



# Black disk, maximal Odderon and unitarity

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## ABSTRACT

We argue that the so-called maximal Odderon contribution breaks the 'black disk' behavior of the asymptotic amplitude, since the cross section of the events with Large Rapidity Gaps grows faster than the total cross section. That is the 'maximal Odderon' is not consistent with unitarity.

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## 1. Introduction

Recently the TOTEM collaboration at the LHC has published the results of the first measurements at  $\sqrt{s} = 13$  TeV of the  $pp$  total cross section  $\sigma_{\text{tot}} = 110.6 \pm 3.4$  mb [1] and of the ratio of the real-to-imaginary parts of the forward  $pp$ -amplitude,  $\rho = \text{Re}/\text{Im} = 0.10 \pm 0.01$  [2]. Since the latter value appears to be sufficiently smaller than that predicted by the conventional COMPETE parametrization ( $\rho = 0.13$ – $0.14$ ) [3], it may indicate either a slower increase of the total cross section at higher energies or a possible contribution of the odd-signature amplitude. (Note that within the COMPETE parametrization the odd-signature term is described by secondary Reggeons and dies out with energy.) Note that a C-odd amplitude, which arises from the so-called Odderon, and which depends weakly on energy, is expected in perturbative QCD,<sup>1</sup> see in particular [4–6] and for reviews e.g. [7,8]. However the naive estimates show that its contribution is rather small; say,  $\Delta\rho_{\text{odd}} \sim 1 \text{ mb}/\sigma_{\text{tot}} \lesssim 0.01$  [9] at the LHC energies.

On the other hand, it is possible to introduce the Odderon phenomenologically as an object which does not violate first principles and the axiomatic theorems. In fact it was stated in [10] that the new TOTEM result is a definitive confirmation of the experimental discovery of the Odderon in its maximal form.

Recall that the Odderon was first introduced in 1973 [11], and since then it has been the subject of intensive theoretical discussion, in particular within the context of QCD. Indeed, there have been several attempts to prove its existence experimentally (see,

for example, [7,8,12] for comprehensive reviews and references). While the discovery of the long-awaited, but experimentally elusive, Odderon would be very welcome news for the theoretical community, our aim here is to try to check whether the presence of the *maximal* Odderon, with an amplitude  $A^- \propto \ln^2 s$ , which has a real part with high energy behavior similar to that of the imaginary part of the even-signature amplitude  $A^+$ , does not violate unitarity at asymptotically large c.m.s. energy  $\sqrt{s} \rightarrow \infty$ . Here we use the normalization  $\text{Im}A = \sigma_{\text{tot}}$ .

## 2. Multi-Reggeon processes

It was recognized already in the 1960s [13,14] that multi-Reggeon reactions,

$$pp \rightarrow p + X_1 + X_2 + \dots + X_n + p, \quad (1)$$

where small groups of particles ( $X_i$ ), are separated from each other by Large Rapidity Gaps (LRG) (see Fig. 1), may cause a problem with unitarity. Indeed, being summed over  $n$  and integrated over the rapidities of each group, the cross section of such quasi-diffractive production increases faster than a power of  $s$ . This was termed in the literature as the Finkelstein–Kajantie disease (FK), see [15] for a review.

Let us explain the situation using the simple example of Central Exclusive Production (CEP) of only one group of particles, as shown in Fig. 2. Here the double line denotes the amplitude,  $A$ , which describes the interaction across the LRG (in particular, the proton–proton elastic amplitude). Correspondingly, the CEP amplitude for Fig. 2a reads

$$A^{\text{CEP}}(y_1, y_2, t_1, t_2) = A(y_1, t_1) \cdot V \cdot A(y_2 - y_1, t_2), \quad (2)$$

where  $V$  is the vertex factor of central production and the  $y_i$  are the values of the rapidity.

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<sup>1</sup> QCD is the  $SU(N=3)$  gauge theory which contains the spin=1 particle (gluon) and (for  $N > 2$ ) the symmetric color tensor,  $d^{abc}$ . Due to these facts in perturbative QCD there exists a colorless C-odd  $t$ -channel state (formed from three gluons) with intercept,  $\alpha_{\text{odd}}$ , close to 1.

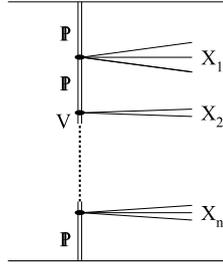


Fig. 1. Multi-Reggeon production.

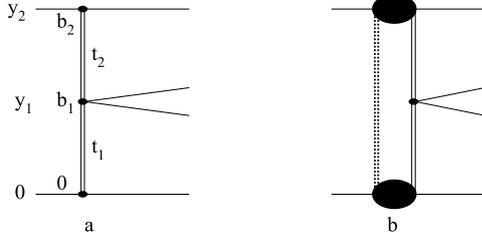


Fig. 2. (a) Central Exclusive Production, (b) the diagram relevant when we discuss the survival probability of the LRG in Section 3.

The full CEP cross section is given by the integral

$$\sigma^{\text{CEP}} = N \int_0^Y dy_1 \int dt_1 dt_2 |A(y_1, t_1) \cdot V \cdot A(Y - y_1, t_2)|^2, \quad (3)$$

where  $N$  is the normalization constant and where we put the upper rapidity  $y_2 = Y = \ln s$ . In case of the maximal Odderon the real part of amplitude  $A(Y)$  grows as  $\text{Re}A = c \ln^2 s = cY^2$ . On the other hand, the  $t$ -slope  $B \propto R^2$ , with the interaction radius limited by the Froissart [16] condition  $R \leq \text{const} \cdot Y$ . That is the integral

$$I = \int dt |A(Y, t)|^2 \sim Y^2 \quad (4)$$

leading to

$$\sigma^{\text{CEP}} = N \int_0^Y dy |I(y) \cdot V^2 \cdot I(Y - y)| \propto Y^5. \quad (5)$$

Thus in such a case, the CEP cross section would grow much faster than the total cross section  $\sigma_{\text{tot}} \sim \ln^2 s = Y^2$ .

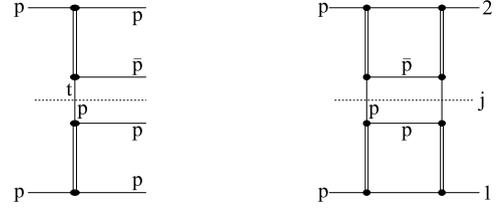
The same result can be obtained in impact parameter,  $b$ , space. Now

$$\sigma^{\text{CEP}} = N' \int_0^Y dy \int d^2 b_1 d^2 b_2 |A(y_1, b_1) \cdot V \cdot A(Y - y, b_2 - b_1)|^2 \propto Y^5. \quad (6)$$

Recall that in  $b$  space the amplitude is limited<sup>2</sup> to  $|A(Y, b)| \leq 2$  by the unitarity equation

$$2\text{Im}A(Y, b) = |A(Y, b)|^2 + G_{\text{inel}}(Y, b) \quad (7)$$

where  $G_{\text{inel}}$  denotes the total contribution of all the inelastic channels. On the other hand the area where the amplitude is large


 Fig. 3. Diagrams for the amplitude (left) and the cross section (right) of  $p\bar{p}$  exclusive production generated by  $t$ -channel unitarity.

( $A \sim O(1)$ ), that is the value of  $\int d^2 b \sim \pi R^2 \propto Y^2$ , increases as  $R^2 \sim Y^2$ .

Summing up the analogous cross sections for processes with a larger number of LRGs (i.e. a larger number,  $n$ , of hadron groups  $X_i$  in Fig. 1) we obtain the cross section which increases faster than the power of  $s$ . Indeed, each additional gap brings a factor  $\ln s$  arising from the integral over the gap size (times the ‘elastic’ cross section which in the Froissart limit increases as  $\ln^2 s_{i,i+1}$ ). The sum of these  $\ln s$  factors leads to the power behavior.

Note that by working in  $b$  space we have a stronger constraint since for each value of  $b$ , that is for each partial wave  $l = b\sqrt{s}/2$  of the incoming proton pair, the ‘total’ cross section,  $\sigma(b)_{\text{tot}}$  must be less than the corresponding CEP contribution.

Actually one will face this FK problem in any model where the elastic cross section does not decrease with energy.

At first sight the simplest way to avoid the FK problem is to say that the production vertex ( $V$  in Fig. 1) vanishes, at least as  $t_i \rightarrow 0$ . However this cannot be true. Indeed, as far as we have a non-vanishing high-energy elastic proton–proton cross section, we can build the diagram on the right side of Fig. 3 from a lower part which is just elastic  $pp$ -scattering and an upper part which corresponds to the proton–antiproton elastic interaction. Such a diagram is generated by the  $t$ -channel two-particle unitarity equation for the amplitude,

$$\text{disc}_t \mathbf{A}_{12} = \sum_j \mathbf{A}_{1j}^* |j\rangle \langle j| \mathbf{A}_{j2}, \quad (8)$$

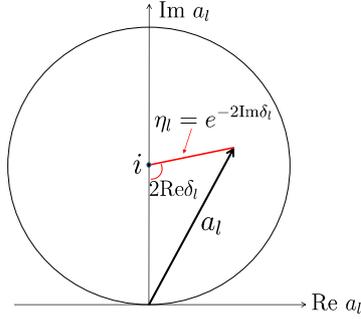
where in our case  $|j\rangle$  is the  $t$ -channel  $p\bar{p}$  state. Note that the contribution of this diagram is singular at  $t = m_p^2$  (where  $m_p$  is the proton mass). There are no other similar terms corresponding to the central exclusive production of a  $p\bar{p}$  pair with the same pole singularity. That is, in the vertex  $V$  of Fig. 2a, there exists at least one subprocess ( $p\bar{p}$  CEP), which cannot be canceled identically.

It is useful to clarify the above argument, since it is a little subtle. We have to distinguish between the momenta transfer squared,  $t_1$  and  $t_2$ , incoming to the vertex  $V$  in Fig. 2a, and the momentum transferred squared inside the vertex  $V$ , denoted by  $t$  in Fig. 3(left). The value of the  $t$  is driven by the transverse momentum,  $p_t$ , of the antiproton. Even if, due to a subtraction in dispersion relation in  $t$  that reconstructs the amplitude, we find at some  $p_t$  point that  $V = 0$ , this will not insure that the total vertex contribution vanishes. We will have  $V \neq 0$  at other  $p_t$  values. Now, to calculate the total CEP cross section, we have to integrate over all available  $p_t$ , so finally we obtain a non-zero contribution of this particular  $p\bar{p}$  subprocess.

### 3. The solution of the FK problem

The only known solution of this multi-Reggeon problem comes from ‘black disk’ asymptotics of the high energy cross sections. In such a case the (gap) survival probability,  $S^2$ , of the events with a LRG, tends to zero at  $s \rightarrow \infty$ , and the value of  $\sigma^{\text{CEP}}$  does not

<sup>2</sup> This is illustrated in Fig. 4 below in terms of the partial wave amplitude  $a_l(s)$ . The plot shows  $|a_l| \leq 2$ .



**Fig. 4.** The partial wave amplitude,  $a_l = i(1 - e^{2i\delta_l})$ , is constrained to lie in the ‘unitarity circle’ centered on  $(0, i)$ . At low energy, for a resonance of orbital momentum  $l$  appearing only in the elastic channel, the energy dependence of the amplitude  $a_l$  follows a counter-clockwise circle of radius  $\eta_l = 1$  with maximum amplitude at  $\delta_l = \pi/2$ . However, for high energy scattering, black disk asymptotics requires  $\text{Re}\Omega(b) \rightarrow \infty$  (see (19)), so  $2\text{Im}\delta_l \rightarrow \infty$  and  $\eta_l \rightarrow 0$ , and hence the amplitude becomes pure imaginary.

exceed  $\sigma_{\text{tot}}$  (for a review of diffractive processes at the LHC see e.g. [17]).

In other words, besides the contribution of Fig. 2a, we have to consider the diagram of Fig. 2b, where the double-dotted line denotes an additional proton–proton (incoming hadron) interaction. This diagram describes the absorptive correction to the original CEP process, and has a negative sign with respect to the amplitude  $A^a$  of Fig. 2a. Therefore to calculate the CEP cross section we have to square the full amplitude

$$|A_{\text{full}}(b)|^2 = |A^a(b) - A^b(b)|^2 = S^2(b) \cdot |A^a(b)|^2, \quad (9)$$

where

$$S^2(b) = |e^{-\Omega(b)}|, \quad \text{with } \text{Re}\Omega \geq 0. \quad (10)$$

Indeed, in terms of  $\mathbf{S}$ -matrix, the elastic component  $S_l = 1 + iA(b)$ , and the unitarity equation (7) reflects the probability conservation condition

$$\sum_n \mathbf{S}_l^* |n\rangle \langle n| \mathbf{S}_l = 1 \quad (11)$$

for the partial wave  $l = b\sqrt{s}/2$ . The solution of unitarity equation (7) reads

$$A(b) = i(1 - e^{-\Omega(b)/2}), \quad (12)$$

or in terms of the partial wave amplitude with orbital moment  $l = b\sqrt{s}/2$

$$a_l = i(1 - e^{2i\delta_l}) = i(1 - \eta_l e^{2i\text{Re}\delta_l}) \quad (13)$$

where

$$\eta_l = e^{-2\text{Im}\delta_l} \quad \text{with } 0 \leq \eta_l \leq 1. \quad (14)$$

The unitarity circle bounding the partial wave amplitude is shown in Fig. 4.

The above discussion shows that  $-\Omega(b)/2$  plays the role of  $2i\delta_l$ . The elastic component of  $\mathbf{S}$  matrix  $S_l = \exp(2i\delta_l)$ . Correspondingly, the probability of inelastic interaction (with all the intermediate states  $n'$  except of the incoming, elastic, state)

$$G_{\text{inel}} = \sum_{n'} \mathbf{S}_l^* |n'\rangle \langle n'| \mathbf{S}_l \quad (15)$$

takes the form

$$G_{\text{inel}}(b) = 1 - S_l^* S_l = 1 - e^{-\text{Re}\Omega(b)}. \quad (16)$$

Within the eikonal model  $\Omega(b)$  is described by the sum of single Reggeon exchanges while the decomposition of the exponent generates the multi-Reggeon diagrams.

The gap survival factor,  $S^2$ , is the probability to observe a pure CEP event, where the LRG is not populated by secondaries produced in an additional inelastic interaction shown by the dotted lines in Fig. 2b. That is according to (16)

$$S^2(b) = 1 - G_{\text{inel}}(b) = e^{-\text{Re}\Omega(b)}. \quad (17)$$

Equation (17) can be rewritten as (see (12), (16))

$$S^2(b) = |1 + iA(b)|^2 = |S_l|^2. \quad (18)$$

In the case of *black disk asymptotics*<sup>3</sup>

$$\text{Re}\Omega(b) \rightarrow \infty \quad \text{and} \quad A(b) \rightarrow i, \quad (19)$$

for  $b < R$ . That is, we get  $S^2(b) \rightarrow 0$ . The decrease of the gap survival probability  $S^2$  overcompensates the growth of the original CEP cross section (Fig. 2a), so that finally we have no problem with unitarity.

Recall that this solution of the FK problem was actually realized by Cardy in [18], where the reggeon diagrams (generated by Pomerons with intercept  $\alpha_P(0) > 1$ ) were considered by assuming analyticity in the number of Pomerons; and also by Marchesini and Rabinovici in [19], where diffractive production processes were discussed for the case of  $\alpha_P(0) > 1$ .

Note that at the moment we deal with a one-channel eikonal. In other words, in Fig. 2 and in the unitarity equation (7), we only account for the pure elastic intermediate states (that is the proton, for the case of  $pp$  collisions). In general, there may be  $p \rightarrow N^*$  excitations shown by the black blobs in Fig. 2b. The possibility of such excitations can be included by the Good–Walker [20] formalism in terms of G–W eigenstates,  $|\phi_i\rangle$ , which diagonalize the high energy scattering process; that is  $\langle \phi_k | A | \phi_i \rangle = A_k \delta_{ki}$ . In this case we encounter the FK problem for each state  $|\phi_i\rangle$ , and we then solve it for the individual eigenstates.

### 3.1. Edge of the disk

This subsection is not crucial for our final result, but it should be mentioned in order to demonstrate the self-consistency of the whole picture.

While the survival factor  $S^2$  solves the FK problem for the central part of the black disk, we still have to address the question of what happens at the edge of the disk, where the optical density is not large? That is, when  $\text{Re}\Omega(b) \sim O(1)$ . For large partial waves, which occur in this domain, we still may have CEP (and other diffractive LRG) cross sections larger than the total cross section corresponding to such  $l$ -waves.

The solution is provided by the fact that actually the constraint on the interaction radius  $R$  is a bit stronger than just  $R \leq c \ln s$ . It was shown in [21,22] that we have to account for the ‘ $\ln \ln s$ ’ correction

$$R = c \ln s - \beta \ln \ln s = cY - \beta \ln Y. \quad (20)$$

In this case, the radius of the CEP interaction shown in Fig. 2a, (that is, before we account for the screening effects of Fig. 2b),

<sup>3</sup> Recall that the word ‘black’ means the *complete* absorption of the incoming state (up to power of  $s$  suppressed corrections). That is,  $\text{Re}\Omega(s, b) \rightarrow \infty$ . ‘Black disk’ means that in some region of impact parameter space,  $b < R$ , the whole initial wave function is absorbed. That is, the value of  $S(b) = 1 + iA(b) = S_l \rightarrow 0$ , i.e.  $A(b) \rightarrow i$ .

$$R_{\text{CEP}} = R(y) + R(Y - y) = cY - \beta [\ln y + \ln(Y - y)]$$

$$< R(Y) = cY - \beta \ln Y, \quad (21)$$

turns out to be smaller than that corresponding to elastic scattering. That is the multi-Reggeon amplitude is placed *inside* the black disk, and its contribution is strongly suppressed by the  $S^2(b)$  factor.

It was shown in [21,23] that the same condition (20) provides the possibility to satisfy the  $t$ -channel unitarity.

#### 4. Maximal Odderon

Now let us consider the situation with the maximal Odderon, where at very high energies the real part of elastic amplitude  $A$  is comparable with its imaginary part, in the sense that the ratio<sup>4</sup>

$$\text{Re}A/\text{Im}A \rightarrow \text{constant} \neq 0. \quad (22)$$

In such a case the elastic amplitude  $A(b)$  has a non-zero real part, which violates the condition  $A \rightarrow i$  at  $s \rightarrow \infty$ . Now the survival factor (18)

$$S^2 = |1 + iA|^2 \geq |\text{Re}A|^2 \neq 0 \quad (23)$$

tends to some non-zero constant. That is we lose the possibility to compensate the growth of the multi-Reggeon (CEP) cross sections by the  $S^2$  factor. Thus these cross sections, which increase faster than the total cross section, will violate unitarity.

Let us consider a dynamical model. Note that the expression for the elastic amplitude (12) is an *exact* solution of the  $s$ -channel two-particle unitarity equation (7), where  $\Omega$  is the two-particle-irreducible amplitude which includes all possible inelastic interactions. That is, in terms of Regge theory, the Odderon contribution must be included into the opacity, and the opacity  $\Omega(b)$  (i.e. the ‘phase’  $\delta_l$ ) should be written at high energies as the sum of the even-signature (Pomeron) and the odd-signature (Odderon) terms

$$\Omega(b) = -i [\text{Pomeron}(b) + \text{Odderon}(b)], \quad (24)$$

where the Pomeron term is mainly imaginary while the Odderon contribution is mainly real. If  $\alpha_P(0) = 1 + \Delta > 1$ , then the Pomeron term increases as the  $s^\Delta$ . That is the exponent  $\exp(-\Omega/2) \rightarrow 0$  and the second term in elastic amplitude (12) vanishes *together with the Odderon* contribution. In other words, in the black disk limit when the value of  $\text{Re}\Omega$  increases and  $\exp(-\Omega/2) \rightarrow 0$  the Odderon contribution dies out. The only chance to have a sizeable Odderon as  $s \rightarrow \infty$  is to collect the contribution from the edge of black disk where the opacity  $\text{Re}\Omega(b) \sim O(1)$  is not large. From this region one may get an Odderon contribution to the forward elastic amplitude  $A_{\text{Odd}}(t=0) \propto \ln s$ ; that is  $\text{Re}A_{\text{Odd}}$  could grow as the area of the ring around the black disk, but certainly it cannot increase as  $\ln^2 s$ .

However even the  $\ln s$  asymptotic behavior is questionable. The point is that the radius of the Odderon induced interaction is most probably smaller than the radius of black disk, generated by the even-signature bare Pomeron. Indeed, the nearest  $t$ -channel singularity of the even-signature amplitude is  $t = 4m_\pi^2$ , while for the Odderon the nearest singularity is at  $t = 9m_\pi^2$ . We cannot build the Odderon state from two pions. The most reasonable appropriate hadron state in the Odderon channel is the  $\omega$  meson. Therefore the growth of the Odderon radius with energy is expected to be

less than the growth of the black disk radius driven by the even-signature amplitude, and the whole Odderon contribution will be ‘absorbed’ (i.e. power of  $s$  suppressed) by the black disk.

Thus in this section we have demonstrated that

- a) the maximal Odderon violates multiparticle  $s$ -channel unitarity,
- b) the Odderon contribution disappears in the black disk limit when  $\text{Re}\Omega \rightarrow \infty$ .<sup>5</sup>

#### 5. Reflective scattering

The same argument can be used to reject the so-called ‘reflective scattering’ asymptotics proposed in [25]. Indeed, in this regime it is assumed that the high energy interaction becomes pure elastic and the amplitude

$$A(b < R) \rightarrow 2i \text{ as } s \rightarrow \infty, \quad (25)$$

(with our normalization fixed by eq. (7)). This means that at very high energies we will have an almost pure elastic interaction with  $G_{\text{inel}} \rightarrow 0$ . In such a case  $S^2 = 1$  (see (18)).

On the other hand,  $t$ -channel unitarity generates the inelastic CEP diagram Fig. 2b with a cross section which increases faster than the elastic cross section. The contribution of such a diagram cannot be suppressed by absorptive effects since now we have  $S^2 = 1$ . That is again we face the FK problem – the cross section of multi-Reggeon processes (in particular CEP) violates the unitarity constraint.

We emphasize that black disk absorption is the only cure of the FK disease. Thus any asymptotic behavior of a high energy cross section, increasing with energy, which does not lead to complete absorption, is not consistent with multi-particle unitarity. In particular, the amplitudes considered in [26–28], should be abandoned, since they do not satisfy the black disk condition.

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<sup>4</sup> Recall that the dotted lines in Fig. 2b denote just the elastic amplitude of proton–proton, or G-W eigenstate, scattering.

<sup>5</sup> A similar conclusion was reached in [24] based on the eikonal model which includes both the bare Pomeron and the bare Odderon poles. It was shown that, even starting with the Odderon with a larger than one intercept (i.e.  $\Delta_{\text{Odd}} = \alpha(0)_{\text{Odd}} - 1 > 0$ ), after eikonalization we get an Odderon contribution ( $\sim s^{-\delta}$ ) which decreases as a power of energy, except for the case when the bare Odderon trajectory coincides with the bare Pomeron trajectory.

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