Using the size and position of drumlins to understand how they grow, interact and evolve

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Abstract

Drumlins are subglacial bedforms streamlined in the direction of ice flow. Common in deglaciated landscapes, they have been widely studied providing rich information on their internal geology, size, shape, and spacing. In contrast with bedform investigations elsewhere in geomorphology (aeolian and fluvial dunes and ripples for example) most drumlin studies derive observations from relict, and thus static features. This has made it difficult to gain information and insights about their evolution over time, which likely hampers our understanding of the process(es) of drumlin formation. Here we take a morphological approach, studying drumlin size and spacing metrics. Unlike previous studies which have focussed on databases derived from entire ice sheet beds, we adopt a space-for-time substitution approach using individual drumlin flow-sets distributed in space as proxies for different development times/periods. Framed and assisted by insights from aeolian and fluvial geomorphology, we use our metric data to explore possible scenarios of drumlin growth, evolution and interaction. We study the metrics of the size and spacing of 36,222 drumlins, distributed amongst 71 flow-sets, left behind by the former British-Irish Ice Sheet, and ask whether behaviour common to other bedform phenomena can be derived through statistical analysis. Through characterising and analysing the shape of the probability distribution functions of size and spacing metrics for each flow-set we argue that drumlins grow, and potentially migrate, as they evolve leading to pattern coarsening. Furthermore, our findings add support to the notion that no upper limit to drumlin size exists, and to the idea that perpetual coarsening could occur if given sufficient time. We propose that the framework of process and patterning commonly applied to non-glacial bedforms is potentially powerful for understanding drumlin formation and for deciphering glacial landscapes.

1. Introduction

Natural processes often organise phenomena into regular and repetitive patterns (e.g. Ball, 1999). This regularity deceptively gives the impression of a simplistic formation process, but pattern-forming processes are often non-linear, with patterns emerging through self-organisation due to complex, and sometimes stochastic, interactions between elements (Pearson, 1993; Werner, 1999; Murray, 2003; Hillier et al., 2016). Aeolian, fluvial and submarine bedforms are often held up as exemplars of natural patterns (Anderson, 1990; Kocurek et al., 2010; Seminara, 2010), occurring as fields of morphologically similar and regularly spaced features (Figures 1A, B and C).

Drumlins are subglacial bedforms elongated in the direction of ice flow (e.g. Clark et al., 2009). The term drumlin is used to define the most common variant in a morphological continuum of subglacial bedforms (Ely et al., 2016), and they are typically 250 to 1000 m long, 120 to 300 m wide and 0.5 to 40 m in relief (Clark et al., 2009; Spagnolo et al., 2012). The formation and development of drumlins remains an unanswered question of great relevance to both geomorphology and glaciology, as the processes which occur at the ice-bed interface govern the mechanics of fast ice flow (e.g. Clarke, 1987; Kyrke-Smith et al., 2015). A plethora of hypotheses have been proposed to explain their initiation (e.g. Smalley and Unwin, 1968; Boulton, 1987; Shaw et al., 1989; Hindmarsh, 1999). However, in this paper we focus on the development of drumlins after they have initiated. Contrary to many prior assertions and analyses that regarded drumlins as randomly positioned individuals or clusters, they have recently been demonstrated to exhibit a regular and repetitive spatial organisation that is characteristic of a patterned phenomenon (Clark et al., 2017; Figure 1D). This is potentially important, because although the concept of patterning is long established in geomorphology (e.g. Anderson, 1990; Werner and Hallet, 1993; Nield and Bass, 2007), drumlins have mostly eluded consideration within this context. Thus, unlike their aeolian, fluvial and marine counterparts, reviews of patterning processes do not generally consider subglacial bedforms within the wider patterning context (e.g. Murray et al., 2014), perhaps due to the immaturity of patterning as a line of enquiry within the subglacial bedform literature. This is likely a consequence of their static and often time-integrated appearance on palaeo-ice sheet beds (e.g. the superimposition of several ice flow patterns), and the alteration of drumlinised landscapes by other geomorphological processes after their exposure, making the identification of any patterning processes which occurred during their formation challenging. Though the concept of subglacial bedforms forming as a field has been considered before (e.g. Smalley and Unwin, 1968; Dunlop et al., 2008; Barchyn et al., 2016), Clark et al. (2017) are the first to demonstrate that patterning is a ubiquitous property of drumlins. Therefore, Clark et al. (2017) opens a new avenue of enquiry for glacial geomorphologists to study drumlins, more akin to how fluvial, aeolian and marine bedforms are considered, whilst simultaneously providing the wider geomorphological community with an interesting, although challenging opportunity to study patterning processes. Here we build upon the work of Clark et al. (2017) by using the size and shape metrics of drumlins formed in different flow events to ask whether behaviour common for bedforms formed by non-glacial geomorphic agents occurs during drumlin formation.

Unlike for fluvial, aeolian or marine bedforms, direct observation of subglacial bedforms beneath (or emerging from) a modern ice mass is logistically challenging and limited to a few examples (Smith et al., 2007; King et al., 2009; Johnson et al., 2010). Furthermore, repeat imaging of active drumlin fields from which bedform formation and pattern evolution could be deciphered has yet to be achieved. Although no longer evolving, the exposed beds of palaeo-ice masses provide numerous examples of drumlin patterns that are likely to preserve information about the processes that created them. This rationale, that the arrangement of a field of bedforms contains information regarding their development, is often adopted to decipher the arrangement of aeolian dunes within a pattern (e.g. Ewing et al., 2006; Ewing and Kocurek, 2010). Recent work on subglacial bedforms has begun to consider how pattern interactions may influence their size-frequency distributions. Hillier et al. (2013) hypothesised that randomness during ice-sediment-water interaction at multiple locations combined with simple rules could explain characteristics of their size-frequency distributions. A statistical model of this was then developed in Fowler et al. (2013) and evaluated in combinations with a variety of models in Hillier et al., (2016). Although exposed drumlin fields are relict, making any identification of pattern-forming interactions challenging, the premise of this paper is that, hidden in the size metrics of relict drumlins, there are measurable properties that provide insight into the time-varying state and evolution of drumlin patterns (Hillier et al., 2016). Here we study the size and spatial arrangement of 36,222 drumlins distributed across 71 different flow-sets of the last British-Irish Ice Sheet (Clark et al., 2009; Hughes et al., 2010; 2014). Unlike previous studies that focussed on the entire database (Clark et al., 2009; Hillier et al., 2013; Fowler et al., 2013), we group drumlins by flow-set prior to analysis. The logic here being that flow-sets or fields represent sets of drumlins at various, as yet unknown, stages of development. This allows us to further examine whether behaviour commonly observed in other natural patterns can be invoked to explain the arrangement and morphology of drumlins.

2. Background and Rationale

Patterns often evolve through interactions between their constituent elements (e.g. Muthukumar et al., 1997; Wootton, 2001). The nature of these interactions leads patterns to evolve in different ways and exhibit different overall behavioural states. Several types of patterning behaviour have been observed in geomorphic systems (Kocurek et al., 2010;

Murray et al., 2014), providing a useful framework for this investigation into drumlins, which we now briefly summarise.

If a pattern remains stable at the wavelength at which it initiates, this is known as simple stabilisation (e.g. Cherlet et al., 2007; Murray et al., 2014). However, if individual elements (e.g. dunes) typically migrate and/or grow laterally, this can lead to merging and/or competition between adjacent elements and thereby reducing the number of elements in the pattern. This is known as 'coarsening' and may stop when a stable wavelength is reached (e.g. Werner and Kocurek, 1999; Coleman et al., 2005; Murray et al., 2014). Alternatively, coarsening could continue perpetually until the pattern is composed of very few or, indeed, a single element (e.g. Andreotti et al., 2009; Murray et al., 2014). For drumlins, we use a quantification of their preserved dimensions and spacing (i.e. metrics) to address the following four questions pertinent to their development within a pattern:

i) Do drumlins stabilise at an initial scale? In the scenario of simple stabilisation, drumlins are seeded (or initiated) throughout the landscape and remain in their original position without interactions occurring.

ii) Do drumlin patterns evolve through the growth of their elements (drumlins)? Phases of drumlin growth might cause neighbouring drumlins to merge and become amalgamated, i.e. coarsening.

iii) Do drumlins migrate? Migration of drumlins could lead to collisions and coarsening.

iv) If there is evidence for (ii) and/or (iii), do drumlin patterns evolve toward a stable coarsened state or do they perpetually coarsen? Growth (ii) and/or migration (iii) could lead to either temporary coarsening, until a new stable state of the drumlin pattern is achieved, or perpetual coarsening, whereby subglacial conditions favour the continuous evolution of the drumlin pattern.

3. Methods

3.1. Data acquisition

Here we study the size and spacing metrics of relict drumlins, on the premise of Hillier et al. (2016) that analysis of these measurable properties should provide insight into drumlin evolution. As noted above, we are presently unable to fully observe drumlin formation and evolution under ice sheets at appropriate time-scales (although see Johnson et al., 2010; Benediktsson et al., 2016 for small sample sizes), and so we substitute space for time, which is a widely-used concept elsewhere in geomorphology for deciphering landscape evolution (e.g. Paine, 1985; Micallef et al., 2014). We assume that drumlin flow-sets (i.e. groups of drumlins interpreted to have been formed during the same ice-flow phase), and the drumlins contained within them, likely represent different (as yet unknown) stages of drumlin formation, e.g. some are likely to have more mature forms, perhaps due to higher ice velocities or a longer duration of flow, whereas others may contain drumlins that represent a more immature stage of development. It is likely that other factors, such as sediment rheological properties, sediment thickness and subglacial hydrological changes could also influence the rate of drumlin pattern development. Indeed, numerical models of both subglacial bedforms (Barchyn et al., 2016) and aeolian dunes (Eastwood et al., 2011) point to sediment availability as being key to pattern development. These factors will be discussed later. For now, our premise is that different flow-sets represent a diversity of drumlin pattern maturity, but that many factors may influence pattern development. From this premise, we build conceptual models of how patterning interactions may have influenced size and spacing metrics of drumlins within discrete flow-sets, and compare these to measured size and spacing frequency distributions.

In order to decipher how patterning occurs in drumlin fields, we study the size and spacing metrics of drumlins formed beneath the former British sector of the British-Irish Ice Sheet (Hughes et al., 2010). For each drumlin, the length, width and relief have been measured previously (see Clark et al., 2009; Spagnolo et al., 2012). Here we calculated the lateral (across-flow) and longitudinal (along-flow) spacing of each drumlin using the method described in Stokes et al. (2013b). These measures are the shortest Euclidian distances between the centre points of adjacent drumlins with respect to ice-flow direction, which is defined as the average azimuth of the neighbouring 10 drumlins (Ely, 2015). Unlike previous studies, which grouped all drumlins into a single database (e.g. Clark et al., 2009; Hillier et al., 2013; Fowler et al., 2013), we retain the categorisation of drumlins into 100 flow-sets, which were used to build a palaeo-glaciological reconstruction (Hughes et al., 2014). Thus, flow-set numbers and locations are from Hughes et al. (2014) (see Table S1 for more information). For the purposes of this study, we discard 29 flow-sets due to the low number (< 30) of drumlins which they contain, or due to poor preservation (e.g. through cross cutting or post-formational modification). The remaining 71 are analysed here, together containing 36,222 drumlins.

3.2. Data processing

In order to characterise the frequency distribution of each size and spacing metric (length, width, relief, lateral and longitudinal spacing) within each flow-set, we extract parameters based upon the description of our data following both gamma (ϕ, λ and Λ) and lognormal (ϕ and $\bar{\mu}$) distributions (Figure 2) (Hillier et al., 2016). First, using the method-ofmoment estimators detailed in Hillier et al. (2013), we compared our measurements to a gamma distribution using frequency histograms (Figure 2A). The modal value (ϕ) was calculated using the method-of-moments (Hillier et al., 2013):

$$\phi = \frac{\left(\left(\frac{\bar{x}}{s_x}\right)^2 - 1\right)}{\left(\frac{\bar{x}}{(s_x)^2}\right)} \tag{1}$$

where $\bar{x}$ is the mean of the distribution and s_x is the standard deviation of the data. The maximum likelihood estimator of gradient of the positive tail of the distribution (λ) was calculated by removing all data from the original dataset below ϕ to produce a subset dataset k_i (Appendix A of Hillier et al., 2013). λ is then:

$$\lambda = 1 \;/\; \left(\frac{\sum_i (k_i - \phi)}{n}\right) \tag{2}$$

To characterise the shape of the distributions before the mode, we also define Λ, the slope of the distribution before the mode. This involved removing all values above ϕ from the original dataset, to produce K_i, and then applying the equation:

$$\Lambda = 1/\left(\frac{\sum_i (K_i - \phi)}{n}\right) \tag{3}$$

Therefore, Λ is a mirrored version of λ, with a statistical justification equivalent to that for λ (see Hillier et al., 2013). Thus, ϕ describes a modal value per flow-set, and the parameters Λ and λ describe the shape of the distribution of the flow-set. λ and Λ are independent of each other. However, Λ may be influenced by ϕ as we are studying scalar variables.

A second set of parameters based upon the log-normal distribution were derived using the method of Fowler et al. (2013). To calculate these parameters, the distributions were first represented as histograms of frequency intensity (f_i), modified to approximate probability density after Fowler et al. (2013):

$$f_i = \frac{n_i}{n\Delta} \quad (4)$$

where n_i is the number of drumlins in each bin, Δ is the bin width and n is the total number of the sample (Fowler et al., 2013). Bin width was kept the same for each histogram. These frequency intensity histograms (Figure 2B) were visually compared to a normal distribution curve in order to verify their log-normality. Secondly, the mean and variance, $\bar{\mu}$ and $\bar{\sigma}$, respectively, of each spacing and size variable per flow-set were defined by:

$$\bar{\mu} = \frac{1}{n}\sum_i \ln x_i \quad (5)$$

$$\bar{\sigma}^2 = \frac{1}{n-1}\sum_i (\ln x_i - \bar{\mu})^2 \quad (6)$$

3.3. Scenario-testing

Figure 3 outlines several conceptual scenarios of possible drumlin patterning behaviour, halted at different stages of development in different flow-sets. Figure 3A outlines our conceptual model of two different flow-sets which undergo simple stabilisation. In flow-set 1, drumlins are initially formed and stabilise at a single size and distance apart. Different ice-bed conditions in flow-set 2 mean that drumlins are formed and stabilise at a different scale. This is illustrated in Figure 3A as a difference in spacing, but could equally be manifested in drumlin size. Under this scenario, assuming conditions across a flow-set are similar, the spread of the distributions (λ, Λ and $\bar{\sigma}$) is defined by little variability between flow-sets, but average measures (ϕ and $\bar{\mu}$) may vary. In this case, the two flow-sets do not represent different stages of maturity with a common evolution.

Figure 3B outlines a second conceptual model outlining a scenario whereby most drumlins evolve through time by simply growing longer and without migrating, which may lead to coarsening and collisions. At each successive stage drumlins grow longer, but remain the same distance apart, eventually colliding and merging. If this is the behaviour that characterises drumlin pattern evolution across all flow-sets, then different flow-sets, frozen at different stages of growth, may show different levels of coarsening (Figure 3B). A similar coarsening effect could also occur if drumlins grow wider, colliding/merging with lateral neighbouring drumlins or if drumlins migrate at different rates, colliding with their upstream neighbours.

Figure 3C illustrates a conceptual model of drumlins initially forming a set distance apart, from which they then deviate due to migration, with each drumlin migrating at its own pace. This assumes that migration can only occur parallel to ice flow. In this paper, we refer to migration as being the movement of both the stoss and lee of drumlin (i.e. whole drumlin movement), from growth of either side leading to a change in the location of the drumlin centre. In the model, this leads to more variability in the longitudinal spacing of drumlins and general coarsening. Again, in this conceptual model, different stages of this process are imagined to be recorded in different flow-sets which have had different periods of time to evolve to their observed state.

While the previous two cases are examples of pattern evolution that might lead to perpetual coarsening, a further scenario of limited coarsening should also be tested. This is the “significant pattern coarsening *en route* to a saturated wavelength” scenario of Murray et al. (2014: p. 62), whereby interactions between elements cause the pattern to approach a steady-state. Figure 3D illustrates a conceptual model of drumlins initially coarsening (as in Figure 3B), but then reaching a size or spacing beyond which they can no longer grow. In this conceptualisation, as more drumlins reach the size and spacing limit λ would increase, as the formation of a tail in the distribution would be inhibited. Similar thresholds may also exist for spacing, width and relief. This would lead to multiple 'mature' flow-sets at which drumlins have reached a stable length (Figure 3D). Note that all four conceptual models are based upon simplifications of drumlin development. It may be that more complex interactions, or none of the proposed cases, occur and therefore dilute the signal of these simplistic expectations.

4. Results

In order to explore how drumlin size and spacing metrics vary between flow-sets, and potentially decipher evolutionary sequences, the relationships between shape parameters for drumlin length, derived following the method of Hillier et al. (2013), are plotted on Figure 4. Similar relationships were found for width, relief, lateral and longitudinal spacing (Figures S1 to S4). All were found to be significant to $p \leq 0.05$ (f-test). The strongest relationships occur between the parameters ϕ and λ (Figures 4A and S1 to S4), taking the form of negatively-correlated power laws. This indicates that as the size or spacing of drumlins increases, the gradient of the positive tail of the distribution decreases (Figure 4A). Similarly, ϕ and Λ are significantly highly-correlated through negative power-law relationships (Figure 4B). Therefore, as drumlins get larger, or further apart, the gradient before the mode decreases. This relationship is weaker for the width parameter (Figure S1). There are also significant and strong correlations between λ and Λ, this time taking the form of positive power laws (e.g. Figure 4C). For each variable, Λ is always greater than λ, indicating that the probability distributions are always positively skewed. Therefore, the gamma-distribution metrics indicate that size and spacing parameters change in a predictable manner between flow-sets.

To determine whether drumlin flow-sets are different from one another, which would undermine the assumption that they represent different stages of drumlin evolution, we performed analysis of variance (ANOVA) tests. Although inter-flow-set variation occurs, these ANOVA tests show that there is no significant separation of flow-sets into different groups or single entities (e.g. Figure 3A), despite the wide range of modal values for different flow-sets (~220-740 m for length, ~100-350 m for width and ~4-12 m for relief). This means that we find no highly-distinct flow-sets. Instead, they all show subtle variations from each other. This is despite flow-sets being formed separately in time in space, and exhibiting a range of different characteristics such as their total area, sedimentary substrate, or varying degrees of underlying topographic influence.

To explore whether the log-normal distribution (commonly used to infer growth) can describe the size and spacing of drumlins within flow-sets, we plot histograms of each metric against log-normal distribution curves (Figure 5). Log-normal distributions have previously been found for whole drumlin populations (Fowler et al., 2013), yet their applicability to the metrics of individual flow-sets is unknown. Such distributions are often taken to infer that stochastic phases of growth have occurred (Limpert et al., 2001; Fowler et al., 2013). Figure 5 shows how the frequency distributions of flow-sets are approximately log-normal for the variables of length, width and relief. The shape of the distributions for these three variables remains approximately log-normal, even for flow-sets which contain low numbers of drumlins, despite the smaller sample size (Figure 5). However, the distributions of lateral and longitudinal spacing do not fit a log-normal shape (Figure 5). Therefore, $\bar{\mu}$ and $\bar{\sigma}$ are poor descriptors for the two spacing variables.

To ascertain how the shape of the distributions differs between flow-sets and, in turn, establish whether any patterning interactions have occurred (Figure 3), Figure 6 plots $\bar{\mu}$ against $\bar{\sigma}$ for length, width and relief. We found significant positive correlations ($p \leq 0.05$, F-test) for length and width, indicating that as drumlin length or width increases on average, the spread of the distribution also increases (Figure 6A and B), although this relationship is weaker for width. However, no such relationship was found for relief (Figure 6C).

To further visualise differences and the potential evolution in the shape of the frequency distributions for each flow-set, Figure 7 overlays the frequency distributions of 5 flow-sets (see also Figure S5) chosen due to their large sample size and to characterise the range of observed drumlin size metrics. For length and width, as the modal value increases, its amplitude gets smaller and the spread increases (Figure 7). Conversely, modal relief remains similar between flow-sets (Figure 7).

5. Discussion

We now return to the conceptual models and four questions posed in Section 2, and discuss the extent to which scenarios described in Section 3.3 and illustrated in Figure 3 are supported by our results.

5.1. Do drumlins stabilise at an initial scale?

One simple explanation for drumlin patterns might be that they stabilise at a set scale and spacing, *without pattern coarsening*. Under this scenario, the varying shape of drumlin size and spacing distributions between flow-sets would indicate that drumlins at different locations were 'printed' at different scales onto the landscape. However, rather than producing flow-sets with significantly different size and spacing metrics, we find no separation of flow-sets into different statistical populations. Instead, the size and spacing metrics of drumlins co-vary in an apparently continuous manner between flow-sets (Figures 4, S1 and S2). When characterised as gamma (Figure 4) or log-normal (Figure 7) distributions, drumlin size distributions indicate that when the modal value increases, the distribution becomes more widely spread. Such distributions could arise from varying conditions at the ice-bed interface, or differences in process leading to a wider spread of the drumlin size, shape and spacing within a flow-set as a result of simple stabilisation. However, and although the mechanics of the following systems are different, similar variations in distribution shape have been observed for stages of a time evolutionary sequence for other phenomena (see Limpert et al., 2001), such as raindrop size (e.g. Srivastava, 1971), grain size in volcanic rocks (Fowler and Scheu, 2016) and crystal size during crystallisation (e.g. Teran et al., 2010; Ng, 2016). Furthermore, we find it unlikely that differences in the conditions at the ice-bed interface and/or spatial variations in processes would lead to the smoothly co-varying metrics that we observe here. Hence, we interpret our results as showing that drumlin size and spacing evolves during development rather than from near-instantaneous ‘printing’ at a set scale, and we rule out simple stabilisation as a cause of differences in drumlin metrics between flow-sets (Figure 3A). Indeed, we note that simple stabilisation is rarely the case for natural patterns, which more often than not require evolution - growth, shrinking, interaction - to yield a specific geometry or arrangement (see Murray et al., 2014; Clark et al., 2017 and references therein), and are thus autogenic in nature (Clifford et al., 1993; Werner et al., 2003; Pelletier et al., 2004), i.e., the pattern that emerges does so due to interactions from elements within the pattern itself.

5.2. Do drumlins grow and/or shrink?

Drumlin metrics amalgamated from multiple flow-sets display log-normal or exponential frequency distributions (Hillier et al., 2013; Fowler et al., 2013). This has been used to infer that drumlins grow and/or shrink over time, through analogy with other phenomena which display log-normal distributions (Limpert et al., 2001) and through replicating the distributions predicted by statistical models (Hillier et al., 2016). Previous analyses have based this interpretation on amalgamated samples from multiple ice-flow events (Hillier et al., 2013; 2016; Fowler et al., 2013), which likely formed under a wide variety of subglacial conditions. Rather than growing and/or shrinking over time, an alternative explanation for the log-normal distribution of these multiple flow-set datasets is that this simply reflects stochastically distributed subglacial parameters (e.g. effective pressure, ice velocity, sediment thickness) across the multiple flow-sets from which the metrics are extracted (Dunlop et al., 2008; Hillier et al., 2013; Fowler et al., 2013; Barchyn et al., 2016). Our analysis of samples from individual flow-sets builds on the analysis of Hillier et al., (2016), who studied the size metrics of a single flow-set and an amalgamated sample of multiple flow-sets, and argued that the characteristics of drumlin frequency distributions could be linked to subglacial parameters, if drumlin growth rate is known. Our work indicates that drumlin size metrics are consistently positively skewed with near log-normal distributions (Figures 5 and 7). This lends strong support to the hypothesis that drumlins grow (or shrink) over time within the same flow-set, where one might suggest that similar subglacial conditions have influenced drumlin development. While the positively skewed and log-normal distributions demonstrate that either growth and/or shrinkage have occurred, the fact that drumlin patterns are not the result of simple stabilisation, but are achieved through drumlin interactions, indicates that these landforms must either grow and/or migrate.

If we assume that drumlins originate as small features (see Hillier et al., 2016), and interpret different drumlin flow-set size and shape distributions as displaying an evolutionary-sequence (e.g. Figures 3 and 7), then our data support the notion that there is a general tendency for drumlins to get longer and wider as they develop (e.g. Figures 4, 6 and 7). This is consistent with the ‘cone-shaped scatter plot’ of drumlin length and width in Clark et al., 2009 (see their Figure 10), and statistical models of drumlin formation (Fowler et al., 2013; Hillier et al., 2016). The tight apex of the cone at lower values (and the lack of datapoints below them) was taken as a fundamental initiation scale (100 metres) from which drumlins were inferred to grow in length and width in various proportions to yield the overall cone-shaped scatter. Consistent with the idea that drumlins originate as small features, Dowling et al. (2016) propose that a set of small drumlins formed within a few years, suggesting that drumlins initiate small and would naturally grow if given enough time (in this case they formed as ice was quickly retreating from the region). Therefore, we interpret the flow-sets with low modal size values as containing less mature drumlins.

Although the frequency distributions suggest that drumlin flow-sets which have had more time to evolve contain longer, wider drumlins (e.g. Figure 7), we cannot rule out that this general trajectory of drumlin growth is interspersed with phases of shrinking, i.e. erosion (e.g. Smith et al., 2007; Hillier et al., 2016). Drumlin shrinking may even be an important mechanism in drumlin pattern evolution, causing to the eventual eradication of some drumlins within a pattern which would lead to pattern coarsening. Furthermore, purely bedrock forms exist, which require an entirely erosional mechanism, but the extent to which they are analogous to drumlins formed of unconsolidated sediments is less clear.

When drumlins are longer and wider within a flow-set, the spread of length and width is also larger (Figures 4, 6 and 7). We interpret this as showing that, rather than all drumlins growing uniformly in a flow-set, some drumlins grow longer and wider than others. Such behaviour is replicated in statistical (e.g. Fowler et al., 2013; Hillier et al., 2016) and numerical models (Barchyn et al., 2016). This could be related to either a variable sediment supply controlling drumlin growth (e.g. Rattas and Piotrowski, 2003; Ó Cofaigh et al., 2013), or drumlins initiating at different times during pattern development and thereby leading to a mixed age population (e.g. Stokes et al., 2013b), or a combination of the two. The preferential growth of some drumlins, at the expense of others, may lead to pattern coarsening (see Section 5.5).

Drumlin spacing varies between flow-sets in a manner similar to the variable length (Figures 4, S3 and S4), but the shapes of the distributions of spacing are not log-normal (Figure 5). Therefore, drumlin spacing does not vary in the same manner as drumlin size metrics. Perhaps the non-log-normal spacing distributions are a consequence of factors independent from drumlin growth initially determining their placement. This is consistent with a mix of regularly-seeded drumlins and ‘clones’ anchored to more randomly distributed bedrock perturbations described by Clark (2010), which would produce non-log-normal distributions. Another possibility is that drumlin spacing is altered due to post-formational modification. That spacing is not uniform between flow-sets and varies at different locations indicates that the scenario of drumlins growing longer without changing lateral spacing was not detected (e.g. Figure 3B). This inference is based upon the assumption that simple stabilisation does not account for the observed patterns in frequency distributions (see Section 5.1.). Instead, changes to the arrangement of drumlins may occur during pattern development, with potential mechanisms for this being growth, erosion leading to the eradication of some drumlins, amalgamation of one or more drumlins, and migration.

We envisage that drumlins (i.e. their relief) can grow in both net erosional and depositional settings (see Clark, 2010; Stokes et al., 2013a). Where mobile sediment is readily available, accretion in both the stoss and lee of a drumlin may occur (e.g. Dardis et al., 1984; Fowler, 2009; Knight, 2016). Indeed, drumlins have often been reported to have lee-side cavity infills (e.g. Dardis and McCabe, 1983; Dardis et al., 1984; Fisher and Spooner, 1994; Stokes et al., 2011; 2013a; Spagnolo et al., 2014b), indicating that lee-side deposition would be an obvious mechanism via which drumlins could grow (Barchyn et al., 2016). Where erosion co-exists with deposition, it is likely that sediment is transported to, and deposited at, the lee of a drumlin, perhaps at the expense of relief and width (e.g. Stokes et al., 2013a; Eyles et al., 2016).

5.3. Do drumlins migrate?

This question is difficult to solve using relict drumlins, but the earlier (Section 5.2.) results on drumlin growth are helpful because these could also be partially or fully achieved by drumlin migration. Our expectation of drumlin migration (that the spread of drumlin along-flow/longitudinal spacing metrics would increase) in Figure 3C is supported by our results (Figure 4). However, the spread in longitudinal spacing metrics could be attributed to preferential growth of some drumlins and destruction of others. Although our data cannot resolve whether or not drumlin patterns evolve through migration, a number of observations from the literature are relevant to this issue. For example, some drumlins are known to be composed of bedrock or gravel cores (Schoof, 2007; Stokes et al., 2011; Dowling et al., 2015), indicating that the drumlin is 'anchored' on a core which is unlikely to have moved during formation, though the surrounding sediment may have changed shape. However, other drumlins contain sedimentary structures (e.g. large deformation structures) which suggest they could have migrated (Boulton, 1987; Hart, 1997; Knight, 2016), and numerical models predict that migration can occur (e.g. Chapwanya et al., 2011; Barchyn et al., 2016). If migration or growth leads to collision, we would expect to find examples of relict drumlins about to collide, or paused in mid-collision. Such arrangements are readily observed for migratory dunes (Hersen and Douady, 2005; Kocurek et al., 2010), with numerical models of bedforms suggesting that collisions are a key factor for regulating bedform size distribution (Barchyn et al., 2016).

Figure 8 shows potential examples of drumlins possibly about to collide. Although perhaps not as common in drumlin fields, similar typologies to those on Figure 8 have been observed and modelled for colliding barchan dunes (e.g. Hugenholtz and Barchyn, 2012; Parteli et al., 2014), and here we suggest that similar arrangements may be caused by drumlin migration and collision. We further note that growth and migration may provide feasible mechanisms to explain compound or fused drumlin typologies (e.g. Knight, 1997) and are consistent with observations of downstream changes in drumlin density or 'packing' (Clark and Stokes, 2001). Collisions may therefore be a mechanism by which pattern coarsening occurs in drumlin fields. Further sedimentological and morphological studies are required to examine if and how any such mobile drumlins interact.

5.4. Do drumlins patterns evolve toward a stable coarsened state?

If drumlin patterns evolve, a logical question is when do they stop evolving; do they reach a final form or steady state? None of the frequency distributions observed here conform to our expectation of stabilisation outlined in Figure 3D. Indeed, that all the distributions of the size variables are approximately log-normal, and modal drumlin size changes from flow-set to flow-set, suggests that size metrics continue to evolve (Figure 6). A similar result was found in the statistical modelling of Hillier et al. (2016). This apparent lack of an upper limit for drumlin length is consistent with the idea of a subglacial bedform continuum (Aario, 1987; Rose, 1987; Ely et al., 2016), whereby drumlins merge into mega-scale glacial lineations (e.g. Stokes et al., 2013b; Spagnolo et al., 2014a; 2016; Barchyn et al., 2016). That relief does not show the evolutionary sequence displayed by length and width, but maintains a log-normal shape with a comparable mean in all flow-sets (Figure 6 and 7) is intriguing. Perhaps relief is prevented from growing higher by some, yet unknown, glaciological cause, leading to early stabilisation of drumlin relief. One possibility is that this limit is imposed by sediment supply, and is reached when all available sediment is contained within drumlins preventing further upward growth.

5.5. Drumlin Pattern Evolution

Interpretation of our analysis of drumlin pattern interactions is summarised in Figure 9. We find that drumlins do not stabilise at their initial scale (Figure 9-T1). It is more likely that they initiate at a length-scales of around 100 m (Clark et al., 2009) from which they then evolve and coarsen (Figure 9-T2). Coarsening is achieved through a mixture of drumlins growing longer (and bumping into each other) and migrating, with some drumlins growing at the expense of others (Figure 9-T3). New drumlins are continually formed during the formation period, if accommodation space and sediment becomes available. Growth of drumlins, especially in a lengthwise direction, leads to perpetual coarsening until conditions change radically (e.g. a change in basal thermal regime from warm-based to cold-based) or deglaciation occurs (Figure 9-T4). This picture of an evolving ice-bed interface is consistent with numerical and statistical modelling (Chapwanya et al., 2011; Fowler et al., 2013; Barchyn et al., 2016; Hillier et al., 2016) and repeat geophysical imaging of cognate subglacial bedforms beneath modern ice masses, which indicates changes at decadal timescales (e.g. Smith et al., 2007; King et al., 2009).

That pattern development occurs in drumlin fields, implies that interactions between drumlins is required to create the ordered landscapes and pattern characteristics observed (see also Clark et al. 2017). In dunes, these bedform interactions (or ‘communications’) are facilitated by interactions at the fluid, bedform and grain scale (Kocurek et al., 2010). For example, flow separation in the lee of dunes can cause scouring in the stoss of upstream dunes (Endo and Taniguchi, 2004). For drumlins, a lack of observations leaves the mechanisms of communication unclear, but several possible candidates exist. At the fluid scale, the drag induced by the sliding of ice over drumlins (see Schoof, 2002) may alter the sediment entrainment rates via shear stress alteration. At the bedform scale, the development of lee-side drumlin cavities (e.g. Fowler, 2009) may regulate drumlin growth and migration (Barchyn et al., 2016), in turn controlling the size and spacing of resultant drumlins. Sediment supply variations may also control inter-drumlin interactions and drumlin positioning. At the simplest level, sediment must be available for bedforms to initially form, and be either supplied to or recycled within the system for patterning interactions to occur. Inter-drumlin interactions determined at the grain scale may occur due to the streaming of sediment around the flanks of a drumlin, caused by the development of drumlin relief (e.g. Boyce and Eyles, 1991). This may cause spatial variability in sediment availability, whereby sediment deflected around upstream drumlins leads to an increased supply downstream of an inter-drumlin area. The presence of a cavity would also introduce a region in the lee of a drumlin where sediment supply is limited, thus inhibiting drumlin growth directly downstream. Which, if any, of the above is the cause of drumlin interaction should be the topic of further study.

That patterning interactions occur in drumlin fields also has implications for drumlin formation hypotheses, of which there are many (see Clark, 2010 for a recent review). The order and predictability of patterns seems at odds with polygenetic formation hypotheses of subglacial bedforms, whereby differences in sediment composition are inferred to mean that a large number of equifinite, processes are responsible for subglacial bedform formation (e.g. Lindén et al., 2008; Sutinen et al., 2010). Möller and Dowling (2016) provide a framework for understanding how sedimentological differences may arise due to a variety of processes, given the morphological similarity of drumlins. Their 'unifying polygenetic' model suggests that differences in boundary conditions (such as sediment thickness and ice velocity) are regulated by a complex set of growth and shaping processes. Here we have shown that such complex processes (growth, migration, collisions) are likely to occur. However, patterning provides a useful framework for understanding these processes. The ubiquity of patterning in drumlin fields that we and Clark et al. (2017) find, suggests that any site-specific sedimentological differences that would otherwise lead to the conclusion of polygenesis, should be considered within the context of the development and interaction of the drumlin pattern. The erodent layer hypothesis (Eyles et al., 2016) proposes that all drumlins are formed by the abrasion of a layer of subglacial sediment into the underlying substrate. This is not necessarily at odds with the observations of patterning presented here; for example, erosional bedforms can also migrate and grow over time (e.g. Richardson and Carling, 2005). It is, however, difficult to reconcile with observations of subglacial bedforms where sediment has accreted over time (e.g. Dardis and McCabe, 1983; Knight, 1997; Spagnolo et al., 2016). The instability hypothesis of drumlin formation views drumlins as the consequence of the coupled flow of ice, water and sediment at the ice-bed interface and allows for both accretion and erosion of bedforms (e.g. Fowler and Chapwanya, 2014). This hypothesis is rooted in the observation that drumlins are arranged in patterns, and is therefore easily suited to explain patterning interactions. Numerical models of ribbed moraine formation demonstrate migration and coarsening occurring (Chapwanya et al., 2011), but have not yet been adapted to produce such interactions for drumlins. However, the instability hypothesis is yet to be supported by sedimentological observations (Spagnolo et al., 2016; McCracken et al., 2016). More generally, the results presented here suggest that advances in understanding drumlin formation should come from numerical modelling that includes the possibility for inter-drumlin interactions (e.g. Barchyn et al., 2016).

6. Summary and Conclusions

Here we study the frequency distributions of the size and shape metrics of 36,222 drumlins from 71 flow-sets of drumlins to seek inferences about patterning interactions. Based on concepts applied to bedform patterns in other fields, we ask whether drumlins i) stabilise at an initial scale; ii) grow; iii) migrate; and, through growth and or migration, iv) evolve to a stable coarsened state or perpetually coarsen. Through examination of the size and spacing metrics of drumlins per flow-set, and using space-for-time substitution, our interpretation is that:

i) Drumlins do not exhibit simple stabilisation, i.e. they are not fixed by a wavelength that is determined during their initial growth.
ii) Drumlins likely grow over time: a process which leads to pattern coarsening.
iii) There is potential for drumlins to migrate during pattern development, which likely contributes to pattern coarsening.
iv) A lack of an observable upper limit on drumlin geometry (with the possible exception of their relief) leaves the potential for perpetual coarsening to occur.

The approach of using size, shape and positioning metrics of relict drumlins to infer dynamic inter-drumlin interactions as they grow is, of course, challenging. Nevertheless we suggest that the patterning that exists in drumlin fields (Clark et al., 2017) arises from drumlin growth and migration, leading to pattern coarsening. This has important ramifications for modelling the formation of subglacial bedforms, and demonstrates that it is misleading to consider drumlins as a collection of individual landforms. Rather, models should ideally address their spatial interactions within a drumlin field.

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Figure Captions

Figure 1. Examples of landscapes containing bedforms. In each example, note the regular and repetitive placement of individual bedforms across the landscape. (A) Sand dunes located within the White Sands Dune Field, USA; Lidar data, hill-shaded from the north-east (Baitis et al., 2014; downloaded from opentopo.sdsc.edu). (B) Submarine dunes on the Irish Sea floor; bathymetric elevation data, hill-shaded from the north-east (data from https://jetstream.gsi.ie/iwdds/map.jsp). (C) Fluvial dunes on the Mississippi river bed, New Orleans; bathymetry elevation data, hill-shaded from the north-west (downloaded from http://www.mvn.usace.army.mil/Missions/Engineering/ChannelImprovementandStabilizationProgram/2013MBMR.aspx). (D) Drumlins located North of Barnoldswick, England; Nextmap digital elevation model (DEM) hill-shaded from the north-west.

Figure 2. Example frequency plots of drumlin lengths for flow-set 9 showing: A) the definition of gamma-based parameters ϕ, λ and Λ; and B), the definition of log-normal based parameters $\bar{\mu}$ and $\bar{\sigma}$. Both were derived to summarise the probability distribution functions of size and spacing metrics (length, width, relief, lateral and longitudinal spacing) per flow-set.

Figure 3. Expected influence on drumlin size and spacing metrics for different patterning behaviours; notably the position of the mode and the spread and skew of the distributions. A) Simple stabilisation at different scales leads to differences in spacing metrics between two different flow-sets. B) Lengthwise growth leads to coarsening. Different stages of this process should be recorded at different flow-sets. C) Migration would lead to an increased spread of along-flow spacing metrics. D) Stabilisation of drumlin length. If drumlins reach a

length beyond which they cannot grow, once a threshold is reached, the histogram's positive tail will steepen. Eventually all drumlins reach the growth-limit and stop growing. (SLR = stable length reached).

Figure 4. The best-fit power-law relationships between gamma-based shape parameters (ϕ = mode, λ = post-modal slope, Λ = pre-modal slope) for length across different drumlin flow-sets. Similar relationships were found for all other variables (Figures S1 to S4).

Figure 5. Examples of frequency histograms for derived variables compared to a log-normal distribution (black line) for 3 different-sized flow-sets. A) Flow-set 15, n = 471, in blue; B) Flow-set 45, n = 1407, in orange; C) Flow-set 65, n = 152, in green. The three examples were chosen because of their different sample sizes.

Figure 6. Scatter plots between $\bar{\mu}$ and $\bar{\sigma}$ for (A) length, (B) width, (C) relief.

Figure 7. Distributions of length, width and relief variables for 5 flow-sets. See Hughes et al. (2014) for numbering and location and Figure S5 for individual histograms.

Figure 8. Examples of drumlins which may be mid-collision due to either growth or migration. Potential collisions are labelled *C*. Approximate palaeo-ice flow direction denoted by white arrow. Data from Nextmap DEM. (A) Instances of touching drumlins within a flow-set; about to coalesce? (B) Elongate drumlins which sometimes appear to touch drumlins

further downstream; wholesale migration to a collision or just downstream growth? (C) Small drumlins appearing to collide into the stoss side of a larger (slower moving?) drumlin. Note the patch of drumlins encircled in a dashed line which may have evolved to a similar arrangement if they had had longer to migrate. (D) Small drumlins which appear to have collided into the stoss side of much larger drumlins.

Figure 9. Summary of potential drumlin patterning behaviour. Each box represents a different stage in the evolution of the same pattern and dashed lines represent intermediate stages. At T1, drumlins begin to evolve immediately, without any simple stabilisation. At T2 preferential growth of one drumlin leads to the amalgamation of a drumlin in its lee, whilst other drumlins grow and migrate. These processes continue in T3, and new drumlins are formed in the space provided by erosion of other drumlins. These processes continue until the bedforms are frozen in position by deglaciation (T4), which could occur at any intermediate stage.

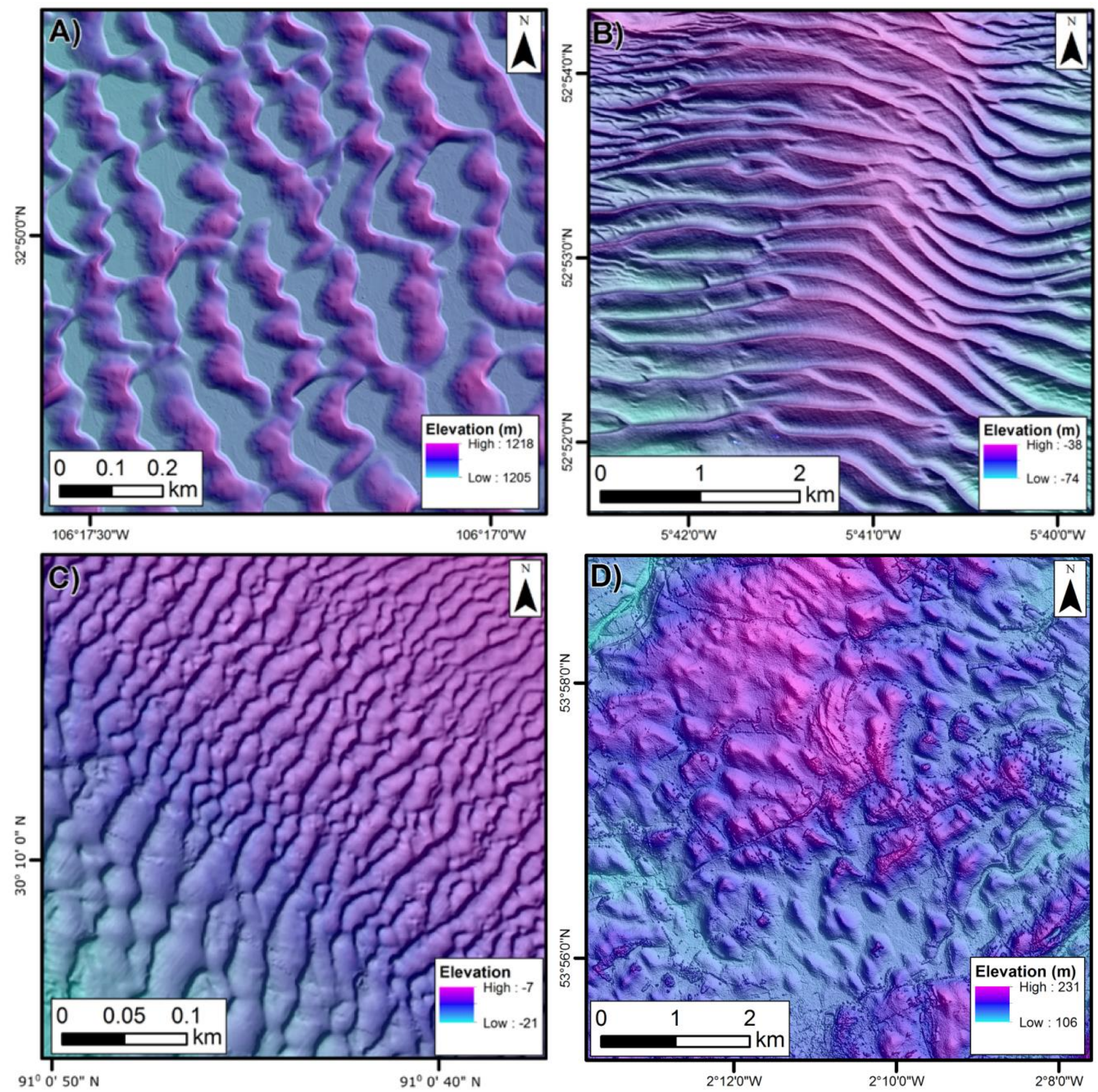
A)
N
32°50'0"N
Elevation (m)
High : 1218
Low : 1205
0 0.1 0.2
km
106°17'30"W
106°17'0"W
B)
N
52°54'0"N
52°53'0"N
52°52'0"N
Elevation (m)
High : -38
Low : -74
0 1 2
km
5°42'0"W
5°41'0"W
5°40'0"W
C)
N
30° 10' 0" N
Elevation
High : -7
Low : -21
0 0.05 0.1
km
91° 0' 50" N
91° 0' 40" N
D)
N
53°58'0"N
53°56'0"N
Elevation (m)
High : 231
Low : 106
0 1 2
km
2°12'0"W
2°10'0"W
2°8'0"W

Figure 1

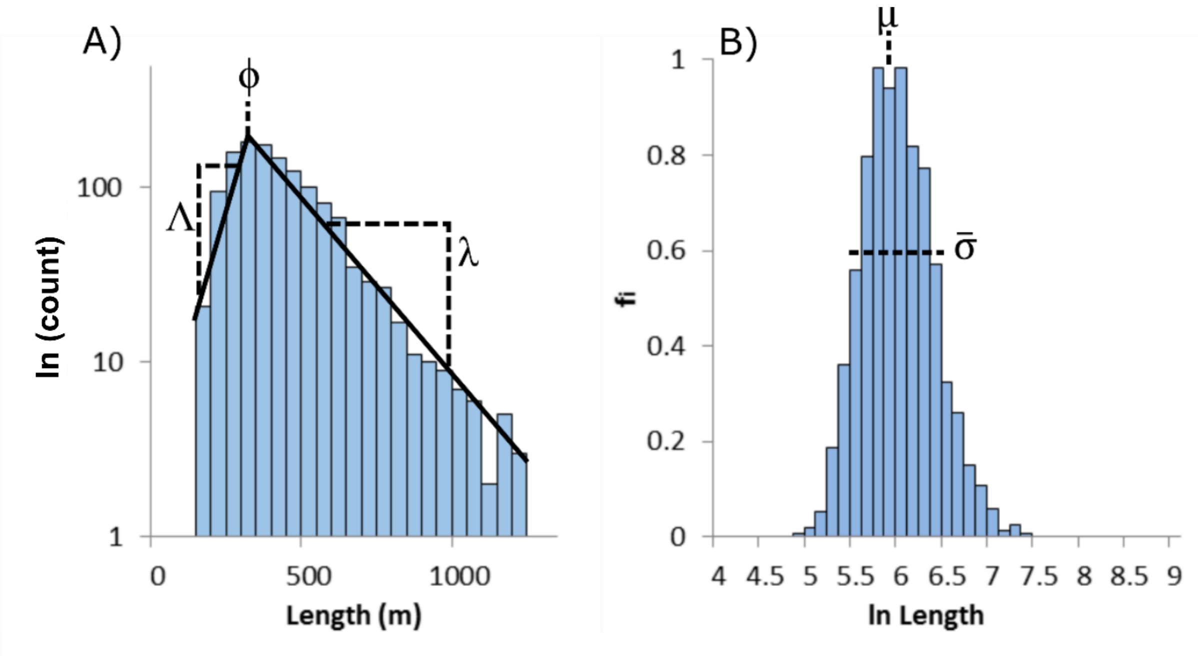
A)
ϕ
Λ
λ
ln (count)
100
10
1
0
500
1000
Length (m)
B)
μ
σ̄
fi
1
0.8
0.6
0.4
0.2
0
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
ln Length

Figure 2

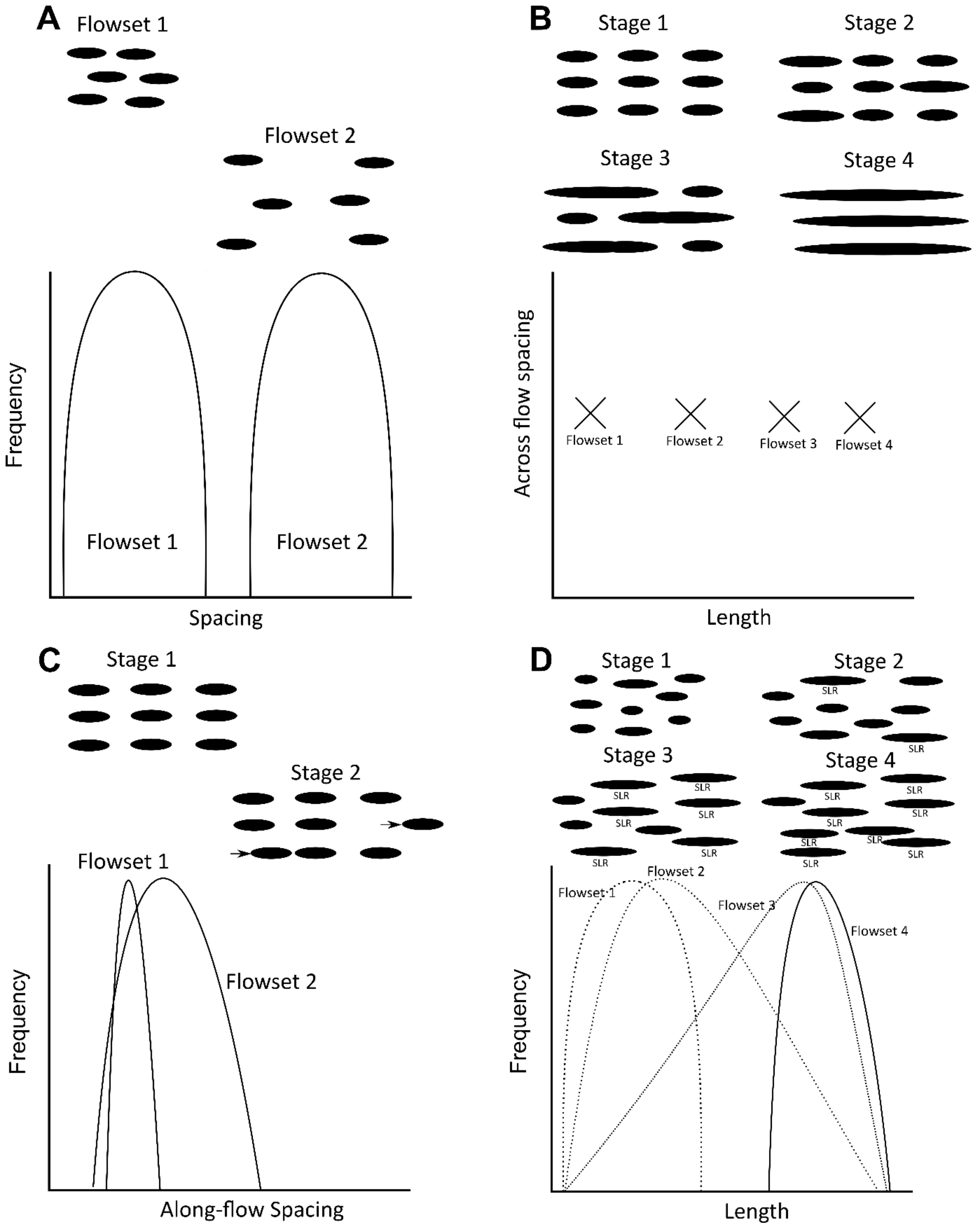
A
Flowset 1
Flowset 2
Frequency
Flowset 1
Flowset 2
Spacing
B
Stage 1
Stage 2
Stage 3
Stage 4
Across flow spacing
Flowset 1
Flowset 2
Flowset 3
Flowset 4
Length
C
Stage 1
Stage 2
Flowset 1
Flowset 2
Frequency
Along-flow Spacing
D
Stage 1
Stage 2
SLR
Stage 3
Stage 4
Flowset 1
Flowset 2
Flowset 3
Flowset 4
Frequency
Length

Figure 3

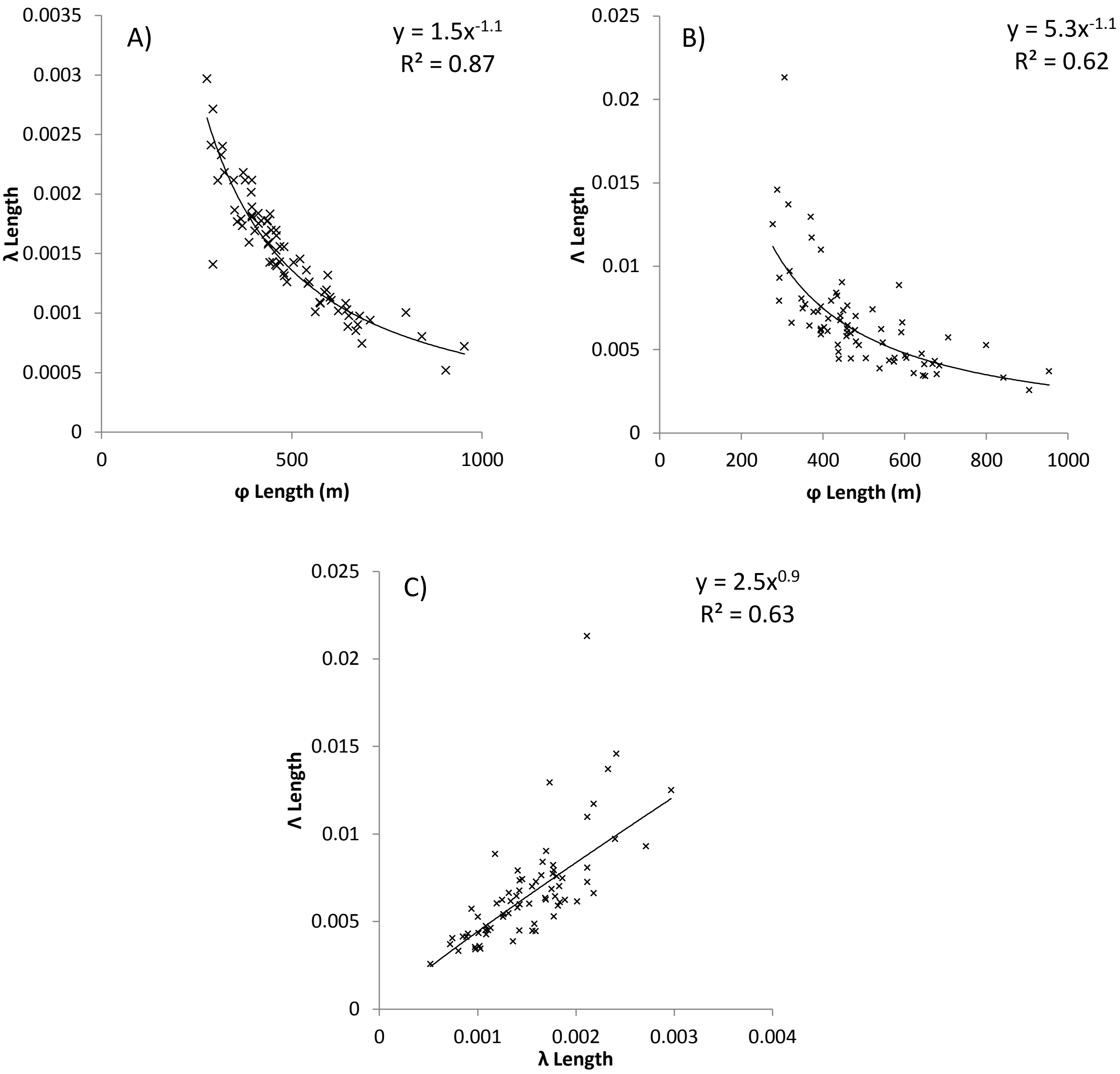
A)
y = 1.5x^-1.1
R² = 0.87
λ Length
φ Length (m)
0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
0
500
1000
B)
y = 5.3x^-1.1
R² = 0.62
Λ Length
φ Length (m)
0.025
0.02
0.015
0.01
0.005
0
0
200
400
600
800
1000
C)
y = 2.5x^0.9
R² = 0.63
Λ Length
λ Length
0.025
0.02
0.015
0.01
0.005
0
0
0.001
0.002
0.003
0.004

Figure 4

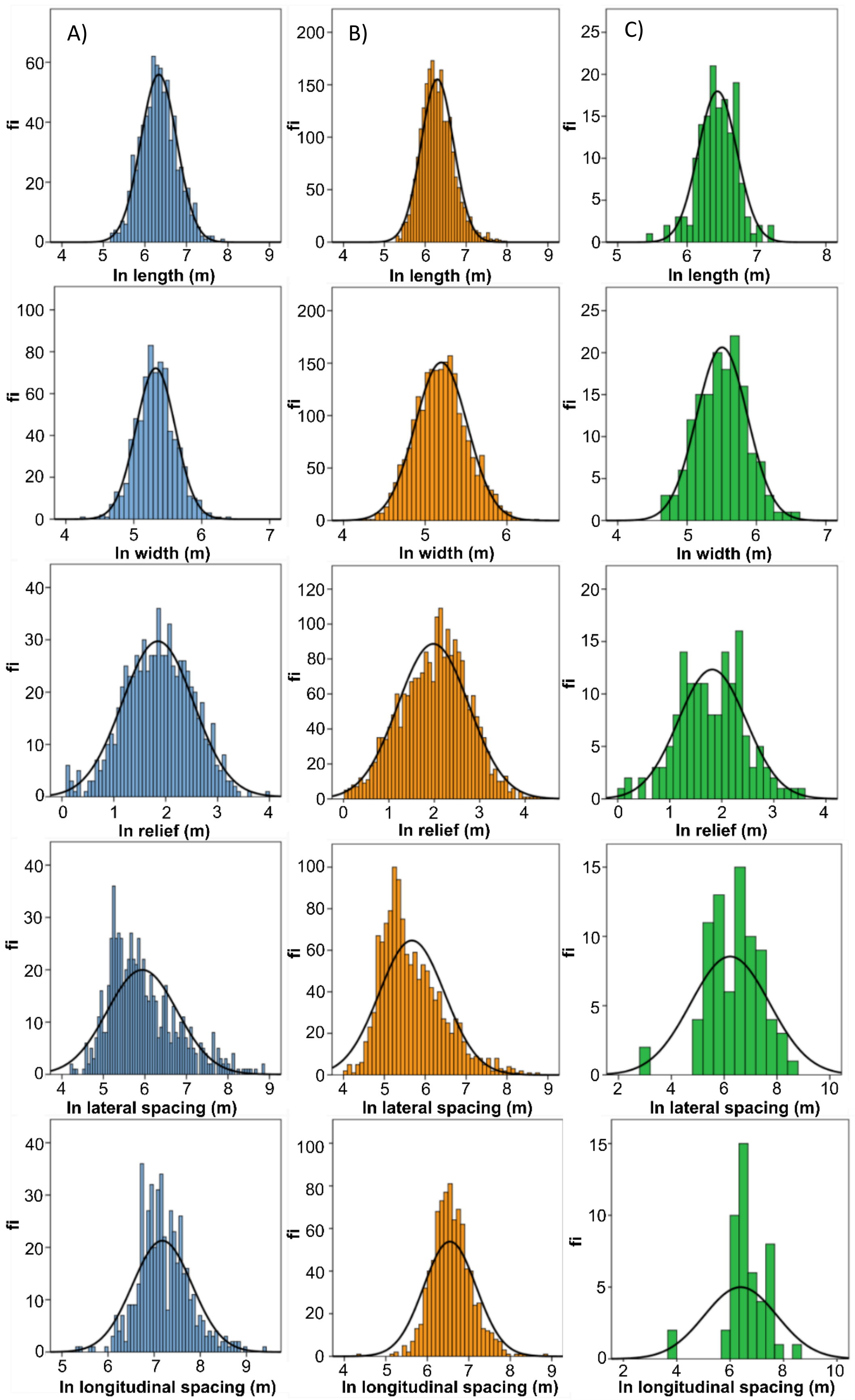
A)
B)
C)
fi
ln length (m)
ln width (m)
ln relief (m)
ln lateral spacing (m)
ln longitudinal spacing (m)

Figure 5

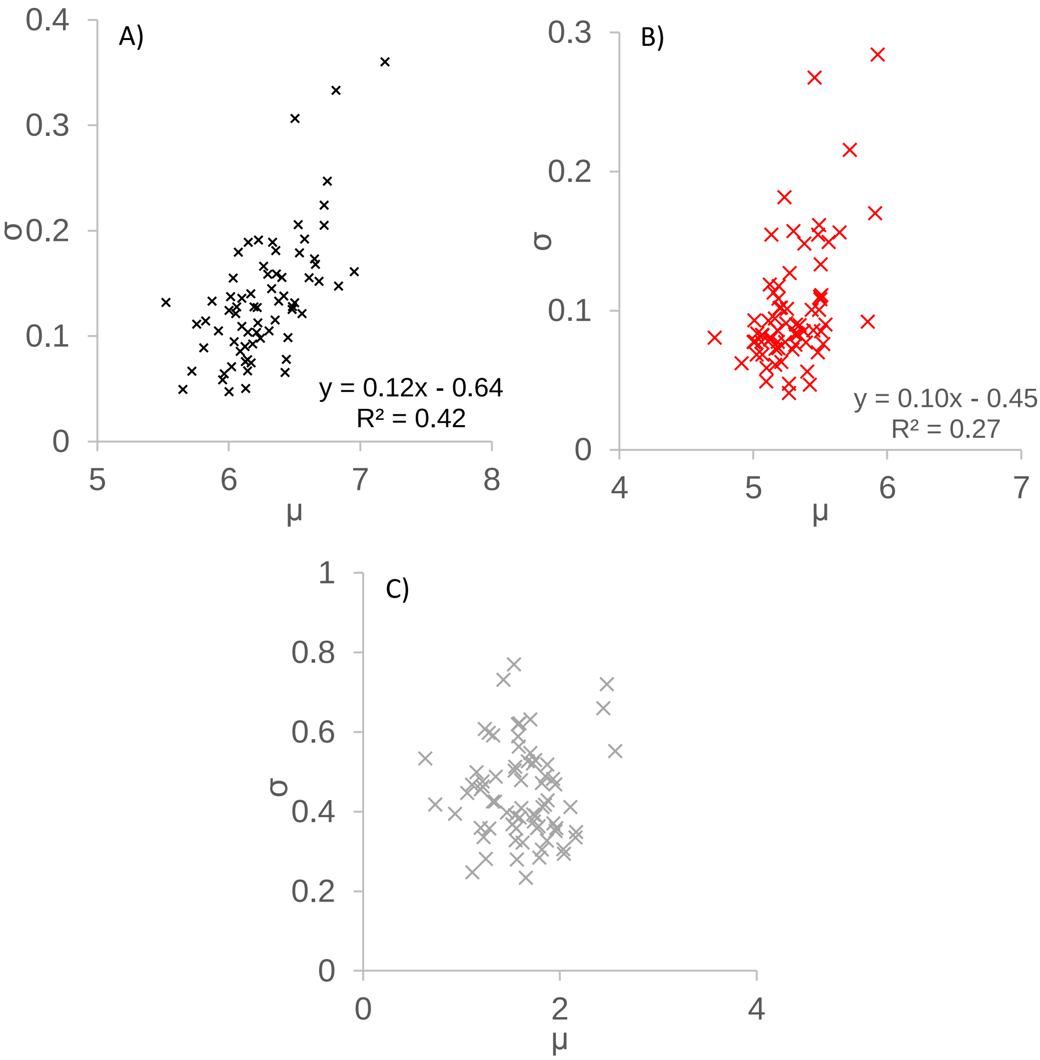

Figure 6

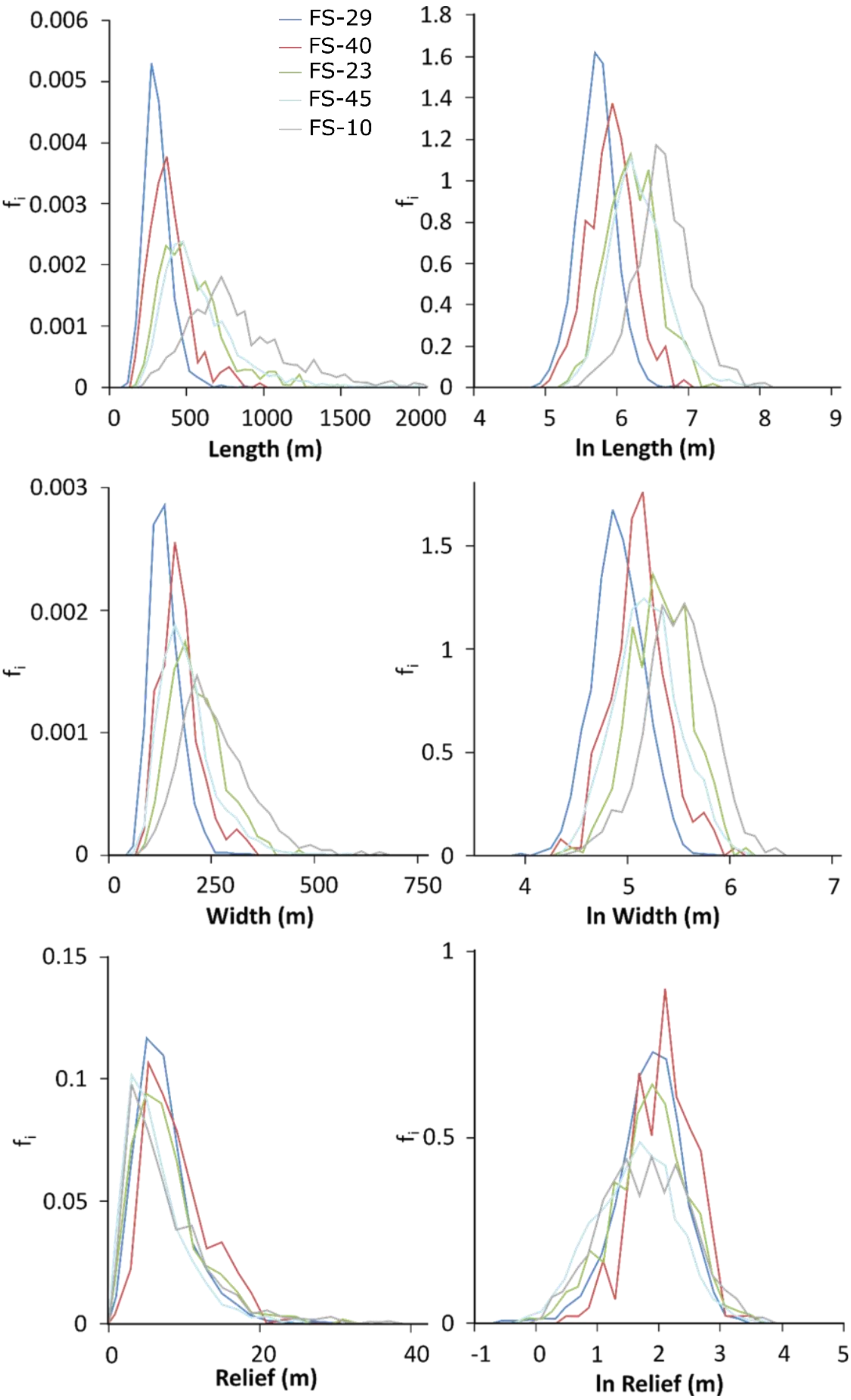
FS-29
FS-40
FS-23
FS-45
FS-10
f_i
0.006
0.005
0.004
0.003
0.002
0.001
0
0
500
1000
1500
2000
Length (m)
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
4
5
6
7
8
9
ln Length (m)
0.003
0.002
0.001
0
0
250
500
750
Width (m)
1.5
1
0.5
0
4
5
6
7
ln Width (m)
0.15
0.1
0.05
0
0
20
40
Relief (m)
1
0.5
0
-1
0
1
2
3
4
5
ln Relief (m)

Figure 7

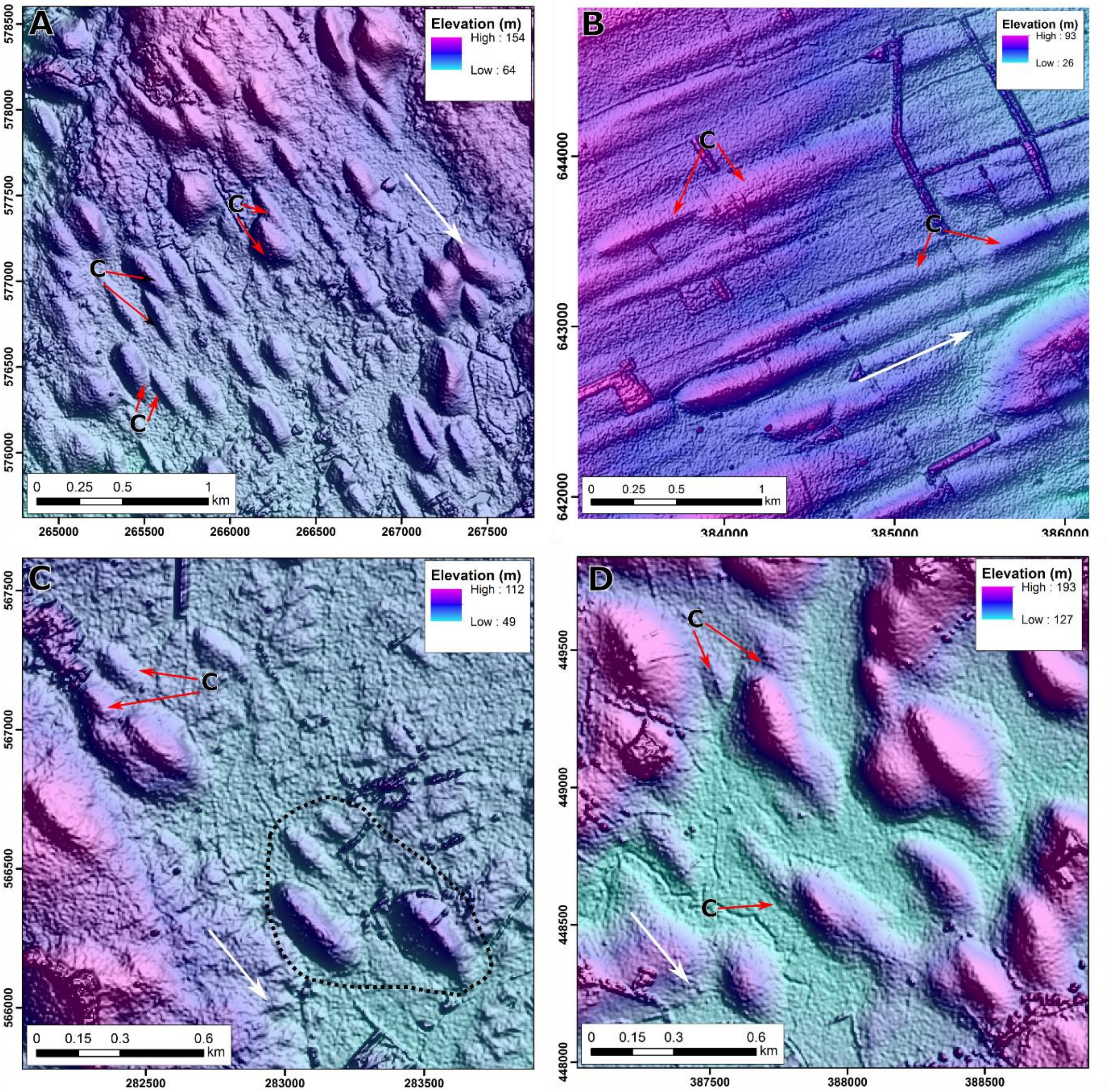
A
Elevation (m)
High : 154
Low : 64
C
C
C
0 0.25 0.5 1 km
B
Elevation (m)
High : 93
Low : 26
C
C
0 0.25 0.5 1 km
C
Elevation (m)
High : 112
Low : 49
C
0 0.15 0.3 0.6 km
D
Elevation (m)
High : 193
Low : 127
C
C
0 0.15 0.3 0.6 km

Figure 8

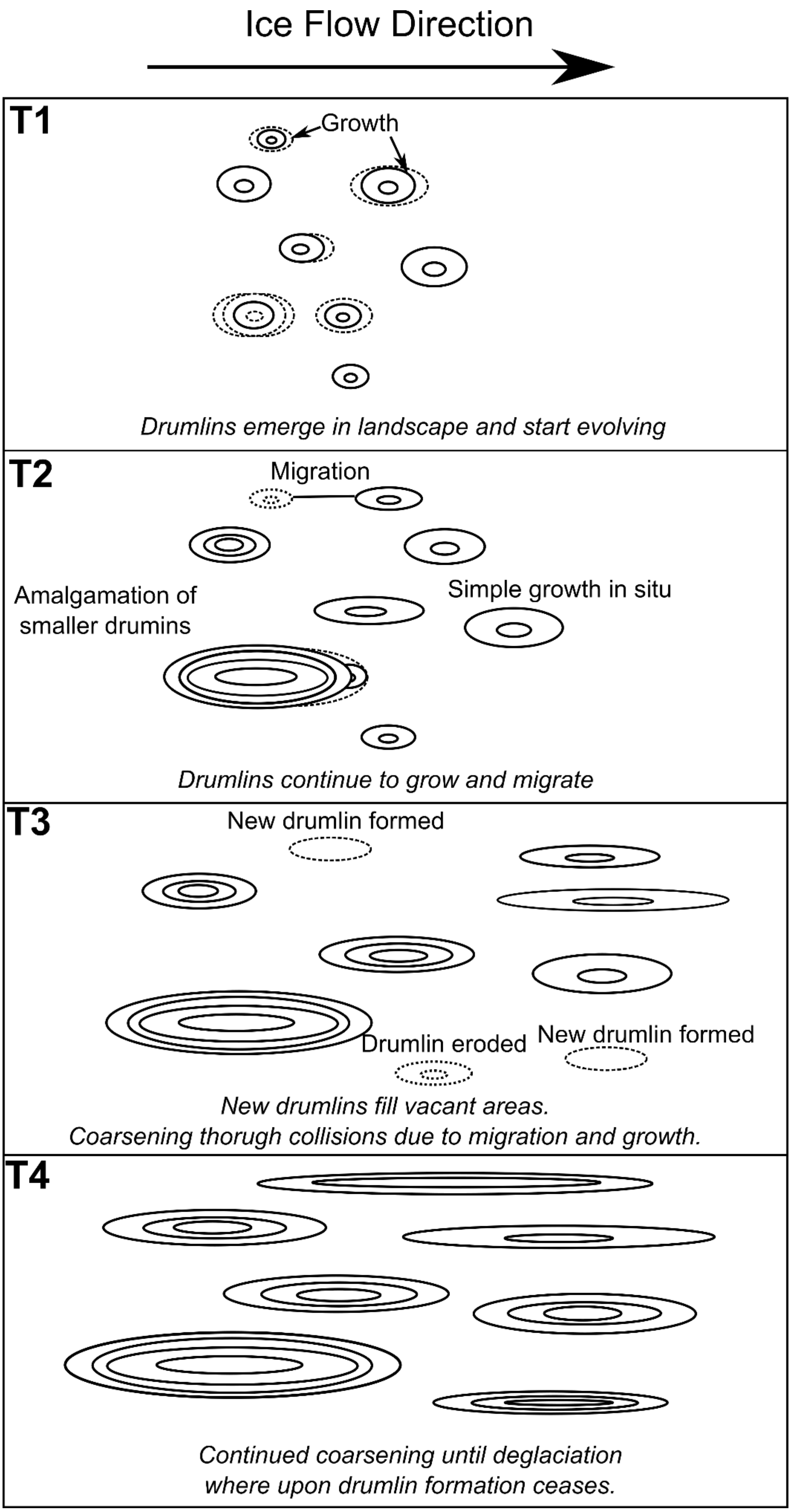
Ice Flow Direction
T1
Growth
Drumlins emerge in landscape and start evolving
T2
Migration
Amalgamation of smaller drumins
Simple growth in situ
Drumlins continue to grow and migrate
T3
New drumlin formed
Drumlin eroded
New drumlin formed
New drumlins fill vacant areas.
Coarsening thorugh collisions due to migration and growth.
T4
Continued coarsening until deglaciation where upon drumlin formation ceases.

Figure 9