On the Sub-Optimality of Entry Fees in Auctions with Entry*

Ángel Hernando-Veciana†

Universidad Carlos III de Madrid

*I am thankful to Javier Álvarez, Dan Levin, Antonio Morales, Fernando Vega-Redondo, and seminar participants at Universidad de Alicante and at the 2002 European Economic Association meeting for their helpful comments. Any remaining error is my sole responsibility. This work has been financially supported by the Spanish Ministry of Science and Technology through the CICYT project SEJ 2004-02172/ECON, and by the Instituto Valenciano de Investigaciones Econmicas under the name of "(Sub-)Optimal Entry Fees".

†Address: Ángel Hernando-Veciana, Departamento de Economía. Universidad Carlos III de Madrid, c/ Madrid, 126. 28903 Madrid, Spain. Phone: + 34 + 91 624 9602. Fax: + 34 + 91 624 9329. Email: angel.hernando@uc3m.es.

Abstract

We study a variation of Myerson's (1981) model in which we allow for uncertainty about the number of bidders. In our set-up, an appropriate reserve price in a standard auction maximizes the auctioneer's expected revenue. However, entry fees can be optimal only under some special conditions. Basically, there must be some homogeneity in bidders' beliefs about the number of bidders and the auctioneer must know, to some extent, these beliefs.

JEL Classification Numbers: D44.

Keywords: optimal auction, exogenous number of bidders, reserve price, entry fee.

1 Introduction

In real-life, auctioneers often use reserve prices to enhance their expected revenue, however, entry fees are very seldom used, at least in what we can term small or popular auctions, such as traditional auction houses like Christie's or Sotheby's, newborn internet auctions like eBay or Yahoo, or some second hand markets organised through auctions like those in London for second hand cars or second hand homes. Another illustration of this fact is that the thorough study of Cassady (1967) about auctioning dedicates several pages to the effect of a reserve price but no comment on entry fees.

This observation, however, contrasts with the theoretical analysis of auctions. Myerson (1981), for instance, proves that under certain assumptions an appropriately chosen entry fee does as well as a given reserve price in terms of auctioneer's expected revenue. Moreover, Milgrom and Weber (1982), Engelbrecht-Wiggans (1993), or Levin and Smith (1994) show that once we introduce some variations in Myerson's (1981) model,[1] the former indifference is broken in favour of entry fees.

In our paper, we provide an alternative variation of Myerson's (1981) model in which reserve prices are *optimal* in the sense of maximizing the auctioneer's expected revenue but, in general, entry fees are not. The distinct feature of our model is that the bidders and the auctioneer have some exogenous uncertainty about the total number of bidders. Cassady (1967) and McAfee and McMillan (1987) have argued that this is a realistic assumption for many real-life auctions, mostly those

[1] An exception is the model by Waehrer, Harstad, and Rothkopf (1998) which we comment on at the end of the Introduction.

we mention above.

The intuition of our result can be understood from Myerson's (1981) Revenue Equivalence Theorem: he shows that under certain assumptions, in particular a fixed number of bidders, any auction that only allocates the good to the bidder with the highest value if above an *optimal cut-off* is revenue maximizing. Many auction formats allocate the good to the bidder with highest value who enters the auction. In this case optimality usually reduces to choosing a reserve price and an entry fee that induces the *optimal level of entry*, i.e that bidders enter the auction if and only if their value is above the optimal cut-off.

We show that a similar result also holds true when there is some exogenous uncertainty about the number of bidders. Thus, it is easy to see that an appropriate auction set-up, e.g. a second price auction, with a reserve price equal to the optimal cut-off and no entry fee, maximizes the auctioneer's expected revenue. However, auctions with entry fees are not optimal in general.

To understand our result note that in our model, as in Myerson's (1981), there is always an entry fee that induces a given bidder to enter the auction if and only if her value is above the optimal cut-off. However, this entry fee depends on the bidder's beliefs about the actual number of bidders. The more bidders she expects to meet in the auction, the less inclined she will be to pay the entry fee. Thus, if there is some heterogeneity in these beliefs, we cannot find an entry fee that induces all bidders simultaneously to take the optimal entry decisions. Moreover, even if the bidders hold the same beliefs, the auctioneer will not be able to compute the optimal entry fee unless he knows the bidders' beliefs.

We also argue that the use of an entry fee induces bidders to acquire private information about the number of bidders, which typically generates heterogeneity on bidders' beliefs and auctioneer's uncertainty about the bidders' beliefs. Thus, if the cost of information acquisition is sufficiently low, we cannot expect entry fees to be optimal.

From a technical point of view, the most closely related paper is by McAfee and McMillan (1987). They also characterise the optimal auction in a model in which bidders have uncertainty about the number of bidders. McAfee and McMillan, however, do not compare auctions with reserve prices and entry fees in terms of expected revenue. Moreover, our model generalises McAfee and McMillan's results to the case in which bidders have private information about the number of bidders.

Levin and Ozdenoren (2004) have another technically related model. They also study a model in which bidders have exogenous uncertainty about the number of bidders. Their model, however, differs in that they do not assume a common prior. Moreover, they do not consider the use of entry fees or reserve prices.

Some other papers compare entry fees and reserve prices. The most closely related models are in the papers by Engelbrecht-Wiggans (1993), and by Levin and Smith (1994). In both cases there is variation in the number of bidders, although endogenous. The former paper points out that an entry fee can implement full surplus extraction, and the latter one shows that this is not the case for reserve prices, and thus that an appropriate entry fee strictly dominates any reserve price. However, what they call an entry fee should more appropriately be called an inspection fee: bidders only learn their value after paying it. Hence, an entry fee can extract the bidders' surplus because bidders have no private information at the time they pay the inspection fee. A reserve price cannot do so well because it is paid only once the bidders have their private information and thus, the auctioneer cannot extract all the bidders' informational rents.

Milgrom and Weber (1982) show that affiliation among the private information of bidders about their values can also make entry fees be revenue superior to reserve prices. Note that in our model we assume the independent private value model to avoid the effects in Milgrom and Weber's model interfering with our results.

On the other hand, Waehrer, Harstad, and Rothkopf (1998) show that a risk averse auctioneer strictly prefers an appropriate reserve price to any entry fee. Our model, however, differs in that Waehrer, Harstad, and Rothkopf (1998) do not allow for uncertainty about the number of bidders and that we assume that the auctioneer is risk neutral.

The paper is organised as follows. In Section 2 we present the set of assumptions that describe the basic features of our model. We move in the third section to our main results: characterisation of the optimal auction, optimality of reserve prices and sub-optimality of entry fees. In the fourth section, we consider the incentives of bidders to acquire information. Section 5 concludes.

2 The Model

We study a generalisation of a model by McAfee and McMillan (1987). In their model an auctioneer puts up a single unit of a non-divisible good for sale. There is a set of *potential bidders* indexed by the natural numbers $\mathbb{N}$. They also assume that there is an exogenous random process that picks the set of *active bidders*, which we denote by B, from the set of potential bidders. These are the bidders that can participate in the auction, if they find it profitable. The active bidders' preferences are characterized by a von Neumann-Morgenster utility function equal to $v_i - p$ when Bidder i gets the good and pays p, and equal to $-p$ when she does not get the good but pays p. We shall refer to v_i as Bidder i's *value.* Each bidder's value is assumed to be private information and to follow an independent distribution F with support $[0, 1]$ and density f.

Our model differs from McAfee and McMillan's (1987) in that we assume that each active bidder, say Bidder i, receives a private signal S_i, with support $\mathcal{S}_i$, informative of the set of active bidders B. Note that in the model by McAfee and McMillan (1987) bidders may have different (hierarchies of) beliefs about the set of active bidders B because the fact of being active gives Bidder i information about B, in particular the event $\{B \ni i\}$. However, this information is not private since the auctioneer can infer it upon observing the bidder's identity. For simplicity we assume that the support $\mathcal{S}_i$ of each of the signals S_i is countable. Note that this means that the set $\prod_{i \in \mathbb{N}} \mathcal{S}_i$ is also countable.

We denote by $\Pr[.]$ the probability measure that describes the common prior that generates B and the signals S_i's. We also denote by $\Pr[.|.]$ the probability of the event on the left side of the vertical bar conditional on the event on the right side of the vertical bar. Finally, we denote by $\mathrm{E}[.]$ and $\mathrm{E}[.|.]$ the corresponding expected values associated to the common prior. We shall assume that $\mathrm{E}[N|A] < \infty$ for any event A with positive probability (with respect to the probability measure Pr), and where N denotes here and in what follows the cardinality of B.

We also assume that the distribution of B and the S_i's is independent of the distribution of the bidders' values. This assumption is a direct generalisation of the assumption of McAfee and McMillan (1987) that the distribution of B is independent of the distribution of the bidder's values.

Finally, we shall restrict ourselves, as McAfee and McMillan (1987) do in their analysis of optimal auctions, to what Myerson (1981) called the *regular* case. This is the case in which the function $v - \frac{1-F(v)}{f(v)}$ is strictly increasing in $v \in [0, 1]$. This assumption is quite standard in auction theory and it is satisfied by many distribution functions (e.g. the uniform). We shall denote by v^* the unique solution to $v^* - \frac{1-F(v^*)}{f(v^*)} = 0$ and assume for simplicity that v^* exists and belongs to the interval $(0, 1)$.

The model described above has three important assumptions. The first one is that we assume the independent private value model. Private value is not essential for our results (they also hold in an independent common value model), however, independence plays an important role. It avoids the effects shown by Milgrom and Weber (1982) interfering with our analysis, as we discuss in the Introduction.

The second important assumption is that we assume symmetry across bidders in their beliefs about the values of the other bidders. This assumption plays an important simplifying role. Basically, it allows us to avoid the difficulties inherent in the analysis of asymmetric auctions. However, this assumption may seem to be in conflict with the fact that we allow for heterogeneity in bidders' beliefs with respect to the number of active bidders.

The third important assumption is that we assume that the bidders' information about the set of active bidders is orthogonal to their information about the other bidders' values. This assumption greatly simplifies the analysis. Moreover, had we not made this assumption, we would require additional restrictions to ensure that there exists homogeneity in bidders' beliefs about the other bidders values, see the previous paragraph.

3 Optimal Auctions

In this section we show that a standard auction, see below, with an appropriate reserve price is optimal, in the sense of maximising the auctioneer's expected revenue, but in general, standard auctions with a strictly positive entry fee are sub-optimal.

Formally, we define a *standard auction* with a reserve price r and an entry fee e as an auction in which bidders must take a participation decision and a bid decision as follows. If a bidder does not enter the auction, she pays nothing but cannot submit a bid. If a bidder enters the auction, she pays the entry fee e and has the opportunity of submitting a bid no lower than the reserve price r after observing the number of bidders who have also entered the auction. The bidder that wins the auction is chosen according to some predetermined rule among those bidders who submit bids, if any. A bidder who enters the auction and does not get the good only pays the entry fee e. The bidder who submits the winning bid pays a price on top of the entry fee e. This price is fixed by a predetermined rule somewhere between the winner's bid and the reserve price.

The family of auction mechanisms that we just described include the first price auction and the second price auction with reserve price and entry fee, but they exclude some types of all pay auctions.

It is convenient for our analysis to define formally a second price auction with a reserve price. This is a standard auction in which each bidder submits simultaneously and independently either one bid above the reserve price r or no bid at all. The bidder who submits the highest bid, if any, gets the good and pays, on top of the entry fee, the highest bid of the other bidders, or if there is no other bid, the reserve price r. Note that if the entry fee is equal to zero, $e = 0$, it is weakly dominant for an active bidder with value v to submit a bid equal to her value if her value is greater than the reserve price r, and to submit no bid otherwise.

Next, we characterize the set of optimal auctions:

Proposition 1. *An auction is optimal if and only if:*

(i) The good is allocated to the active bidder with highest value if higher than v^*, and otherwise, the auctioneer retains the good.

(ii) Any active bidder with a value 0 gets zero expected utility for any realization of her private signal S_i.

Proof. Suppose, first, that the auctioneer could observe the realization of the private signal S_i of each active bidder. This case is equivalent to an alternative model in which there is no signal and the set of potential bidders is equal to $\prod_{i \in \mathbb{N}} \mathcal{S}_i$ rather than $\mathbb{N}$. This alternative model satisfies the assumptions of McAfee and McMillan (1987). Thus, we can apply their Theorem 4 to show that the optimal auction in our alternative model must satisfy the conditions in our proposition. But a second price auction with a reserve price $r = v^*$ and no entry fee, $e = 0$, satisfies these conditions and it does not require that the auctioneer observes the bidders' private signals. Consequently, these conditions must also characterize the optimal auction when the auctioneer does not observe the bidders' private signals. ■

Proposition 1 implies a version of the Revenue Equivalence Theorem at least for optimal allocations. It basically says that any two auction games that implement the allocation in point (i) and satisfy point (ii) give the same (maximum) expected revenue to the auctioneer.

It is easy to find a standard auction with a reserve $r = v^*$ and no entry fee which satisfies the conditions of Proposition 1 in equilibrium, e.g. a second price auction. We could conjecture by analogy with the analysis of Myerson (1981) that entry fees appropriately chosen may also be optimal. We show below that this is not always the case.

Proposition 2. *Any optimal standard auction must have a zero entry fee unless:*

(a) "Active bidders have homogeneous beliefs" in the sense that for any $i, j \in \mathbb{N}$ and such that $Pr[i \in B]{\cdot}Pr[j \in B] > 0$,

$$E[F(v^*)^{N-1}|S_i, B \ni i] = E[F(v^*)^{N-1}|S_j, B \ni j], \ \textit{a.s.}$$

(b) "Bidders do not have private information with respect to the auctioneer" in

the sense that for each potential bidder $i \in \mathbb{N}$, such that $Pr[i \in B] > 0$,

$$E[F(v^*)^{N-1}|S_i, B \ni i] = E[F(v^*)^{N-1}|B \ni i], \text{ a.s.}$$

Proof. Suppose an optimal standard auction with reserve price r and a strictly positive entry fee e. Since condition (i) in Proposition 1 must be satisfied in equilibrium, any active Bidder $i \in B$, and for any realization of her signal S_i, must enter the auction if and only if her value is greater than v^*. Thus, by continuity,[2] if Bidder i has value v^*, then she must get zero expected utility in the auction for any realization of her private signal.

If Bidder i enters the auction, she pays the entry fee e. Moreover, by condition (i) in Proposition 1, she wins with value v^* if and only if no other active bidder has a value above v^*. The probability of this event conditional on Bidder i's information is equal to $\mathrm{E}[F(v^*)^{N-1}|S_i, i \in B]$. But, if no other active bidder has a value above v^*, Bidder i will face no competition in the auction and thus, in equilibrium, she wins at the minimum price, i.e. the reserve price r.

From the arguments in the last two paragraphs we can derive the following necessary condition for optimality:

$$e = (v^* - r)\mathrm{E}[F(v^*)^{N-1}|S_i, B \ni i], \text{ a.s.} \tag{1}$$

This condition can be satisfied simultaneously by all active bidders for $e > 0$ only if (a) holds. Similarly, the above condition can be satisfied by all types of any given active bidder for $e > 0$ only if (b) holds. ■

[2]It is easy to show that the interim expected utility of a bidder must be continuous with respect to her value.

The reason why there is no optimal entry fee with generality is somewhat subtle. It is true that we may find an entry fee that induces a given bidder to take the entry decisions required for optimality. Hence, we could think of appealing to the version of the Revenue Equivalence Theorem in Proposition 1 to claim that an optimal entry fee must exist. However, this is not the case in general. The reason is that an entry fee is an upfront payment for the option to submit a bid whose profitability depends on the number of other bidders. Thus, a bidder's participation decision depends on her beliefs about the number of other active bidders. Consequently, if these beliefs differ across bidders in some sense, we cannot find an entry fee that induces simultaneously the optimal level of entry of each bidder. Another difficulty is that the auctioneer needs to know what the bidders' beliefs are about the number of bidders, in order to be able to compute the entry fee that induces the optimal level of entry.[3]

4 Acquisition of Information and Optimal Entry Fees

In this section, we show that the sub-optimality of entry fees may arise endogenously. The fact that the auctioneer uses an entry fee provides incentives to bidders to acquire information about the number of active bidders which usually makes the use of entry fees sub-optimal.

To prove this claim we provide a variation of the model in the previous section.

[3] We might think that the auctioneer can avoid this problem by fixing a "contingent" entry fee that varies with the number of bidders that submit bids. To some extent, this is actually what a reserve price does in equilibrium: an entry fee that it is paid only if one bidder enters the auction.

Basically, we assume that active bidders do not observe any private signal unless they incur a cost $c > 0$. Moreover, and for the sake of simplicity, we assume that this information acquisition decision is not observable and it is taken after the announcement of the auction mechanism by the auctioneer, but before the bidder can observe her value.

We also assume that if no active bidder acquires her private signal the conditions in Proposition 2 are met. In particular this means that the following symmetry condition must hold for any $i, j \in \mathbb{N}$ such that $\Pr[i \in B]{\cdot}\Pr[j \in B] > 0$:

$$\mathrm{E}[F(v^*)^{N-1}|B \ni i] = \mathrm{E}[F(v^*)^{N-1}|B \ni j], \text{ a.s.}$$

With a slight abuse of notation, we denote each of the active bidders' signal by S_i. For simplicity, we assume that the support of each S_i has only two points $\{\underline{s}, \overline{s}\}$ and that S_i is informative of the number of active bidders in the sense that:[4]

$$\mathrm{E}[F(v^*)^{N-1}|B \ni i, S_i = \underline{s}] < \mathrm{E}[F(v^*)^{N-1}|B \ni i, S_i = \overline{s}]. \tag{2}$$

To start the analysis we assume that the auctioneer announces a standard auction with a reserve price r^* and an entry fee e^* which are optimal under the assumption that no active bidder acquires her private signal. From the arguments in the proof of Proposition 2 we can deduce that (r^*, e^*) must satisfy:

$$e^* = (v^* - r^*)\mathrm{E}[F(v^*)^{N-1}|B \ni i]. \tag{3}$$

[4]For instance, it is sufficient for Equation 2 that the distribution of N conditional on $S_i = \underline{s}$ first order stochastically dominates, in strict sense, the distribution of N conditional on $\mathrm{S}_i = \overline{s}$.

If no active bidder acquires private information, the auction is optimal. Thus, we would want to check whether a given active bidder finds it profitable to acquire her private information when all the other bidders do not acquire theirs. If this were the case, there would be no equilibrium in which active bidders do not acquire private information, and thus, we could conclude that entry fees cannot be optimal.

Lemma 1. *An active bidder i gets strictly greater utility when she acquires private signal S_i than when she does not if: the auctioneer uses a standard auction with a reserve price r^* and an entry fee $e^* > 0$ that satisfy Equation (3), all the other bidders do not acquire their private signals, and c is sufficiently small.*

Proof. Suppose the conditions of the lemma hold. Equation (2) and the law of iterated expectations imply that:

$$\mathrm{E}[F(v^*)^{N-1}|B \ni i, S_i = \underline{s}] < \mathrm{E}[F(v^*)^{N-1}|B \ni i] < \mathrm{E}[F(v^*)^{N-1}|B \ni i, S_i = \overline{s}],$$

which together with Equation (3) implies:

$$(v^* - r^*)\mathrm{E}[F(v^*)^{N-1}|i \in B, S_i = \underline{s}] - e^* < 0 < (v^* - r^*)\mathrm{E}[F(v^*)^{N-1}|i \in B, S_i = \overline{s}] - e^*.$$

The latter equation means that if Bidder i acquires signal S_i and enters the auction with a value v^* (or close to v^* by continuity), she gets strictly negative expected utility if $S_i = \underline{s}$ and strictly positive expected utility if $S_i = \overline{s}$. Thus, if c is sufficiently low, Bidder i can strictly improve by secretly acquiring the signal S_i and revising her entry strategy when her value equals v^* (or it is close to v^*). ∎

A consequence of this proposition is that in equilibrium active bidders must acquire their private signals with positive probability. In this case, the conditions required in Proposition 2 for optimality of entry fees would not hold and thus:

Corollary 1. *If c is low enough, a standard auction can be optimal only if it has no entry fee, $e = 0$.*

5 Conclusions

In this paper we have shown that when there is uncertainty about the number of bidders the auctioneer can achieve his maximum expected utility with an appropriate auction (e.g. a second price auction) with a reserve price. We have also shown that entry fees are, however, sub-optimal mainly due to two reasons: heterogeneity in bidder's beliefs about the number of active bidders, and auctioneer's uncertainty about the bidders' beliefs and hence, the optimal entry fee, if any.

References

Cassady, R. (1967): *Auctions and Auctioneering.* University of California Press, Berkeley and Los Angeles, California.

Engelbrecht-Wiggans, R. (1993): "Optimal Auctions Revisited," *Games and Economic Bahaviour*, 5, 227–239.

Levin, D., and E. Ozdenoren (2004): "Auctions with Uncertain Numbers of Bidders," *Journal of Economic Theory*, 118(2), 229–251.

Levin, D., and J. L. Smith (1994): "Equilibrium in Auctions with Entry," *The American Economic Review*, 84(3), 585–599.

McAfee, R. P., and J. McMillan (1987): “Auctions with a Stochastic Number of Bidders,” *Journal of Economic Theory*, 43, 1–19.

Mertens, J. F., and S. Zamir (1985): “Formulation of Bayesian Analysis for Games with Incomplete Information,” *International Journal of Game Theory*, 10, 619–632.

Milgrom, P., and R. Weber (1982): “A Theory of Auctions and Competitive Bidding,” *Econometrica*, 50, 1089–1122.

Myerson, R. B. (1981): “Optimal Auction Design,” *Mathematics of Operation Research*, 6(1), 58–73.

Waehrer, K., R. M. Harstad, and M. H. Rothkopf (1998): “Auction Form Preferences of Risk-Averse Bid Takers,” *Rand Journal of Economics*, 29(1), 179–192.