PRIZE-BASED MECHANISMS FOR FUND-RAISING: THEORY AND EXPERIMENTS

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Abstract

We study the optimal design of mechanisms for the private provision of public goods in a setting in which donors compete for a prize of commonly known value. We discuss equilibrium bidding in mechanisms that promote both conditional cooperation and competition (i.e. the lottery and the all-pay auction with the lowest-bid payment rule) and rank their fund-raising performance vis-á-vis their standard (pay-your-own-bid) counterparts. The theoretically optimal mechanism in this model is the lowest-price all-pay auction – an auction in which the highest bidder wins the prize and all bidders pay the lowest bid. The highest amount for the public good is generated in the unique, symmetric, mixed-strategy equilibrium of this auction. In the laboratory, the theoretically optimal mechanism generates the highest level of donations with three bidders but not with two bidders.

JEL Classification: D44, D64

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"The safety net will be stretched thin in some places and eliminated entirely in others. For the functions government no longer will be able to provide, we must turn to neighbors, private charities, faith-based organizations, and other local programs. Our communities, more than ever, will be asked to step up."

— Chris Gregoire, Washington state governor, December 15, 2010

1 Introduction

With the steady growth of the philanthropic market and the decline of government funding for various areas of public life (e.g. arts, culture, public media, higher education, hospital services, environmental protection, etc.), the design of mechanisms for the private financing of public goods has substantially gained in importance. Most of the "solutions" to the public good under-provision problem rely on taxation and subsidy schemes to counterbalance free-riding incentives (Clarke, 1971; Groves, 1973; Groves and Ledyard, 1977; Walker, 1981). These schemes are not suitable for charitable fund-raising due to their reliance on coercive power; private organizations can use rewards but not punishment or coercion to elicit contributions.

Mechanisms in which prizes are awarded to donors – such as auctions and lotteries – have traditionally been employed to raise funds for charitable causes,[1] and more recently

[1] Auctions and lotteries are an essential part of the annual events of large philanthropic organizations such as the gala of the Robin Hood Foundation in New York City (Anderson, 2007), or the Winter Festival of the Naples Children and Education Foundation (Sullivan, 2016). Smaller organizations also routinely use incentive-based mechanisms on-line and on-site to raise funds. Among the most commonly used formats are the silent and live winner-pay charity auctions (Carpenter, Holmes and Matthews, 2010b; Popkowski Leszczyc and Rothkopf, 2010). In the past decade new consulting companies have emerged which offer assistance to small charities with various aspects of the fund-raising process including the items to be sold, venue, logistics, and mechanisms to be used. An example of such a consulting firm is the company Fundraising Auctions, which provides consulting services in the United Kingdom and Ireland. It supplies charitable organizations with the items to be auctioned, offers assistance with the logistics of the fund-raising event, provides advise on the fund-raising mechanism to be used, and offers software systems for the implementation of interactive

private organizations and government entities have begun experimenting with alternative fund-raising designs by adopting novel allocation and payment rules. Within the class of prize-based all-pay mechanisms, a variety of new designs have emerged, both static (e.g. lowest and highest unique price auctions) and dynamic (e.g. penny auctions).[2] Given the wide range of mechanisms used in practice, a new literature has developed that seeks to understand, both theoretically and experimentally, which mechanisms work best under different equilibrium and informational assumptions.

In this paper we contribute to this literature by posing the questions of (a) how much funds can be raised with award-based mechanisms, and (b) which method is best at raising funds. We explore, theoretically and experimentally, how changes in the allocation and the payment rule of the fundraising mechanism affect donations in a commonly studied complete information framework.

The potential of prize-based mechanisms to alleviate free-rider problems in public good provision has been illustrated by Morgan (2000) who compared lotteries with voluntary contributions in a complete information setting. In the voluntary contribution mechanism, when choosing their donations, contributors do not take into account that their individual contributions confer a benefit to all other agents. The failure to internalize this positive

winner-pay online auctions (see fundraising-auctions.co.uk).

[2]In the lowest unique price auction, for example, bidders pay a fee to place a bid while the winner is determined as the bidder with the lowest unmatched bid. Eichberger and Vinogradov (2015) present an equilibrium analysis of this mechanism and comment that it been used by TV and radio broadcasters not only as a marketing instrument but also to sell flats, houses and luxury cars. Some of these media companies, e.g. "London Capial FM" operate such prize-based mechanisms to support charitable causes. The lowest unique prize auction combines features of both an auction and a lottery as the highest contributor is not necessarily the winner. Östling, Wang, Chou, and Camerer (2011) term it "the lowest unique positive integer" and report that it has been used in Sweden by the government-owned gambling monopoly Svenska Spiel. Penny auctions, also called bidding fee auctions, have also become quite popular. They are dynamic formats in which a bidder pays a fee to incrementally increase the standing bid in an ascending price auction.

externality leads to systematic under-provision of the public good relative to the social optimum. A lottery, in which a prize is awarded to one of the contributors, creates an additional negative externality because an increased contribution by one bettor (a larger number of raffle tickets purchased) reduces the winning chances of the other bettors. Morgan (2000) makes the point that this negative externality (partially) offsets the public good externality and serves to narrow the gap between private benefits to donors and social benefits. As a result, in the lottery, equilibrium total contributions net of the prize are greater than the total donations in the voluntary contribution mechanism. Morgan and Sefton (2000) test these insights with laboratory experiments, confirming the superiority of lotteries over voluntary contributions.

Building on these insights, the more recent theoretical literature has considered charitable fundraising as a mechanism design problem in which the organizer can choose from a set of mechanisms. Goeree, Maasland, Onderstal and Turner (2005) and Engers and McManus (2007) derive optimal fund-raising mechanisms in the independent private value auction model using tools from the optimal auction design literature. Among the most significant theoretical insights of this literature is that all-pay mechanisms, such as a lottery and an all-pay auction generate more revenue than the traditional winner-pay auctions when the auction revenue is used to finance a public good. In an all-pay auction, topping a rival's winning bid does not eliminate their contribution to the public goods. It is therefore less costly for bidders to increase their bids in all-pay auctions – a property leading to the revenue superiority of all-pay auctions as fund-raising mechanisms. For the independent private value model, Goeree et al. (2005) show that the lowest-bid all-pay auction, augmented by an appropriately chosen entry fee and a reserve price, is an optimal mechanism. In this mechanism, the prize is awarded to the highest bidder and all bidders contribute an amount specified by the lowest bid submitted.[3]

While to the best of our knowledge the lowest-bid all-pay auction has so far not been used in practice, its performance has been analyzed in the laboratory. Carpenter, Holmes and Matthews (2010a) allow for endogenous participation in an incomplete information setting similar to the one analyzed in Goeree et al. (2005) and compare a variety of mechanisms. Participation in the lowest-bid all-pay format is slightly higher than that in the own-bid all-pay auction and slightly lower than that in the lottery with the differences being rather small. Orzen (2008) compares bidding in the lowest-bid all-pay auction, the lottery, the voluntary contribution mechanism (without a prize), and the lowest common denominator[4] in a symmetric complete information model in which bidders compete for a prize of commonly known value. He runs experiments with four participants all of whom have a given budget and decide how much of it to contribute to the public good. In his setting the sum of the marginal per capita return of contributions to the public good exceeds the marginal cost of an individual contribution so that the efficient allocation can be attained with various mechanisms. In particular, all players donating their entire budget emerges a Nash equilibrium not only in the lowest-bid all-pay auction but also in the lowest common denominator in which no prize is allocated. Indeed, donating their entire budget is as a weakly dominant strategy for players in both mechanisms. Orzen (2008) finds that subjects, in accordance

[3]For the case of two participating bidders, this mechanism is known as the "war of attrition" and its equilibria have been previously studied in the contest theory literature (see e.g. Milgrom and Weber, 1985). Goeree et al. (2005) show that this mechanism maximizes expected total contributions in the symmetric independent private value model with $n \geqslant 2$ bidders.

[4]The lowest common denominator employs the lowest-bid payment rule, but does not allocate a prize to contributors.

with theory, donate the highest amounts in these two mechanisms. In this setting, the observed behavior in the experiment approximates Nash equilibrium as subjects increase their contribution levels with experience.

In the present model, in contrast, we consider the alternative case in which the sum of the marginal per capita return of contributors is not sufficient to cover the cost of public good provision. In our setting we allow for the existence of bystanders: individuals who benefit from the public good but are unable contribute. In this setting, as we will show, no mechanism exists which implements the efficient allocation, and we study the revenue generating properties of prize-based mechanisms with different allocation and payment rules.

While we focus on an environment in which the marginal per capita return is constant (as in Goeree et al., 2005; Engers and McManus, 2007; Orzen, 2008; and Corazzini et al., 2010) we allow the mechanism designer to choose from the set of all sealed bid voluntary participation mechanisms. A mechanism in our model is generally defined by an *allocation rule* specifying how the prize will be awarded and a *payment rule* specifying how donations are determined as a function of the announced contribution levels. Participation in these mechanisms is voluntary in the sense that bidders cannot be asked to pay more than their bids and, thus, agents who bid zero do not contribute. In this setting the mechanism designer solves the problem of combining the allocation and the payment rule in such a way as to maximize the private benefit that donors receive from their contributions to the public good.

In our model there are $n \geqslant 2$ donors with a budget B who decide how much to contribute to a public good and how much to keep for private consumption. There are a total number of N beneficiaries from the public good (including the donors) with a marginal per capita return of α such that $\alpha N > 1 > \alpha n$. That is, we analyze a setting in which contributions are socially desirable, yet, the joint benefit of the public good to the donors – the individuals who have a budget and can contribute – does not cover the cost of public good provision. The analysis of optimal fund-raising mechanisms in this setting is important for several reasons.

First, such a setting is a common occurrence in practice, especially in charity fund-raising where the proceeds go to the benefit of others. Second, the mechanism design question is relevant from a theoretical standpoint because the free-riding problem cannot be resolved with trivial mechanisms. Indeed, in the present theoretical setting, players would not donate unless they are given the chance of winning a prize of a positive value.[5] Third, as we will show, when budgets are sufficiently large, mechanisms that produce the efficient outcome do not exist, and we focus on mechanisms that generate contributions closest to the social optimum. Finally, when the number of donors is small relative to the beneficiaries, and the incentives for free-riding are particularly strong, we can test empirically to what extent, and in which circumstances, prize-based mechanisms can solve the free-rider problem.

In our analysis we pay particular attention to six prize-based all-pay mechanisms which are obtained by combining three allocation rules and two payment rules. We allocate the prize by an auction, a lottery, or using a random assignment regardless of contribution amounts (i.e. Tullock contests with contest parameters ∞, 1 and 0), and we require bidders to pay either their own bid or the lowest bid submitted. While the Nash equilibria of mechanisms with the own-bid payment rule (the standard all-pay auction, the lottery and the voluntary contribution mechanism) have been been analyzed in the literature, no results are available

[5]For $\alpha n > 1$ there are simple mechanisms that implement the first-best outcome even without a prize. Asking all bidders to donate their entire budget B, and threatening not to provide the public good otherwise is a mechanism that implements the first-best outcome. Using the "lowest common denominator" is another way to implement the first-best outcome even without a prize (see Orzen, 2008).

for equilibrium bidding in the lowest-bid auction and the lowest-bid lottery. Yet, these mechanisms are interesting from a theoretical standpoint as they provide incentives both for competition and for conditional cooperation. We analyze here the equilibria of these two formats.

We show that the lowest-bid all-pay auction has a unique symmetric equilibrium which is given in mixed strategies. We derive the cumulative distribution function of the equilibrium mixed strategy in a closed form and show that this mechanism generates the highest expected revenue among all voluntary participation mechanisms. That is, the optimality result demonstrated by Goeree et al. (2005) for the incomplete information model carries over to the complete information (common value) setting. Expected revenue in the lowest-bid all-pay auction equals $V/(1-\alpha\, n)$ while the standard own-bid all-pay auction generates only $V/(1-\alpha)$, and the lottery generates the even lower amount of $(n-1)V/[n(1-\alpha)]$.[6]

The symmetric equilibrium in the lowest-bid lottery is also given in mixed strategies. While we could not derive the equilibrium mixed strategy in a closed form, we obtained a functional equation for its cumulative distribution function and showed that it has full support over an interval with upper bound given by the bidders' endowment. Further, we demonstrate that in every equilibrium bidders donate their entire endowment with a positive probability – a property that, as we show, holds also for the lowest-bid all-pay auction.

We further demonstrate that, when the budget is sufficiently low, the lowest-bid lottery has a unique equilibrium in which bidders donate their entire budget. For this case we

[6]For the analysis of the all-pay auction see, e.g. Hillman and Samet (1987); the lottery has been analyzed extensively in the contest literature, see e.g. Konrad (2009) for an overview of the recent literature. A summary of these theoretical findings and a derivation of the symmetric mixed strategy equilibrium in the own-bid all-pay auction with a budget constraint is provided in Orzen (2008).

obtain further ranking results. In particular, the lowest-bid lottery generates higher revenue than the standard lottery, but a lower revenue that the lowest-bid auction. Interestingly, the ranking between the own-bid all-pay auction and the lowest-bid lottery is sensitive to the benefit conferred by the public good and the number of bidders. For a sufficiently high level of the marginal per capita return, and a sufficiently high number of bidders, the lowest-bid lottery outperforms the own-bid all-pay auction.

The rest of the paper is organized as follows. In the next section we explain how our study contributes to the existing literature on the fundraising performance of prize-based mechanisms. In Section 3 we present our theoretical framework, analyze the equilibria of the lowest-bid lottery and the lowest bid auction, and derive a revenue ranking of the studied mechanisms. In Section 4 we present our experimental design and in Section 5 our experimental results. In the final section we present a summary of our main results, point to limitations of our study, and identify areas for future research.

2 Contribution to the literature

Most closely related to our study is the experiment by Orzen (2008) who finds that the mechanisms with the lowest-bid payment rule (the lowest-price auction and the lowest common denominator in which no prize is awarded) generate higher revenues than the lottery and the own-bid all-pay auction. Orzen (2008) attributes the success of the lowest-bid payment rule to its ability to promote "conditional cooperation" and conjectures that such manipulations of the incentive structure could substantially increase revenues for private charitable initiatives.

We complement Orzen's (2008) analysis by allowing for the existence of bystanders –

individuals without a budget who cannot contribute to the public good, yet benefit from the contributions of others. That is, we consider a setting in which the collective benefit of the public good to the donors is lower than their cost. In this theoretical setting, positive contributions to the public good can only be elicited through the use of prize-based mechanisms. Such mechanisms indeed have the potential to generate revenue in excess of the value of the prize.

We study the equilibria of the lowest-bid lottery and the lowest-bid auction and compare their performance vis-á-vis the own-bid all-pay auction and the own-bid lottery with laboratory experiments. We use groups of $N = 4$ agents and vary the number of bidders who have budgets and can contribute by considering treatments with $n = 2$ and $n = 3$ active bidders.[7] We find that the relative performance of the lowest-bid all-pay auction critically depends on the number of active donors. Among the six mechanisms, the theoretically optimal mechanism is superior in the laboratory with $n = 3$ active bidders but generates comparatively low revenues with $n = 2$ donors. To gain insights into the behavioral forces behind these results, we ran additional treatments of the lowest-bid all-pay auction in which we elicited beliefs from participants about the bidding behavior of their fellow donors. In the case of $n = 2$ active bidders subjects have lower expectations about the contribution level of their fellow donors compared to the $n = 3$ case. This point to the limitation of the lowest-bid payment rule as a way of promoting conditional cooperation, especially in cases in which

[7] The complete information model presents an experimental setting where only strategic uncertainty (uncertainty regarding bidding behavior) is present and there is no uncertainty about payoffs or other aspects of the environment. This helps us to draw inferences about impact of the allocation and payment rules on bidding behavior. As Gneezy and Smorodinsky (2006) argue, examining behavior in a complete information setting is an attempt to "differentiate between patterns of behavior induced by the mechanism itself and patterns induced by the complexity that is usually found in a 'real-world' environment." Our focus on the complete information version of Goeree et al.'s (2005) framework allow us to take advantage of this approach here.

the donors benefit from the public good relatively little compared to their cost. Yet, in a scenario in which the gap between public good benefit and cost is relatively small, the lowest-bid payment rule outperforms all other mechanisms.

Our paper also relates to the literature on fund-raising with lotteries and all-pay auctions. Lange, List and Price (2007) extend Morgan's (2000) model to allow for risk aversion and heterogeneity in the marginal per capita return of donors. They show that, when agents are sufficiently risk averse or sufficiently heterogeneous in the way they value the public good, multiple-prize lotteries are optimal. Their laboratory experiments provide further evidence that both single-prize and multiple-prize lotteries generate more revenue than voluntary contributions.

Corazzini, Faravelli and Stanca (2010) compare the fund-raising potential of the lottery, the all-pay auction, and the voluntary contribution mechanism in a setting of incomplete information regarding income levels. While the all-pay auction is the theoretically optimal mechanism in this setting, they find that the lottery generates higher revenue in the laboratory; both incentive-based mechanisms outperform the voluntary contribution mechanism theoretically and empirically. In a more recent door-to-door experiment, Onderstal, Schram and Soetevent (2013) also compare the own-bid all-pay auction with the lottery, and two versions of a voluntary contribution mechanism. Controlling for various factors related to the solicitor, they find that the auction generated the least revenue despite theoretically being the superior mechanism.

Our results accord with the recent advancements in the contest literature which highlight the reasons for the underperformance of the standard lottery and suggest various ways to enhance competition in this format. Faravelli and Stanca (2012) argue that the smoothness of the payoff function associated with the randomness of the allocation prescribed by the lottery is responsible for its underperformance relative to the deterministic contest (i.e. the own-bid all-pay auction). They show that, by restricting the number of tickets sold, the decreasing marginal utility in the lottery can be eliminated so that the lottery contest can produce (almost) full dissipation.[8] In accordance with this intuition, we show that switching from a lottery to an auction would enhance revenue also under the lowest-bid payment rule. Other ways to enhance revenue in the lottery, in particular when bettors have different valuations, includes offering discounts on multiple tickets (Damianov, 2015) or setting individualized prices for tickets sold to different contestants (Franke, Kanzow, Leininger and Schwartz, 2014; Franke and Leininger, 2014).

Another implication of our model is that revenue in the lottery can be enhanced not only by switching to the auction allocation rule but also by switching to the lowest-bid payment rule. Intuition for this latter result can be developed either by analyzing the equilibrium structure or by exploring the incentives for bidders to increase their bids. As we show, in the equilibria of both the own-bid and the lowest-bid auction, bidders randomize continuously over an interval with a lower bound of zero. As zero bids belong to the mixed strategy support in both auctions, the payment rule allows a straightforward calculation of the symmetric equilibrium expected payoff of bidders. In particular, zero bids leads to lower expected payoff in the lowest-bid all-pay auction because a zero bid eliminates not only the chances for winning the prize, but also all contributions of the other bidders. As we will argue, lower expected payoffs of bidders in our setting correspond to higher expected

[8]We would like to thank an anonymous reviewer for drawing our attention to this strand of the contest literature.

donations. Therefore, as we will show, the donations in the lowest-bid auction exceed the ones in the own-bid all-pay auction.

3 Theoretical model and analysis

A number of $n \geqslant 2$ participants in a fund-raising event decide how to divide their budget B between contribution to a public good and personal consumption. The value of the public good for each participant is a constant fraction α of total funds raised for the public good, where $\alpha < \frac{1}{n}$. In this theoretical setting, using a voluntary contribution mechanism results in zero donations because of free-riding incentives.[9]

3.1 Mechanisms

To alleviate the free-riding problem, a charitable organization awards a prize V to one of the donors. The organization chooses the mechanism according to which the prize V will be awarded, and we analyze the level of contributions that can be raised by different allocation and payment rules. In the fund-raising mechanisms considered, each participant $i = 1, 2, \ldots, n$ announces her willingness to contribute to the public good, x_i. The mechanisms consist of an *allocation rule* $P_i(x_i, x_{-i})$ which specifies the relationship between the announced contribution levels of donors and their probabilities of winning the prize, and a *payment rule* $C_i(x_i, x_{-i})$ which specifies how the actual donations are determined based

[9]When $\alpha \geqslant \frac{1}{n}$, the free-rider problem can easily be resolved with simple arrangements according to which all contributors agree to donate a certain amount, and if at least one of them does not contribute, the public good is not provided (see Bagnoli and Lipman, 1989; Bagnoli and McKee, 1991; Orzen, 2008). One notable disadvantage of these mechanisms is that they do not generate donations when $\alpha < \frac{1}{n}$. This case appears to be quite relevant in the practice of fund-raising because the set of potential beneficiaries from the public good is much larger than the original set of donors. In fact, the beneficiaries of charitable donations are often outside the set of the contributors. We therefore focus on the case $\alpha < \frac{1}{n}$, although the mechanisms we explore solve the free-riding problem in the alternative case as well.

on the announced contribution levels, where $x_{-i} = (x_j)_{j \neq i}$. If donors are risk neutral and participate in a mechanism with an allocation rule $P_i(x_i, x_{-i})$ and a payment rule $C_i(x_i, x_{-i})$, the expected payoff of donor i is given by

$$\Pi_i(x_i, x_{-i}) = [\, B - C_i(x_i, x_{-i}) \,] + \alpha \cdot \textstyle\sum_{j=1}^{n} C_j(x_j, x_{-j}) + P_i(x_i, x_{-i}) \cdot V.$$

The first term presents the payoff from private consumption. The second term captures the benefit from the public good, and the third term defines the expected benefit from winning the prize.

In our experiment we study the performance of the following six mechanisms constructed by combining three allocation rules P_i and two payment rules C_i:

$$P_i(x_i, x_{-i}) = \begin{cases} \frac{1}{n} & \text{(random [RND])} \\ \frac{x_i}{\sum_{j=1}^{n} x_j} \text{ if } \sum_{j=1}^{n} x_j > 0 \text{ and } \frac{1}{n} \text{ otherwise} & \text{(lottery [LOT])} \\ \frac{1}{|\operatorname{argmax}_{j=1,\ldots,n} x_j|} \cdot \mathbb{1}_{[i \in \operatorname{argmax}_{j=1,\ldots,n} x_j]} & \text{(auction [AUC])} \end{cases}$$

and

$$C_i(x_i, x_{-i}) = \begin{cases} x_i & \text{(own-bid [OWN])} \\ \min_{j=1,\ldots,n} x_j & \text{(lowest-bid [LOW])} \end{cases}$$

Thus, the considered six mechanisms include two versions of the voluntary contribution mechanism (OWN–RND and LOW–RND), the standard lottery and the lowest-bid lottery (OWN–LOT and LOW–LOT) as well as the the standard all-pay auction and the lowest-bid all-pay auction (OWN–AUC and LOW–AUC). Next we present the analysis of symmetric equilibria.

3.2 Equilibrium analysis

We begin with a strategic equivalence argument which allows us to analyze all mechanisms in a setting without a public good.

Lemma 1 (**Strategic equivalence**). *Mechanisms with the own-bid payment rule in a public good setting are strategically equivalent to mechanisms in a setting without a public good in which a prize of the amount* $\frac{V}{1-\alpha}$ *is allocated. Mechanisms with the lowest-bid payment rule in a public good setting are strategically equivalent to mechanisms in a setting without a public good in which a prize of the amount* $\frac{V}{1-\alpha\, n}$ *is allocated.*

Proof. We show here that the claim holds for the lowest-bid payment rule. The proof for the own-bid payment rule is analogous. The payoff of participant i in the lowest-bid mechanisms is given by

$$\begin{aligned}
\Pi_i^{\text{LOW}}(\alpha, V)(x_i, x_{-i}) &= B - \min_j x_j + \alpha\, n \cdot \min_j x_j + P_i(x_i, x_{-i}) \cdot V \\
&= \alpha\, n \cdot B + (1 - \alpha\, n) \cdot (B - \min_j x_j + P_i(x_i, x_{-i}) \cdot \tfrac{V}{1-\alpha\, n}) \\
&= \alpha\, n \cdot B + (1 - \alpha\, n) \cdot \Pi_i^{\text{LOW}}(0, \tfrac{V}{1-\alpha\, n})(x_i, x_{-i}),
\end{aligned}$$

which is an affine transformation of the setting without a public good and a prize of $\frac{V}{1-\alpha\, n}$.

□

As indicated previously, the voluntary contribution mechanisms generate expected revenue of zero because bidders lack incentives to compete. Orzen (2008) derives the equilibria of the own-bid all-pay auction and the lottery by taking into account the budget constraint.[10] Orzen's (2008) results can be summarized as follows.[11]

Proposition 1 (**OWN–AUC and OWN–LOT (Orzen, 2008)**). *The own-bid auction has a unique symmetric equilibrium. When* $B > \frac{V}{n\,(1-\alpha)}$ *the equilibrium is given in mixed*

[10]While Orzen (2008) works directly with the original payoff functions, an alternative way to derive the equilibria of the mechanisms with the own-bid payment rule is by using Lemma (1) to transform the considered setting into an alternative one without a public good but with an inflated prize, and then invoking results from the existing contest literature (e.g. Hillman and Samet 1987; Konrad 2009).

[11]While we consider the case $\alpha < \frac{1}{n}$ here, Orzen's (2008) analysis applies for the current setting as well.

strategies. Bidding zero belongs to the support of the mixed strategy distribution. Expected revenue equals $\frac{V}{1-\alpha}$. *When* $B \leqslant \frac{V}{n\,(1-\alpha)}$ *bidders donate their entire budget.*

The lottery has a unique symmetric equilibrium given in pure strategies. When $B > \frac{n-1}{n} \cdot \frac{V}{n\,(1-\alpha)}$ *bidders donate* $\frac{n-1}{n} \cdot \frac{V}{n\,(1-\alpha)}$ *and expected revenue equals* $\frac{n-1}{n} \cdot \frac{V}{(1-\alpha)}$. *When* $B \leqslant \frac{n-1}{n} \cdot \frac{V}{n\,(1-\alpha)}$ *bidders donate their entire budget.*

Next we consider the lowest-bid formats. Let bidder i play the pure strategy $x_i = x$ while all other bidders play a mixed strategy given by the cumulative distribution function F. Further, for notational convenience, let us denote the strategy space of bidder j by $S_j = [0, B]$. The expected payoff of bidder i in the lowest-bid lottery is given by

$$\mathbb{E}_{x_{-i}}[\Pi_i^{\text{LOW–LOT}}(0, \tfrac{V}{1-\alpha\, n})(x, F)] =$$

$$B - \left[\int_0^x y\, d(1-(1-F(y))^{n-1}) + (1-F(x))^{n-1} \cdot x \right]$$

$$+ \frac{V}{1-\alpha\, n} \cdot \int\limits_{\times_{j \neq i} S_j} \frac{x}{x + \sum_{j \neq i} x_j} dF(x_{-i}) \tag{1}$$

where $F(x_{-i}) = F(x_1) \cdots F(x_{i-1}) \cdot F(x_{i+1}) \cdots F(x_n)$ is the joint probability distribution of the bids of all bidders except bidder i. The two terms in the squared brackets represent the expected payment of bidder i from playing the pure strategy x. The first term is the expected payment when at least one of the players bids below x, which equals to the lowest-order statistic of all other bidders, conditional on this lowest-order statistic being below x. The second term is the probability that all other players bid above x, multiplied by the payment x. The last term in the above expression is the expected gain from winning the prize (probability of winning multiplied by adjusted value of the prize).

Similarly, the expected payoff of bidder i in the lowest-bid auction is given by

$$\mathbb{E}_{x_{-i}}[\Pi_i^{\text{LOW-AUC}}(0, \tfrac{V}{1-\alpha n})(x, F)] =$$

$$B - \left[\int_0^x y\, d(1-(1-F(y))^{n-1}) + (1-F(x))^{n-1} \cdot x\right]$$

$$+\frac{V}{1-\alpha n} \cdot F(x)^{n-1} \tag{2}$$

We begin with the analysis of the lowest-bid lottery.

Proposition 2 (**LOW–LOT**). *The lowest-bid lottery has a symmetric equilibrium.*

(A) High budget constraint: $B > \frac{n-1}{n} \cdot \frac{V}{n\,(1-\alpha n)}$. *In a symmetric equilibrium players employ mixed strategies. The cumulative distribution function $F(x)$ satisfies the functional equation*

$$\frac{V}{1-\alpha n} \cdot \int\limits_{\times_{j\neq i} S_j} \frac{\sum_{j\neq i} x_j}{(x + \sum_{j\neq i} x_j)^2} dF(x_{-i}) = (1-F(x))^{n-1} \tag{3}$$

for all x in its support. F(x) has a full support on an interval $[b, B]$ where $0 \leqslant b < B$. The equilibrium mixed strategy has a mass point at B, i.e. bidders contribute their entire budget with a positive probability.

(B) Low budget constraint: $B \leqslant \frac{n-1}{n} \cdot \frac{V}{n\,(1-\alpha n)}$. *In the unique symmetric equilibrium bidders contribute their entire budget.*

Proof. See Appendix A. □

While for Case (A) of the proposition we do not have sufficient information on the structure of equilibrium in order to calculate the expected revenue, we know that the revenue is strictly less than $n\,B$.[12] As we will see in the subsequent analysis, this information will be sufficient to establish that the LOW–LOT generates less revenue than the LOW–AUC format. For Case (B) the revenue equals $n\,B$ which allows us to establish revenue ranking of this mechanism vis-á-vis all other mechanisms. In the following proposition we describe the symmetric equilibrium of the LOW–AUC format.

Proposition 3 (**LOW–AUC**). *The lowest-bid auction has a unique symmetric equilibrium.*

(A) High budget constraint: $B > \frac{V}{n\,(1-\alpha\,n)}$. *The equilibrium is given in mixed strategies. Bidders randomize continuously on the interval* $[0, b]$ *according to the cumulative distribution function* $F(x)$ *and donate their entire budget with a probability of* $1 - F(b)$. *The function* $F(x)$ *is determined by the unique solution to the differential equation*

$$F'(x) = \frac{1-\alpha\,n}{V} \cdot \frac{(1-F(x))^{n-1}}{(n-1)F(x)^{n-2}} \tag{4}$$

with an initial condition $F(0) = 0$. *The cutoff value* b *is determined by the unique solution to the equation*

$$\frac{V}{1-\alpha\,n} \cdot \Big[\, \frac{1-F(b)^n}{n\,(1-F(b))} - F(b)^{n-1} \Big] = [1-F(b)]^{n-1}(B-b).$$

Expected revenue equals $\frac{V}{1-\alpha\,n}$.

(B) Low budget constraint: $B \leqslant \frac{V}{n\,(1-\alpha\,n)}$. *In the unique equilibrium bidders donate their entire budget.*

Proof. See Appendix A. □

The differential equation presented in equation (4) can further be used to analyze the mixed

[12] The equilibrium probability distribution and expected revenue in this case can be calculated using numerical methods.

strategy distribution function. In Proposition 5 presented in Appendix A, we derive a closed form solution for the inverse of the cumulative distribution function $F(x)$ for any number of bidders.

A notable implication of the equilibrium analysis is that expected revenue in the lowest-bid all-pay auction increases in the number of active bidders n, while revenue in the own-bid all-pay auction is independent of n. The increasing revenue is a consequence of the shill bidding effect inherent to the lowest-bid payment rule: when the lowest bidder increases his bid, he raises not only his own contribution, but also the contribution of the other bidders. As demonstrated in Lemma 1, this effect is stronger when the number of active bidders is greater. The differences in incentives created by the two payment rules can also be observed by comparing the consequences of submitting a zero bid in the auction. While a zero bid eliminates the contributions of the other bidders under the lowest-bid payment rule, it has no effect under the own-bid rule. As zero belongs to the equilibrium mixed strategy support under both payment rules, the expected revenue (E) in the two mechanisms is given by the following equations:

$$V + \alpha \cdot n \cdot E^{\text{LOW–AUC}} - E^{\text{LOW–AUC}} = 0 \;\Leftrightarrow\; E^{\text{LOW–AUC}} = V/(1 - \alpha\, n)$$

$$V + \alpha \cdot n \cdot E^{\text{OWN–AUC}} - E^{\text{OWN–AUC}} = (n-1) \cdot E^{\text{OWN–AUC}} \;\Leftrightarrow\; E^{\text{OWN–AUC}} = V/(1 - \alpha)$$

The left hand-side gives the payoff of a player under the equilibrium strategy while the right hand-side represents the payoff of bidding zero. As in a symmetric mixed strategy equilibrium these payoffs are the same, we obtain the expected revenue in the two formats without the need to derive the exact form of their mixed strategy equilibria.

3.3 Expected revenue ranking

From the derived results it is straightforward to establish the following revenue ranking of mechanisms: OWN–RND = LOW–RND < OWN–LOT $\leqslant$ OWN–AUC $\leqslant$ LOW–AUC with the strict inequality applying for sufficiently high values of B. It remains to discuss where the LOW–LOT format fits in this ranking. We first consider Case (B) specified by the inequality $B \leqslant \frac{n-1}{n} \cdot \frac{V}{n\,(1-\alpha\,n)}$. For this case the LOW–LOT mechanism strictly outperforms OWN–LOT when $B > \frac{n-1}{n} \cdot \frac{V}{n\,(1-\alpha)}$ and generates the same revenue when $B \leqslant \frac{n-1}{n} \cdot \frac{V}{n\,(1-\alpha)}$. Further for $\frac{V}{n\,(1-\alpha\,n)} \geqslant B > \frac{n-1}{n} \cdot \frac{V}{n\,(1-\alpha\,n)}$ we can establish that the LOW–AUC format in which all bidders donate B strictly outperforms the LOW–LOT format in which bidders play a mixed strategy (see Proposition 2). Finally, we find that the LOW–LOT format can generate more or less revenue than the OWN–AUC format depending on the parameter values.

Let us consider next the case $B \in \left(\min\{\frac{n-1}{n} \cdot \frac{V}{(1-\alpha\,n)}, \frac{V}{1-\alpha}\}, \max\{\frac{n-1}{n} \cdot \frac{V}{(1-\alpha\,n)}, \frac{V}{1-\alpha}\}\right)$. For this case, the revenue in the LOW–LOT format is strictly greater than the revenue in the OWN–AUC format when $\frac{n-1}{n} \cdot \frac{V}{(1-\alpha\,n)} > \frac{V}{1-\alpha} \Leftrightarrow \alpha > \frac{1}{n^2-n+1}$ and strictly lower when $\frac{n-1}{n} \cdot \frac{V}{(1-\alpha\,n)} < \frac{V}{1-\alpha} \Leftrightarrow \alpha < \frac{1}{n^2-n+1}$. An intuition for this finding can be gained by considering that, when the public good has a greater marginal per capita return, the shill bidding effect associated with the lowest-bid rule more than compensates for the lower level of competition associated with the randomness in the allocation of the prize.

3.4 Optimality of the lowest-bid auction

In the previous section we established that the LOW–AUC is the best performing mechanisms from the ones we study. Here we will show that from the class of voluntary participation mechanisms, no other mechanism can generate more revenue. To develop intuition for this result, note that in the studied mechanisms each participant submits a bid, and the payment rule of the mechanism determines the size of the donations of all participants depending on the bids submitted. A bidder cannot be asked to pay more than their expressed willingness contribute, and a participant who bids zero does not make a contribution to the public good. In that sense, contributions in the mechanism are voluntary. The penalty for non-contributors consists of being assigned zero probability for winning the prize and possibly in a reduction of the overall provision of the public good. Note that among all voluntary mechanisms, the lowest-bid all-pay auction entails the most severe penalty to non-contributors as it eliminates all the contributions of the other players and assigns a zero chance for winning the prize. Thus, the lowest-bid all-pay auction should generates the largest amount of donations as it is most effective at "punishing" non-contributors. The next proposition supports this intuition.

Proposition 4 (**Optimality of LOW–AUC**). *Among all mechanisms which transfer a total value V to bidders, the lowest-bid auction generates the highest expected revenue in its symmetric equilibrium.*

Proof. We denote by $\varphi_i := \varphi_i(F_i, F_{-i})$ the transfer that a voluntary participation mechanism prescribes to bidder i in a symmetric mixed strategy Nash equilibrium.[13] Similarly, we denote by $C_i := C_i(F_i, F_{-i})$ the expected contribution of bidder i in the symmetric equilibrium of a voluntary mechanism. Recall that in a voluntary mechanism a bidder who submits a bid of zero is not required to pay. We now establish an upper bound on the total revenue generated by any mechanism using the voluntary participation constraint. For each i, this

[13] For mechanisms in which a single prize is awarded, this transfer equals $\varphi_i(F_i, F_{-i}) = P_i(F_i, F_{-i}) \cdot V$.

constraint is given by $C_i \leqslant \varphi_i + \alpha \cdot \sum_{j=1}^{n} C_j$. Summing over all i we obtain

$$\sum_{i=1}^{n} C_i \leqslant \sum_{i=1}^{n} \varphi_i + \alpha\, n \cdot \sum_{j=1}^{n} C_j \iff \sum_{j=1}^{n} C_j \leqslant V + \alpha\, n \cdot \sum_{j=1}^{n} C_j \iff \sum_{j=1}^{n} C_j \leqslant \tfrac{V}{1-\alpha\, n}.$$

Thus, due to the participation constraint, no higher revenue can be generated by any voluntary mechanism. □

4 Experimental design

We conducted a laboratory experiment to compare the fund-raising performance of the considered six mechanisms. In order to explore the potential role of competition, we added the number of active agents (i.e. agents who have a budget and can contribute to the public good) as another treatment variation. In total our experiment consisted of twelve treatments (two for each mechanism).

Between April 2009 and January 2011, students from [**blinded for review purposes**] were recruited to participate in an experiment on economic decision-making in which money can be earned. During the experiment, subjects were seated behind isolated computer terminals, via which the experiment was run. After subjects read the instructions, answered the control questions correctly, and eventual clarifying questions were answered, the z-Tree software (Fischbacher, 2007) was started.[14] It was made clear that they would be paid in cash at the end of the session and they earned on average 20.28 US dollars for a session lasting approximately 75 minutes.

For each of our twelve treatments, one experimental session was conducted with twenty different subjects. In order to allow these subjects to familiarize themselves with the mech-

[14]Instructions for one of the treatments are included in Appendix B.

anism they are participating in, we conducted sessions of twenty rounds. At the beginning of each round, subjects were randomly assigned to groups of size four. To limit supergame effects, reciprocity, possibilities to collude, or other types of interdependence of choices, subjects were not aware of whom they were grouped with, but they did know that the group composition changed every round. Depending on the treatment, either two or three randomly chosen subjects in each quadruple were assigned the role of *active agent* while the remaining agent(s) were assigned the role of *passive agent*. The active agents had a budget of 100 tokens, and had to specify an amount they are willing to contribute (their bid) to the public good, whereby any integer between 0 and 100 could be chosen. The passive agents had no budget and benefited only from the donations of the active agents.

In the treatments with own-bid payment rule, the actual contribution for each active bidder corresponded to her own willingness-to-contribute, while in the treatments with lowest-bid payment rule, the actual contribution for an active agent was determined by the lowest willingness-to-contribute specified by the active agents in her group. For each active agent, the actual contribution was subtracted from the given budget. The benefit from the public good, both for active and passive agents, equaled the sum of the actual contributions multiplied by the marginal per capita return of 0.3.

In addition to these earnings, a prize of 20 tokens was allocated to one of the active agents in a group. In the random treatments, the prize was randomly assigned to one of the active agents. In the lottery treatments, the probability of winning the prize was proportional to the agent's willingness-to-contribute. In the auction treatments, the prize was awarded to the agent with the highest willingness-to-contribute (ties were resolved at random).

After each round of play, subjects received information on the willingness-to-contribute of all active agents, their own contribution (in case the subject was an active agent), the total amount of contributions, whether they won the prize, and their entire payoff. In order to make sure that subjects took notice of this feedback, and hence become aware of the outcomes of their decisions, they were asked to record part of it on paper. The payoff at the end of each session was determined by a random selection of one of the twenty rounds. So, while participants experienced both the roles of an active agent and a passive agent during the course of the 20 rounds, the final payoff was determined on the basis of their role in the round selected for payment. These tokens were converted into dollars at a rate of 0.15 US dollars per token.

In the experiment, the marginal per capita return was chosen in such a way that all active agents donating their entire budget was socially optimal when we take into account the payoffs of both the active and the passive agents. Each token donated to the public good generated a return of 1.2 tokens for the group of four. For the sub-society of the active bidders, however, the benefit of providing the public good did not cover the cost.[15] Exploring this scenario is of particular interest because in reality the donors are often only a subset of the beneficiaries of the public good. Even more importantly, in this scenario the free-riding problem cannot be resolved without prize-based mechanisms or, more generally, mechanisms that transfer some value to the donors. These types of mechanisms are the particular focus of our analysis.

Table 1 presents the expected actual contribution levels per donor in the limiting logit equilibrium of the discrete version of the mechanisms that were implemented in the experi-

[15]With $\alpha = 0.3$ the total return of 1 token invested in the public good was 0.6 of a token in the case of $n = 2$ active players and 0.9 of a token in the case of $n = 3$ active players.

ment. This limiting logit equilibrium, as introduced by McKelvey and Palfrey (1995), is the unique Nash equilibrium found when assuming that the random utility component is extreme value distributed and letting the respective noise parameter diverge. For the mechanisms that we are able to solve analytically on the continuous domain (all except the lowest-bid lottery), the expected actual contributions in this limiting logit equilibrium are quite close to that of the analytical solution for the continuous domain.[16]

		RND	LOT	AUC
OWN	$n = 2$	0.00	8.00	14.00
	$n = 3$	0.00	6.00	9.12
LOW	$n = 2$	0.00	19.93	24.03
	$n = 3$	0.00	64.50	66.31

Table 1: Individual (expected) actual contributions according to Nash equilibrium behavior.

The numbers in the table conform to the revenue ranking established in our theoretical analysis. First, the mechanisms that entail competition for the prize (via either all-pay auctions or lotteries) generate larger total donations compared to the random assignment of the prize. Second, comparing the all-pay auction with the lottery, we find that the auction is more effective in generating donations. Third, the lowest-bid payment rule generates more revenue than the own-bid payment rule for the lottery and the all-pay auction.

5 Experimental results

In this section we investigate whether there is empirical evidence for the established theoretical differences in revenue in the six treatments. We begin with a discussion of the issue of dependence of observations in our data (Section 5.1) and then present summary statistics

[16]To solve for the limiting logit equilibrium, we used the QRE-solver of Gambit (see McKelvey, McLennan and Turocy, 2014; http://www.gambit-project.org). For reasons of computational complexity we used a grid twice as rough as that used in the experiment.

and graphically illustrate the dynamics of contributions over rounds (Section 5.2). Further we analyze the impact of the allocation rule (Section 5.3) and the payment rule (Section 5.4) on individual contributions. Finally, we explore whether the lowest-bid all-pay auction is the optimal mechanism and describe the results of additional experiments that we conducted in order to understand the reasons for the discrepancies that we observed between theoretical predictions and behavior in the experiment (Section 5.5).[17]

5.1 Unit of observation and (in)dependence

To compare the performance of the mechanisms, we use the average individual actual contribution as the unit of observation. As subjects interact repeatedly, subject-level observations may lack independence. Yet, our experiment is particularly designed to limit the degree of such dependence. First, subjects are randomly rematched in five groups of four over twenty rounds, so that the probability of each subject interacting with the same players as in the previous round is rather small. Second, subjects are randomly assigned to roles (active or passive agent) within groups, which further reduces the probability that the same subject(s) interact in two consecutive rounds. Third, identities of subjects are not revealed before or after decisions are made. Fourth, payments are determined according to an ex-post randomly selected round, which limits supergame incentives and reasoning. All four features of the design serve to confine possible dependence across individual observations.

In addition, as in Corazzini et al. (2010), we investigate the issue of dependence em-

[17]While the presence of bystanders is a distinctive feature of our setting, the number of bystanders differs between our treatments. Understanding the pure effect of the presence and the number of bystanders requires that the number of active agents is held fixed which is beyond the scope of this paper. Such a question would require a design where the number of bystanders is varied while keeping the number of donors fixed, with treatment variations where either the marginal per capita return α or the social benefit of donation αN is kept fixed (while staying within the condition $\alpha n < 1 < \alpha N$ for all treatments).

pirically. Spearman rank correlation tests for the null hypothesis of independence between the willingness-to-contribute of each subject and the average willingness-to-contribute of the subjects who were in the same group in the previous period, show significant correlation at the 10% level only in 12.9% of the cases (31 out of 240).[18] Further, in the significant cases, 74.2% of the correlation coefficients are positive and 25.8% are negative, indicating that there is no systematic pattern in the relationship between the contributions of subjects in one period and of their group members from the previous period.

On the basis of both the features of our experimental design and the results of the Spearman tests, we consider the dependence across individual decisions to be negligible so that we can draw inferences using subjects as the unit of observation.

While individual willingness-to-contribute decisions may be considered independent, and this directly implies independence of individual actual contributions for the treatments with the own-bid payment rule, the actual individual contributions in the treatments with the lowest-bid payment rule are strongly dependent. To attribute the contribution of a bidders to their willingness-to-contribute and not the lowest bid submitted in the group, we constructed individual *expected* actual contributions for these treatments.

First, we construct for each treatment an empirical distribution of all bids observed in all rounds. Then, for each subject, and each round in which this subject is active, we calculate the expected actual contribution of this individual on the basis of her own willingness-to-contribute in this round, assuming that the other donors in her group randomize according

[18]Corazzini et al. (2010) performs these tests on 5% level. Since subjects are passive in some rounds, we have less observations per subject. For that reason we decided to take a more conservative test. Despite using a more conservative test, we can report a lower percentage of correlation (13% versus 15%). The respective percentage on 5% level is in our case 5.8% (14 out of 240).

to the empirical distribution. The individual expected actual contribution of the subject is given by the average of these expected actual contributions over all rounds in which this individual is active.

In order to control for the effect of experience on decisions and outcomes, we also construct the individual expected actual contributions on the basis of the data gathered in the first half and in the second half of the experiment. For these variables, both the empirical distributions used to construct the expected actual contributions and the averaging over the individuals' expected actual contributions are restricted only to the first or to the second half of the rounds.

Tables 7–9 in Appendix D present, for each treatment, the average values of the three key variables (that is, willingness-to-contribute, actual contribution, and expected actual contribution) and the correlations between them. For the mechanisms with the own-bid payment rule, these averages are identical. For the mechanisms with the lowest-bid payment rule, we observe that the actual contributions are very close to the expected actual contributions. Furthermore, the amounts that subjects are willing to contribute almost perfectly correlate with the expected actual contributions. Under the random allocation rule and the lowest-bid payment rule, the constructed synthetic measure of the expected actual contribution can be interpreted as the pure attitude of bidders to contribute that is analogous to the pure attitude to contribute expressed in the bids under the own-bid payment rule.

5.2 Summary statistics and timing effects

Figure 1 shows the dynamics of the average individual (expected) actual contributions over rounds in the different treatments. The first row of graphs corresponds to the own-bid payment rule and the second row to the lowest-bid payment rule. The three columns of graphs correspond to the random, the lottery and the auction allocation rule, respectively. In each graph the dashed curve corresponds to the treatment with two bidders and the solid curve to that with three bidders.

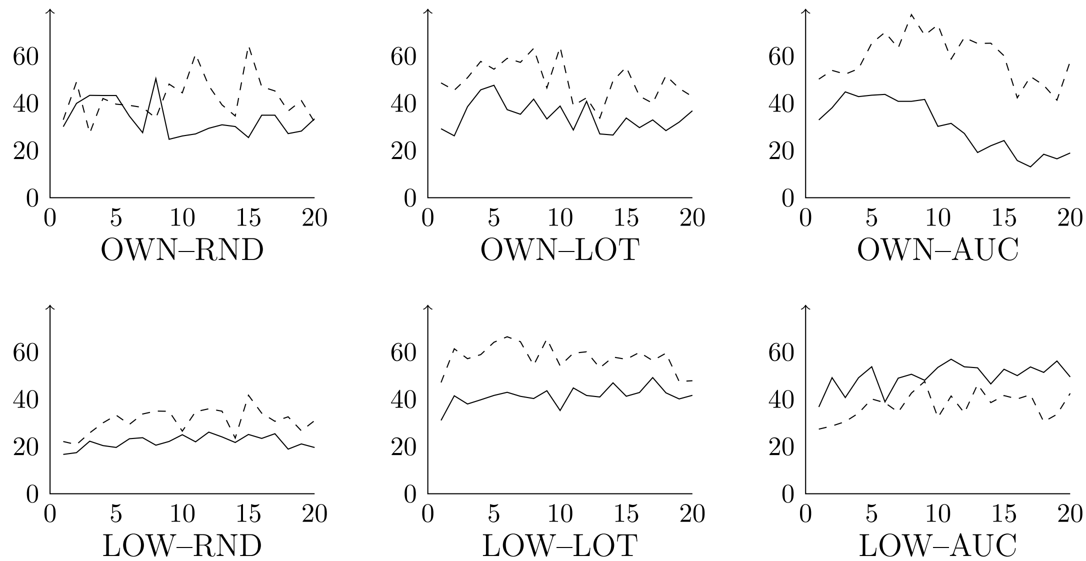

Figure 1: Average individual (expected) actual contributions over rounds in the different treatments. The first row of figures corresponds to the own-bid payment rule; the second to the lowest-bid payment rule. The first column corresponds to the random allocation rule; the second to the lottery; the third to the auction. Dashed curves correspond to the treatments with two bidders; solid curves to three bidders.

The mechanism with the own-bid payment rule and the prize being randomly allocated is strategically equivalent to a voluntary contribution mechanism. Usually contributions in this mechanism exhibit a declining trend. As an example, see Lange et al. (2007) who consider a setting with four donors and a marginal per capita return of 0.3 in a stranger matching environment. While our initial contribution levels are comparable to that reported in Lange et al. (2007), we do not observe a steep decline in contributions in our experimental data (see top-left graph). A likely reason for the sustained contributions over time is that in our experiment subjects are randomly assigned in each round into the passive or the active role

– an aspect which presumably creates stronger pro-social preferences.

Table 2 presents for each treatment the average individual (expected) actual contribution in the first and the second half of the experiment, and the result of a Wilcoxon test for equality of these values.[19] The statistical tests reveal that individual actual contributions do not change over time in the mechanisms with own-bid payment rule when there are two donors and decrease when there are three donors. Apart from the lottery with two donors we find an increase in the individual expected actual contributions in the treatments with the lowest-bid payment rule.

		RND	LOT	AUC
OWN	$n = 2$	41.78 $\sim_{.4103}$ 43.29	53.84 $\sim_{.2322}$ 46.60	63.88 $\sim_{.4440}$ 53.93
	$n = 3$	37.29 $>_{.0333}$ 29.71	36.70 $>_{.0545}$ 30.59	39.95 $>_{.0002}$ 20.11
LOW	$n = 2$	27.95 $<_{.0111}$ 34.90	61.01 $\sim_{.4209}$ 53.17	35.04 $<_{.0251}$ 42.28
	$n = 3$	20.06 $<_{.0006}$ 24.80	37.81 $<_{.0022}$ 47.21	42.48 $<_{.0002}$ 57.24

Table 2: Individual (expected) actual contributions in the first half versus that in the second half of the experiment.

Because differences between the first and the last ten rounds are observed in eight out of the twelve treatments, when presenting the results, we decided to refer to the last ten rounds. For the sake of completeness, all tables show the averages and the test results also for the first ten rounds and for all rounds.

5.3 Impact of allocation rule

Table 3 presents pairwise comparisons between the three allocation rules while holding fixed the payment rule on the basis of all rounds, the first ten rounds, and the last ten rounds.

[19]For this table, and the tables to follow, we constructed the values for the treatments with lowest-bid payment rule on the basis of the empirical distribution over willingness-to-contribute decisions in the respective rounds. Graphs of the distribution of individual (expected) actual contributions in the different treatments on the basis of all decisions in the first ten and last ten rounds are presented in Figures 2 and 3 in Appendix C.

For each comparison between two treatments, the average individual (expected) actual contributions (over the respective rounds) for each of the treatments and the result of a Mann-Whitney test are presented in this table.[20]

Number of donors	Payment rule	Allocation rule comparison	1–20	1–10	11–20
$n = 2$	OWN	RND vs. LOT	$42.31 \sim_{.4487} 49.76$	$41.78 \sim_{.1104} 53.84$	$43.29 \sim_{.6456} 46.60$
		RND vs. AUC	$42.31 <_{.0989} 57.68$	$41.78 <_{.0043} 63.88$	$43.29 \sim_{.3039} 53.93$
		LOT vs. AUC	$49.76 \sim_{.2503} 57.68$	$53.84 <_{.0764} 63.88$	$46.60 \sim_{.4093} 53.93$
	LOW	RND vs. LOT	$30.57 <_{.0000} 55.27$	$27.95 <_{.0000} 61.01$	$34.90 <_{.0013} 53.17$
		RND vs. AUC	$30.57 <_{.0935} 37.79$	$27.95 <_{.0834} 35.04$	$34.90 \sim_{.1230} 42.28$
		LOT vs. AUC	$55.27 >_{.0011} 37.79$	$61.01 >_{.0000} 35.04$	$53.17 >_{.0398} 42.28$
$n = 3$	OWN	RND vs. LOT	$33.61 \sim_{.9569} 33.81$	$37.29 \sim_{.7454} 36.70$	$29.71 \sim_{.9784} 30.59$
		RND vs. AUC	$33.61 \sim_{.5885} 29.84$	$37.29 \sim_{.6651} 39.95$	$29.71 \sim_{.1676} 20.11$
		LOT vs. AUC	$33.81 \sim_{.6849} 29.84$	$36.70 \sim_{.6553} 39.95$	$30.59 \sim_{.1850} 20.11$
	LOW	RND vs. LOT	$22.03 <_{.0000} 41.88$	$20.06 <_{.0000} 37.81$	$24.80 <_{.0003} 47.21$
		RND vs. AUC	$22.03 <_{.0000} 49.30$	$20.06 <_{.0000} 42.48$	$24.80 <_{.0000} 57.24$
		LOT vs. AUC	$41.88 <_{.0547} 49.30$	$37.81 \sim_{.1595} 42.48$	$47.21 <_{.0416} 57.24$

Table 3: Impact of allocation rule.

On the basis of the last ten periods, we find that, in both treatments with two and with three donors, the allocation rule has no significant impact on actual contributions if the own-bid payment rule is adopted. These findings differ from the results reported in the literature comparing lotteries with voluntary contributions. Morgan and Sefton (2000), Lange et al. (2007), Corazzini et al. (2010) and Orzen (2008) find that the lottery generates more contributions than the voluntary contribution mechanism. The latter two studies report in addition that the auction outperforms the voluntary contribution mechanism. These results may be due to differences between the experimental designs, such as the number of donors, informational conditions, the presence of bystanders, etc.

[20]Given that we compare three allocation rules, we can reduce the number of tests by running Kruskal-Wallis tests that take as null hypothesis equality across all variations. The Kruskal-Wallis tests reject equality across all variations (with p-values below .01) for the lowest-bid treatments regardless of number of donors and phase considered and for the own-bid treatment only in the first half with two donors. In all other instances the tests do not reject equality across all variations (with p-values above 0.20).

With respect to the comparison between the lottery and the auction, the evidence from previous experimental studies is mixed. Within an independent private value set-up, Schram and Onderstal (2009) observe that the auction generates higher revenues than the lottery, while Corazzini et al. (2010) find that the lottery outperforms the auction, in contrast to their theoretical prediction. In a setting that is more closely related to ours, Orzen (2008) finds no difference between the auction and the lottery.

If the lowest-bid payment rule is adopted, the random allocation rule is dominated by both the lottery and the auction (no significant differences are observed between the random allocation rule and the auction in the treatment with two donors). Under this payment rule, the lottery dominates the auction in the treatment with two donors, and is dominated by the auction in the treatment with three donors.

The constant marginal per capita return in our setting makes the fund-raising mechanisms analyzed here isomorphic to mechanisms in an environment without a public good but with an inflated prize (see Lemma 1). This affords direct comparison of our results to the experimental literature on rent-seeking contests. In particular, the auction and the lottery with the standard own-bid payment rule are studied experimentally in equivalent settings. Gneezy and Smorodinsky (2006) report systematic overbidding in the all-pay auction in the early stage of their experiment which is stronger in treatments with more bidders. However, contributions decline over time and in the later rounds total contribution levels do not depend on the number of bidders – an observation consistent with theory. We observe here similar overbidding in early rounds and a similar intertemporal pattern of declining contribution levels and convergence between treatments with different number of agents (see Figure 1 and Table 3 presented here and Figure 1 in Gneezy and Smorodinsky 2006). Davis and Reilly (1998) compare rent dissipation in the auction and the lottery and document systematic overbidding in both mechanisms with a greater rent dissipation observed in the auction. We also observe overbidding, but do not find significant differences between mechanisms (see Table 3). One possible explanation for this difference is that in our treatments the existence of passive agents and the pro-social behavior of active agents outweigh the differences in competition between the lottery and the auction.

5.4 Impact of payment rule

Table 4 shows pairwise treatment comparisons between payment rules while holding fixed the allocation rule. For each comparison between two treatments, the average individual (expected) actual contributions (over the respective rounds) for each of the treatments and the result of a Mann-Whitney test are presented in this table.

Number of donors	Allocation rule	Payment rule comparison	1–20	1–10	11–20
$n = 2$	RND	OWN vs. LOW	$42.31 >_{.0398} 30.57$	$41.78 >_{.0173} 27.95$	$43.29 \sim_{.2674} 34.90$
	LOT	OWN vs. LOW	$49.76 \sim_{.5338} 55.27$	$53.84 \sim_{.2111} 61.01$	$46.60 \sim_{.8924} 53.17$
	AUC	OWN vs. LOW	$57.68 >_{.0215} 37.79$	$63.88 >_{.0002} 35.04$	$53.93 \sim_{.3039} 42.28$
$n = 3$	RND	OWN vs. LOW	$33.61 >_{.0200} 22.03$	$37.29 >_{.0002} 20.06$	$29.71 \sim_{.8924} 24.80$
	LOT	OWN vs. LOW	$33.81 \sim_{.1368} 41.88$	$36.70 \sim_{.8076} 37.81$	$30.59 <_{.0074} 47.21$
	AUC	OWN vs. LOW	$29.84 <_{.0010} 49.30$	$39.95 \sim_{.7867} 42.48$	$20.11 <_{.0000} 57.24$

Table 4: Impact of payment rule.

On the basis of the data over the last ten rounds, the lowest-bid payment rule does not raise significantly more contributions than the own-bid payment rule when the prize is randomly allocated (though, on the basis of the first ten rounds it produces significantly lower contributions). For the mechanisms where the prize is allocated via competitive bidding, we find that the lowest-bid payment rule yields significantly more revenue than the own-bid payment rule when three donors compete, but not when two donors compete.

5.5 Optimality of the lowest-bid all-pay auction

On the basis of the analysis in the previous two sections, we find that the lowest-bid all-pay auction is the optimal mechanism when three donors are competing for the prize. This result conforms to Orzen's (2008) finding that the lowest-bid all-pay auction is optimal in a setting without bystanders. Orzen attributes the success of the lowest-bid auction to its ability to promote "conditional cooperation." In the case of two active bidders, however, we find the lowest-bid auction is no longer optimal. It generates donations that are not statistically different from the donations under the random assignment of the prize. Our experiment is similar to Orzen's in the aspect that the budget is high relative to the prize that can be earned. Thus, both experiments can be considered as complementary to each other when viewed as attempts to study the performance of mechanisms relying on "conditional cooperation." To gain understanding of whether, and for what reason the conditional cooperation breaks down when only two active bidders are donating, we conducted additional experiments with this mechanism in [**blinded for review purposes**]. These additional experiments were run in a set-up identical to those run in [**blinded for review purposes**] (same number of subjects, same instructions, etc.), except for one difference: once subjects have made their willingness-to-contribute decision, we ask them (1) how much they expect their actual contribution to be and (2) how likely in their view it is that they win the prize.[21]

Table 5 presents the performance throughout the different rounds in the [**blinded for

[21] We did not provide additional payments to subjects based on the belief they reported for two main reasons. First, the domain of feasible beliefs is dependent on the expressed willingness to contribute, and this could have impacted the bids submitted by subjects. Second, incentivizing subjects would necessitate the use of scoring rules that assign payments to point predictions submitted by subjects which, in the case of three active agents, would have to be constructed on the basis of two submitted bids. An alternative would have been to ask subjects directly to predict the bids submitted by the other active agents, yet this would create an identification problem concerning the two aspects we are interested in.

review purposes] sessions and the [**blinded for review purposes**] sessions. While donations seem to be a bit lower in [**blinded for review purposes**] in comparison to [**blinded for review purposes**], we see that the rather good performance of the lowest-bid all-pay auction with three donors and the rather poor performance with two donors is replicated in the [**blinded for review purposes**] experiments; even the small increase in contributions from the first half to the second half of the experiment is replicated.

Number of	1–20		1–10		11–20	
donors	Loc1	Loc2	Loc1	Loc2	Loc1	Loc2
$n = 2$	37.79	28.51	35.04	27.08	42.28	30.51
$n = 3$	49.30	46.90	42.48	45.35	57.24	48.63

Table 5: Average individual (expected) actual contributions in the lowest-bid all-pay auction. Loc1: [**blinded for review purposes**] sessions; Loc2: [**blinded for review purposes**] sessions.

For each willingness-to-contribute decision we compare the stated beliefs on actual contributions and winning probabilities with the actual (expected) values on the basis of the empirical distribution. We term the difference between believed and actual contribution values "overestimation of actual contribution" and the difference between believed and actual winning probabilities "overestimation of winning probability." Table 6 presents the share of instances in which beliefs reflect an overestimation of actual contributions and winning probabilities. We observe that when only two donors are contributing, they underestimate each other's bids and overestimate their probability of winning relative to the case of three contributing donors. Thus, the belief in conditional cooperation that sustains the high contribution levels in the case of three donors weakens once the benefit of joint contributions becomes small relative to cost.[22]

[22] In the case of three active donors for each token donated each bidder receives 0.9 tokens whereas in the case of two active bidders the benefit is only 0.6 tokes for each token contributed.

Number of donors		Actual contribution 1–20	1–10	11–20	Winning probability 1–20	1–10	11–20
$n = 2$	overestimation	47.00	45.00	46.00	60.00	63.00	60.00
	correct	10.00	10.00	10.00	0.50	0.00	0.00
	underestimation	43.00	45.00	44.00	39.50	37.00	40.00
$n = 3$	overestimation	60.33	52.67	72.00	32.00	46.67	17.33
	correct	14.33	13.33	15.33	0.00	0.00	0.00
	underestimation	25.33	34.00	12.67	68.00	53.33	82.67

Table 6: Overestimation of actual contributions and winning probabilities. The numbers in the table present the share of instances in which beliefs reflect an overestimation, are correct or reflect an underestimation.

6 Conclusion

In this paper we study the performance of prize-based mechanisms for fund-raising in a framework which is amenable to changes in the allocation and the payment rule of the fund-raising mechanism. Our complete information setting allows us to experimentally study the patterns of behavior that are due to the variation of the mechanism itself rather than the complexity of the economic environment. We show that, in our complete information setting, the lowest-bid all-pay auction is the theoretically optimal fund-raising mechanism, and we derive the symmetric mixed strategy equilibrium distribution of this mechanism. We further study equilibrium bidding in the lowest-bid lottery – a mechanism in which the winning chances are proportional to the announced contribution levels, yet actual contributions are determined by the lowest-bid rule.

We find that, in the laboratory, the theoretically optimal mechanism generated the highest expected revenue with three donors. With two donors, however, contributions are disappointingly low and on par with the donations in the voluntary mechanism; the lottery generates more revenues with the difference being statistically significant. We ran additional treatments to gain insights into this bidding behavior by eliciting beliefs from participants about the expected behavior of their fellow bidders, whereby we measured the way beliefs deviated from the empirical distribution of actual bids made in the experiment. We find that, with three active agents, bidders over-estimate the bidding of their fellow bidders – a belief which sustains high levels of contributions in the optimal auction. Our results point to the limits of optimal mechanism design for fund-raising: the theoretically optimal mechanism outperforms the other mechanisms when three active participants support one participant without a budget, but is not behaviorally optimal when two active participants support two participants without a budget.

Our analysis opens various opportunities for further theoretical and experimental research. Of direct relation to the current experimental findings is the question of whether the discrepancy between theoretical predictions and experimental results can be reconciled with alternative models which account for pro-social behavior. One explanation would be provided by the theoretical framework proposed in Corazzini et al. (2010) which posits that subjects bear a psychological cost of contributing less than socially optimal levels. This approach has been successfully used to explain some pervasive discrepancies observed in experiments, including the positive contributions in the voluntary contribution mechanism, the dominance of the lottery over the all-pay auction (see Corazzini et al., 2010) and other notable effects observed in lottery treatments as documented in Morgan and Sefton (2000) and Orzen (2008). Taken to our framework, this approach presumably has the potential to explain the reversal of the ranking of the auction and the lottery under the lowest-bid payment rule as we move from two to three active bidders.[23]

[23]Such an approach may underscore the importance of the shilling effect inherent in the lowest-bid pricing rule for the ranking of fund-raising mechanisms. As me move from a setting with two to a setting with three bidders, the private cost of increasing the bid of the lowest bidder decreases from $1-2\alpha$ to $1-3\alpha$ and the

Another aspect of importance is understanding how the ranking of mechanisms changes as we alter some key assumption of the model. An assumption which is probably most often violated in reality is the symmetry of bidders. In this paper, we assume symmetry across four dimensions: budgets of active bidders, individual benefits from the public good, value of the prize, and equilibrium behavior. Relaxing any of these symmetry assumptions, albeit theoretically challenging, may generate new insights. Recent work by Bos (2011) shows that the dominance of the own-bid all-pay auction over the lottery does not generally hold if bidders are not symmetric in the way they value the prize and the public good. How to design optimal mechanisms when such asymmetries are present remains an open question.

Another extension is to allow for endogenous participation by donors. In a field experiment, Carpenter, Holmes and Matthews (2008) observe that, in contrast to theory, the winner-pay first-price auction generates more revenue than the own-bid all-pay auction (and the winner-pay second-price auction) – a finding that the authors attribute to endogenous participation: individuals feel more attracted to participate in mechanisms that they are familiar with. In a more recent theoretical work, Carpenter, Holmes and Matthews (2010a) derive the symmetric Bayesian Nash equilibria for the above mechanisms in a scenario where bidders are facing mechanism-specific entry costs. How to design optimal fund-raising mechanisms with endogenous participation, however, is yet to be explored – even in symmetric settings.

Finally, our model considers the fund-raising activity in isolation of future fund-raisers.

social benefit increases from 2α to 3α. Thus, for some values of the parameter α, a bidder who believes that the other agent(s) bid high, would bid very high in an auction with three bidders and very low in an auction with two bidders when social preferences are taken into consideration. Such drastic changes in contribution levels would not be observed in the lottery as the item is not always assigned to the highest bidder.

Many fund-raising efforts are, however, repeated events in which behavioral spill-overs play a role. Using field experiments, Landry, Lange, List, Price and Rupp (2010) find that previous donors are more likely to give than those who are asked for a first time to contribute, and explore factors that keep donors committed to the cause. One important conclusion that Landry et al. (2010) draw is that donors initially attracted via economic mechanisms – such as auctions, lotteries, seed money, matching grants, etc. – are more likely to continue to contribute in the future than the ones attracted by "non-mechanism" factors (e.g. the appearance of the solicitor). Thus, theoretical and experimental work on optimal economic mechanisms for fund-raising that explicitly accounts for the recurrence of fund-raising events presents another fruitful avenue for future research.

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A Proofs and additional result

Proof of Proposition 2 (LOW–LOT)

The existence of a symmetric equilibrium follows from Theorem 1 in Becker and Damianov (2006) who show that any symmetric game with a continuous payoff function has a symmetric (mixed strategy) equilibrium. Applying Lemma 1, we consider the setting without a public good and a prize of $\frac{V}{1-\alpha n}$.

Part (A).

We proceed in three steps.

Step 1. *Equation (3) holds for any open interval (c,d) that belongs to the support.* For all $x \in (c,d)$ the expected payoff $\mathbb{E}_{x_{-i}}[\Pi_i^{\text{LOW–LOT}}(0, \frac{V}{1-\alpha n})(x,F)]$ is constant. Hence

$$\frac{\partial}{\partial x}\,\mathbb{E}_{x_{-i}}[\Pi_i^{\text{LOW–LOT}}(0, \tfrac{V}{1-\alpha n})(x,F)] = 0. \tag{5}$$

Let us define the marginal revenue of bidder i in the lowest-bid lottery by

$$\begin{aligned} MR^{\text{LOW–LOT}}(x,F) &= \frac{\partial}{\partial x}\left[\frac{V}{1-\alpha n}\cdot \int\limits_{\times_{j\neq i} S_j} \frac{x}{x+\sum_{j\neq i} x_j} dF(x_{-i})\right] \\ &= \frac{V}{1-\alpha n}\cdot \int\limits_{\times_{j\neq i} S_j} \frac{\sum_{j\neq i} x_j}{(x+\sum_{j\neq i} x_j)^2} dF(x_{-i}) \end{aligned}$$

and the marginal cost by

$$\begin{aligned} MC^{\text{LOW–LOT}}(x,F) &= \frac{\partial}{\partial x}\left[\int_0^x y\, d(1-(1-F(y))^{n-1}) + (1-F(x))^{n-1}\cdot x\right] \\ &= x\cdot \tfrac{d}{dx}[1-(1-F(x))^{n-1}] + (1-F(x))^{n-1} - (n-1)(1-F(x))^{n-2}F'(x)\cdot x \\ &= x\cdot (n-1)(1-F(x))^{n-2}F'(x) + (1-F(x))^{n-1} - (n-1)(1-F(x))^{n-2}F'(x)\cdot x \\ &= (1-F(x))^{n-1}. \end{aligned}$$

Equation (5) can be represented as

$$MR^{\text{LOW–LOT}}(x,F) = MC^{\text{LOW–LOT}}(x,F) \tag{6}$$

with the interpretation that, for all x in the mixed strategy equilibrium support, marginal revenue of increasing the bid of a player should be equal to the marginal cost. From the derived expressions for the marginal revenue and the marginal cost we obtain that equation (6) is equivalent to

$$\frac{V}{1-\alpha\, n} \cdot \int\limits_{\substack{\times \, S_j \\ j \neq i}} \frac{\sum_{j\neq i} x_j}{(x + \sum_{j\neq i} x_j)^2} dF(x_{-i}) = (1 - F(x))^{n-1}$$

which corresponds to equation (3) stated in the proposition. In the next step we show that the support of the mixed strategy distribution cannot have gaps, i.e. all open intervals in $[b, B]$ have a positive probability mass. More formally, we establish the following claim.

Step 2. *For any c and d where $b \leqslant c < d \leqslant B$, if $[b, c] \cup [d, B]$ belongs to the mixed strategy support then (c, d) also belongs to the support of the mixed strategy equilibrium.* Assume by contradiction that $\mathbb{E}_{x_{-i}}[\Pi_i^{\text{LOW–LOT}}(0, \frac{V}{1-\alpha\, n})(x, F)] = k$ for $x \in [b, c] \cup [d, B]$ and $\mathbb{E}_{x_{-i}}[\Pi_i^{\text{LOW–LOT}}(0, \frac{V}{1-\alpha\, n})(x, F)] < k$ for $x \in (c, d)$. Observe that, since $\frac{\sum_{j\neq i} x_j}{(x+\sum_{j\neq i} x_j)^2}$ is decreasing in x it follows that $MR^{\text{LOW–LOT}}(x, F)$ is decreasing in x. As $MC^{\text{LOW–LOT}}(x, F)$ is constant for $x \in (c, d)$, because $F(x)$ is constant (note that we assumed no probability mass in this interval) and $MR^{\text{LOW–LOT}}(x, F) = MC^{\text{LOW–LOT}}(x, F)$ for $x = c$ it follows that $MR^{\text{LOW–LOT}}(x, F) < MC^{\text{LOW–LOT}}(x, F)$ for $x \in (c, d)$. Therefore,

$$\mathbb{E}_{x_{-i}}[\Pi_i^{\text{LOW–LOT}}(0, \tfrac{V}{1-\alpha\, n})(c, F)] > \mathbb{E}_{x_{-i}}[\Pi_i^{\text{LOW–LOT}}(0, \tfrac{V}{1-\alpha\, n})(d, F)],$$

a contradiction.

Step 3. *The symmetric equilibrium has a mass point at B.* Assume by contradiction that there is no mass point and B. It follows that

$$MC^{\text{LOW–LOT}}(B,F) = (1-F(B))^n = 0$$

Note that

$$MR^{\text{LOW–LOT}}(B,F) = \frac{V}{1-\alpha\, n} \cdot \int\limits_{\times_{j\neq i} S_j} \frac{\sum_{j\neq i} x_j}{(B+\sum_{j\neq i} x_j)^2} dF(x_{-i}) > 0$$

a contradiction to condition (6).

Part (B).

We proceed in two steps.

Step 1. *All players donating their entire budget B is an equilibrium profile.* When bidder i plays the pure strategy $x \leqslant B$ and all other bidders play B the payoff of bidder i is given by

$$\Pi_i^{\text{LOW–LOT}}(0, \tfrac{V}{1-\alpha\, n})(x,B) = B - x + \tfrac{x}{x+(n-1)B} \cdot \tfrac{V}{1-\alpha\, n}.$$

The first-order condition for the best response of bidder i is given by

$$\tfrac{\partial}{\partial x}\Pi_i^{\text{LOW–LOT}}(0, \tfrac{V}{1-\alpha\, n})(x,B) = \tfrac{(n-1)B}{(x+(n-1)B)^2} \cdot \tfrac{V}{1-\alpha\, n} - 1 = 0.$$

Requiring that the first-order condition holds for $x = B$ gives $B = \frac{n-1}{n} \cdot \frac{V}{n\,(1-\alpha\, n)}$. For $B \leqslant \frac{n-1}{n} \cdot \frac{V}{n\,(1-\alpha\, n)}$ the inequality $\frac{\partial}{\partial x}\Pi_i^{\text{LOW–LOT}}(0, \frac{V}{1-\alpha\, n})(x,B) > 0$ holds for all $x < B$ and therefore the strategy profile in which all bidders donate B is a symmetric pure strategy equilibrium profile. For $B > \frac{n-1}{n} \cdot \frac{V}{n\,(1-\alpha\, n)}$ it is straightforward that the symmetric equilibrium is not given in pure strategies. Indeed, a symmetric pure strategy profile cannot constitute an equilibrium because, by increasing their bid, bidders increase their probability of winning but not their payment.

Step 2. *No other symmetric equilibrium exists.* Assume that there is another equilibrium with a cumulative distribution function $G(x)$. The distribution $G(x)$ should satisfy the properties given in Part (A), that is, it has support over the interval $[b, B]$ where $0 \leqslant b < B$ and satisfies equation (6). Let us denote the sum of the donations of all players except player i by $y := \sum_{j \neq i} x_j$ and let $\varphi(x, y) := \frac{y}{(x+y)^2}$. Note that $\varphi(x, y)$ decreasing in x for all $x \in [b, B]$ and observe that $\frac{\partial}{\partial y}(\varphi(x, y)) = \frac{x-y}{(x+y)^3}$. It follows that $\varphi(x, y)$ is decreasing in y for $y > x$. Hence, for $y \in [b, (n-1)\,B)$ it follows that

$$\varphi(b, y) > \varphi(b, (n-1)\,B) > \varphi(B, (n-1)\,B) = \frac{(n-1)\,B}{(n\,B)^2}.$$

For the marginal revenue the following inequalities hold

$$MR^{\text{LOW–LOT}}(b, G) = \frac{V}{1-\alpha\,n} \cdot \int_{(n-1)\,b}^{(n-1)\,B} \varphi(b, y) dL_G(y) > \frac{V}{1-\alpha\,n} \cdot \frac{(n-1)\,B}{(n\,B)^2}$$

where $L_G(y)$ the probability distribution function of $y = \sum_{j \neq i} x_j$, (i.e. $L_G(y)$ is the convolution of $G(x)$). Note that for $B \leqslant \frac{n-1}{n} \cdot \frac{V}{n\,(1-\alpha\,n)}$ the inequality $\frac{V}{1-\alpha\,n} \cdot \frac{(n-1)\,B}{(n\,B)^2} \geqslant 1$ holds, and hence $MR^{\text{LOW–LOT}}(b, G) > 1 \geqslant (1 - G(b))^n = MC^{\text{LOW–LOT}}(b, G)$, a contradiction to condition (6).

■

Proof of Proposition 3 (LOW–AUC)

Applying Lemma 1, we consider the setting without a public good and a prize of $\frac{V}{1-\alpha\,n}$. It is easy to see that no symmetric equilibrium in pure strategies exists because each bidder has an incentive to increase her bid and secure the prize without paying more. Thus, we consider the mixed strategy extension of the game in which each player i chooses a cumulative distribution function $F_i(x)$ over the set of pure strategies. Let us also denote by $\phi_i(\tilde{x}) = F_i(\tilde{x}) - \lim_{x \uparrow \tilde{x}} F_i(x)$ the size of a mass point placed at bid $\tilde{x}$. We proceed now in six steps.

Step 1. *There are no mass points in the symmetric equilibrium distribution (except at the budget constraint B).* Assume that there exists an atom in the symmetric equilibrium distribution, i.e. there is a mass point at bid $\tilde{x}$. With a probability of $\phi_i(\tilde{x})^{n-1}$ there is a tie at this bid in which case bidder i wins the prize only with a probability of $1/n$. Consider a deviation according to which bidder i shifts the mass $\phi_i(\tilde{x})$ from $\tilde{x}$ to $\tilde{x} + \epsilon$. The total probability of winning the prize will increase by at least $(1 - \frac{1}{n}) \cdot \phi_i(\tilde{x})^{n-1}$, while the payment will increase by no more than ϵ (observe that the payment function is continuous). For a small enough ϵ the deviation is profitable.

Step 2. *The lower bound of the support of the symmetric equilibrium is zero.* Assume on the contrary that the lower bound is $\ell > 0$. Because the distribution is atom-less, with a bid of ℓ, bidder i pays ℓ, but the chance of winning the prize is zero. So, a bid of zero is a profitable deviation.

Step 3. *In the symmetric equilibrium each bidder contributes on average the amount $\frac{V}{n\,(1-\alpha\,n)}$.* As the bid of zero is in the support of the mixed strategy equilibrium, and the payoff is B when this bid is played, the expected payoff of each bidder in a symmetric equilibrium must be B. Let E be the expected payment of each bidder (assuming symmetric equilibrium strategies). The expected payoff of the equilibrium mixed strategy equals the expected payoff of each strategy in the support, in particular the bid of zero. Observe now that with a bid of zero the chance of a bidder to win the prize is zero, and total contributions equal zero as well. Thus, if according to the symmetric equilibrium distribution function each bidder donates on average E, the following equation holds for E

$$\tfrac{V}{n} + \alpha \cdot n \cdot E - E = 0 \qquad \Longleftrightarrow \qquad E = \tfrac{V}{n\,(1-\alpha\,n)}.$$

Step 4. *Derivation of the symmetric equilibrium mixed strategy distribution.* Let us denote the symmetric mixed strategy cumulative probability distribution function by F. As the expected payoff is constant for all x in the support, we have

$$\frac{\partial}{\partial x}\,\mathbb{E}_{x_{-i}}[\Pi_i^{\text{LOW–AUC}}(0, \tfrac{V}{1-\alpha\,n})(x, F)] = 0.$$

This is equivalent to

$$MR^{\text{LOW–AUC}}(x, F) = MC^{\text{LOW–AUC}}(x, F) \tag{7}$$

where

$$MR^{\text{LOW–AUC}}(x, F) = \frac{\partial}{\partial x}\left[\frac{V}{1-\alpha\,n} \cdot F(x)^{n-1}\right] = \frac{V}{1-\alpha\,n}(n-1)F(x)^{n-2}F'(x)$$

and

$$\begin{aligned} MC^{\text{LOW–AUC}}(x, F) &= \frac{\partial}{\partial x}\left[\int_0^x y\, d(1-(1-F(y))^{n-1}) + (1-F(x))^{n-1}\cdot x\right] \\ &= MC^{\text{LOW–LOT}}(x, F) = (1-F(x))^{n-1}. \end{aligned}$$

Substituting these results in equation (7) and rearranging terms we obtain that the equilibrium distribution function satisfies the differential equation

$$F'(x) = \frac{1-\alpha\,n}{V} \cdot \frac{(1-F(x))^{n-1}}{(n-1)F(x)^{n-2}} \tag{8}$$

with an initial condition $F(0) = 0$.

Step 5. *The symmetric equilibrium is unique.* Assume by contradiction that there are two symmetric equilibria, given by the cumulative distribution functions $F(x)$ and $G(x)$. Using the standard definition of stochastic dominance, we say that $F(x)$ first degree stochastically dominates $G(x)$ if $F(x) \leqslant G(x)$ for all x with strict inequality for some x. Since both distributions are assumed to be equilibria, then it cannot be the case that $F(x)$ first degree dominates $G(x)$ or vice versa. Indeed, if this were the case, then the lowest-order statistic of $F(x)$, given by the distribution $1-(1-F(x))^n$ would stochastically dominate the lowest-order statistic of $G(x)$, given by $1-(1-G(x))^n$. As expected revenue is n times the expected value of the lowest-order statistic, both distributions will lead to different expected revenues. This contradicts the result we established that the expected contribution in every mixed strategy equilibrium equals $\frac{V}{1-\alpha n}$ (see Step 3). As both functions, being equilibrium distributions, are continuous, and $F(0) = G(0) = 0$, the distribution functions must cross at least one more time. Let us assume that $y > 0$ is the minimum point at which they cross again, i.e. $F(y) = G(y)$, and without loss of generality let us assume that $F(x) \leqslant G(x)$ for $0 \leqslant x \leqslant y$. Then, in the interval $[0, y]$, on average, one of the distributions will result in a higher payment for the bidders than the other. This is, however, not possible, because the expected payoff is zero at all points in the support, and at both points the probability of winning the item is the same. That is, the same increase in probability should be gained by the same increase in the expected payment. Thus, there is only one symmetric mixed strategy equilibrium.

Step 6. *Effect of the budget constraint B.* To prove that the strategy profile described in the proposition is indeed an equilibrium, we need to show that all bids that belong to the support yield the same expected payoff for a bidder, given that the other bidders follow the equilibrium strategy. Observe that all bids in $[0, b]$ generate the same probability of winning the prize as in the case without a budget constraint, and the expected payments of these bids are also the same. It remains to show that the bid B generates the same payoff as the bid b. Note that the only case in which a bid B will win the item and a bid b will not is when there is at least one other bidder who bids B. If several bidders bid B there will be a tie. Let bidder i submit a bid of B. The additional probability for this bidder to win the item (compared to the situation in which he bids b) is given by the following binomial expression describing the probabilities of a tie for bidder i and any number of his rivals ranging from 1 to $(n-1)$:

$$\sum_{j=1}^{n-1} \tbinom{n-1}{j}(1-F(b))^j F(b)^{n-1-j} \tfrac{1}{j+1}.$$

A standard manipulation of the above expression yields

$$\frac{1-F(b)^n}{n(1-F(b))} - F(b)^{n-1}.$$

Thus, the additional expected gain from bidding B instead of b is

$$\frac{V}{1-\alpha\, n} \cdot \Big[\frac{1-F(b)^n}{n(1-F(b))} - F(b)^{n-1} \Big]. \tag{9}$$

With a bid of B bidder i pays the same as with the bid b when at least one of his rivals bids below B. When all other bidders bid B, then bidder i pays B. Thus, the additional cost of raising the bid from b to B is

$$(1-F(b))^{n-1}(B-b). \tag{10}$$

The condition given in the proposition equates the expressions (9) and (10) and ensures that the benefit of raising the bid from b to B corresponds to the cost. To see that bids in the interval (b, B) do not lead to a higher payoff for a bidder, observe that the winning probability and the expected payoff with these bids are the same as with the bid b. The uniqueness of the symmetric equilibrium established in Step 5 guarantees that there is a unique b which solves the equation given in the proposition.

■

Additional result

Proposition 5. *In the case $n = 2$ the symmetric equilibrium of the lowest-bid all-pay auction takes the form*

$$F(x) = 1 - e^{-c \cdot x}$$

with $c = \frac{1-\alpha\, n}{V}$.

In the case $n > 2$ the inverse of the cumulative distribution function takes the form

$$F(y)^{-1} = \frac{(n-1)}{c} \cdot \Big[\tfrac{1}{n-2}\big(\tfrac{y}{1-y}\big)^{n-2} - \tfrac{1}{n-3}\big(\tfrac{y}{1-y}\big)^{n-3} + \ldots - \tfrac{1}{2}\big(\tfrac{y}{1-y}\big)^{2} + \tfrac{y}{1-y} - \ln\big(\tfrac{1}{1-y}\big) \Big]$$

when n is odd, and

$$F(y)^{-1} = \frac{(n-1)}{c} \cdot \Big[\tfrac{1}{n-2}\big(\tfrac{y}{1-y}\big)^{n-2} - \tfrac{1}{n-3}\big(\tfrac{y}{1-y}\big)^{n-3} + \ldots + \tfrac{1}{2}\big(\tfrac{y}{1-y}\big)^{2} - \tfrac{y}{1-y} + \ln\big(\tfrac{1}{1-y}\big) \Big]$$

when n is even.

Proof. Equation (8) is an autonomous equation. Denoting $y = F(x)$ we obtain

$$\frac{dy}{dx} \; = \; c \cdot \frac{(1-y)^{n-1}}{(n-1)y^{n-2}}$$

or

$$\frac{(n-1)y^{n-2}}{c \cdot (1-y)^{n-1}} \cdot dy \; = \; dx,$$

where $c = \frac{1-\alpha\, n}{V}$. Integration with $z = \frac{y}{1-y}$ yields

$$x + K \;=\; \frac{(n-1)}{c} \cdot \int \frac{y^{n-2}}{(1-y)^{n-1}}\, dy \;=\; \frac{(n-1)}{c} \cdot \int \frac{z^{n-2}}{z+1}\, dz.$$

If n is odd we obtain

$$\begin{aligned}
x + K &= \tfrac{(n-1)}{c} \cdot \int \tfrac{z^{n-2}+1-1}{z+1}\, dz \\
&= \tfrac{(n-1)}{c} \cdot \int \big(z^{n-3} - z^{n-4} + \ldots - z + 1 - \tfrac{1}{z+1}\big)\, dz \\
&= \tfrac{(n-1)}{c} \cdot \big[\, \tfrac{z^{n-2}}{n-2} - \tfrac{z^{n-3}}{n-3} + \ldots - \tfrac{z^2}{2} + z - \ln|z+1|\, \big] \\
&= \tfrac{(n-1)}{c} \cdot \big[\, \tfrac{1}{n-2}(\tfrac{y}{1-y})^{n-2} - \tfrac{1}{n-3}(\tfrac{y}{1-y})^{n-3} + \ldots - \tfrac{1}{2}(\tfrac{y}{1-y})^2 + \tfrac{y}{1-y} - \ln(\tfrac{1}{1-y}))\, \big].
\end{aligned}$$

If n is even we obtain

$$\begin{aligned}
x + K &= \tfrac{(n-1)}{c} \cdot \int \tfrac{z^{n-2}-1+1}{z+1}\, dz \\
&= \tfrac{(n-1)}{c} \cdot \int \big(z^{n-3} - z^{n-4} + \ldots + z - 1 + \tfrac{1}{z+1}\big)\, dz \\
&= \tfrac{(n-1)}{c} \cdot \big[\, \tfrac{z^{n-2}}{n-2} - \tfrac{z^{n-3}}{n-3} + \ldots + \tfrac{z^2}{2} - z + \ln|z+1|\, \big] \\
&= \tfrac{(n-1)}{c} \cdot \big[\, \tfrac{1}{n-2}(\tfrac{y}{1-y})^{n-2} - \tfrac{1}{n-3}(\tfrac{y}{1-y})^{n-3} + \ldots + \tfrac{1}{2}(\tfrac{y}{1-y})^2 - \tfrac{y}{1-y} + \ln(\tfrac{1}{1-y})\, \big].
\end{aligned}$$

In both cases the initial condition $y(0) = 0$ gives $K = 0$. $\square$

B Experimental instructions (LOW–AUC–3)

Welcome to our experiment. Please read these instructions carefully. They are the same for every participant. Please do not talk with other participants and remain quiet during the entire experiment. Please turn off your cell phone and don't switch it on until the end of the experiment. If you have a question, please raise your hand and we will come to you. The entire experiment will take around 60 minutes.

The experiment will consist of 20 rounds. In each round you will be assigned to a group of 4 participants. The other 3 members of your group will be selected randomly from all experimental participants in each round.

In each round your computer screen will indicate whether you are an *active* or a *passive* participant. There will be 3 active members and 1 passive member in your group each round, and this assignment will also be random.

Procedure in each round

If you are an active participant, you will have a budget of 100 ECU (Experimental Currency Units) in this round. At this point you will need to specify the highest amount you are willing to contribute to the group account (or your *willingness-to-contribute*). *Your actual contribution will be equal to the lowest willingness-to-contribute specified by an active member (yourself or someone else).* The actual contributions of the active members will be taken from their private accounts and transferred to the group account. The funds in the group account will generate a return of 1.2 ECU for each ECU available in the group account, and will be split equally among the 4 members of the group. Hence, each member will receive

0.3 ECU for each ECU actually contributed to the group account.

If your willingness-to-contribute is the highest, you will receive an additional prize of 20 ECU. In case you specified the highest willingness-to-contribute and another active member specified the same willingness-to-contribute, the 20 ECU prize will be awarded randomly to one of you. Hence, your total earnings in this round will be the sum of:

- 0.3 times the total amount in the group account,
- the amount that remains in your private account,
- another 20 ECU if you win the prize.

If you are a passive participant you will not have a budget in this round. Your earnings will equal 0.3 times the total amount that the three active group members actually contributed to the group account. At the end of each round you will learn the *willingness-to-contribute* specified by each active bidder, and whether you won the prize. Please write down in your answer sheet 1) the total amount of ECU contributed to the group account, and 2) the total amount of ECU you earned in this round.

Example of contributions and earnings:

Group member:	1	2	3	4
Role:	Active	Active	Active	Passive
Budget:	100	100	100	0
Willingness to contribute:	20	40	40	–
Actual contribution:	20	20	20	–
Group account:		$20 + 20 + 20 = 60$		
Earnings				
Groups account ($0.3 \times 60 = 18$):	18	18	18	18
Private account:	80	80	80	0
Chance of winning the 20 ECU prize:	0%	50%	50%	–

Payments

At the end of the experiment we will randomly select one out of the 20 rounds. The total amount of ECU you have earned in this round will be converted to US Dollars at the exchange rate of $0.15 for 1 ECU. To this amount we will add another $7.50 and the entire amount will be paid to you in private and in cash.

That is all about the rules. They are the same for every participant. If you have any question, raise your hand and we will come to you. Before starting the experiment, please answer the questions on the following page to make sure that you understood all the rules.

C Additional graphs

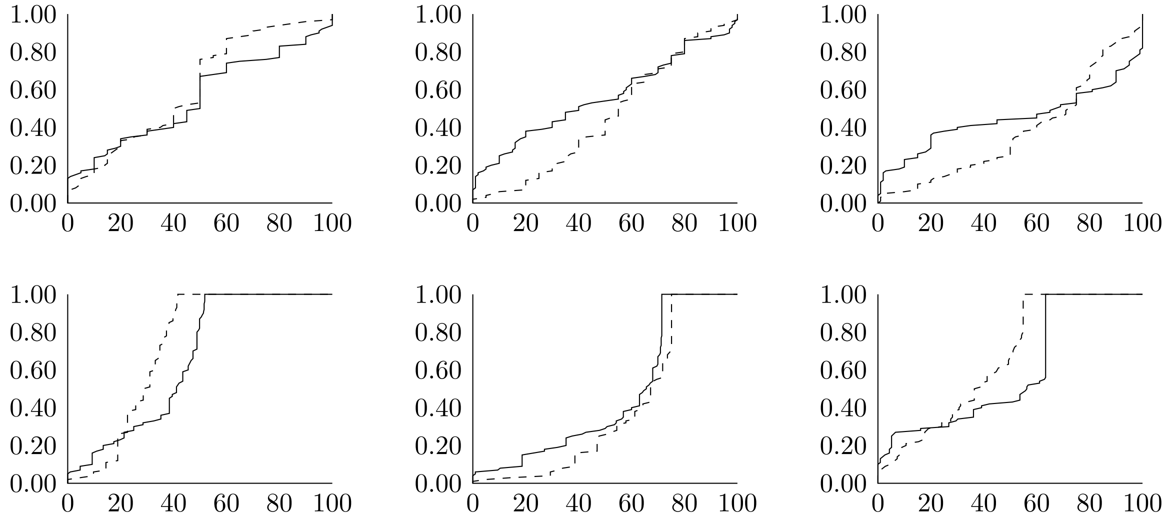

Figure 2: Distribution of individual (expected) actual contributions in the different treatments with two donors. The first row of figures corresponds to the own-bid payment rule; the second to the lowest-bid payment rule. The first column corresponds to the random allocation rule; the second to the lottery; the third to the auction. Dashed curves are based on all decisions in the first ten rounds; solid curves those in the last ten rounds.

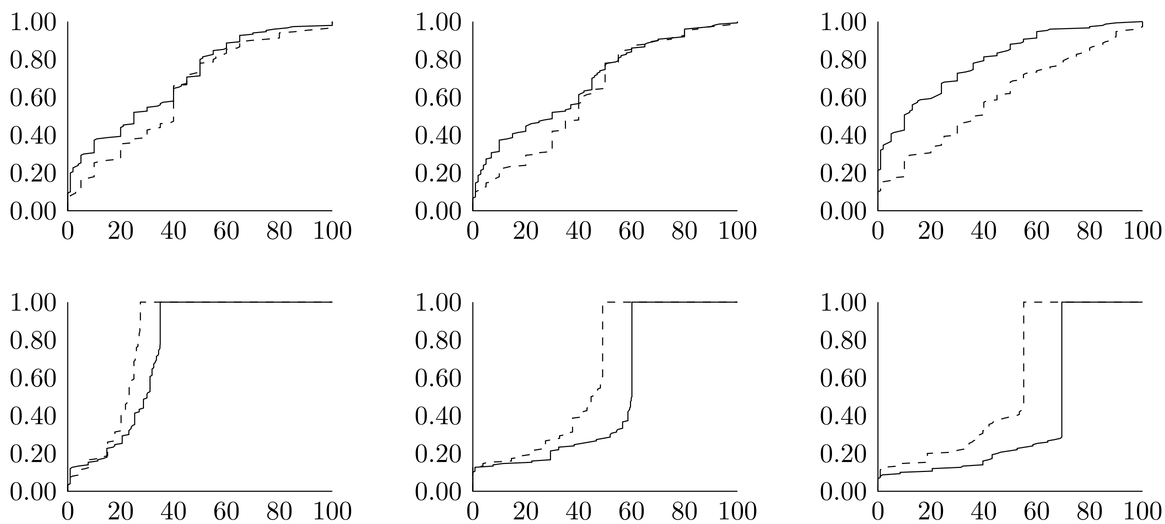

Figure 3: Distribution of individual (expected) actual contributions in the different treatments with three donors. The first row of figures corresponds to the own-bid payment rule; the second to the lowest-bid payment rule. The first column corresponds to the random allocation rule; the second to the lottery; the third to the auction. Dashed curves are based on all decisions in the first ten rounds; solid curves those in the last ten rounds.

D Additional tables

Number of donors	Payment rule	Allocation rule	Will. to contr.	Act. contr.	Exp. act. contr.	Corr. exp. act. contr. with will. to contr.	Corr. exp. act. contr. with act. contr.
$n = 2$	OWN	RND	42.15	42.15	42.15	1.0000	1.0000
		LOT	49.52	49.52	49.52	1.0000	1.0000
		AUC	59.34	59.34	59.34	1.0000	1.0000
	LOW	RND	46.71	31.34	30.86	0.9532	0.6318
		LOT	73.26	57.92	57.62	0.9895	0.6593
		AUC	59.14	39.76	37.35	0.9941	0.6729
$n = 3$	OWN	RND	33.23	33.23	33.23	1.0000	1.0000
		LOT	34.51	34.51	34.51	1.0000	1.0000
		AUC	30.29	30.29	30.29	1.0000	1.0000
	LOW	RND	49.79	21.12	21.91	0.8881	0.4664
		LOT	72.81	42.54	41.37	0.9916	0.5210
		AUC	78.30	51.15	49.66	0.9958	0.5411

Table 7: Average values of the key variables and correlations between them in the different treatments on the basis of the data in all twenty rounds.

Number of donors	Payment rule	Allocation rule	Will. to contr.	Act. contr.	Exp. act. contr.	Corr. exp. act. contr. with will. to contr.	Corr. exp. act. contr. with act. contr.
$n = 2$	OWN	RND	39.44	39.44	39.44	1.0000	1.0000
		LOT	54.68	54.68	54.68	1.0000	1.0000
		AUC	62.93	62.93	62.93	1.0000	1.0000
	LOW	RND	41.62	28.06	28.05	0.9217	0.6289
		LOT	75.10	61.84	61.22	0.9854	0.6718
		AUC	54.90	37.60	34.37	0.9849	0.6894
$n = 3$	OWN	RND	36.32	36.32	36.32	1.0000	1.0000
		LOT	37.37	37.37	37.37	1.0000	1.0000
		AUC	39.91	39.91	39.91	1.0000	1.0000
	LOW	RND	44.89	20.20	19.84	0.8427	0.5053
		LOT	68.56	38.96	36.33	0.9828	0.5520
		AUC	73.49	44.46	42.06	0.9925	0.5406

Table 8: Average values of the key variables and correlations between them in the different treatments on the basis of the data in the first ten rounds.

Number of donors	Payment rule	Allocation rule	Will. to contr.	Act. contr.	Exp. act. contr.	Corr. exp. act. contr. with will. to contr.	Corr. exp. act. contr. with act. contr.
$n = 2$	OWN	RND	44.86	44.86	44.86	1.0000	1.0000
		LOT	44.36	44.36	44.36	1.0000	1.0000
		AUC	55.75	55.75	55.75	1.0000	1.0000
	LOW	RND	51.80	34.62	34.93	0.9681	0.6355
		LOT	71.42	54.00	54.24	0.9933	0.6475
		AUC	63.37	41.92	41.24	0.9984	0.6555
$n = 3$	OWN	RND	30.15	30.15	30.15	1.0000	1.0000
		LOT	31.65	31.65	31.65	1.0000	1.0000
		AUC	20.66	20.66	20.66	1.0000	1.0000
	LOW	RND	54.69	22.04	24.80	0.9213	0.4372
		LOT	77.05	46.12	47.44	0.9966	0.4859
		AUC	83.11	57.84	58.45	0.9979	0.5202

Table 9: Average values of the key variables and correlations between them in the different treatments on the basis of the data in the last ten rounds.