same definition (EVANS & COX 1995, EVANS 2006). These results are analysed here, followed by comparisons with differently produced data sets.

Distributions of cirque size and shape can be used to address several geomorphological questions. A particular question in the development of cirques by glacial erosion is whether there is an upper limit on cirque size: EVANS (2010) suggested that cirques are scale-specific, with upper and lower limits to their size. Another question concerns allometry, the variation of shape with size and age: EVANS (2009) reported work confirming the static allometry of glacial cirques in several European and British Columbian areas. Morphometric data can also be used to address the question whether the form of mountain cirques is produced essentially by deep-seated rock avalanches (TURNBULL & DAVIES 2006), or by glacial erosion: this will be addressed elsewhere. This challenge does, however, underline the importance of cirque floor extent and low gradient in supporting a glacial origin, as the headwall of a large rock avalanche scar may be indistinguishable from that of a glacial cirque.

The focus of this paper is on the problems of generating consistent data. Initially considered are cirque size and shape for nine well-studied regions in three countries. For the nine regions we present a series of graphs portraying every value, as well as means, medians and quartiles: these are produced by the Stata program `stripplot` (COX 2016), implementing the ideas of PARZEN (1979). This thorough graphical presentation not only permits a better comparison of size and shape across the nine regions; it also highlights some data problems that were not otherwise obvious. We suggest that simply reporting means, maxima and minima is insufficient for a useful comparative analysis. Finally, the discussion is broadened to compare results, mainly for cirque size, from a broad range of published studies.

2. *Study areas and Methods*

Tests for differences in cirque size and shape between nine regions with complete inventories are undertaken: three divisions of Romania, three in Britain (Wales and England), and three adjacent ranges in southwest British Columbia (B.C.). The first six data sets were produced by Evans, who has studied all nine regions in the field. The Romanian coverage was produced by Marcel Mîndrescu following the same definitions, with checks by Evans (MÎNDRESCU & EVANS 2014). The Lake District (England) data are from EVANS & COX (1995), but with two deletions (EVANS 2015); those for the two divisions of Wales are from EVANS (2006); and the British Columbian data were used in EVANS (2010). Each data set is based on detailed fieldwork, air photo interpretation, and large-scale topographic maps. Checking is most complete for the Lake District, with all cirques visited in the field: the data set has been available, used by many people since 1995, and no errors in measurements have been found. The Welsh and Romanian data are well-edited, whereas the British Columbian data are older and subject to revision. Although much can now be achieved with Google Earth, we suggest that field acquaintance remains valuable in distinguishing cirques from features with different origins.

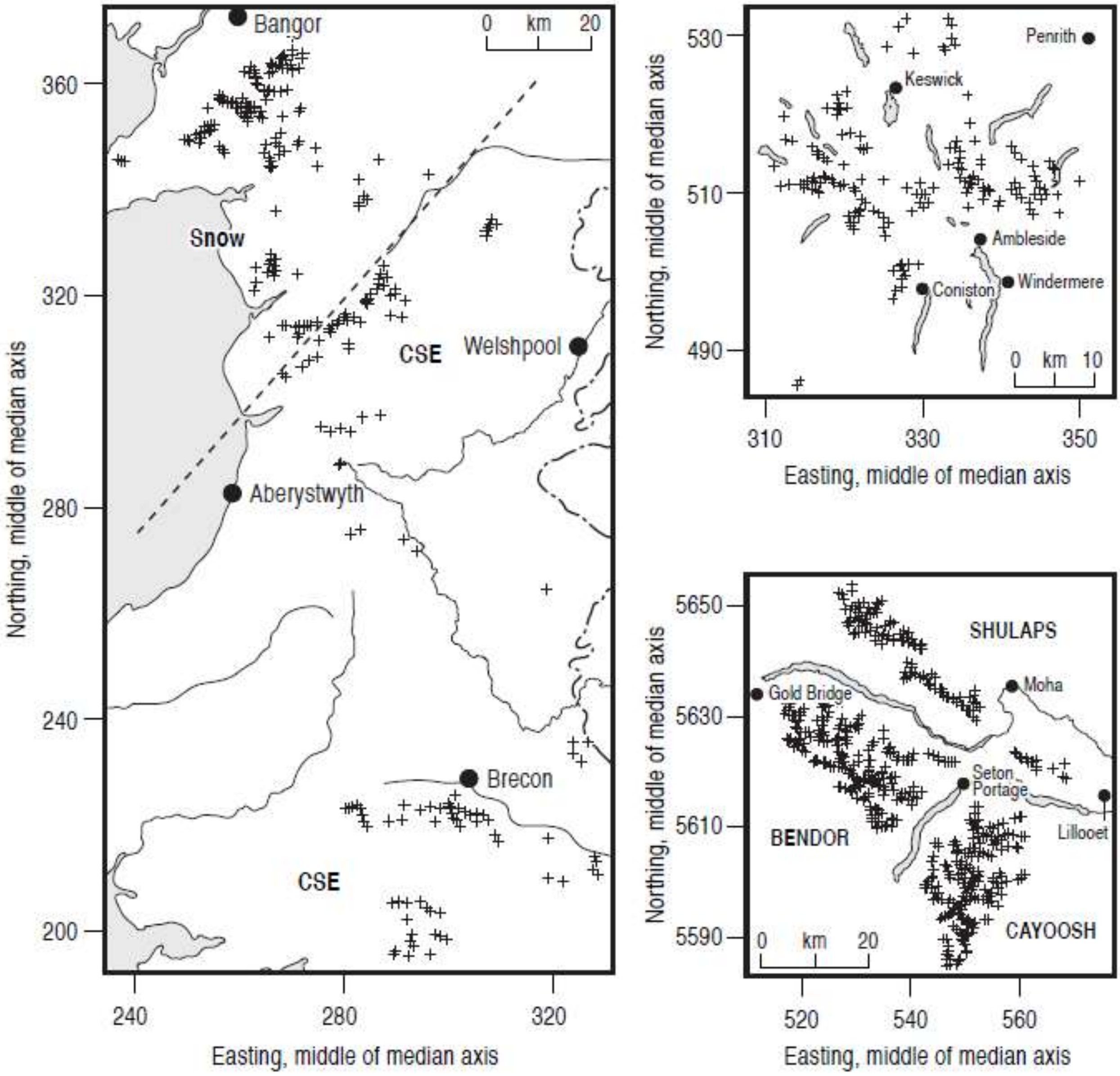

Figure 1 Cirque distributions in Wales (left), Lake District (top right), and British Columbia (bottom right). Scales vary. The axes are labelled in kilometres, every 40 km for Wales, and every 20 km for Lake District and British Columbia. Grid references are on national grids, UTM in British Columbia and the Ordnance Survey variant of UTM in Britain. In Wales, Snow (Snowdonia) and CSE (Central, Southern and Eastern Wales) are separated by the dashed line. All cirques within each map outline are shown, except in British Columbia where there are many cirques in ranges southeast of Cayoosh, southwest of Bendor, and (a few) along the northern border.

We thus have a cluster sample (figs. 1 and 2), with complete coverage of Romania, of Wales and the Lake District, and of three contiguous mountain ranges in British Columbia (Cayoosh, Bendor and Shulaps, labelled Cay, Ben and Shu respectively). The Bendor Range is separated from the Cayoosh Range by

Anderson Lake and from the Shulaps by Carpenter Lake (fig. 1). The Cayoosh Range extends to Seton Lake, Cayoosh Creek and the pass at c.1987 m and grid 547.7 km W, just north of the west end of Duffey Lake, leading to Haylmore Creek. The Bendor Range extends from McGillivray Pass in the west to Mission Pass in the east: a preliminary map of cirques was published in Derbyshire & Evans (1976). The Shulaps Range lies between Yalakom and Bridge Rivers and Tyaughton Creek, extending north to Mud Lakes and the upper reach of Churn Creek: we also include Mission Ridge across the Bridge River Canyon, between Mission Pass, the lower Bridge River, and Seton Lake. Note that the northern tip of the Shulaps Range, beyond 51.03°N, 5653 km in UTM, is not included.

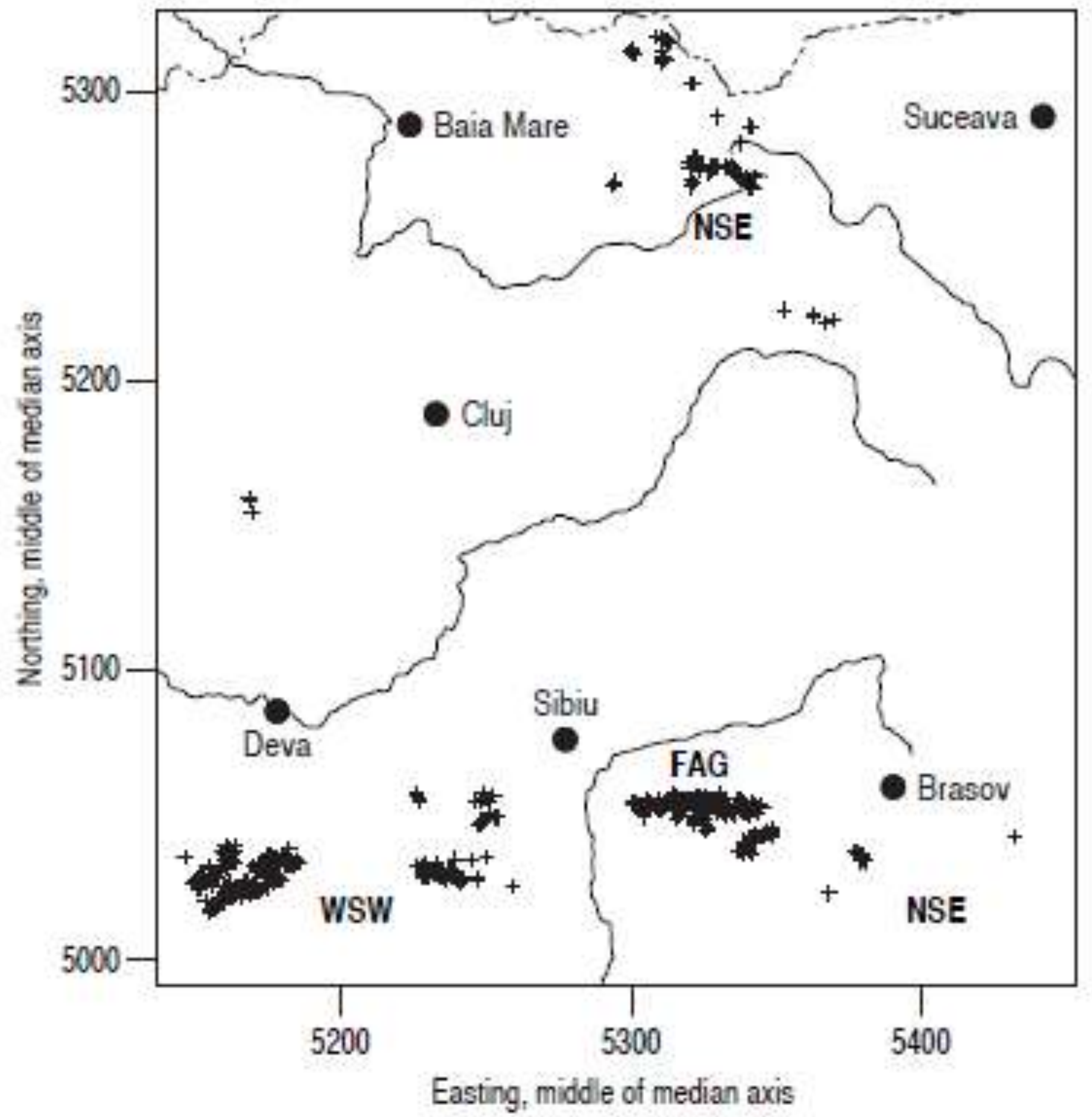

Figure 2 Cirque distribution in Romania; axes are labelled in kilometres, every 100 km on Romanian UTM grid. The Făgăraş Mountains (FAG) have a compact cluster of cirques: all ranges farther west are in region WSW (West and Southwest Romania): all to the east, including the Iezer cluster immediately southeast, are in NSE (Northern Romania and Southeast Carpathians).

Wales is divided into the old volcanic and metamorphic terrain of the northwest ('Snowdonia', including the Harlech Dome, Arenigs and Cadair Idris) and the mainly sedimentary or weakly metamorphosed terrain of central, southern and eastern Wales (labelled CSE), from the Corris area, Arans and Berwyns to the Brecon Beacons and South Wales coalfield. In Romania (fig. 2), the threefold division is achieved by separating the largest glaciated range, the Făgăraş Mountains with 206 of the 631 cirques, from the mountains to the west, labelled WSW (west and southwest Romania), and from those to the east and north, labelled NSE (northern Romania and the southeast Carpathians). The Iezer Mountains are close to the Făgăraş but lithologically and probably tectonically distinct, and are included in NSE. Thus we define nine regions with between 117 and 293 cirques each. Source map styles vary, especially between the three countries, but all definitions have been checked by Evans.

The *quantile-box plots* (PARZEN 1979) shown here (figures 3 to 9) combine *quantile plots* showing all values in order from smallest to largest, plotted against cumulative probability (or equivalently rank) for each group (WILK and GNANADESIKAN 1968; CLEVELAND 1993), with boxes showing median and quartiles, as in dispersion diagrams (CROWE 1933). Quantile plots are thus kin with hypsometric curves (CLARKE 1966). The design here makes explicit a principle of *box plots* that half the data points belong inside the boxes and half outside (TUKEY 1977). We further add longer horizontal lines showing means. The aim of a quantile-box plot is thus to show not only broad differences between distributions in level (central tendency), spread (dispersion) and shape, but also fine structure such as straggling tails, possible outliers and granularity or other grouping of values.

Table 1. Definitions of variables used here, mainly selected from EVANS AND COX (1995).

Variable (units)	***Abbreviation***	***Definition***
Length (m)	*L*	Length of the median axis, from the focus (middle of the cirque threshold) to the crest, dividing the cirque into two equal halves, left and right.
Width (m)	*W*	Maximum dimension measured at right angles to the median axis.
Amplitude (m)	*A*	Difference in altitude between highest and lowest points on median

		axis: axial height range.
Height range (m)	*H*	Difference in altitude between highest point on crest and lowest point on floor.
Wall height (m)	wallht	Greatest range in altitude on headwall along any single slope line.
Max. gradient (°)	maxgrad	Maximum headwall gradient over 30 m vertically (30.48 or 40 m in British Columbia; 50 m in Romania), from contour spacing.
Min. gradient (°)	mingrad	Minimum floor gradient over 10 m vertically (20 m in British Columbia). 0° if large lake or bog.
Plan closure (°)	planclos	Change in azimuth (generalised over 100 m of contour to ignore minor gullies) along contour at mid-height between highest and lowest points in cirque.
Profile closure (°)	profclos	Difference between maximum and minimum gradients.
Axial gradient (°)	axgrad	arctan(amplitude/length)
Width/length	WidLen	Ratio, dimensionless
Length/height range	LenHeight	Ratio, dimensionless

3. *Size analyses*

Measured components of size are two horizontal dimensions (length and width), and three ways of defining the vertical dimension (Table 1): these are defined in diagrams in Evans & Cox (1995) and Mîndrescu & Evans (2014). Length is defined from a median axis, starting from the middle of the cirque threshold (down-valley limit of the floor) and dividing the cirque into two equal parts, left and right. Width is the greatest length at right angles to this axis. Note that width thus defined can be greater or smaller than length. Amplitude is the axial height difference, between the intersections of the median axis with the cirque crest and with the middle of the threshold. Height range is the difference in altitude between the highest point (on the crest) and the lowest point (on the threshold). Wall height is the greatest height difference along any slope line on the headwall. As all these size variables are positively skewed, it is useful to portray the median as well as the mean (which is always higher) (figs. 3-6: see also Evans & Cox 2015). The effects of skewness (Cox 2010) are avoided by performing all further analyses on logarithms of these size variables, reducing moment-based skewness (per region) to between −0.75 and +0.63 (from initial values between +0.52 and +4.30).

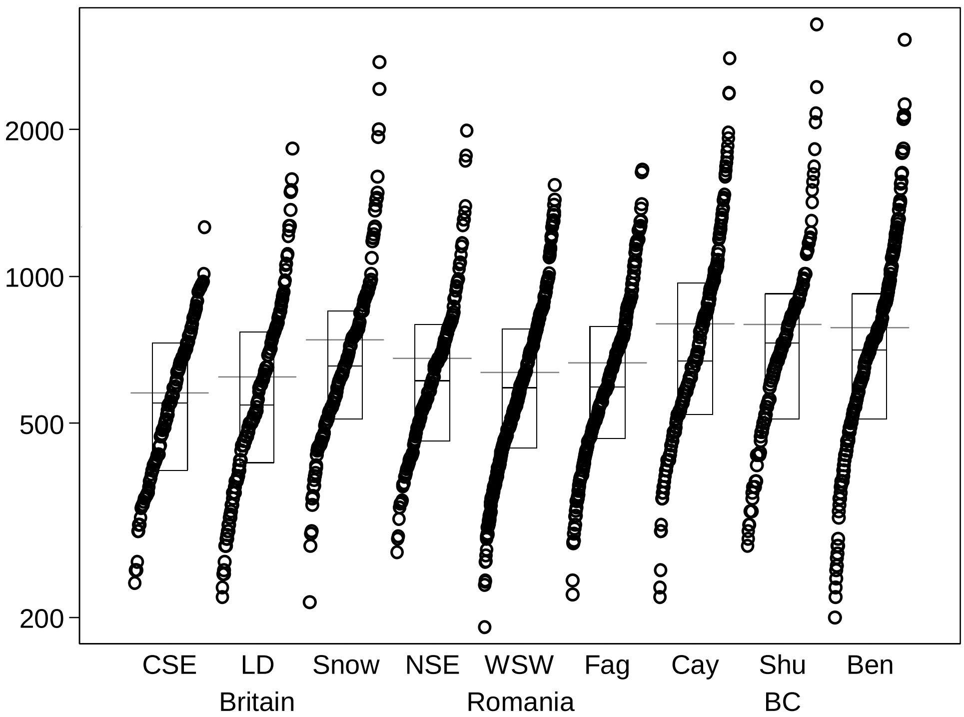

Fig. 3 Length (logarithmic scale): values in ranked order with medians and quartiles as boxes; longer lines show (arithmetic) means. Note that median of logarithm = logarithm of median.

Table 2. Median dimensions (m) and numbers of cirques.

Region	***Number***	***Length***	***Width***	***Amplitude***	***Height range***	***Wall height***
N & SE Romania	132	610	666	270	300	200
Făgăraş	206	592	652	280	330	215
W & SW Romania	293	591	644	240	280	180
Lake District	156	545	600	230	261	200
Snowdonia	143	655	720	242	285	222
Wales C, S & E	117	550	685	185	210	140
Cayoosh	198	670	625	305	381	270
Bendor	222	705	670	312	395	285
Shulaps	126	730	670	310	360	260

TOTAL	**1593**	**625**	**656**	**260**	**310**	**210**

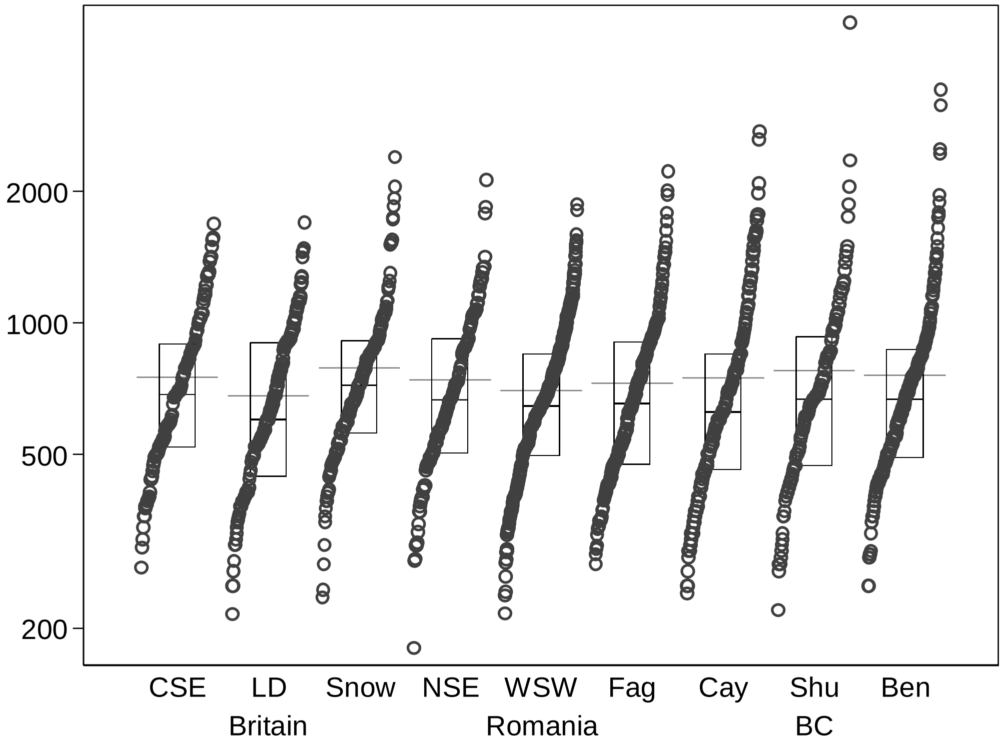

Fig. 4 Width (logarithmic scale): values in ranked order with medians and quartiles as boxes; longer lines show means.

Note that length is greatest for the three British Columbian regions, and least for the Lake District and CSE Wales (Table 2 and fig. 3). Width varies little but is greatest for Snowdonia and least for the Lake District (fig. 4): all three B.C. ranges have greater maxima. Vertical dimensions are strongly inter-correlated, and much more variable between regions than are length and width. All three vertical variables are greatest in British Columbia and least in CSE Wales (figs. 5 & 6): within Britain, Snowdonia > Lake District > CSE Wales, and within Romania, Făgăraş > NSE > WSW. In B.C., Bendor has the greatest height ranges and Shulaps has the lowest.

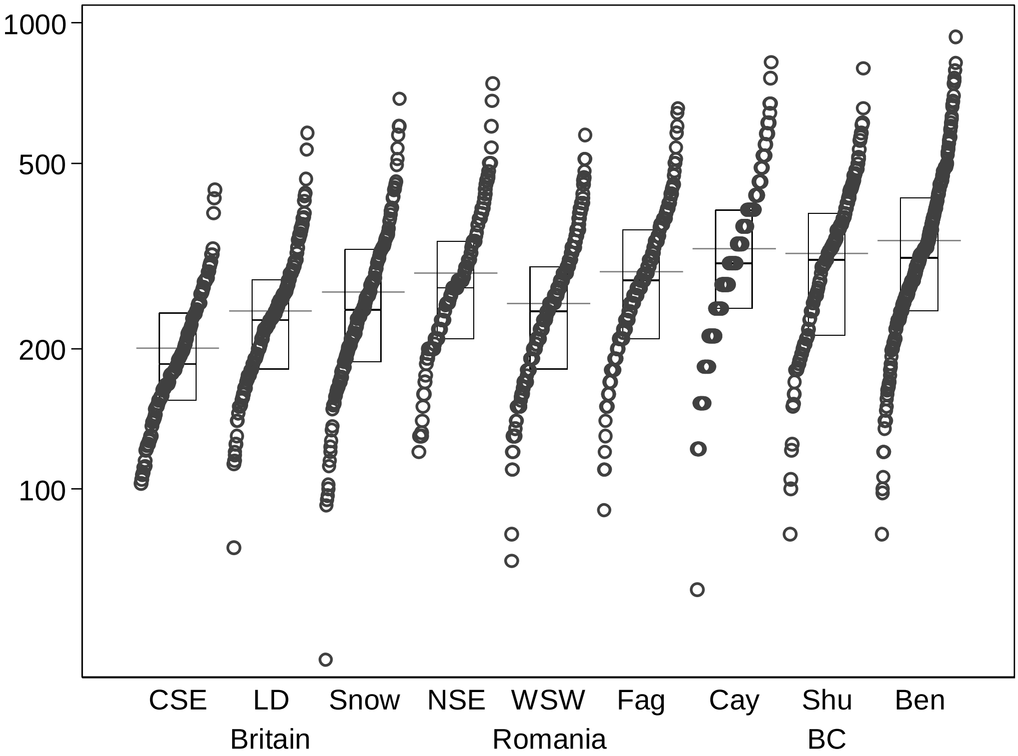

Fig. 5 Amplitude (logarithmic scale): values in ranked order with medians and quartiles as boxes; longer lines show means. Note the discretisation (rounding) of values for Cayoosh, due to the coarse (30.48 m) contour interval on the maps used.

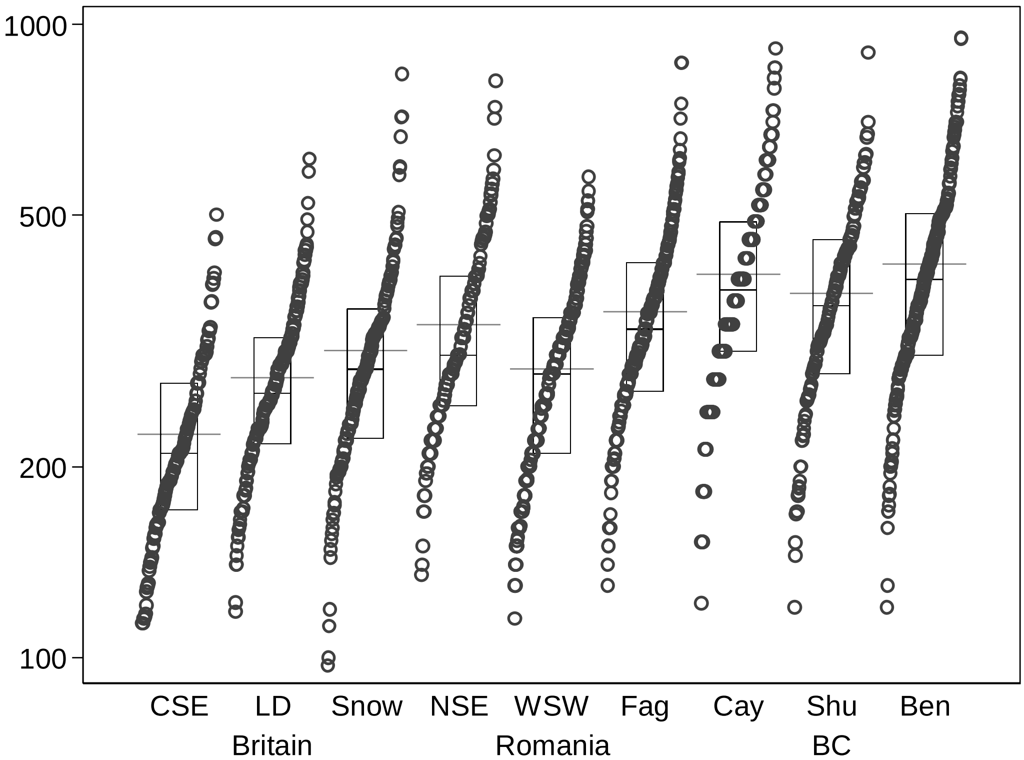

Fig. 6 Height range (logarithmic scale): values in ranked order with medians and quartiles as boxes; longer lines show means. Note the discretisation (rounding) of values for Cayoosh, due to the coarse (30.48 m) contour interval on the maps used.

Table 3. Analysis of variance results for variance accounted for by the division into nine regions. All except width are highly significant (P<0.0001). The SD (overall standard deviation) is given for comparison with the RMSE (root mean square deviation) within regions, demonstrating that most of the scatter is intra-regional. Logarithms are used for the first five variables (dimensions). Gradients and closures are in degrees.

Variable	***F***	***R^2***	***Adjusted R^2***	**RMSE**	**SD**
Length	9.37	.045	.040	0.188	0.192
Width	1.52	.008	.003	0.192	0.192
Amplitude	26.77	.119	.115	0.166	0.177
Height range	48.39	.196	.192	0.155	0.172
Wall height	42.10	.175	.171	0.182	0.200
Max gradient	80.51	.289	.285	8.970	10.620
Min gradient	20.71	.095	.090	5.390	5.650
Plan closure	9.67	.047	.042	48.110	49.150
Profile closure	55.80	.220	.216	11.400	12.870
Axial gradient	13.39	.063	.059	6.510	6.710
Width/length	12.82	.061	.056	0.365	0.375
Length/height range	24.73	.111	.106	0.643	0.681

Analysis of variance between and within the nine regions produced highly significant differences (P<0.0001) for all size dimensions except width (for which P = 0.1452) (Table 3). Judging by either F ratio or R^2, between-region contrasts were greatest for vertical dimensions: height range, wall height and amplitude. These results were obtained with logarithmic transformation of all five variables: similar results are obtained without transformation. R^2 values are in the same rank order as F values, which is inevitable given the use of nine regions throughout. Despite their high significance levels, R^2 values are fairly weak, 0.192 to 0.115 for the verticals and 0.040 for length (after adjustment for model parameters fitted). An alternative way of evaluating the relevance of the 9-fold regional classification to each variable is to compare RMSE (root mean square error of residuals) with SD (overall standard deviation): both are in the same units. The modest reductions (up to 10%) again show relatively weak effects of classification: most of the variation is within each region.

Table 4. Median gradient and shape variables (°, except last two columns) and numbers of cirques, by region. Variable names are abbreviated from those in Table 3: see also Table 1.

Region	***Number***	***Maxgrad***	***Mingrad***	***Planclos***	***Profclos***	***Axgrad***	***WidLen***	***LenHeight***
N & SE Romania	132	48	10.2	137	37.6	23.8	1.10	2.04
Făgăraş	206	55	8.7	145	46.5	24.6	1.03	1.81
W & SW Romania	293	51	7.5	134	42.3	22.5	1.11	2.10
Lake District	156	63	7.1	123	56.0	22.7	1.10	2.09
Snowdonia	143	65	3.5	121	61.5	20.6	1.07	2.29
Wales C, S & E	117	56	5.2	110	50.0	20.0	1.27	2.52
Cayoosh	198	68	9.7	135	55.6	25.4	0.91	1.88
Bendor	222	63	8.2	124	50.6	24.4	0.97	1.79
Shulaps	126	53	7.3	100	45.4	23.3	0.97	2.01
TOTAL	**1593**	**57**	**7.7**	**128**	**49.2**	**23.1**	**1.05**	**2.03**

4. *Shape and gradient analyses*

Table 4 gives median values for gradient and shape variables. The gradient and closure variables show minimal skewness (−0.47 to +1.00 per region) and means and medians are similar, but medians are given for compatibility with the two ratios (skews +0.49 to +1.65) and with Table 2. Maximum and minimum gradients are calculated from minimum and maximum contour spacings, measured manually (Table 1). Profile closure is the difference between them. Plan closure is the change in orientation of the mid-height contour (generalized over 100 m lengths to remove gullies) as it passes through the cirque: it expresses the degree to which a cirque 'bites' into the relief. Axial gradient approximates the surface slope of a former glacier just filling the cirque: it is obtained by dividing amplitude by length of median axis and taking the arctangent. All five of these variables are expressed in degrees. Finally two ratio variables are included as these are favoured by some other authors (e.g. FEDERICI & SPAGNOLO (2004) and GÓMEZ-VILLAR et al. (2015). Width/length (*W/L*) measures elongation, inversely: given the above definitions it can be either greater or less than the value of unity for circular plan forms. Length/(height range) (*L/H*) is a reciprocal measure of overall gradient, differing from axial gradient in that the highest point can be anywhere on the cirque crest, and not necessarily on the median axis: thus it has less relation to possible former glaciers.

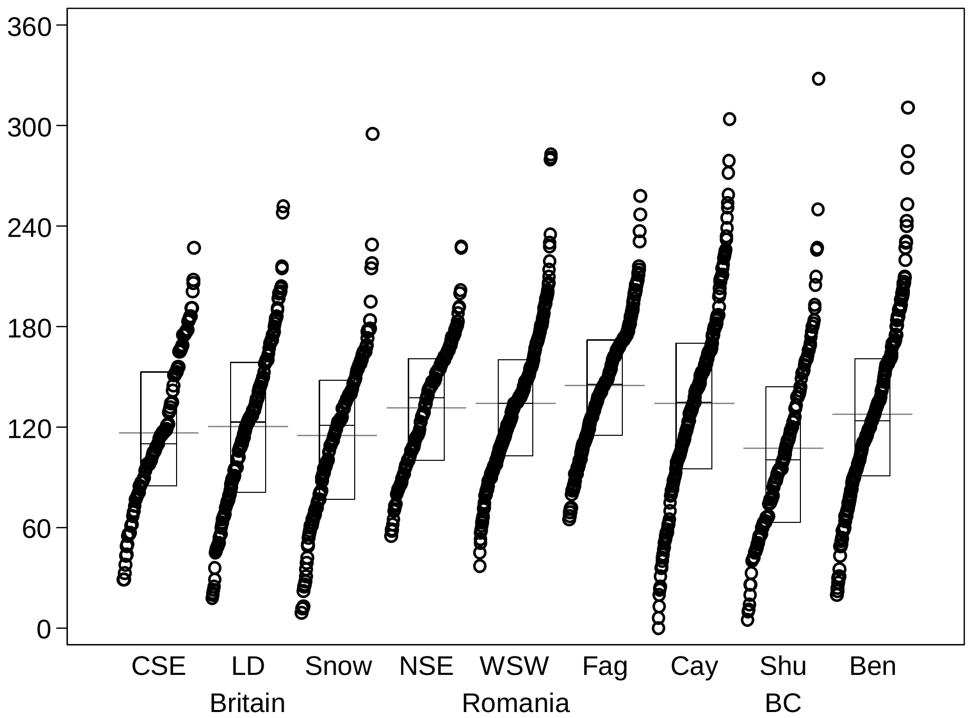

Fig. 7 Plan closure: values in ranked order with medians and quartiles as boxes; longer lines show means.

Median plan closure (fig. 7) varies between 100° (Shulaps) and 145° (Făgăraş): these differences are highly significant, like those for length. British Columbia covers a broader range of values, but overall Romania has the higher plan closures, followed by Cayoosh and Bendor. Closure might be expected to increase during occupation by cirque glaciers, rather than by out-flowing valley glaciers. The British regions have low values, but Shulaps has the smallest plan closures. Within Romania, Făgăraş has the largest plan closures. There is little difference between the three British regions.

Minimum gradient has a secondary mode at zero (fig. 8), for cirques with lakes or bogs (given full bathymetry and subsurface information, these would have negative gradients, i.e. upstream slopes). On average, British cirques have the lowest floor gradients: Snowdonia has the gentlest floors, while NSE Romania and Cayoosh have the steepest. The contrasts between the three British regions are real, but the 10 m contour interval (c.i.) of the maps used in Britain probably permits more extreme values of minimum and maximum gradient than in areas where contour intervals were greater. Hence, the small differences between countries are doubtful. Snowdonia (34%) and WSW Romania (23%) have the most zero-gradient floors: Cayoosh has none, which reflects a different measuring procedure, using contour spacing even where a large lake is present. Because of the high relief in British Columbia, numerous features with floors sloping at more than 21° throughout have been accepted as cirques, but one Bendor outlier must be a data error.

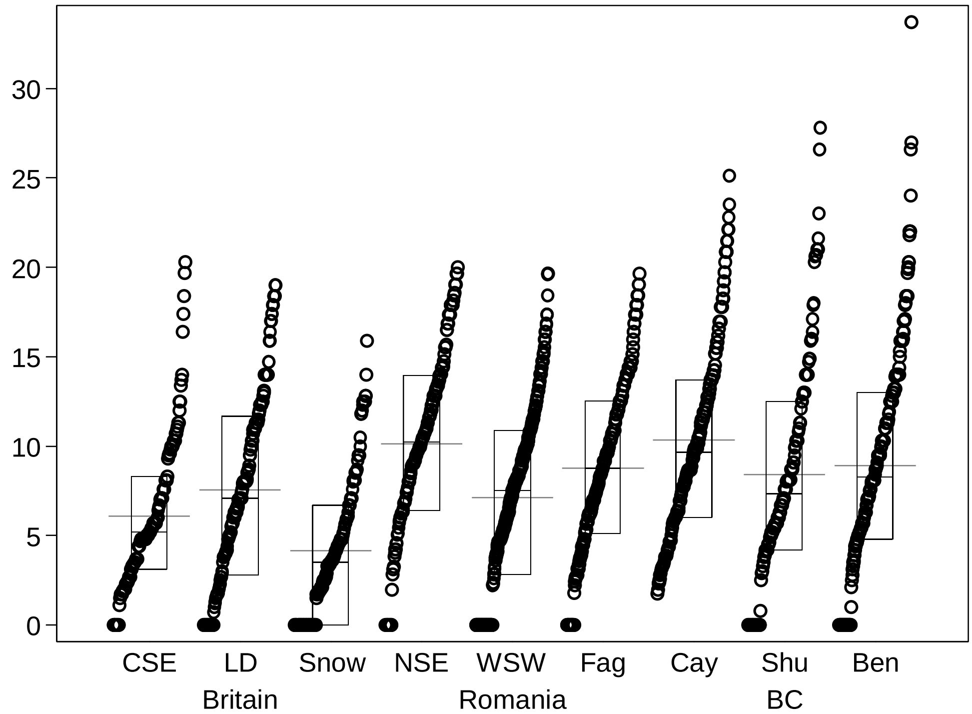

Fig. 8 Minimum gradient: values in ranked order with medians and quartiles as boxes; longer lines show means.

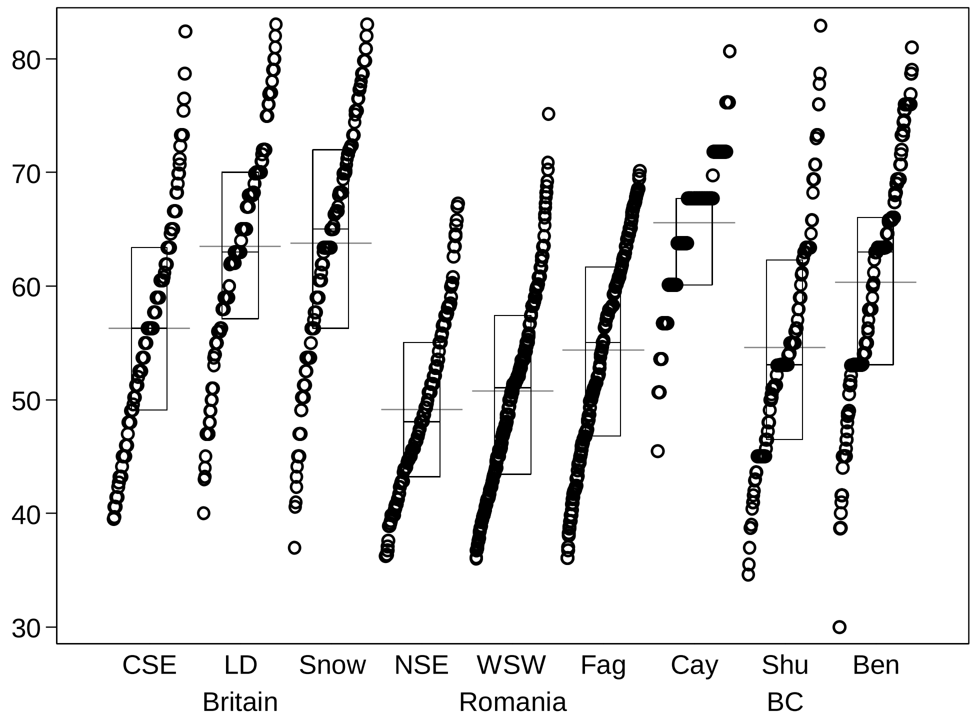

Fig. 9 Maximum gradient: values in ranked order with medians and quartiles as boxes; longer lines show means.

Maximum gradient (fig. 9) tells a different story: the 1:25,000 map scale and measurement over 50 m vertically (two contour intervals) in Romania lead to underestimation relative to British 10 m c.i. 1:10,000 maps where measurement was over 30 m (three contour intervals). Likewise, the British Columbian data are from poorer-scale maps (1: 31,680 and 30.48 m c.i. for Cayoosh, the oldest data set; 1: 20,000 and 20 m c.i. for Bendor and Shulaps), and maximum gradients are underestimated. The Cayoosh data also show discretisation (granularity or rounding), that is distinct repeated values which are artefacts of the coarse contour interval, and the median is identical numerically to the upper quartile. Bendor and Shulaps are affected more subtly: 45.0° 53.1°, 63.4° and 76.0° are especially frequent because they are produced by 40 m contour spacings at the rounded values of 40, 30, 20, and 10 m respectively. Comparisons are reliable, however, for measurements from the same map series, i.e. mainly within countries. In Britain, maximum gradient for CSE Wales has a much lower mean than the other two regions, and in British Columbia, Shulaps has likewise: in both cases this is credible because rock types are weaker than in the two nearby regions. In Romania means are lowest in NSE, which includes weaker rocks in the Câlimanii and Maramureş.

Problems revealed here in comparing maximum gradients from contour maps will not be relevant to future work measuring slope gradient from DEMs. Comparability will be achieved, however, only when the DEMs are of comparable accuracy and resolution.

Profile closure is controlled mainly by maximum gradient: Snowdonian cirques are best-developed (with especially gently-sloping floors), followed by Lake District and Cayoosh, while NSE Romania has the poorest. It is likely, again, that Romanian gradients are distorted by relatively poorly-contoured maps, mainly at 1:25,000, with contours interrupted at cliff symbols.

CSE Wales has the highest *W/L* and *L/H* ratios, while the British Columbian ranges have the lowest, accompanied by Făgăraş for *L/H*. This reflects the poor showing of CSE Wales on length and especially on height range.

Analysis of variance (Table 3) demonstrates highly significant differences between regions for all seven shape variables (gradient, closure and ratio variables). Differences, as shown by F ratio and R^2 values, are greatest for gradient variables and thus for profile closure, and for the Length/Height ratio which is an inverse gradient measure. Maximum gradient has an adjusted R^2 of 0.285, and its RMSE is 15.5% less than its SD. Results for size measures, however, are more reliable than for gradient measures (including profile closure).

The 12 size and shape variables may thus be ranked in order of inter-regional contrast, measured by F in Table 3, as: Maximum gradient; Profile closure; Height range; Wall height; (*gap*); Amplitude; (Length/Height range) ratio; Minimum gradient; (*gap*); Axial gradient; (Width/Length) ratio; Plan closure; Length; Width. The first six all include a vertical dimension, and it is clearly this that varies most between regions. Minimum gradient and shape measures come next, followed by Length and (insignificant) Width. Within this three-country, nine-region data set, maximum gradient and vertical dimensions, starting with Height range, have the greatest between-region variation.

5. *Further data*

The three clusters above were selected as study areas for their feasibility and accessibility, and they are obviously not representative of cirques globally. They cover intrusive, old volcanic, metamorphic and sedimentary rock areas in old crystalline massifs and young orogenic belts, but not young volcanic areas or the highest-relief mountains. Spatial comparisons can be broadened by considering published results from various authors (Table 5), although the global coverage remains uneven and unrepresentative.

In selecting data for Table 5, we have focussed on sizeable data sets with the exception of Bohemia and Iran, which cannot be sensibly merged with other data. Thus we have not subdivided the W-C. Yukon or N.E. U.S.A. data, and have grouped the Greece data into two regions. More exhaustive tables are presented in BARR & SPAGNOLO (2015) and in MITCHELL & HUMPHRIES (2015: Table DR1): but note that these include some regions with fewer than 20 cirques (in the latter, three with fewer than 10), and both old and new data for some overlapping regions.

BARR & SPAGNOLO (2013) tabulated cirque size means from various authors, for 16 areas, although 5 of these had fewer than 40 cirques. These further data sets show a greater range of sizes (see also Table 5) than the nine comparable regions. Excluding the special case of Antarctica, those with more than 40 cirques ranged in mean length from 295 to 1687 m, more than five-fold, and much more varied than the 577 to 798 m (545 to 730 m in medians) here in Table 2. Their mean widths varied from 467 to 954 m, two-fold and considerably more than the 681 to 797 m (600 to 720 in medians) here. Mean height ranged from 236 to 442 m (a 209 m value refers to wall height): this is somewhat greater than the 225 to 419 m (210 to 395 m in medians) here. Barr and Spagnolo's tabulated size ranges for individual cirques were 100 to 4000 m in length (compared with 191 to 3280 here), 125 to 3100 m in width (180 to 4870 here), and 57 to 1328 m (97 to 953 here) in height range. The greatest subjectivity probably concerns recognition of cirques 100 to 200 m long or wide, and those <100 m in height.

Table 5. Mean cirque size data (m) from other authors (* = mean amplitude). 1974 and 1995 refer to the Evans and Cox papers on definition of cirques and cirque variables. GE = Google Earth, AP = aerial photographs (note: most authors made ancillary use of AP). Map scales for the previous nine regions are given in section 4.

region	***number***	***Length***	***Width***	***Height range***	***source***	***Map scale***	***Uses 1974***	***Uses 1995***
Kintail-Affric-Cannich, W. Scotland	231	625	586	(276*)	GORDON 1977 (simple cirques)	10k	YES	-
N. Scandin-avia transect	537	845	888	400	HASSINEN 1998	50k	NO	NO
High Tatra	116	570	550	311	KŘÍŽEK & MIDA 2013	10 m DEM	NO	YES
Bohemia	27	788	700	272	KŘÍŽEK et al. 2012	25k	NO	YES
Maritime Alps, Italy	432	672	663	355	FEDERICI & SPAGNOLO 2004	25k	NO	YES
E. Pyrenees,	1071	489	482	(223*)	DELMAS et al.	25k	YES	YES

France					2014 (simple)			
C. Pyrenees, Spain	206	519	691	364	GARCIA-RUIZ et al. 2000	50k	NO	YES
S.W. Asturias & WPE	70	487	594	255	RUIZ-FERNANDEZ et al. 2009	25k	NO	YES
W. Picos de Europa	59	295	467	294	RUIZ-FERNANDEZ et al. 2009	25k	NO	YES
N.E. USA	49	1687	954	442	DAVIS 1999	24k	YES	YES
W.-C. Yukon	331	802	736	214	NELSON & JACKSON 2003	50k	YES	YES
Kamchatka	3520	868	992	421	BARR & SPAGNOLO 2013 (>0.05 KM^2)	30 m DEM	YES	YES
Fiordland, N.Z.	1296	855	882	463	RICHTER 2006 (>0.1 KM^2)	25 m DEM	YES	YES
Westland, N.Z.	480	1069	961	580	RICHTER 2006 (>0.1 KM^2)	25 m DEM	YES	YES
Ben Ohau Ra., N.Z.	90	489	536	216	BROOK et al. 2006	50k	NO	YES
N. Greece	166	530	737	289	BATHRELLOS et al. 2014	50k	NO	NO
S. Greece	99	376	460	173	BATHRELLOS et al. 2014	50k	NO	NO
Outer Vestfirðir N.W. Iceland	100	515	752		PRINCIPATO & LEE 2014	50k & 10 m DEM & GE	NO	NO
Zardeh Kuh, Zagros, Iran	28	880	805	338	SEIF & EBRAHIMI 2014	10 m DEM & GE	YES	YES
U. Sil, S. Cantabria	67	625	707	277	GÓMEZ-VILLAR et al. 2015	50k & AP	YES	YES
Montaña Central, S. Cantabria	89	468	655	237	GÓMEZ-VILLAR et al. 2015	50k & AP	YES	YES

Okoa Bay, Baffin I.	165	1053	833	257	ANDREWS & DUGDALE 1971	125k & AP	-	-

Fig. 10 plots mean values of width against length for the nine regions discussed above and the further 22 regions tabulated in Table 5. The three Romanian regions (NSE, Fag, WSW) plot very close together, as do the three British Columbian regions (Cay, Ben and Shu); the British regions (LD, CSE, Snow) are slightly more varied. The 22 extra regions are more widely dispersed, especially in length, but they show the expected positive relation between means of width and of length. Three of them might be regarded as outliers: even on a logarithmic scale, the mean length of cirques in the north-eastern U.S.A. (NEU: Katahdin, Longfellow, White, Green, Adirondack and Catskill Mountains) is much greater than in any of the other data sets, both absolutely and relative to width. Southern Greece (SGr: Sterea Hellas, Peloponesus and Crete) and Western Picos de Europa (WPE) have the narrowest and, especially, the shortest cirques.

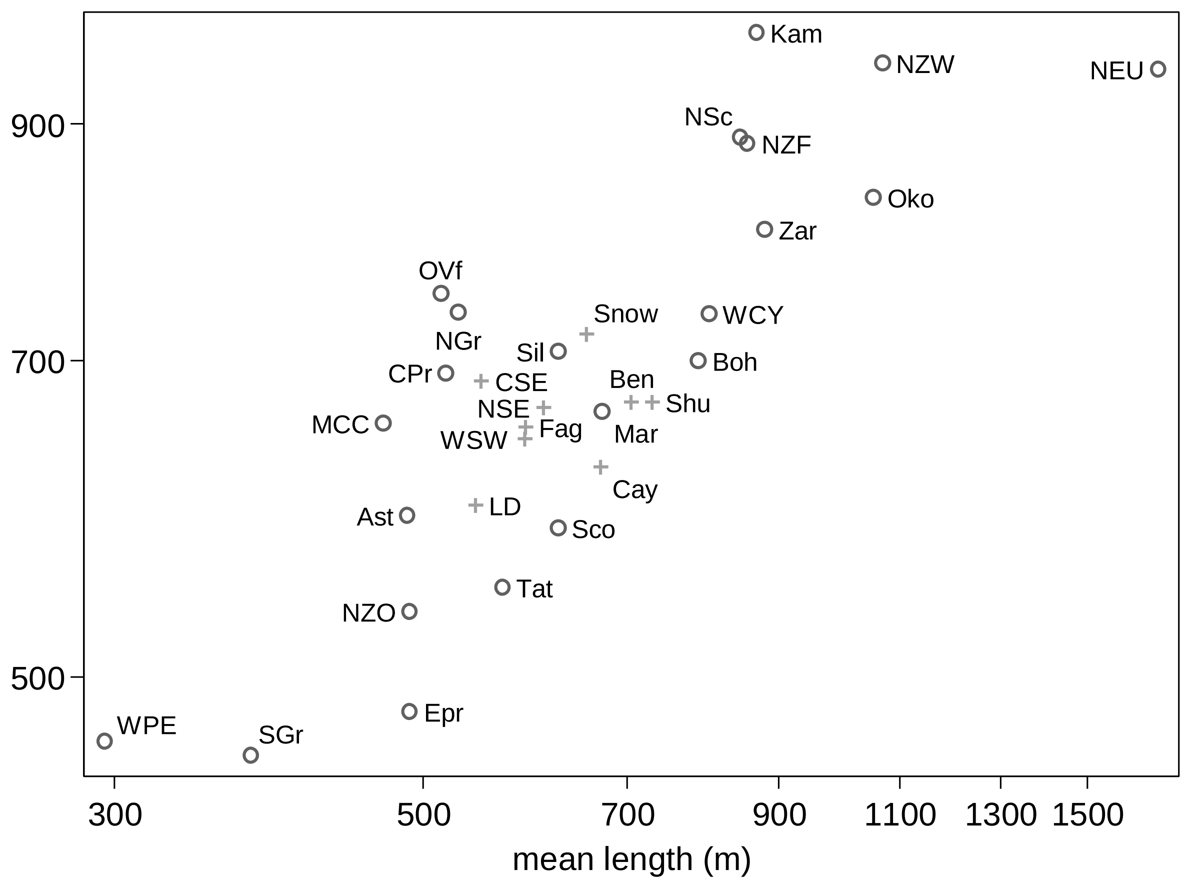

Fig. 10 Horizontal size of cirques for 9 regions (grey dots) and 22 further regions from the literature (crosses). Mean width is plotted against mean length, both on logarithmic scales labelled in metres.

Ast Asturias, Spain; Ben Bendor Range, B.C.; Boh Bohemia; Cay Cayoosh Range, B.C.; CPr Central Pyrenees, Spain; CSE Central, Southern and Eastern Wales; Epr Eastern Pyrenees, France; Fag Făgăraş, Romania; Kam

Kamchatka, Russia; LD Lake District, England; Mar Maritime Alps, Italy; MCC Montaña Central, S. Cantabria, Spain; NEU Northeast USA; NGr North Greece; NSc Northern Scandinavia transect; NSE Northern Romania and Southeast Carpathians; NZF New Zealand Fiordland; NZO Ben Ohau Range, New Zealand; NZW New Zealand Westland; Oko Okoa Bay, Baffin I., Canada; OVf Outer Vestfirðir, NW Iceland; Sco West Scotland; Shu Shulaps Range, B.C.; Sil Upper Sil, S. Cantabria, Spain; SGr South Greece; Sno Snowdonia, Wales; Tat Tatra, Poland and Slovakia; WCY West-central Yukon; WPE West Picos de Europa, Spain; WSW West and Southwest Romania; Zar Zardeh Kuh, Zagros, Iran.

Fig. 11 relates mean cirque height range to mean length. Again the nine regions show limited variation, despite covering three different tectonic environments. This vertical dimension is greater in British Columbia than in Romania, and Britain has lower values especially in CSE Wales. The 21 extra regions are more varied, with high height ranges in New Zealand (due probably to exclusion of the smaller cirques) and unusually low height ranges in southern Greece (SGr): these regions plot on the general trend, of height range increasing with length. Two regions plot off this trend and well away from the others: N.E. USA (NEU) because of its high mean length, and western Picos de Europa (WPE) because of its short lengths. West-central Yukon (WCY) has high lengths for its limited height ranges. It may be that where cirque thresholds are not pronounced, there is greater subjectivity in establishing the down-valley limits, and thus length may be subject to greater operator variance than height range and width. Further investigation is needed to explain some of these variations between authors and regions.

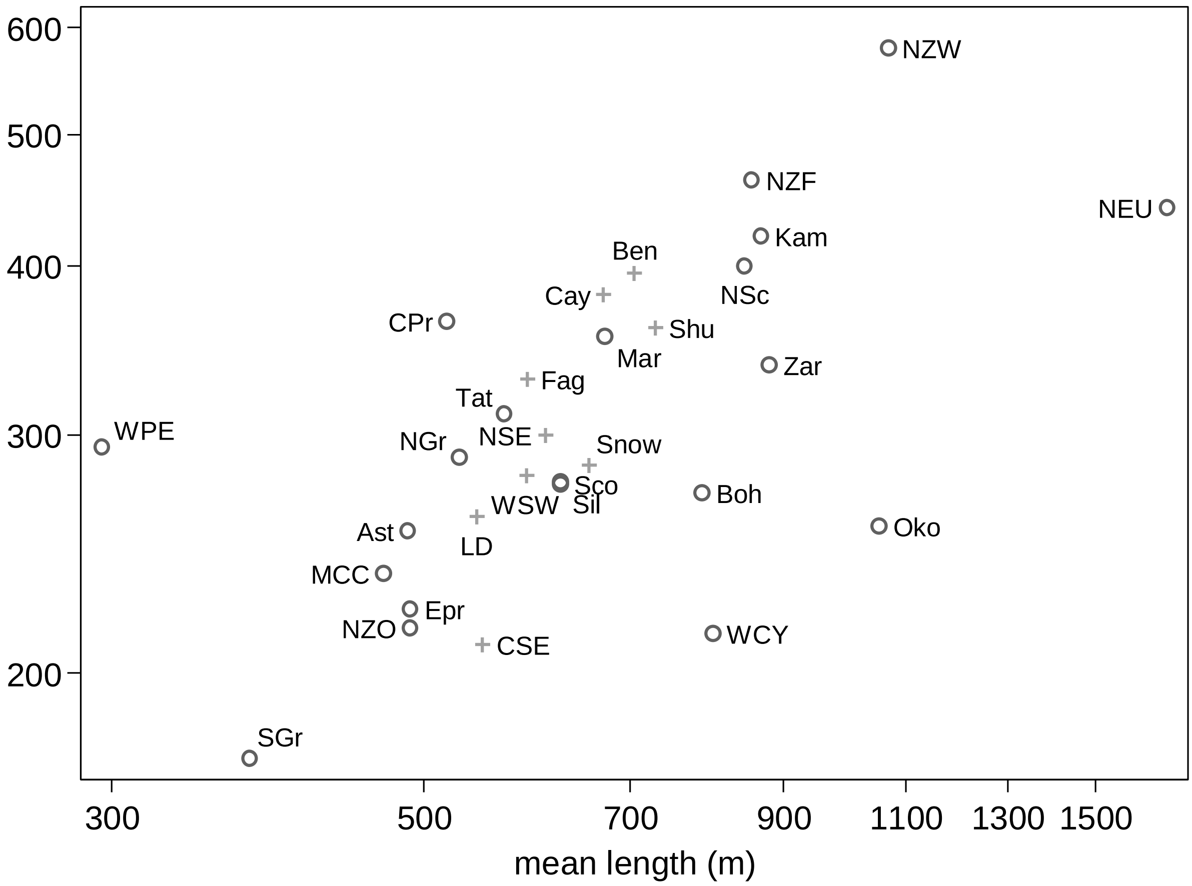

Fig. 11 Cirque mean Height range and Length, both on logarithmic scales labelled in metres, for 9 regions (grey dots) and 21 further regions from the literature (crosses). Abbreviations as in Fig. 10. Note: Sco and Sil are almost identical here – see Table 5.

Also in Table 5, cirques in the western Picos de Europa (RUIZ-FERNANDEZ et al. 2009) have a remarkably low mean length (295 m), the same as their 294 m mean height range. These are steeper than cirques elsewhere, possibly because this is a high-relief limestone massif. GARCIA-RUIZ et al. (2000) also have some very steep cirques, as high as long, in the central Spanish Pyrenees (CPr).

ANDREWS & DUGDALE (1971) provided the first thorough analysis of cirque morphometry. For the Okoa Bay area of Baffin Island, they measured median cirque length as 1053 m, width as 833 m and height range as 257 m. As they included only well-developed cirques, the medians for all cirques must be lower than these. Still, they plot within the trend in fig. 10: their high length in relation to height range (fig. 11) may relate to exclusion of cirques smaller in plan.

The greater contrasts between regions in Table 5 (than in Table 2) might be because a greater variety of regions has been included. However, three of the greatest mean height ranges and widths come from studies based on satellite imagery or (RICHTER 2006) on automatic cirque identification: Richter included only features >0.1 km^2 in area, giving 35 cirques in the Ben Ohau Range where BROOK et al. (2006) found the 90 plotted as NZO. Fieldwork and use of higher-resolution DEMs or maps identifies smaller cirques, giving smaller average sizes. Only complete inventories are likely to be comparable.

6. *Difficulties in comparing older and partial results*

Many early papers on cirques were concerned mainly with distribution: those without statistics for length and width are not included in Table 5, but see the bibliography in BARR & SPAGNOLO (2015). Partial but useful data are available for example in ZIENERT (1967), excluding marginal forms and cirques with poor thresholds. These give median height ranges of 215 and 135 m for cirques on crystalline rocks and on sandstone respectively, in the Schwarzwald, Germany; and 250 and 180 m similarly in the Vosges, France: these would plot near the bottom of fig. 11.

MITCHELL & HUMPHRIES (2015) measured and collated data on cirque relief for 51 regions, giving an overall mean of region means of 346 m, with a standard deviation between regions of 107 m and a range from 135 to 644 m (see their supplementary Table DR1). They measured cirque relief to the 'highest adjacent peak', which produces values sometimes higher than height range within a cirque. 6249 individual cirques have a mean relief of 382 m (standard deviation 150 m) and 8% have a relief above 600 m: but note that this analysis is confined to 'mostly ice-free mountains'. Only their lowest values, for subdivisions of west-central Yukon, and the two highest values of 644 m for Glacier National Park, Montana, and 592 m for Mount Kenya, slightly extend the range in fig. 11. Apart from Glacier N.P., mean relief in the original data of MITCHELL & HUMPHRIES (2015) for 18 regions based on the USGS National Elevation Dataset varies from 258 to 447 m, showing consistency within-researcher and within-data type.

Antarctica is a special case, where very long-continued glaciation may have developed larger cirques. Thus the 56 mapped in the 'Dry Valleys' by ANIYA & WELCH (1981) have a mean length of 2116 m, mean width of 1679 m, and mean height range of 515 m: they dominate compiled graphs of cirque length, as in DELMAS et al. (2014) and BARR & SPAGNOLO (2013), but are excluded here. HAYNES (1995, 1998) mapped 1666 'alpine valley heads' (avoiding the term cirque) in the Antarctic Peninsula, largely from 1: 250,000 maps: 13% are over 3.5 km wide and one reaches 34 km wide. Clearly the extensive ice cover gives difficulties in recognizing individual cirque floors, and more detailed topographic maps and maps of the subglacial surface are awaited.

It might be expected that cirques around the world's highest mountain (Everest, Qomolangma, Sagarmatha) have been eroded vigorously for a considerable time period, and should thus be larger than those in areas of more marginal local glaciation. There are difficulties given the presence of thick glaciers masking cirque floors, but these do not hinder measurement of cirque width and length. Preliminary measurements from the 1: 50,000 'National Geographic' 1986 map of Everest show that the mean width of 35 cirques around Mount

Everest is 2.23 km (median 2.0 km). The largest cirque, with Lhotse Glacier, is 4.6 km wide, followed by the Western Cwm at 3.75 km: both are 3.9 km long. This is not out of line with the largest cirques elsewhere: what are lacking, on this highest terrain, are small cirques. Perhaps small cirques were eliminated as neighbouring larger cirques grew. On the nearby but lower Nuptse-Dingboche ridge 22 cirques average 727 m wide (median 625 m), which is comparable to the nine regions in Table 2. It seems that widths and lengths around 4 km are the limiting dimensions for mid-latitude glacial cirques, developing from previously fluvial topography.

DERBYSHIRE & EVANS (1976) reported data from J. PETERSON for Tasmania, where 325 cirques have a median area of 0.46 km^2, slightly larger than those in the Bendor Range; their median height range is 240 m. MASSAGLIA (1996) measured the areas of 1543 cirques in northwest Italy, with median areas between 0.137 and 0.299 km^2 on six different rock types. She gave ratios between length, width and height range, but not the original linear measurements. TRENHAILE (1976) gave mean areas for valley-head and valley-side cirques in seven Ranges of eastern British Columbia and the Alberta Rockies, varying from 1.79 to 4.20 km^2; these are high, implying lengths and widths well over 1 km.

GRAF (1976) gave the mean widths and amplitudes (but not lengths) for all cirques with glaciers and for a sample of 30 ‘empty’ cirques in each of eight Ranges in the American Rockies. Mean widths varied from 743 to 1459 m for occupied cirques and from 471 to 1103 m for the empty ones; mean amplitudes varied from 276 to 574 m and 200 to 321 m respectively. He gave also the mean length/width ratios, varying from 0.70 to 1.33 for occupied and 0.77 to 1.24 for empty.

SAUCHYN & GARDNER (1983) measured 54 open rock basins in the Kananaskis area of the Alberta Rockies. Among these, 9 larger features were classified as ‘open cirques’: their mean length was 743 m, width 553 m and height range 700 m. Although their plan size is comparable to several regions in fig. 10, their height ranges being greater than widths, and close to lengths, rules them out as glacial cirques. (45° axial gradients leave little room for cirque floors: they prevent rotational flow. Admittedly this applies also to the W. Picos de Europa cirques.)

VILBORG (1977, 1984) made a very thorough survey of all cirques in central and northern Sweden. The 1977 report on Lapland gave limited results for width and height range, but the 1984 publication on central Sweden gave statistics for five grades of cirque (not easily mapped onto the EVANS & COX 1995 grades), with mean lengths for the first three grades (definite cirques) of 1300, 1150 and 725 m respectively; mean widths of 1350, 1075 and 850 m; and mean amplitudes of 335, 280 and 170 m. The numbers of cirques are 26, 62 and 278 respectively, so overall averages would be closer to the third values and compare well with those in fig. 11. Vilborg’s fourth grade is ‘strongly demolished’ and the fifth is ‘slightly concave, with steep walls’, of various origins.

EMBLETON & HAMANN (1988) made comparisons between 169 British cirques and 133 Austrian, measured from 1:25,000 maps. Their study was confined to “cirques with clearly developed basins or back-tilted floors” in four Scottish, two Welsh, one English (Lake District) and three Austrian regions. Absence of the split between these regions hinders any comparison with Evans’ results for Wales or the Lake District, but the number for Britain suggests that both grade 1 (classic) and grade 2 (well-developed) cirques have been included (74 in Wales and 42 in the Lake District, in Evans’ data). (Note: the other grades are 3, definite; 4, poor; and 5, marginal: EVANS & COX 1995.) Embleton and Hamann’s British length-height ratio of 2.98 (18.6°) compares with 17.4° for grade 1 and 20.0° for grade 2 in the Lake District (EVANS & COX 1995). 166° for plan closure equals that for Lake District grade 1 cirques. Their 41.1° for profile closure (complement of their backwall-floor angle), however, is poorer than the 46.6° for Lake District grade 5 (marginal): this may be because they measure only along the cirque long axis, whereas Evans and Cox took maximum and minimum gradients located anywhere in a cirque. This suggests the importance both of having identical definitions of variables, and of comparing either complete inventories, or those with the same threshold grade. Embleton and Hamann’s comparison of well-developed cirques cannot be applied to total British and Austrian cirque populations.

Results from cluster samples cannot be extrapolated beyond the precise region that was studied. Comparison between studies is hindered by publication of mean values with no indication of spread. This prevents checking for censoring of smaller or less-developed forms. Either standard deviation or interquartile range is preferred, and availability of maxima and minima permits checking for unreasonable values (either measurement errors or the inclusion of dubious forms). Of course full data sets should be published or made available.

7. *Future work*

(a) It is hoped that further studies will produce comparable data sets for regions in different world climates and degrees of glacier cover. These should be measured from 10k or 25k maps, with contour intervals of 20 m or better, or from DEMs with grid spacing 20 m or better. Cirque definitions should be checked on Google Earth or air photographs.

(b) For comparability, all cirques should be included, definitely all above c. 200 x 200 m in L x W (0.032 – 0.04 km^2 in area), and of grade 5 or better. It is useful if grade, or degree of confidence in recognition as a cirque, is given, together with indication of any lower cut-off size.

(c) Large data sets can be produced based on remote sensing and DEM-algorithms. Ideally these should be calibrated against detailed regional field mapping of sample areas.

(d) We need further replicated studies, with different researchers defining cirques in the same region, following on the comparisons made by EVANS & COX (1974) and ARDELEAN et al. (2013), but with experimental designs that separate researcher variance from that due to data resolution.

(e) The effect of different definitions of thresholds for cirque recognition (cf. cirque grade) should be investigated and the form of excluded features ('not-quite cirques') should be measured. Measurement of slope gradients on floor and headwall provides further assurances on the quality of features included.

(f) The list of variables used here, dating back to the 1980s when contour maps and aerial photographs were the main data sources, can now be extended. DEMs should be used to generate frequency distributions of altitude, gradient, aspect, plan and profile curvature within each cirque, providing more thorough sampling and greater objectivity. This provides a 'general geomorphometry' for each (EVANS 1987). The quality of both floor and headwall can be measured more thoroughly. A further important quality, not yet measured quantitatively, is the roundness (as opposed to V-shape) of contours on the headwall.

(g) The quality or degree of development of glacial cirques, measured subjectively by 'Grade' and quantitatively by plan closure, high maximum gradient and low minimum gradient, can now be expressed on a broader quantitative basis where detailed DEMs are available. DEM-based measures of development include high standard deviation of gradient, low hypsometric integral (expressing concavity), high skewness of altitude, high percentages of slopes <20° (floor) and >35° (clear headwall), and low vector strength of aspects of slopes steeper than 20°.

(h) A further request is that reports should cover level and spread of dimensions and direct measures, before moving on to ratios and indices.

(i) In these ways we may eventually be in a position to do broad international comparisons and advanced spatial and statistical analyses of large, representative data sets.

8. *Conclusions*

Three definite conclusions emerge from this analysis.

First, when a clear, consistent operational definition is applied, differences in cirque size and shape are small between regions, compared with variations within each region. Nevertheless, differences between regions in length, all vertical dimensions, gradients and closures can be highly significant: from the 9-region data set, only width does not differ significantly between regions.

Second, differences are greatest for vertical dimensions and maximum gradients, due especially to contrasts in tectonic setting. Data for maximum gradient, however, are more sensitive to data quality (map scale and contour interval, or DEM resolution) than those for other variables.

Third, greater variations emerge when results from different authors are compared – as might be expected. Means of cirque populations can be compared where measured by the same author from similar data sources, but those from different authors cannot as yet be taken as real differences between regions. Real differences are expected, but most comparisons are not as yet securely based. Sampling bias, varying definitions and varying sources for altitude data are probably greater hindrances than measurement accuracy. It is likely that in the near future a broad sampling of cirque form in different mountain ranges will be possible, with comparable definitions and measurements permitting contrasts between ranges to be reliably estimated so that attempts can be made to explain differences. We maintain here, however, that this has not yet been achieved, and further measures are needed to reach this desirable goal.

It is hoped that the results in Tables 2 and 4 provide a starting point for a consistent multi-regional data set to which future measurements of cirques can be related. Broad global comparisons of cirque form remain an aspiration.

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