

Transient skin effect in power electronic applications

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Abstract: This study describes transient electromagnetic phenomena in electrical conductors that are connected to modern power electronic circuits and therefore subjected to current controlled pulses with fast rise times and high peak current ratings. Electromagnetic field diffusion is discussed and a brief review of alternating current skin effect phenomena is presented before it is generalised to transient regimes of operation. It is shown that the magnetic field diffuses into the conductor from the outside on the initiation of a current pulse and therefore the axial current density distribution inside the conductor changes with time under a transient current due to the well-known electromagnetic relations – the current begins at the conductor surface and diffuses inwards. The implications of such behaviour are briefly discussed in the context of modern power electronics and its applications.

1 Introduction

Power electronic applications have expanded considerably over the past decade. The desire for greater electrification and control of transportation [1, 2] and industrial processes, coupled with the development of new renewable energy conversion technologies and high voltage direct current power transmission [3, 4] have contributed significantly to the expansion of research effort in this field. New semiconductor device, packaging and control technologies have also contributed [5].

Much focus in the literature is on the power electronic circuit topology or control, as would be expected, with the effort in the electromagnetic domain related to these circuits focused on electromagnetic interference and its mitigation [6] or the AC skin effect and its mitigation [7]. Each application, circuit and device has its own design issues, which need to be solved – such work is of great importance. This paper complements the existing literature in the electromagnetic domain relating to power electronics and its applications where pulsed DC currents and voltages are required, for example, in some special scientific experiments, modern DC power networks and their transient behaviour [8], such that may be experienced in the new field of hybrid electric aircraft power systems [9]. Transient electromagnetic behaviour of conductors attached to power electronic devices is explored.

This transient electromagnetic behaviour can be found explicitly in applications such as electromagnetic launchers, pulsed power supplies for fusion power experiments or in power system fault scenarios.

2 Conventional AC skin effect

Traditionally, skin effect is described in the frequency domain [10]. AC steady-state theory considers the time harmonic form of Maxwell's equations, where the time harmonic equations contain $j\omega$ in place of the time derivatives, greatly simplifying the analysis. The resulting equations for a cylindrical are in the form of Bessel's differential equation and are solved using standard methods to give the traditional steady-state AC skin depth

$$\delta_{AC} = \sqrt{\frac{2\rho}{\omega\mu}} \quad (1)$$

This equation describes to the extent to which the current density distribution penetrates the surface of the conductor at high frequency in the continuous AC steady-state wire (see Fig. 1) – this theory is well known. This effect is usually explained by induced

eddy currents and is mitigated by various methods including stranding and transposition within special wire types designed for AC operation [11, 12]. The steady-state AC skin effect also limits the maximum useful sizing of a conductor for a particular conducting material and frequency. At power frequencies (50/60 Hz) it is known that using a cylindrical bus bar with a diameter >10 mm is technically a waste of copper in terms of current carrying capacity. In steady-state AC circuits that contain more than a single electrical frequency (but remain periodic), analytical equations for the AC skin effect can be found [13]. In DC circuits, it is usually assumed that the skin effect can be neglected as it is usually considered a purely AC phenomenon. The aim of this paper is to investigate the case of skin effect in DC power electronic circuits where current transients exist.

3 Transient regime

The transient effect highlighted in this paper is in contrast to conventional thoughts that in DC circuits the current density distribution is uniform through the conductor. This paper is concerned with the time domain where discontinuous functions (Heaviside step) cannot be represented by simple harmonic (or Fourier) components, thus in the transient DC case the $j\omega$ simplification is not applicable. Here, we consider a rectangular bus bar as shown in Fig. 2, this is representative of a low impedance bus bar in an electrical network that may be controlled by power electronics or may under certain circumstances be subjected to transient fault currents.

This copper bus bar is then considered to be subjected to a current pulse described by the Heaviside function

$$I(t) = \begin{cases} 0 & , t < 0 \\ I_{max} & , t \geq 0 \end{cases} \quad (2)$$

The step function representing a fast current controlled pulse of a maximum value $I_{max} = 10$ A at $t \geq 0$. This pulse is depicted in Fig. 3. While this pulse is unrealistic in terms of its instantaneous rise from zero to maximum, it represents a worst-case transient scenario for illustration of the electromagnetic phenomena in the electrical conductor. In reality, the rate of rise of the current is limited and its rise can follow linear or exponential relationships, while these are not considered further in this paper, they may be of interest in some applications.

4 Electromagnetic diffusion

Electromagnetic phenomena are governed by the Maxwell equations, written in differential form as

$$\begin{aligned}\nabla \cdot \mathbf{D} &= \rho \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= \frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\end{aligned}\quad (3)$$

Vectors in this paper are written short hand; $\mathbf{F} = \mathbf{F}(\mathbf{r}, t)$, a function of position \mathbf{r} and time t . The transient DC case can be described in terms of a parabolic partial differential equation, of a similar form to the heat transfer diffusion equation [14]. This can be derived from the set of equations in (3) by taking the curl equations, neglecting displacement current and forming an equation in terms of the magnetic flux density $\mathbf{B}(\mathbf{r}, t)$ only. The resulting equation is [15]

$$\nabla^2 \mathbf{B} = \frac{1}{\mathfrak{D}} \frac{\partial \mathbf{B}}{\partial t} \quad (4)$$

This is an electromagnetic diffusion equation and it governs the electromagnetic behaviour inside the electrical conductor. Thus, the magnetic field in a transient DC scenario is shown to be described by (4) where \mathfrak{D} is a magnetic diffusion coefficient of the conductor defined by;

$$\mathfrak{D} = \frac{1}{\sigma \mu} \quad (5)$$

Here σ is the electrical conductivity and μ is the magnetic permeability, which are determined by the conductor material used. Equation (4) describes how the magnetic field diffuses into the conductor on the initiation of the current pulse described by (2). Notice there that there is no mention of $j\omega$ anywhere and that these equations are purely in the time domain, thus, the transient regime of operation is appropriately represented. Equation (4) is readily solved by the finite element method (FEA) as the magnetic diffusion equation is implicitly encoded into Maxwell's equations. Using low-frequency 2D FEA, the transient skin effect in a 20 mm \times 10 mm unit length rectangular conductor subjected to a 10 A current pulse is explored. The magnetic field diffusion and current density distribution are now reported.

5 Magnetic field penetration

The penetration of the magnetic field is described by the electromagnetic diffusion equation, subject to the material properties of the conductor in equation and the boundary/initial conditions of the problem. The equation $\nabla^2 \mathbf{B} = (1/\mathfrak{D})(\partial \mathbf{B}/\partial t)$ is solved inside the conductor subject to zero initial electromagnetic field or current density in the conductor at $t=0$ and at which time the total current pulse as described in Fig. 3 is injected into the conductor. In rectangular coordinates (x, y) the diffusion equation is

$$\frac{\partial^2 \mathbf{B}}{\partial x^2} + \frac{\partial^2 \mathbf{B}}{\partial y^2} = \sigma \mu \frac{\partial \mathbf{B}}{\partial t} \quad (6)$$

Use is made of the following relation $\mathbf{B} = \nabla \times \mathbf{A}$. The magnetic flux lines, contours of the magnetic vector potential \mathbf{A} are used to visualise the penetration of the magnetic field over a time from $t=0=t_0$ to $t=\text{steady state}=t_3$. The solution is obtained using a time stepping transient technique. The solution period is 2 ms with time step of 1 μs to ensure sufficient resolution of the diffusion behaviour. Fig. 4 shows the 2D FEA results at times of $t_0=1 \mu\text{s}$, $t_1=0.2 \text{ ms}$, $t_2=0.5 \text{ ms}$ and $t_3=2 \text{ ms}$.

At the initial time instant the magnetic field is wholly outside the conductor – this satisfies the initial condition that there is no

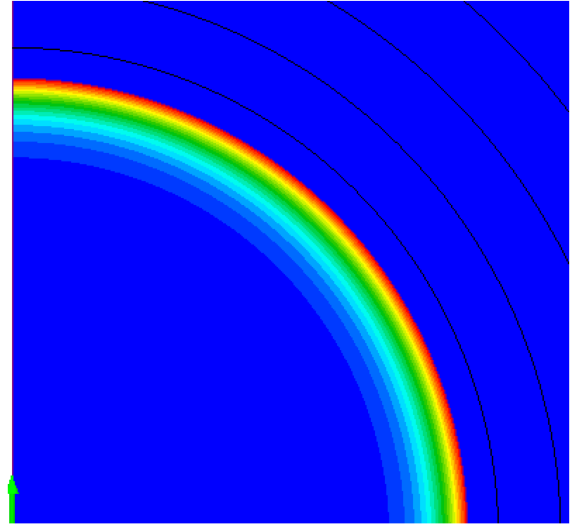


Fig. 1 High frequency AC skin effect in a cylindrical wire – the peak current density is on the surface (red)

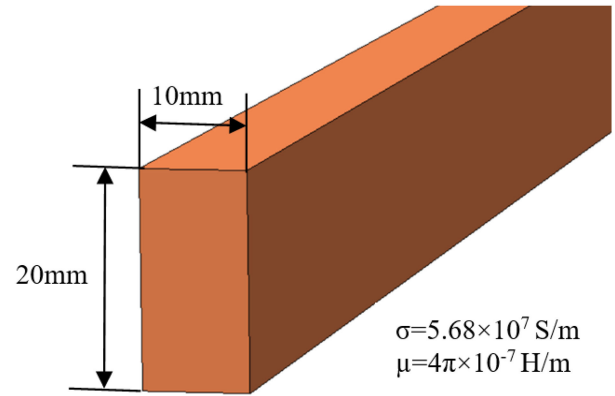


Fig. 2 Example copper bus bar of unit length

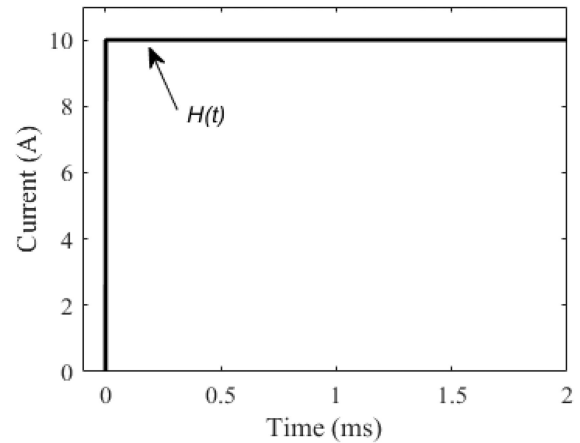


Fig. 3 Step total current applied to bus bar

magnetic field in the conductor on the introduction of the current pulse and it cannot, in zero time, be present at the centre of the conductor, as dictated by the diffusion equation. As time progresses, the magnetic field lines progressively penetrates the conductor and by $t=2 \text{ ms}$ the magnetic field has fully penetrated the conductor and steady-state DC conditions apply. The analysis starts with the magnetic flux density field (magnetic vector potential) as the electromagnetic relations then permit the computation of the other fields of interest. From the Maxwell equations, the following equation can be derived from Faradays law

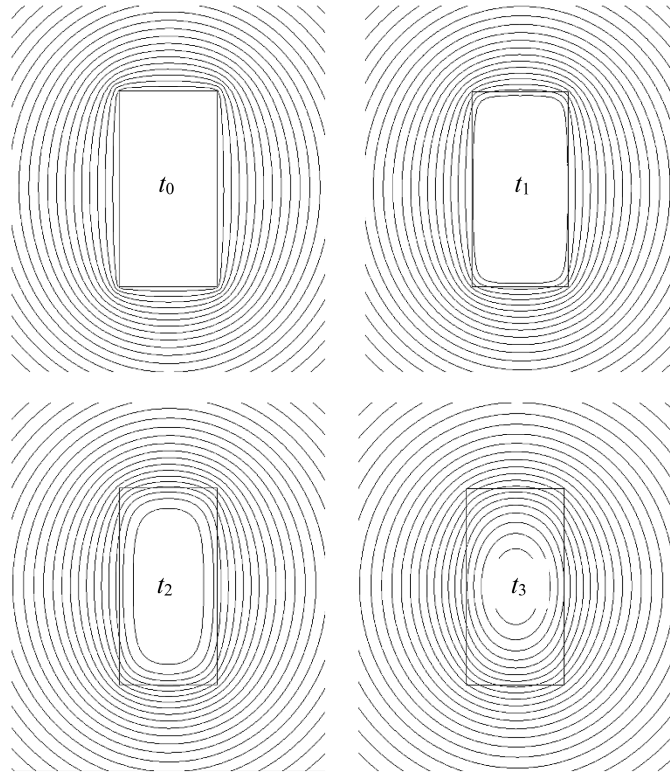


Fig. 4 Magnetic field penetration into the example bus bar (2D cross section) where the time: $t_3 > t_2 > t_1 > t_0$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \quad (7)$$

As such, the axial electric field inside the conductor can be readily computed and perhaps more importantly, the electric current density can then be computed

$$\mathbf{J} = \sigma \mathbf{E} = \sigma \frac{\partial \mathbf{A}}{\partial t} \quad (8)$$

Based on the diffusion of the magnetic field into the conductor, it is evident that the current density also follows this pattern. The current density diffusion can be obtained directly from the Maxwell equations.

6 Current density distribution

From the Maxwell curl equations, it is readily shown that the current density inside the conducting region is described by

$$\frac{\partial^2 \mathbf{J}}{\partial x^2} + \frac{\partial^2 \mathbf{J}}{\partial y^2} = \sigma \mu \frac{\partial \mathbf{J}}{\partial t} \quad (9)$$

This is of the same form of the magnetic field diffusion (and the equations for \mathbf{E} and \mathbf{A} also). From (9), it can be shown that the axial current density may be described in terms of an infinite sum of sine and cosine terms, as with heat conduction problems [15], which are governed by equations of the same form. There are additional conditions to consider in the application of the diffusion equation to the time evolution of the electric current density distribution. The current density distribution in the conductor must obey the following equation:

$$I_{\text{total}}(t) = \iint \mathbf{J} \cdot d\mathbf{S} \quad (10)$$

where the surface integral must be taken over the surface area of the conductor in which the total current I_{total} is flowing – this is the function represented by (2). The solution for the current density must also be finite over the surface area within the conducting boundary. Again, the solution is obtained using a time stepping

transient technique. The solution period remains 2 ms with time step of $1 \mu\text{s}$ to ensure sufficient resolution of the diffusion behaviour. Fig. 5 shows the 2D FEA results at times of $t_0 = 1 \mu\text{s}$, $t_1 = 0.3 \text{ ms}$, $t_2 = 0.6 \text{ ms}$ and $t_3 = 2 \text{ ms}$. These time instants illustrate the transient current distribution.

From the figure, it is seen that as the pulse begins to rise from zero to 10 A the current density distribution is located only at the surface (transient skin effect) which then diffuses inwards from the edge to reach a uniform current density when DC conduction mode has been achieved (steady state) at t_3 . Therefore, it is clear that under a DC current transition, the current density in the conductor is not instantaneously uniform and a transient skin of electric current appears in the first instance before diffusing inwards until uniform density is reached. This should perhaps not be surprising as the rise of current in a wire describes by the DC transient case is analogous to that of the first half cycle of sinusoidal (frequency domain) excitation. The difference between this DC case and the AC case is that in the AC case the current density does not have time to reach uniform density before the exciting signal is reversed and repeated, periodically, hence the manifestation of the AC skin effect – the current density does not have time to diffuse to the centre. To show the change in distribution over the time period, Fig. 6 shows the fictitious line in the centre of the conductor used to extract the current density profile for each time point. Fig. 7 shows the current density profiles (along the y -axis), clearly showing the skin effect at current initiation and the following diffusion effect towards uniform electric current density.

Despite the simplicity of this explanation, the fact that transient skin effects exist seems to have eluded books and research papers – heading straight for the frequency domain approach, which can be forgiven as most classical applications

involve feeding sinusoidal periodic alternating currents (electrical machines, power lines, transformers), however, this transient skin effect should be of interest in modern power electronic applications and should be investigated further. Of course, as with both vector fields outlined here (\mathbf{B} , \mathbf{J}), if the conductor was sufficiently small, the diffusion could be neglected, as with the high frequency AC skin effect.

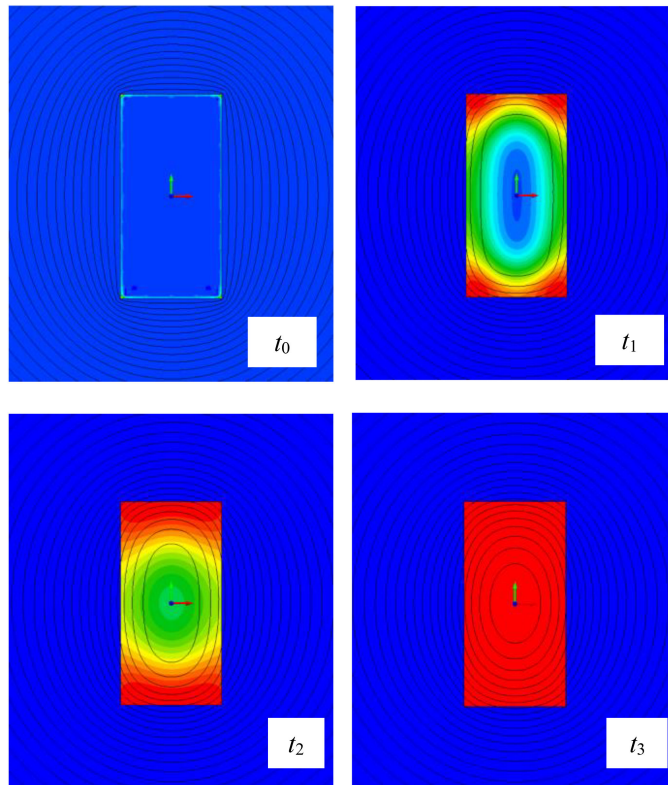


Fig. 5 Axial current density distribution in the example bus bar (2D cross section) where the time: $t_3 > t_2 > t_1 > t_0$. Red = high current density; Blue = zero current density

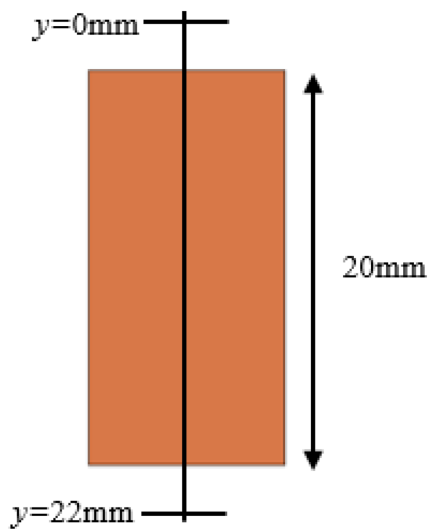


Fig. 6 Bus bar field line plot location along the y-axis

7 Some implications

For current pulses of finite rise time, it is intuitive to suggest that the current would rise linearly in the conductor with a uniform current density also rising linearly. The time-domain diffusion equation approach presented here shows this not to be the case. The behaviour of conductors operating in a pulse-width-modulation-type application, as would be expected in power electronic systems, demands special consideration. The uniform current density approximation will only valid for slow transients, rather than the high dI/dt transients in modern applications. As with the AC steady-state skin effect analysis, it is known that there is an associated change in conductor impedance with frequency. Here, it must follow that there is to be a change in conductor impedance with time, i.e. the impedance of the conductor is time varying until it reaches the steady-state value.

The transient skin effect may have significant implications, the magnitude of which depends on the transient are subject to further

investigation. We can say, based on analogy with AC skin effect theory:

- Increased circuit resistance (within a transient period) and increased joule heating is likely
- The conductor inductance is modified (within a transient period)
- A transient voltage associated with this transient impedance is also likely leading to discrepancies in the assumed dynamic response of a given circuit (unexpected and increased damping)

Thus, it is immediately clear that certain situations will experience the transient skin effect;

- Some switched mode power electronic circuits
- Both DC and AC power system transients
- Pulsed DC applications

8 Conclusions

This paper has highlighted the need to consider time-domain transient electromagnetic behaviour when fast current pulses (leading to steady-state DC) is employed in power electronic circuits. It was shown that the governing equation, the electromagnetic diffusion equation is implicitly encoded in Maxwell's equations and the form is the same as the heat conduction equation. All the electromagnetic fields follow this diffusion equation, where for particular interest in power electronic applications, it is shown that the electric current density distribution due to a fast DC current pulse leads to a transient skin effect. This is analogous to that of the AC skin effect (likely affecting the conductor impedance and circuit response), but has time to reach uniform current density due to the nature of the pulse signal. Example results on a rectangular bus bar from 2D FEA are presented to illustrate the phenomena. The ensuing implications are briefly discussed and further investigations into the applicability and severity of these are the subject of further work.

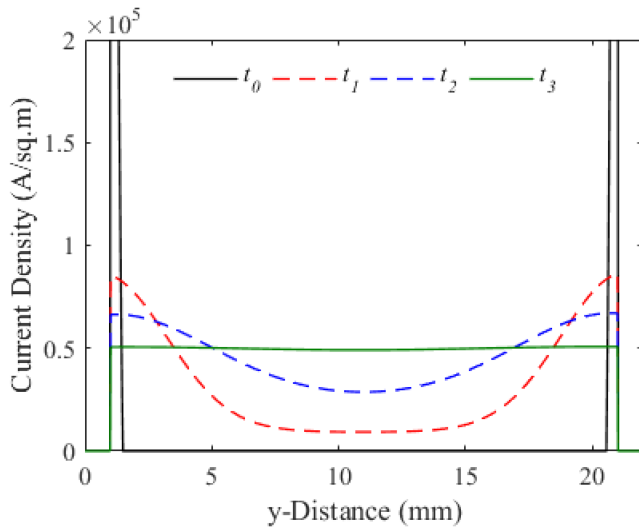


Fig. 7 Current density distribution curves

9 Acknowledgments

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