## Absence of Quantum Criticality and Bulk 3D Magnetism in Green Dioptase

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Abstract –Green dioptase is a naturally occurring antiferromagnetic mineral that in recent years has been suggested as a candidate to exhibit quantum fluctuations at both above and below  $T_{\rm N}$ . Our work uses muon spectroscopy to study the dynamic and static properties of the magnetism in zero-applied field. We observe the antiferromagnetic transition through tracking out the evolution of the muon precession frequency as a function of temperature.  $T_{\rm N}$  is calculated to be 15 K and the critical order parameter of the transition matches with that of a 3D Heisenberg system. We also note that no evidence for any quantum magnetic fluctuations are observed either above or below  $T_{\rm N}$ .

Introduction. – Gemstones have fascinated people 1 and societies for centuries, where they have garnished and 2 adorned various precious items such as jewellery. The natural origin of gemstones has also presented scientists with the challenge of understanding them and essentially studying nature's playground. Green Dioptase is one such precious mineral that forms dark green crystals with the empirical formula CuSiO<sub>3</sub>·H<sub>2</sub>O and has found favour within 8 the crystal healing community. Green dioptase crystallises with  $R\bar{3}$  symmetry with  $Si_6O_{18}$  rings intersected by  $Cu^{2+}$ 10 ions. The Cu ions form quasi 1D helical chains that run 11 along the *c*-direction within the crystalline axes (see Fig-12 ure 1). Each Cu ion has two intra-chain nearest neighbours 13 and an additional Cu ion on the adjacent chain [1,2], thus 14 each Cu ion only has three other Cu nearest neighbours. 15 The water molecules sit in positions between the silicate 16 rings where one proton of each water forms a bent O-H 17 bond with an O on the silicate ring as elucidated by NMR 18 measurements [3]. 19

The magnetic properties of green dioptase have been a point of recent discussion. It was first measured by Wintenberger *et al.* [4] where a broad hump was seen with a maximum at 50 K and an approximate fit to Curie-Weiss law suggested that the sample was antiferromagnetic in



Fig. 1: Crystal structure of green dioptase selectively cut to illustrate the nature of the quasi-1D Cu-O chains along the c or z-direction of the unit cell. Note that Cu ions are blue, O ions are red and Si ions are green in the image.

nature. From neutron diffraction data, the antiferromag-25 netic structure could be solved confirming the nature of 26 the low temperature magnetism [2]. ESR measurements 27 showed a shift in the resonance frequency at 50 K where 28 there was a maximum in the magnetic susceptibility, but 29 there was the clear onset of an antiferromagnetic ground 30 state with an energy gap of 350 GHz ( $\sim 17$  K) [5]. Re-31 cent reports have, however, suggested that green dioptase 32 is an ideal candidate to show quantum critical behaviour 33 since the inter- and intra- chain exchange energies may 34 be similar leading to frustration and a dynamic magnetic 35 ground state [6]. Further work was conducted that sug-36 gested that strong quantum fluctuations were present in 37 the broad maximum in the magnetic susceptibility but 38 theoretical models suggested that the system is 3D and 39 not frustrated [7]. Very recently, inelastic neutron scatter-40 ing measurements at temperatures well below  $T_{\rm N}$  showed 41 that the magnetic structure is a spiralled AFM along the 42 spin chains [8] but there was no evidence for any quantum 43 fluctuations. 44

There is still some ambiguity in the magnetic behaviour 45 of green dioptase where additional techniques are required. 46 To this end we employ muon spectroscopy ( $\mu$ SR) to help 47 study any potential magnetic quantum fluctuations or 48 static order in zero-applied field.  $\mu$ SR is a technique that is 49 sensitive to both dynamic and static magnetic behaviour. 50 In the past,  $\mu$ SR has proved useful for showing that at low 51 temperatures magnetic systems have strong dynamic char-52 acter from frustration and quantum critical fluctuations 53 [9–12]. Using  $\mu$ SR, we have been able to show that green 54 dioptase orders at approximately 15 K where there is no 55 evidence of magnetic fluctuations below  $T_{\rm N}$ . Above  $T_{\rm N}$ , 56 the muon spin relaxation measurements show the muon 57 couples only to nuclear moments, however there is also no 58 evidence of electronic fluctuations. 59

Experimental. – The green dioptase crystals were purchased from Crystal Classics [13] sourced from Namibia. Crystals of green dioptase were then ground up to be used within measurements where X-Ray diffraction and X-Ray fluorescence spectroscopy was used to confirm all samples were phase pure.

Magnetic susceptibility measurements were performed 66 on a Quantum Design SQUID VSM with fields capable of 67 up to 7 T. Muon spin relaxation/rotation ( $\mu$ SR) measure-68 ments were conducted on the EMU instrument at the ISIS 69 Neutron and Muon Source. The  $\mu$ SR technique involves 70 implanting a spin polarised ensemble of positive muons in 71 the sample and monitoring the time evolution of the the 72 muon spin polarisation through the asymmetry in the for-73 ward and backward directions of the positron decay that 74 occurs with a half-life of 2.2  $\mu$ s. Thus the technique is 75 sensitive to processes on the MHz time scale. The muon 76 is a local probe, sampling a radius of approximately 2 nm 77 and upon implantation in the sample is sensitive to both 78 local nuclear and electronic moments. If the local, inter-79 nal magnetic field is homogeneous at the muon site, the 80



Fig. 2: Temperature dependence of the ZFC and FC magnetic susceptibility where the data were taken in an applied field of 5 T. The solid lines are fits to the data.

muon ensemble will precess coherently and this precession frequency,  $\nu = \gamma_{\mu} \cdot B_{\text{int}}$  where  $\gamma_{\mu}$  is the gyromagnetic ratio of the muon and  $B_{\text{int}}$  is the internal magnetic field at the muon site. If however, the internal field experienced by the muon becomes more inhomogeneous, then the field distribution increases and this leads to a dephasing of the muon ensembles spin and one sees a damping or relaxation of the precession signal.

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Results and Discussion. - Magnetic susceptibil-89 ity measurements have previously been reported for green 90 dioptase [4, 7]. Our data shows similar behaviour where 91 there is a gradual increase in the susceptibility with a 92 maximum at approximately 50 K. There is then a de-93 crease and an obvious change in slope occurs below 25 K, 94 which is likely to be the emergence of the magnetically 95 ordered phase. This type of behaviour has been seen 96 within other 1D systems [14]. A fit to the high temper-97 ature data to Curie-Weiss law showed good agreement, 98 however the value of  $\theta = -98.1(7)$  K, suggesting an an-99 tiferromagnetic (AF) ground state. The frustration pa-100 rameter,  $f = -\theta/T_{\rm N}$  is calculated to be 6.6, which shows 101 there is a fairly strong suppression of order. Given the 1D 102 nature of the sample and the S = 1/2 moments on the Cu 103 ions, a better approximation may be the Bonner-Fisher 104 model [15], which is a mathematical expansion that de-105 scribes increasing interactions along an antiferromagneti-106 cally coupled 1D S = 1/2 chain. The value of J, the ex-107 change energy extracted from this fit is 45.89(7) K, where 108 there is good agreement between the fit and temperatures 109 above 75 K. However, as the temperature reaches 50 K 110 and the susceptibility goes through the maximum, the fit 111 breaks down and cannot fit the broad peak. At 50 K, there 112 may be increasing inter-chain interactions that lead to to 113 a break down in the 1D behaviour of the sample. Since 114 the sample has been reported to be in an AF ground state 115 at the lowest temperatures, this would be accompanied by 116 the opening of a spin gap and so it seems sensible to model
the low temperature susceptibility with an activated behaviour such as;

$$\chi = A \exp(-E_{\rm a}/k_{\rm B}T) \tag{1}$$

where A is the pre-exponent and  $E_{\rm a}$  is the activation or 120 spin gap energy and thus a measure of J. From the fits 121 the estimate of the spin gap is 43.4(8) K. This is an indi-122 cation of the exchange energy associated with the ordered 123 phase and this is close to the intra-chain exchange energy 124 calculated from the fits to the Bonner-Fisher model. This 125 could also be an indication that the inter- and intra-chain 126 interactions are very close in value and this may put the 127 system on the edge of a 1D to 3D behaviour. 128

In order to further study the nature of this low tempera-129 ture (T < 50 K) transition  $\mu$ SR was used. This presents a 130 unique zero-field (ZF) probe and has advantages for study-131 ing critical behaviour of materials. The raw data can be 132 seen in Figure 3. The high temperature data were taken 133 between 30 and 50 K where there was no apparent differ-134 ence between the muon spectra. This allowed for a high 135 temperature spectra with high statistics to be collected 136 and analysed. The raw data resemble that observed by 137 Cottrell *et al.* [16] who attributed such a low frequency 138 oscillation in the baseline to the muon binding to a de-139 protonated oxygen and with some anisotropy in the po-140 sition of the nuclear moments relative to the muon site. 141 In our case, the high temperature data could be fitted 142 using a summation of 2 components; the first is a the-143 ory developed by Meier [17] that accounts for the muon 144 spin relaxation from dipolar interactions with surround-145 ing nuclei with an exponential damping and the second 146 is an additional Gaussian relaxation. This implies that 147 there are two muon stopping positions within the sample 148 and with a ratio of 0.66:0.34 between component 1 and 149 2 respectively, the baseline is low at 1.32%. The strong 150 anisotropic homogeneous dipolar coupling from the first 151 component produces oscillations in the tail of the relax-152 ation with a field at the muon site of 7.46(2) G and a 153 damping of 0.094(2) MHz. As for the second Gaussian 154 component, this has a damping of 0.162(3) MHz, and so 155 the broader field distribution represents a site with more 156 field inhomogeneity. More work will be conducted on the 157 high temperature behaviour of the sample and reported 158 elsewhere [18], however, there is no evidence that there 159 are any electronic fluctuations above  $T_{\rm N}$ . 160

For the low temperature (T < 30 K) data high statis-161 tics spectra were collected in order to resolve the heavily 162 damped oscillation at short times, as can be seen in Figure 163 3 and indicates the presence of magnetic order in the sam-164 ple. On first glance, it is clear that there is a large drop in 165 initial asymmetry that is congruent within the onset of a 166 bulk magnetically ordered state. This drop in the asym-167 metry is due to large internal fields causing many of the 168 muons to dephase outside of the experimental time scale. 169 Although at high temperatures there appears to be two 170



Fig. 3: Raw data from the muon spectroscopy experiments. The high temperature data were co-added between 30 and 50 K to present a high statistics spectra to fit to since there was no apparent change in the relaxation with temperature. The low temperature data is shown at 13 K where the sample is within the critical region.

muon sites, this is harder to detect in the lower tempera-171 ture data and it is therefore likely that the two sites have 172 very similar frequencies or one has a frequency outside our 173 time window making them hard to resolve, especially given 174 that the raw data have high statistics. Therefore, in order 175 to parametrise and gain a value for the critical parame-176 ter a reasonable fit was achieved using a single oscillatory 177 component: 178

$$G(t) = A_1(\cos(\omega t)\exp(-\lambda_1 t)) + A_2\exp(-\lambda_2 t) + A_B \quad (2)$$

where  $A_n$  is the asymmetry of the relative components, 179  $\omega$  is the frequency of the muon spin precession that is re-180 lated to the internal field at the muon site by  $\omega = \gamma_{\mu} B$ ,  $\lambda_{\rm n}$ 181 is the relaxation that describes the field distribution and 182  $A_{\rm B}$  is the baseline. In the ordered phase, the baseline is 183 around 7.4% and the asymmetries are fairly constant as is 184 the value for  $\lambda_2$  at 0.32 MHz.  $\lambda_1$  shows a gradual increase 185 with decreasing temperature, which can be seen in the 186 supplementary information. Below 9 K, the damping ap-187 pears, within error, to be constant. The gradual rise in  $\lambda_1$ 188 is likely due to the increasing field broadening due to the 189 highly anisotropic 1D system; as the electronic moments 190 order, these will be along the 1D chains and the muon will 191 be in a position of an inhomogeneous field distribution. 192

The temperature dependence of the muon spin rotational frequency from fits to Equation 2 can be seen in Figure 4. The frequency *vs.* temperature data could be fit within the critical region with the equation

$$\omega = D(1 - T/T_{\rm C})^{\beta} \tag{3}$$

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where D is the frequency at 0 K and  $\beta$  is the critical exponent. From the fits solely in the critical region,  $\beta = 0.37(1)$ , which is similar to what is expected for a



Fig. 4: The frequency of the muon precession as a function of temperature. The solid line is a fit to the data using Equation 3 to get the critical exponent and corresponding  $T_{\rm C}$ . Inset: The longitudinal field dependence of the muon polarisation associated with the baseline observed at temperatures above and below the  $T_{\rm C}$ .

<sup>200</sup> 3D Heisenberg bulk system. The value of  $T_{\rm C}$  equates to <sup>201</sup> 14.96(6) K, and this is approximately where we observe <sup>202</sup> the kink in the magnetic susceptibility at low tempera-<sup>203</sup> tures. Therefore at this point the system falls into a 3D <sup>204</sup> ordered ground state.

To further confirm this, a longitudinal field (LF) sweep 205 was conducted, where the application of an LF decouples 206 the polarisation of the muon spin from the surrounding 207 environment, thus at higher fields you will always recover 208 to a polarisation of 1. The data can be seen in the inset 209 of Figure 4, where there is a clear difference between the 210 low and high temperature LF sweeps. At 20 K, the muon 211 polarisation is fully recovered at 40 G, which is typical of a 212 muon - nuclear moment dipole interaction. There is no in-213 dication of dynamics of nuclear or electronic moments on 214 the muon time scale otherwise; it would require a larger 215 field to decouple the muon spin. At low temperatures, 216 much larger fields are needed to begin decouple the muon 217 spin from the internal fields, there may be a change of 218 slope at 300 G, with the mid-point of the curve at  $\sim 150$  G 219 (2.03 MHz). There is then a slow increase in the polarisa-220 tion, which might suggest that the second muon stopping 221 site sits in a region where the internal fields are too high 222 for us to measure with the experimental technique. It 223 should be noted that at the higher fields (> 1 kG) there 224 is no relaxation and the spectra are essentially a flat line; 225 therefore there is no evidence of electronic fluctuations on 226 the muon time scale. 227

If there was observation of quantum critical behaviour, then the electronic moments would be dynamic and the muon spin relaxation measurements would reflect this. Evidence, for example, would be no dramatic loss in asymmetry on going through  $T_{\rm N}$ ; one would simply expect either there to be no apparent relaxation since the muon

would be in the motionally narrowed state, or a relaxation 234 but no missing asymmetry, provided the fluctuations were 235 within the experimental time window. One would also ex-236 pect to see a relaxation even in applied fields, where, once 237 the nuclear moments had been decoupled, the electronic 238 fluctuations would not be perturbed by the applied field 239 and so the muon will not be decoupled from the dynamics. 240 Instead we observe no evidence of any strong relaxation 241 that would suggest the sample is in a dynamic state col-242 orredwithin the experimental time-scale. The muon spec-243 tra resemble that of a system that has magnetically or-244 dered where the baseline has shifted higher in asymmetry 245 to approximately 1/3, which is expected when in the or-246 dered state, where one has a x/3 + y/3 + z/3 average; a 247 purely dynamic state would not be accompanied by this 248 missing asymmetry. The onset of the oscillatory compo-249 nent is a clear indication of a bulk ordered phase, albeit 250 with a significant damping. The calculated critical expo-251 nent associated with the transition matches well with that 252 expected for a 3D Heisenberg magnet and given previous 253 work [2, 8], it is certainly antiferromagnetic in nature. 254

Concluding Remarks. – In summary, we have stud-255 ied the nature of the antiferromagnetic transition in green 256 dioptase using magnetic susceptibility and muon spec-257 troscopy. The order parameter associated with the tran-258 sition, obtained from the temperature dependence of the 259 muon spin rotational frequency, showed the sample orders 260 to a 3D Heisenberg ground state. We find that there is 261 no evidence of quantum critical behaviour above or be-262 low  $T_{\rm N}$ . Instead, below  $T_{\rm N}$  the sample appears to enter a 263 static state magnetic state on the time scale of the muon 264 measurement. 265

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