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On Heaviside's Contributions to Transmission Line Theory: Waves, Diffusion and Energy Flux

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Summary

This paper surveys some selected contributions of Oliver Heaviside FRS (1850-1925) to classical electromagnetic theory and electrical engineering science. In particular, focusing on his contributions to the development of electrical transmission line theory and his deep insight into the 'physical' nature of the physical phenomena relating to 19th Century telegraphic problems. Following a brief historical introduction to the life of Heaviside to put his achievements in context, we explore his contributions to the reformulation of Maxwell's equations and the understanding of electromagnetic wave propagation along the external region of transmission lines. This leads naturally to his researches regarding the electromagnetic diffusion process inside of the line conductors and his subsequent realisation that the circuital parameters, usually assumed constant, are not always so. Finally, taking both these internal and external viewpoints of the conductors, his important work regarding the flow of energy described by his 'Energy current' concept is presented.

Introduction

Oliver Heaviside FRS (portrait in Fig. 1) was born on the 18th May, 1850 at 55 King Street (now known as Plender Street, Camden), London and died in Torquay on 3rd February, 1925, at the age of 74. Perhaps one of the most remarkable aspects of Heaviside's life was that he left school at a young age, having received no formal education beyond the age of 14 – thus, he would have left with little knowledge beyond algebra and trigonometry. Despite this lack of advanced mathematical and scientific training, he was able to make very many significant scientific achievements to rival the most revered of his university-educated peers in the great Victorian era of scientific discovery. His trajectory from schoolboy to an eminent *Electromagnetician* (as prescribed by his friend G. F. C. Searle FRS), can be thought of as an embodiment and direct result of the well known 'self improvement' philosophy, which was a staple part of the Victorian middle-class culture in which he was brought up. In Heaviside's case, this seemed to be fuelled by indefatigable intellectual curiosity and the desire for deep knowledge and understanding of the unknown – he was a self-taught electrical engineer, physicist and mathematician, finding his own way through the new electrical engineering science of the 19th Century. Today, contemporary degree qualified electrical engineers, physicists and mathematicians are likely to know little of Heaviside or his works and his significant influence in the field of Electromagnetic Theory.

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It can be argued that his work is vastly underappreciated. This is surprising as he made major and lasting contributions not only to classical physics, but also to that of both pure and applied mathematics, as well as countless contributions to the electrical engineering science that underpins most modern electrical/electronic technology. The *Institution of Electrical Engineers Heaviside Centenary Volume* provides a good account of his outstanding works in these areas, including some important findings from his unpublished notebooks [1]. H. J. Josephs, a Post Office Research Station engineer gives valuable insight into those unpublished works held at the Archives of the Institution of Engineering and Technology (IET), showing how Heaviside develop new methods to solve indefinite integrals, linked with rather tricky electrical circuit problems. The Centenary Volume also includes a sketch of Heaviside's personality by his friend; Cavendish Professor G. F. C. Searle FRS.

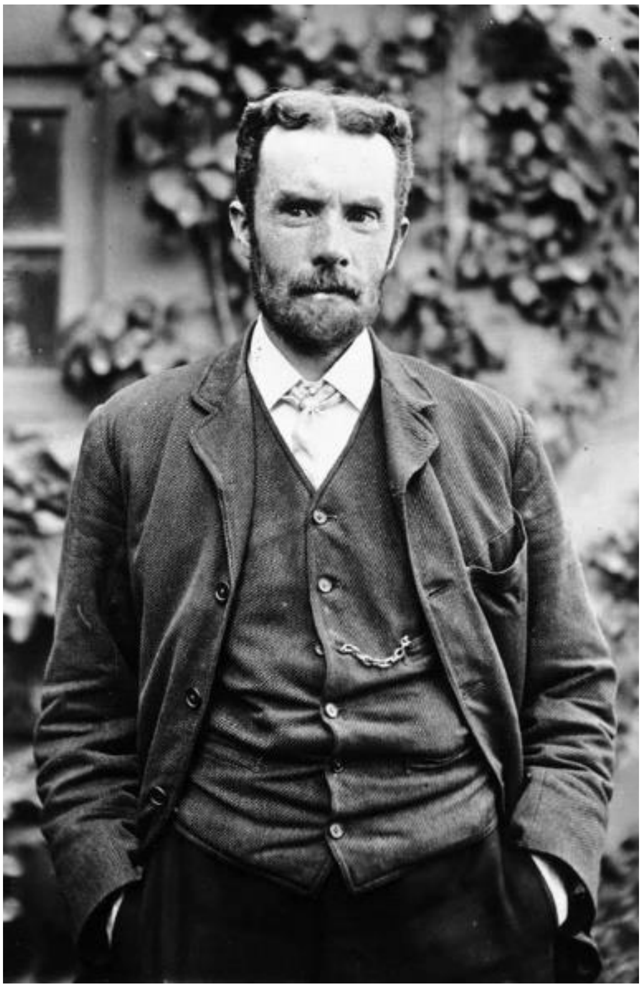

Figure 1 – Portrait of Oliver Heaviside FRS [Public Domain]

Heaviside's approach in his work was that of the 'physical'; he sought to understand the physical nature of the electrical and electromagnetic phenomena, despite his heavy use of mathematics. Much of his fundamental and most important work on electromagnetic theory, transmission lines, and electric circuit theory is contained within his two volumes of ‘Electrical Papers’, published in 1892. It is true to say that the scope of his work was wide-ranging, at first focused on the great practical telegraphic and telephonic problems of the late 19th Century, leading to the invention of inductive loading coils and a patent for the coaxial cable [2]. Later he explored (in his three volumes of Electromagnetic Theory, published from 1893 onwards, *125 years ago*), gravitational and electromagnetic analogies, waves from moving sources and even skirted around superconductivity with his comments on low-temperature conductors.

An engineer, but he was also a mathematical pioneer, an early adopter of the vector analysis and calculus, developing it to suit his engineering and physics problems. More than fifty pages of the Philosophical Transactions of the Royal Society are devoted to his development of these vector methods. Using these methods he made a most substantive and very important contribution by recasting James Clerk Maxwell’s twenty-two quaternionic equations into the four coupled partial differential equations we know today [3]. Maxwell had published his two-volume work ‘A Treatise on Electricity and Magnetism’ in 1873 [4]. Heaviside first came across this seminal work as part of his ‘self-study’ regime while he was living in Newcastle upon Tyne in late 1873, while working for the Great Northern Telegraph Company (his only paid work, which he left in 1873 at the age of 23 to concentrate on his studies). A description of this encounter is best kept in his own witty words, as he recollected in a personal letter later in life [5];

> *“I remember my first look at the great treatise of Maxwell's when I was a young man... I saw that it was great, greater and greatest, with prodigious possibilities in its power... I was determined to master the book and set to work. I was very ignorant. I had no knowledge of mathematical analysis (having learned only school algebra and trigonometry which I had largely forgotten), and thus my work was laid out for me. It took me several years before I could understand as much as I possibly could. Then I set Maxwell aside and followed my own course. And I progressed much more quickly... It will be understood that I preach the gospel according to my interpretation of Maxwell.”*

Following Maxwell, cutting his own path forward through electromagnetic theory, he explored the fringes of relativistic electrodynamics in 1888 [6, 7] and studied the physical implications of hyper-light particle motion, leading him to the theoretical prediction of electromagnetic radiation under these conditions in 1888-89 [8]. This radiation was verified experimentally by Cherenkov, for which his team received the Nobel Prize in Physics in 1958. Heaviside first studied a generalized form of ‘skin effect’ in conductors in 1885 [9], he published in 1902 a theory of global electromagnetic wave propagation utilizing a conducting region in the atmosphere, now known as the Heaviside-Kennelly layer [10]. These remarkable achievements, among many others, led to many accolades including election as a Fellow of the Royal Society in 1891, and in 1908 an Honorary Member of the Institution of Electrical Engineers (IEE). In 1922, three years before his death, he was awarded the very first Faraday Medal by the IEE. These achievements are outstanding for a self-taught, lower middle class, independent researcher and the son of a wood engraver from Stockton-on-Tees. Heaviside was a remarkable man, an original thinker with brilliant mathematical powers and physical insight.

It is beyond this scope of this paper to explore deeper into the life of Oliver Heaviside. For an excellent and in-depth account of his life, illuminating his relationships with eminent scientists of the age and with the 'establishment', the reader is directed to the excellent books on the subject [7, 11,12]. Here we are concerned mainly with his achievements in the field of electromagnetic theory in relation to transmission lines, this brief introduction serving to 'set the scene' for those not familiar with the life and work of Heaviside and to provoke the reader to explore deeper into the brief and limited descriptions of his contribution herein. It is not claimed here that the contributions within are comprehensive; they merely scratch the surface of Heaviside's work until around 1900. The focus of this paper is on his studies on electromagnetic wave propagation, electromagnetic diffusion, and energy flow.

Electromagnetic Waves

Heaviside made many original contributions to the theory of electromagnetic wave propagation through his desire to develop a comprehensive theory of the transmission line and its application to 19th Century telegraphic problems. As pointed out by Baron Jackson of Burnley FRS [1], Heaviside published over 'one and a half million words' on his researches, in which Burnley gives a detailed overview of some of these contributions. Presented here are those original contributions that are perhaps the most important in the development of his comprehensive theory, setting the scene for his subsequent work.

Heaviside's Equations – Duplex Forms

Most of Heaviside's step change contributions begin with his interpretation of Maxwell's theory of the electromagnetic field as he discovered it in 1873 [3,4]. The so-called *Maxwell's* equations, a set of partial differential equations that describe electromagnetic field interactions and which are a cornerstone of classical physics, developed by Maxwell from 1861 onwards, are immediately recognizable to physicists, electrical engineers, and most mathematicians today [13]. In differential form and modern notation they can be written;

$$\begin{gathered}
\nabla \cdot \mathbf{D}(\mathbf{r},t) = \rho \\
\nabla \cdot \mathbf{B}(\mathbf{r},t) = 0 \\
\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} \\
\nabla \times \mathbf{H}(\mathbf{r},t) = \boldsymbol{J}(\mathbf{r},t) + \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t}
\end{gathered} \tag{1}$$

Here **D** is the electric displacement field, ρ the electric charge density, **B** the magnetic flux density, **E** the electric field, **H** the magnetic field intensity and finally $\boldsymbol{J}$ the electric current density; all being a function of space **r** and time t. These equations, along with the constitutive relations describe electromagnetic phenomena. However, these equations are not *really* from Maxwell. The original electrodynamic questions equations from 1873 were written in a different language, that of the mathematics of Quaternions, an extension of complex numbers developed by Sir William Hamilton FRSE. This language obscured the physics according to Heaviside, and he viewed the mathematical presentation as '*hard to read and lengthy in execution'* [14]. Much preferring his Vector methods (alongside J. W. Gibbs), Heaviside was first to recast the 22 Maxwell equations into this Vector algebra form, as is common today in almost all textbooks on the subject. This pioneering work made the theory of electromagnetism more accessible and readily able to be applied to solve practical problems such as those Heaviside sought to solve, as a practical 'telegrapher'. It must be noted that Heaviside had not changed the content of Maxwell's work, but was "*preach*[ing] *the gospel according to* [his] *interpretation of Maxwell*", albeit in a different language. However, this achievement is not trivial [3], it was a long road to the form we are now familiar with.

As if this recasting was not significant enough, Heaviside then turned his attention to the 'symmetry' of the equations when electric charges and conduction currents are present. The curl equation for the $\mathbf{E}$ field is not symmetric with the curl equation for the $\mathbf{H}$ field. Heaviside presented his vectorial 'duplex form' of the equations in 1884 [9] where he introduced hypothetical magnetic conduction currents $\boldsymbol{G}$ and magnetic charge ρ_m to preserve this symmetry as found with the 'free space' Maxwell equations. In modern notation;

$$\begin{aligned} \nabla \cdot \mathbf{D}(\mathbf{r},t) &= \rho_e \\ \nabla \cdot \mathbf{B}(\mathbf{r},t) &= \rho_m \\ -\nabla \times \mathbf{E}(\mathbf{r},t) &= \boldsymbol{G}(\mathbf{r},t) + \frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} \\ \nabla \times \mathbf{H}(\mathbf{r},t) &= \boldsymbol{J}(\mathbf{r},t) + \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} \end{aligned} \tag{2}$$

He realized that these artifacts had no experimental evidence and said there is 'probably no such thing' [9] but maintained that the duplex equations (Eq. 2) should be presented for completeness, symmetry and some *analytical advantage*. While in reality, the magnetic charge and its magnetic conduction current go to zero, if one considered both an electric and magnetic conduction current and associated heat losses when an electromagnetic wave propagates and that they could be somehow balanced, the wave would travel distortion-free [15]. A major result and leap forward into the theoretical understanding of how to improve communications systems which was to have a profound impact on his later work regarding transmission lines; later work was based on the application of his reformulation of Maxwell's original equations. The work led him to explore various aspects of electromagnetic wave propagation, including spherical waves and harmonics, electromagnetic radiation from faster-than-light (in the medium) particles and attenuation of electromagnetic waves in seawater throughout his volumes of *Electromagnetic Theory* (published between 1893 and 1912). His works are notoriously difficult to read due to the volume, difficulty and his own notation used in his mathematical work – this is easily be seen at a glance of any volume of Electrical papers or Electromagnetic Theory. It will also be noticed that there are very few diagrams amongst the mathematics.

The Telegrapher's Equations

With his new duplex equations, Heaviside turned to solving some practical problems. Long distance telegraphy with 'good' signal rates was a significant technical challenge; many companies involved in the commercialization of this technology struggled for faster communication over longer distances. The signal was found to be 'smeared out' at the receiving end, reducing the possible signal rate. Eminent scientists such as Lord Kelvin had made some headway in obtaining a theoretical understanding [16], which was in the form of a 'heat diffusion equation' (written here for voltage V only);

$$\frac{\partial^2 V}{\partial x^2} = RC\frac{\partial V}{\partial t} \tag{3}$$

Where x is the space co-ordinate, t is time and R & C and the electrical resistance and capacitance respectively. Heaviside revolutionized the theory of signaling along wires utilizing 'electromagnetic waves' in his early theoretical work on the subject [17]. Lord Kelvin had missed a crucial component in the physical model, his 'Telegraph equation' – *electromagnetic inductance*. Limiting the model to electrostatic capacitance and electrical resistance missed this element of Maxwell's theory and hence had fell short of the 'wave' description. Heaviside, in his article on 'The Extra Current' [18] and later more explicitly [19], derived the now familiar differential equations that describe either the voltage V and current I on an electrical transmission line as a function of distance x and time t – which have the form of a damped wave equation;

$$\frac{\partial^2 V}{\partial x^2} = RC\frac{\partial V}{\partial t} + LC\frac{\partial^2 V}{\partial t^2} \tag{4}$$

Here L is the inductance; the form of the equation is a 'damped wave equation'. Heaviside realized the nature of this equation and readily determines that the speed and characteristics of the signaling depend on the nature of the circuit parameters (now known as the propagation/attenuation constants determined by line parameters R, L & C). Subsequently he used these now familiarly termed Telegrapher's Equations' to derived closed-form solutions for numerous electric circuit configurations in his volumes of Electrical Papers, including producing results relating to faults in cables and signalling through telegraph circuits, undoubtedly inspired by his time as a paid Telegraph Clerk at the Great Northern Telegraph Company [11,12] in Demark and Newcastle upon Tyne. This deep insight falls out of Heaviside's interpretation of Maxwell's theory and led to his comprehensive understanding of the practical engineering problems of the time, reconceptualising the fundamentals of telegraphic propagation of signals.

Distributed Elements & the Heaviside Condition

The signal on a transmission line can become 'distorted', as Victorian telegraphs had found out at an early stage. Heaviside's Telegraphy equations gave insight into their behavior; here he utilized the concept of 'distributed circuit elements' for the first time, effectively developing the 'distributed element transmission line model' in the mid-1880's. He investigated the effect of 'continuously distributed resistance' along with a line on wave propagation [20] and moved on to consider uniformly distributed leakage-conductance and the other circuit parameters. This work was of great importance, and it ultimately led him to two major advances in the field – one theoretical and one practical.

The first discovery is the mathematical condition necessary for '*distortionless* propagation' of a telegraphic signal (but not *attenuation-less*), obtained in 1887 [21] in his work '*Electromagnetic Induction and it's Propagation'*. Then secondly, using this mathematical condition in conjunction with his 'distributed element model' and operational calculus – he proposed how one could achieve this condition with a practical telegraph line [22]. The condition that Heaviside derived for distortionless transmission was that the line parameters must obey the following equality;

$$\frac{L}{C} = \frac{R}{G} \tag{5}$$

Here R, C and L are the electrical resistance, capacitance and inductance respectively; now G is the electrical leakage conductance between the 'go' and 'return' conductors of the transmission line. Heaviside showed that if the equality were satisfied, this would lead to a condition of a 'distortionless circuit' as the 'compensation' of each of the circuit elements respective effects creates the necessary conditions. This equality is the origin of the *loading coil* as he realized that in real telegraph lines, typically $\frac{R}{G} \gg \frac{L}{C}$ and as such, this insight prompted him to suggest an increase in L to fulfil the equality [23] within the constraints of the technical and engineering economic factors known to him. His practical method (1893) was to use discrete inductors (distributed inductive elements) at intervals along the line [22], but he did not patent this idea, and consequently, others did and probably made a substantial monetary gain in the process [11, 12].

From this volume of work, a large cross-section of modern electrical engineering science can be said to be derived. It forms the foundation for analyzing DC circuits through to microwave and THz wave propagation. Coupled with his recasting of electromagnetic theory into the vector equations and his development of the Operational Calculus (not discussed here), Heaviside's work on transmission line modeling and distortionless conditions present a comprehensive toolset. This toolset was developed for solving both hypothetical (insightful) and practical engineering and physics problems, that has 'propagated' into the modern day with great success, having widespread significance and impact on the profession.

Electromagnetic Diffusion

Heaviside's studies of electromagnetic wave propagation and the transmission line brought him to *electromagnetic diffusion*. It is apparent that he sought an answer to the following question: 'How does electric current or magnetic flux penetrate and flow in media?' He addressed this question using his expert command of Maxwell's theory and some very challenging mathematics from around 1883. Heaviside used his vector form of Maxwell's equations to derive the following partial differential equation for the magnetic field **H** [24]**;**

$$\nabla^2 \mathbf{H}(\mathbf{r},t) = \overbrace{\sigma\mu\frac{\partial \mathbf{H}(\mathbf{r},t)}{\partial t}}^{\text{Diffusion}} + \underbrace{\varepsilon\mu\frac{\partial^2 \mathbf{H}(\mathbf{r},t)}{\partial t^2}}_{\text{Wave}} \tag{6}$$

Where σ is the electrical conductivity, μ the magnetic permeability and ε the electric permittivity. From this equation, it is clear that two different processes are involved in the distribution of magnetic field, one wave-like and one diffusion-like. He writes in his long running article 'Electromagnetic Induction and its Propagation' [24], "*discard the last term when the wire … is in question and discard the previous when the dielectric is in question*". Heaviside generalised the physical picture of the transmission line to include both wave and diffusion terms, reconceptualising telegraphic propagation in the process. From here, he set up problems to investigate the phenomena, mostly using his 'coaxial cable' configuration when analysing electric conductors, but his interest was also in the penetration of magnetic flux into highly permeable cores. Perhaps this interest was influenced by the use of *iron* wires, rather than copper, in some 19$^{\text{th}}$ Century telegraph circuits. However, this was to lead him towards the important results regarding 'skin effect' and guide his research toward considering the 'energy flux' in the dielectric, which is the focus of the next section.

The 'Heat Like' Behaviour of Magnetic Cores & Electric Conductors

The wave effects in the dielectric are ignored in the electrically conducting media and thus the second term on the RHS of Eq. 6 vanishes. The resulting equation is then the same equation used originally by Lord Kelvin [16] to describe the 'heat like' diffusion of electric current in telegraph and submarine cables, neglecting the inductance of such lines. Heaviside changed the viewpoint to that of 'electromagnetic waves', however, of concern was the induction of currents in the originating circuit as well as of those induced in external media.

The first works in which Heaviside made a major attempt at understanding electromagnetic field penetration was in 'The Induction of Currents in Cores' [25]. The penetration of magnetic flux into a magnetic core due to a change in external driving current was investigated in depth. He realizes that periodic currents in an exciting coil cause the magnitude of the magnetic flux density to diminish progressively into the core, he terms this phenomena 'magnetic waves' but recognizes the 'diffusion' behavior [26] and attributes eddy currents as a driving factor. Heaviside went further and calculated the 'core-heat per second' [26], due to the alternating magnetic field in the core. Subsequently, this approach was extended by many authors, with particular attention on the penetration of magnetic flux into laminated steel sheets, e,g, [27], as used in electric motors and generators to reduce eddy current losses.

His work goes on to describe in great mathematical detail the effects of variable 'inductivity' (magnetic permeability) [26], fringing around the topic of magnetic saturation within the core and its effect on the rate and character of magnetic diffusion. This, as with his other special cases, often resulted in daunting looking series equations scattered with Bessel functions of various orders and argument – his mathematical powers being of great utility with these difficult problems.

Following his work on the characteristics of magnetic cores, in [24], Heaviside changed direction on considered longitudinal currents and the circular magnetic field surrounding straight electrical conductors. Based on his previous work, he recognizes at once [25];

> *"The propagation of magnetic force and of electric current (a function of the former) in conductors takes place according to the mathematical laws of diffusion, as of by heat conduction."*

He sets out a complete mathematical treatment of the subject in [26]. In this work he moves straight from the descriptive differential equation into the solution of the current density distribution as a function of space and time involving *Fourier's cylinder functions* (Bessel functions), immediately obtaining a solution for the magnetic field and the magnetic vector potential of a straight conductor. His solution is in general, based on an arbitrary impressed force driving the change - he states that the central part of the wire is the '*last to get its current'* on impression of a driving force and on removal of that force, that same central part is the '*last to get rid of its currents'*. The current begins at the conductor surface and diffuses inwards; almost as if the current 'enters' from the surface when the circuit is switched on. He then makes an astute observation that '*If the steady state is not fully set up before the impressed force is removed, the central part of the wire is less useful as a conductor… [with] sufficient rapid reversals… the central part of the wire is practically inoperative'.*

His study of electromagnetic diffusion, the complementary electromagnetic process to wave propagation, when analyzing transmission lines led him to discuss in very general terms about the now well-known 'skin effect'. His analysis is valid for arbitrarily shaped conductors with any transient impressed force, not only persistent, steady-state AC currents. He presents a complete theory of such electromagnetic phenomena, including associated energies and forces [26]. Some discussion is given to the 'proximity' of the return conductor, and the effect of its distance from the main conductor, translatable to the electromagnetic proximity effect that engineers know today.

The Skin Effect & Circuit Parameters

The effect of current tending to flow on the surface of a conductor was first described by Horace Lamb FRS [29] in 1883; his analysis was limited to the special case of spherical conductors. A full-generalized analysis was first presented by Heaviside in 1885, following his work on electromagnetic diffusion described above. Following the insight into the electromagnetic field distributions, Heaviside worked on translating the field theory to circuit theory. Using his transmission line wave theory, he finds modified expressions for both the resistance and inductance of the lines and discusses the effect of increasing frequency on these parameters [28] due to the change in magnetic and current density field distributions. Correctly recognizing that the resistance is found to be increased and the internal inductance is found to be decreased under these transient circumstances, developing complex series solutions for these circuital parameters and forming the circuital descriptive differential equations with the modified parameters to take into account skin and proximity effects in his transmission line model.

As Nahin points out [12], the papers by Lamb and Heaviside firmly established a mathematical description of the skin effect. This statement is extended by the present author to suggest that Heaviside's contribution laid out proximity effect and expressions for the resistance and inductance of the conductors that experience these transient and persistent electromagnetic effects.

The Energy Flux

Heaviside had made substantial contributions to both the wave phenomenon (The second term on the RHS of Eq. 6) external to the guiding conductors and the slower diffusive electromagnetic phenomena (The first term on the RHS of Eq. 6) internal to those conductors. He thought that the sum of those studies does not form a complete physical picture of the transmission line – once again, he set to work. The wave and diffusion viewpoints he considered are concerned with the electric and magnetic fields only; Maxwell's theory of electromagnetism dictates that those fields must have associated energy densities and thus it follows that transfer of energy must take place somewhere – it is this that what was missing.

The Russian physicist and mathematician Nikolay Umov first postulated a vector in 1874 to describe a generalized view of energy flux in liquid and elastic media [30], this energy flux is the rate of transfer of energy – energy movement within media. In essence, this permits new physical insights into how and where the energy flows spatially and temporally in the fluid.

Lumped Element 'Confinement'

The distributed-element model of the transmission line is concerned with propagating waves of voltage and currents (Eq. 4). The progress of these waves is governed by some forcing function (usually an impressed voltage, V) the load and the line parameters (R, G, C & L), and nothing else. Thus, this approach confines all interested activity to the internals of the conductors themselves and gives no explicit insight into the processes outside of the domain of the wire. The nature of the governing electromagnetic fields is that of distributed quantities ($\mathbf{D}$, $\mathbf{B}$, $\mathbf{E}$, $\boldsymbol{J}$ and $\mathbf{H}$) in all space, the nature of circuit theory, is at best, a 2D drawing with all processed confined to within the drawn lines. This is usually fine in the mathematical analysis of a transmission line problem, but does not give the great 'physical' insight that Heaviside sought, especially when it came to the 'flow' of energy. He sought to escape this confinement and pursued a line of research concerned with the electromagnetic energy densities in the free space surrounding the current carrying conductors.

The Poynting Theorem & the 'Energy Current'

In late 1882, Prof. Henry Poynting [31] independently formulated an electromagnetic variant of the Umov energy flux vector using Maxwell's original quaternionic equations alongside Oliver Heaviside in his vector notation, mid 1884 – this is sometimes termed the 'Energy current' [32, 33]. Hertz attributes both Poynting and Heaviside as the originators of this concept [34], however Heaviside stated that his 'transfer-of-energy' formula was a 'most general form' [35], but credited Poynting with the discovery of the fundamental electromagnetic concept. To get there, Maxwell's theory of Electromagnetism 'localises' the electromagnetic field energy densities. As this energy is localised, it must be able to 'flow' from one location to another. Before Poynting and Heaviside, this was not really understood but its discovery was to provide a different physical viewpoint, opposed to the normal electric current (flow of charge), where the process of energy transfer in situations where the transfer of electromagnetic energy can usefully be viewed in terms of a 'flow'. The so-called Heaviside-Poynting Theorem is found in almost all textbooks on electromagnetism, expressed in modern notation as;

$$\underbrace{\iiint \mathbf{E}\cdot\mathbf{J}\; dV}_{\textbf{Electrical power dissipated}} \quad + \quad \overbrace{\iiint \left[\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t} + \mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}\right] \mathrm{d}V}^{\textbf{Change in electromagnetic energy}} \quad = \quad \underbrace{-\oiint (\mathbf{E}\times\mathbf{H})\cdot d\mathbf{S}}_{\text{Power flowing out of closed surface}} \tag{7}$$

Equation 7 is an energy-conservation law derived directly from Maxwell's equations. The new feature here is the vector on the RHS, inside the surface integral $\oiint (\mathbf{E}\times\mathbf{H})\cdot d\mathbf{S}$ containing the cross product of $\mathbf{E}\times\mathbf{H}$, the electrical and magnetic fields. The resultant vector must therefore be perpendicular to both of these fields – it describes physically the energy transfer due to time-varying electric and magnetic fields, directed perpendicular to the fields; in other words, the directional energy flux (the energy transfer per unit area per unit time) of an electromagnetic field. The derivation and physical interpretation is a stroke of genius, for which we must attribute to Poynting and Heaviside. The terms on the LHS of Eq. 7 represent the power dissipated and the change in electromagnetic energy. Heaviside used the symbol $\mathbf{W}=\mathbf{E}\times\mathbf{H}$ in his work and he describes it with the now informal term 'energy current'. It must be noted that the units of $\mathbf{W}$ are watt per square metre, $W/_{m^2}$ and by dividing through by c^2, the units transform to momentum density. This is an 'electromagnetic momentum', associated with the electromagnetic fields.

It is well-known that this approach is useful in antenna theory and microwave circuits. However, Heaviside extended the use of the theorem to DC electric circuits. In doing so, he reversed the contemporary view of electric current, proposing that the electric and magnetic fields due to the current are the primitives, rather than being a result of the motion of the electronic charge in the conductor. This is a controversial viewpoint and in his Electrical Papers the phrase '*we reverse this*' [36], referring to the 'current in the wire being set up by the energy transmitted through the medium around it' reverberates even to this day. This view is supported by his work on electromagnetic diffusion (previous section) and the nature of the electromagnetic field and current density penetration of an electrical conductor subjected to a step current. As discussed in a modern context by Feynman [37], showing that the electromagnetic momentum is 'required' in order to conserve angular mechanical momentum associated with the energy flux vector $\mathbf{W}$, a detailed historical discussion is presented by Nahin [12]. The 'uniqueness' of the vector $\mathbf{W}$ and the physical existence of mysterious and counter-intuitive circulating ‘energy currents’ emerging from static fields (e.g., a point electric charge with a superimposed magnetic dipole), has been subject of some debate amongst many scientists over a long period of time. Heaviside was the first to consider these issues in 1893 [38]. The reader is directed to [12, 37] for a detailed analysis and discussion on these matters.

The concepts presented here, as derived by Heaviside, complete his physical picture. From wave to diffusion phenomena, they are processes involving energy, and as such, the Heaviside-Poynting theorem can be used to develop a complete understanding of an electric circuit to escape the ‘lumped element confinement’.

A Simple Circuit?

One problem that Heaviside used to illustrate the energy current concept was that of a battery connected by a simple circuit with a resistor load in his 1886-1887 work ‘The Transfer of Energy and Its Application to Wires. *Energy Current*.’ [33] – effectively an analysis of a ‘twin wire transmission line’. The standard approach is to assume that electrical energy flows inside metal wires (confinement), Heaviside’s energy current approach dictates otherwise. Poynting had first published on this arrangement [31], which was criticized by Heaviside due to Poynting’s misconception of the nature of the external electric field surrounding the wires [32, 34]. Poynting considered only a tangential electric field in the axis of the wire, combined with the circumferential magnetic field, resulting in an inwardly radial component of the energy flux density, **W,** only. Thus, giving no energy flux in the direction of the electric current – how does energy get from the battery to load? This is not answered by conventional circuit theory. Heaviside argued that this ‘Poynting component’ is simply the heat lost due to Joule heating in the conductor, however a more prominent component exists outside of the wire due to a radial electric field, the surface charges on the conductors that set up the field and maintain the electric current are responsible for the energy transfer external to the conductors. This complete field picture than presents a ‘map’ of the energy flow. Heaviside shows for the first time that a radial electric field and circumferential magnetic field produces an ‘energy current’, a flow of electromagnetic energy in the space surrounding the electric conductors, directed form the battery along the axis of the conductor towards and entering the load. Figure 2 shows a diagram from Heaviside’s work, illustrating the energy flow from battery (at A-C) to a resistor (at B-D) guided by conductors C-D and A-B.

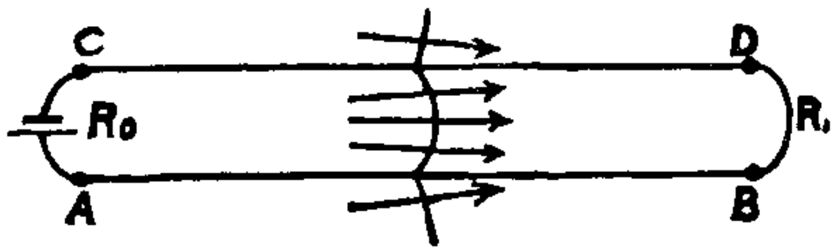

Fig. 2 – Energy flow in a simple circuit, circa. 1892 [33]

This remarkable result is one that is rarely presented, but one in which he gives great detail in his Electrical papers: Vol. II, which has recently been the subject of a rigorous modern mathematical treatment [39], confirming Heaviside's results. It is readily shown that the energy current approach is compatible with the circuit theory approach by applying the integral formulation of the Poynting theorem to the problem, determining the power dissipated in the load resistor as VI, as dictated by conventional 'confined' circuit theory. This 'energy current approach' gives physical insight, but requires detailed knowledge of the electric and magnetic fields surrounding the analysed circuits. It complements his work on electromagnetic wave propagation by analysing the energy associated with the wave and also his work on electromagnetic diffusion where he found that the current in the wire penetrates from the outside surface inwards.

Conclusion

This paper has presented some selected contributions of Oliver Heaviside to Electromagnetic Theory, specifically in relation to transmission line analysis from the wave propagation, electromagnetic diffusion, and energy flow perspectives. The scope of Heaviside's work is vast, contained within his two volumes of Electrical papers and his Three Volumes of Electromagnetic Theory. Heaviside radically changed how engineers and physicists treated transmission line problems and gave practical insight into how the operation of high-frequency landline and subsea telegraphic cables should be analyzed, designed and operated. He went a step further, moving from the circuit theory domain into the full electromagnetic field theory description of such lines, providing advanced in the understanding of the granular electromagnetic detail including the theoretical establishment of transient & persistent skin effects, as well as proximity related phenomena. His contributions of the physical description of energy flow were a major achievement, extending physical insight into not only high-frequency RF and microwave applications but also to understanding DC circuits and the 'transference of energy', localized by Maxwell's theory. A great deal of Heaviside's electrical engineering science underpins modern advances, which continue to this day. Modern applications and theories for THz wave propagation and electric power transient analysis, to name but a few, owe much to Heaviside's 19th Century contributions to electromagnetic theory. However, Heaviside's work is as relevant today as it was in the Victorian period. It has underpinned a vast expanse of modern theory and has application well beyond what he considered at the time or that he could have predicted into the future. Presented here very briefly, are a few modern examples that can be argued to have evolved from Heaviside's work in electromagnetic theory;

Thz Wave Propagation

THz waves (1 to 10THz) are electromagnetic waves of recent research interest. Many modern applications present themselves, such as medical & scientific imaging, process monitoring and quality control in manufacturing. Heaviside's distributed element model and his mathematical analysis of high-frequency lines have paved the way for analysis in this frequency range [40]. The analytical tools utilized are fundamentally based on his work intended for solving the 19th Century telegraphic problems, with appropriate modifications to include additional physics at such frequencies.

Transient Skin Effects

In modern electrical power systems, much research is underway regarding DC systems, rather than the traditional AC approach. In the AC systems, persistent skin effect is taken into account in the design of the cables to operate at power frequencies. With modern power systems being ever more ‘power electronic controlled’, fast switching devices inherently introduce transient electromagnetic effects, and as such, a transient skin effect can be present. This follows Heaviside’s work on electromagnetic diffusion and his generalized viewpoint on the ‘skin effect’. Fault conditions in both AC and DC power systems also are concerned with transient electromagnetic phenomena - the fundamental work having been put forth by Heaviside.

Power Flow in Electrical Machines

The ‘energy current’ or Poynting vector approach to antenna design is well known. It has been shown by the author [41], and others, that this approach complements the traditional approaches (Faraday and Lorentz) in understanding electrical machines (motors and generators). Lorentz Forces are usually calculated to determine the torque produced in a rotary electrical machine; these forces are located at either side of an air gap separating the stationary component from the rotor. These are confined to the respective mechanical components – what happens in the air gap between and how does power flow from one member to another? Application of Heaviside’s approach illuminates a radial flux of energy across the air gap when the motor is loaded, a circulating component wholly tangential to the cylindrical rotor surface exists when the motor is unloaded. This is a unique physical insight into the processes involved in the electromechanical conversion process.

Author’s Final Remarks

The author first came across Heaviside as an undergraduate in his ‘self-study’ regime of wider learning regarding electromagnetism. Reading, with great difficulty!, the volumes of ‘Electrical Papers’ and ‘Electromagnetic Theory, the author became interested in the life and times of Heaviside. This enthusiasm culminated in the co-founding of the North East Electromagnetics Interest Group in 2013 aimed at discussing both his life and work as well as acting for a vehicle for general discussion of electromagnetic theory. The ‘Heaviside Memorial Project’ was launched in 2014, the author, with Prof. Alex Yakovlev at Newcastle University, raised funds through public subscription to restore the grave of Oliver Heaviside and his family in Paignton, Devon. The work completed in August 2014 with an unveiling ceremony at Paignton Cemetery on Saturday 30th August 2014, attended by the local Mayor and MP as well as a distant relative of Heaviside. It is from the North East Electromagnetics Interest Group and the combined interest of the author and Prof Yakovlev that the idea of this Royal Society Special Issue was conceived. Indeed many of the authors in this issue have presented seminars on their chosen topic within electromagnetic theory for the Interest Group.

It is hoped that this Commemorative Issue will assist with the associating the name of Heaviside with his many enduring ideas that are so important and that due recognition of the impact of his work be realized. It is further hoped that his work in his volumes of Electrical Papers and Electromagnetic Theory will still be of interest to the next generator of engineers, physicists, and mathematicians. Just perhaps, there are still a few electromagnetic ‘gems’ that have been missed, some elements of Heaviside’s great insight might remain to be found scattered in his works.

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