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# Exploiting high rate differential algebraic space-time block code in downlink multiuser MIMO systems

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**Abstract:** This paper considers a multiuser multiple-input multiple-output (MIMO) space-time block coded system that operates at a high data rate with full diversity. In particular, they propose to use a full rate downlink algebraic transmission scheme combined with a differential space-time scheme for multiuser MIMO systems. To achieve this, perfect algebraic space-time codes and Cayley differential transforms are employed. Since channel state information (CSI) is not needed at the differential receiver, differential schemes are ideal for multiuser systems to shift the complexity from the receivers to the transmitter, thus simplifying user equipment. Furthermore, orthogonal spreading matrices are employed at the transmitter to separate the data streams of different users and enable simple single user decoding. In the orthogonal spreading scheme, the transmitter does not require any knowledge of the CSI to separate the data streams of multiple users; this results in a system which does not need CSI at either end. With this system, to limit the number of possible codewords, a sphere decoder is used to decode the signals at the receiving end. The proposed scheme yields low complexity transceivers while providing full rate full diversity with good performance. Simulation results demonstrate the effectiveness of the proposed scheme.

## 1 Introduction

Multiple-input multiple-output (MIMO) technology is one of the most important milestones in the development of wireless communications and can be used to increase the spectral efficiency through the spatial multiplexing and improve the link reliability through transmit diversity [1]. The MIMO design trade-offs such as multiplexing, diversity, performance, and complexity in both uncoded and coded MIMO systems play a fundamental role in efficient system planning and deployment [2]. Furthermore, wireless systems require effective transmission techniques to support high data rate and reliable communications. As such, space-time block code (STBC) is a potential transmission technique which can be utilised, as part of multiple antenna systems, to enhance the spatial diversity of the system [3], and it is used in standard systems such as the UMTS standard for mobile wireless, the IEEE 802.16 standard for fixed and nomadic wireless, and the IEEE 802.11 standard for wireless LANs [4].

The transmission of an orthogonal STBC over a MIMO channel in [5, 6] was proposed to achieve full diversity with a low complexity receiver. However, orthogonal STBC suffers from an inability to work with a greater number of antennas at full transmission rates. When decoding complexity is not an issue, one may use non-orthogonal full rate full diversity algebraic STBC [7, 8]. For MIMO systems, there are many previous employed space-time codes that provide a higher rate with full diversity in a trade-off with complexity, such as threaded algebraic space-time block codes, the classic Bell Laboratories layered space-time (V-BLAST) and linear dispersion block codes [9, 10]. However, the minimum determinants of these codes are generally non-zero, but vanish as the spectral efficiency of the signal constellation is increased. The authors of [11] have constructed full rate and full diversity perfect algebraic STBC with a non-vanishing determinant when the spectral efficiency increases.

In the multiuser multiple-input multiple-output (MU-MIMO) downlink, transmit diversity can be applied using downlink transmission techniques, such as the orthogonal spreading multiplexing code. The authors of [12, 13] used this technique to decompose the MU-MIMO channels into parallel single user non-interfering channels, and hence co-channel interference (CCI) was eliminated. Implementing the orthogonal spreading technique at the transmitter (e.g. a base station (BS)) helps maintain simplicity in the receiver, so that simple linear decoding approaches are applicable at the receiving end (e.g. end users). In a coherent scenario, this approach was later considered in [14] as a multiplexing scheme for a MU-MIMO system, and was combined with full rate full diversity algebraic STBC. The proposed method cancels the CCI and provides a substantial gain in terms of full rate and spatial diversity. However, for the decoding process, each receiver still needs to know the channel state information (CSI) to coherently decode the algebraic STBC. In practice, each receiver acquires the composite channel by direct estimation, which leads to increased complexity of the receivers.

The prior focus of the high rate MU-MIMO downlink transmission techniques has been on cases where CSI is available at the receivers and transmitter. However, for some systems, due to the high mobility and the cost of channel training and estimation, CSI acquisition is impossible [15]. One alternative method is to encode the transmitted data differentially using a Cayley differential (CD) transform and to decode differentially without any knowledge of the CSI at the receiver [16]. Our previous work in [17] has dealt with implementing the MU-MIMO downlink transmission of an Alamouti STBC combined with differential modulation, which does not require channel knowledge for decoding. The scheme provides low complexity transceivers while providing good performance. However, the authors of [17] could not provide a comprehensive high rate differential scheme in a downlink scenario.

In this work, the use of the high-rate CD STBC for downlink transmission in a MU-MIMO system is considered. Specifically, we show how to use differential STBC combined with full rate full diversity perfect algebraic STBC. The use of differential STBC in a multiuser scenario simplifies the complexity of the receivers since neither feedback nor the estimation of the CSI is required at the receiver. Furthermore, differential STBC is considered based on the orthogonal spreading technique in order to separate the data streams of multiple users. With the use of orthogonal spreading, the transmitter needs no knowledge of the CSI to design the spreading matrices. Therefore, implementing the orthogonal spreading scheme with the differential STBC will result in a system in which neither the transmitter nor the receiver needs knowledge of the

CSI. At the receiver of each user, a sphere decoder (SD) is implemented for high-rate coherent and differential perfect algebraic STBC to limit the set of candidate symbols to those within a sphere of some radius *d*. The proposed schemes facilitate the multiple user data separations, enhancing full rate full diversity, and achieving low complexity receivers and transmitters through the use of differential STBC. However, the system in this paper has higher computational complexity thanks to its higher rate.

The rest of the paper is organised as follows. Section 2 introduces the system model of STBC MU-MIMO. Section 3 reviews the coherent perfect algebraic STBC for MU-MIMO. Section 4 presents the differential perfect algebraic STBC for MU-MIMO. In Section 5, the computational complexity and rate analysis of the system are derived. In Section 6, the simulation results are shown and, finally, conclusions are drawn in Section 7.

## 2 System model

Consider a MU-MIMO downlink broadcast channel where the BS transmits multiple streams to $K$ users (e.g. mobile stations), as shown in Fig. 1. The BS has $n_\mathrm{t}$ transmit antennas and each user $k$ has $n_\mathrm{r}^k$ receive antennas. We assume that all users have the same number of receive antennas unless otherwise stated. Furthermore, the superscript $k$ is omitted for simplicity. The channel matrix $\boldsymbol{H} \in \mathbb{C}^{n_\mathrm{r} \times n_\mathrm{t}}$ for each user $k$ is a Rayleigh flat fading matrix given by

$$\boldsymbol{H} = \begin{bmatrix} h_{1,1} & \cdots & h_{1,n_\mathrm{t}} \\ \vdots & \ddots & \vdots \\ h_{n_\mathrm{r},1} & \cdots & h_{n_\mathrm{r},n_\mathrm{t}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_1 \\ \vdots \\ \boldsymbol{h}_{n_\mathrm{r}} \end{bmatrix}, \tag{1}$$

where the element $h_{i,j}$ is the channel coefficient between the $j$th transmit antenna and the $i$th receive antenna of user $k$, and $\mathbb{C}^{M \times N}$ denotes the set of $M \times N$ complex matrices. The elements of $\boldsymbol{H}$ are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance, i.e. $\mathscr{CN}(0, 1)$.

For any $k$th user, the $n_\mathrm{t} \times n_\mathrm{r}$ information symbol matrix can be defined as

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \cdots & \boldsymbol{s}_{n_\mathrm{r}} \end{bmatrix} = \begin{bmatrix} s_{1,1} & \cdots & s_{1,n_\mathrm{r}} \\ \vdots & \ddots & \vdots \\ s_{n_\mathrm{t},1} & \cdots & s_{n_\mathrm{t},n_\mathrm{r}} \end{bmatrix}, \tag{2}$$

where $s_{i,j}$, $i = 1, \ldots, n_\mathrm{t}$, $j = 1, \ldots, n_\mathrm{r}$, are the information symbols taken from the constellation set $\mathbb{Z} \in \{\mathrm{QAM}, \mathrm{PAM}\}$. In this study, we consider a class of linear non-orthogonal STBCs that have full rate and full diversity, such as perfect algebraic STBC [7, 11]. A perfect algebraic STBC codeword is a $n_\mathrm{t} \times n_\mathrm{t}$ matrix $\boldsymbol{X}$ whose entries are a linear combination of the input information signals. The spatial and temporal diversity of the codeword $\boldsymbol{X}$ is integrated into the space-time code design, as will be shown in the following sections.

The received signal matrix $\boldsymbol{Y} \in \mathbb{C}^{n_\mathrm{r} \times K n_\mathrm{t}}$ for the $k$th user is given by

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{X}\boldsymbol{V} + \boldsymbol{H} \sum_{j=1, j \neq k}^{K} \boldsymbol{X}_j \boldsymbol{V}_j + \boldsymbol{Z}, \tag{3}$$

where $\boldsymbol{V} \in \mathbb{C}^{n_\mathrm{t} \times K n_\mathrm{t}}$ is the orthogonal spreading matrix for user $k$, $\boldsymbol{Z} \in \mathbb{C}^{n_\mathrm{r} \times K n_\mathrm{t}}$ is an additive white Gaussian noise matrix. Note that the composite transmitted matrix is $\sum_{k=1}^{K} \boldsymbol{X}_k \boldsymbol{V}_k$.

In the orthogonal spreading code matrix, each user is assigned a unique orthogonal spreading code to separate the data of the users at the receivers. To eliminate CCI, the spreading code matrix has to obey the following conditions:

$$\boldsymbol{V}_k \boldsymbol{V}_k^\mathrm{H} = \boldsymbol{I}_{n_\mathrm{t}}, \quad k = 1, \ldots, K, \tag{4}$$

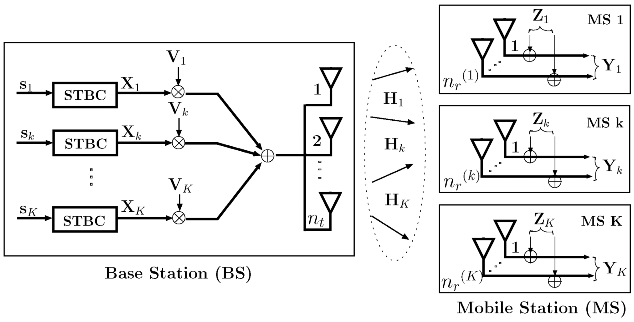


**Fig. 1** *STBC MU-MIMO downlink transmission system*

$$\boldsymbol{V}_j \boldsymbol{V}_k^\mathrm{H} = 0, \quad k, j = 1, \ldots, K, \quad \text{and } j \neq k, \tag{5}$$

where $(\cdot)^\mathrm{H}$ denotes the Hermitian operator. The orthogonal spreading code for each user can be constructed as a sub-matrix of the Hadamard matrix or from a discrete Fourier transform matrix. Hadamard matrices are of interest because of their simplicity [17]. The received signal matrix $\boldsymbol{Y}$ in (3) for the $k$th user is despread by multiplying it with $\boldsymbol{V}^\mathrm{H}$, which yields

$$\hat{\boldsymbol{Y}} = \boldsymbol{Y}\boldsymbol{V}^\mathrm{H} = \boldsymbol{H}\boldsymbol{X} + \hat{\boldsymbol{Z}}, \tag{6}$$

where

$$\hat{\boldsymbol{Y}} = \begin{bmatrix} \hat{y}_{1,1} & \cdots & \hat{y}_{1,n_\mathrm{t}} \\ \vdots & \ddots & \vdots \\ \hat{y}_{n_\mathrm{r},1} & \cdots & \hat{y}_{n_\mathrm{r},n_\mathrm{t}} \end{bmatrix} \tag{7}$$

and

$$\hat{\boldsymbol{Z}} = \boldsymbol{Z}\boldsymbol{V}^\mathrm{H} = \begin{bmatrix} \hat{z}_{1,1} & \cdots & \hat{z}_{1,n_\mathrm{t}} \\ \vdots & \ddots & \vdots \\ \hat{z}_{n_\mathrm{r},1} & \cdots & \hat{z}_{n_\mathrm{r},n_\mathrm{t}} \end{bmatrix}. \tag{8}$$

We now present a brief review of the MU-MIMO high-rate perfect algebraic STBC system, where the CSI is available only at the receiver.

## 3 Review of the coherent perfect STBC for MU-MIMO with downlink transmission

In this section, we consider a coherent scheme where the receiver knows the CSI. The scheme transmits data in linear combination over space and time. The design criterion of perfect STBC is to minimise the maximum pairwise error probability (PEP), where the maximum-likelihood (ML) detection might receive the distorted version $\hat{\boldsymbol{X}}$ of the original transmitted signal $\boldsymbol{X}$, and the PEP is given as [3, 11]

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}}) \le \frac{4^{r n_\mathrm{r}}}{\left(\prod_{i=1}^{r} \lambda_i\right)^{n_\mathrm{r}} \rho^{r n_\mathrm{r}}}, \tag{9}$$

where $r$ is the rank of the codeword difference matrix $(\boldsymbol{X} - \hat{\boldsymbol{X}})$, $\rho$ is the signal-to-noise ratio (SNR) per receive antenna, $\lambda_i, i = 1, \ldots, r$, are the eigenvalues of $(\boldsymbol{X} - \hat{\boldsymbol{X}})(\boldsymbol{X} - \hat{\boldsymbol{X}})^\mathrm{H}$, the minimum value of $r n_\mathrm{r}$ is the diversity gain and the minimum value of $\left(\prod_{i=1}^{r} \lambda_i\right)^{1/r}$ is the coding gain.

### *3.1 Encoding of coherent perfect algebraic STBC for MU-MIMO with downlink transmission*

For coherent perfect STBC, the input symbol vectors, $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_{n_\mathrm{r}}$, are first rotated by the real or complex rotation matrix $\boldsymbol{M} \in \mathbb{C}^{n_\mathrm{t} \times n_\mathrm{t}}$ and then threaded into different layers $l$, where $l = 1, \ldots, n_\mathrm{t}$. In other words, we denote the symbols transmitted in the $l$th layer by $x_{1l}, x_{2l}, \ldots, x_{n_t l}$, i.e. [11]

$$\boldsymbol{x}_l = \boldsymbol{M}\boldsymbol{s}_l. \tag{10}$$

Thus, a layer can be viewed as an array of size $n_t \times n_t$. Any element of this array can be specified by two indices, $(a,t)$, where $a$ denotes the spatial domain and $t$ denotes the temporal domain. Let $l_i$, $1 \le i \le n_t$ denote the $i$th layer. Hence, a layer can be formed such that

$$l_i = \{((t + i - 1)_{n_t}, t) \colon 0 \le t < n_t\}, \tag{11}$$

where $(x)_{n_t}$ denotes $x$ modulo $n_t$ operation. Accordingly, consecutive symbols from the same codeword are transmitted from different transmit antennas in different time slots. This method of transmission maximises the spatial and temporal diversity of the system.

The rotation matrix $\boldsymbol{M}$ (real or complex) is designed to maximise the distance between the symbol vectors to minimise the error rate and is constructed from an algebraic number field $\mathbb{Q}(\theta)$ of degree $n_t$ generated by an algebraic number $\theta$ as in [18, 19].

The perfect algebraic STBC, as proposed in [11], is constructed based on cyclic division algebra theory for the special cases of $n_t = 2, 3, 4, 6$. To thread the symbols into the perfect algebraic STBC, the rotated symbol vectors are applied to the code block by

$$\boldsymbol{X} = \sum_{l=1}^{n_t} \operatorname{diag}(\boldsymbol{M}\boldsymbol{s}_l) \cdot \boldsymbol{e}^{l-1}, \tag{12}$$

where $\operatorname{diag}(\cdot)$ denotes the diagonal of a matrix, the threading matrix $\boldsymbol{e}$ is given as follows:

$$\boldsymbol{e} = \begin{bmatrix} 0 & 0 & 0 & 0 & \gamma \\ 1 & 0 & 0 & \vdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 1 & 0 & 0 \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix} \tag{13}$$

and $\gamma = \sqrt{-1}$ is chosen by using the class field theory that ensures the transmitted code block has a non-vanishing determinant [11]. For multiple users equipped with different numbers of receive antennas, one important design parameter to consider is the number of threads $l$. Therefore, in this study, we assume the total number of layers is limited by the number of receive antennas $n_r$ per user, i.e. $l = n_r$. Hence, the perfect STBC codeword can be rewritten as

$$\boldsymbol{X} = \sum_{l=1}^{n_r} \operatorname{diag}(\boldsymbol{M}\boldsymbol{s}_l) \cdot \boldsymbol{e}^{l-1}. \tag{14}$$

*Example 1:* For $n_t = 4$, number of users $K = 2$, user 1 equipped with $n_r = 3$ and user 2 equipped with $n_r = 1$. Then, the perfect algebraic STBC codeword for user 1 with $l = 3$ layers is in the form of

$$\boldsymbol{X}_{4\times 4} = \begin{bmatrix} x_{11} & 0 & \gamma x_{13} & \gamma x_{12} \\ x_{22} & x_{21} & 0 & \gamma x_{23} \\ x_{33} & x_{32} & x_{31} & 0 \\ 0 & x_{43} & x_{42} & x_{41} \end{bmatrix},$$

of course, a higher-rate code can be implemented by increasing the number of threads per user.

### *3.2 Decoding coherent perfect algebraic STBC for MU-MIMO with downlink transmission*

The sphere decoding approach is one of the most important decoding schemes for high data rate transmission systems over MIMO channels. The SD is basically a distance-based decoder that limits the number of possible codewords by considering only those codewords within a sphere centred at the received signal vector [20]. The $k$th user received spread signal can be expressed in terms of its vectorisation as [10, 14]

$$\begin{aligned} \operatorname{vec}(\hat{\boldsymbol{Y}}^{\mathrm{T}}) &= \operatorname{vec}\big((\boldsymbol{H}\boldsymbol{X})^{\mathrm{T}}\big) + \operatorname{vec}(\hat{\boldsymbol{Z}}^{\mathrm{T}}) \\ &= \mathscr{B}_c \operatorname{vec}(\boldsymbol{S}) + \operatorname{vec}(\hat{\boldsymbol{Z}}^{\mathrm{T}}), \end{aligned} \tag{15}$$

where

$$\operatorname{vec}(\hat{\boldsymbol{Y}}^{\mathrm{T}}) = [\hat{y}_{1,1}, \ldots, \hat{y}_{1,n_t}, \ldots, \hat{y}_{n_r,1}, \ldots, \hat{y}_{n_r,n_t}]^{\mathrm{T}},$$
$$\operatorname{vec}(\hat{\boldsymbol{Z}}^{\mathrm{T}}) = [\hat{z}_{1,1}, \ldots, \hat{z}_{1,n_t}, \ldots, \hat{z}_{n_r,1}, \ldots, \hat{z}_{n_r,n_t}]^{\mathrm{T}},$$
$$\operatorname{vec}(\boldsymbol{S}) = [s_{1,1}, \ldots, s_{n_t,1}, \ldots, s_{1,n_r}, \ldots, s_{n_t,n_r}]^{\mathrm{T}},$$

and $\mathscr{B}_c$ is the new $n_t n_r \times n_t n_r$ effective channel matrix of the coherent perfect STBC which is given by

$$\mathscr{B}_c = \tilde{\boldsymbol{H}} \cdot (\boldsymbol{I}_{n_r} \otimes \boldsymbol{M}), \tag{16}$$

where the $n_t n_r \times n_t n_r$ matrix $\tilde{\boldsymbol{H}}$ is given by

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} \operatorname{diag}(\boldsymbol{h}_1) & \cdots & (\operatorname{diag}(\boldsymbol{h}_1)\boldsymbol{e}^{n_r - 1})^{\mathrm{T}} \\ \vdots & \ddots & \vdots \\ \operatorname{diag}(\boldsymbol{h}_{n_r}) & \cdots & (\operatorname{diag}(\boldsymbol{h}_{n_r})\boldsymbol{e}^{n_r - 1})^{\mathrm{T}} \end{bmatrix}, \tag{17}$$

and $\otimes$ denotes the Kronecker matrix product. The underlying complex system in (15) can be converted into an equivalent real system by separating the real and imaginary parts of the received vector to define the following $2n_t n_r \times 1$ signal

$$\mathscr{Y} = \mathscr{H}_c \mathscr{S} + \mathscr{Z}, \tag{18}$$

where

$$\mathscr{Y} = \left[\Re(\operatorname{vec}(\hat{\boldsymbol{Y}}^{\mathrm{T}})) \quad \Im(\operatorname{vec}(\hat{\boldsymbol{Y}}^{\mathrm{T}}))\right]^{\mathrm{T}},$$

$$\mathscr{S} = \left[\Re(\operatorname{vec}(\boldsymbol{S})) \quad \Im(\operatorname{vec}(\boldsymbol{S}))\right]^{\mathrm{T}},$$

$$\mathscr{Z} = \left[\Re(\operatorname{vec}(\hat{\boldsymbol{Z}}^{\mathrm{T}})) \quad \Im(\operatorname{vec}(\hat{\boldsymbol{Z}}^{\mathrm{T}}))\right]^{\mathrm{T}},$$

and

$$\mathscr{H}_c = \begin{bmatrix} \Re(\mathscr{B}_c) & -\Im(\mathscr{B}_c) \\ \Im(\mathscr{B}_c) & \Re(\mathscr{B}_c) \end{bmatrix}.$$

In (18), we have a simple linear system of equation that may be decoded using the SD technique, which can be implemented to decode the $k$th user symbols $\hat{\mathscr{S}}$ such that

$$\hat{\mathscr{S}} = \arg \min_{S \in \mathbb{Z}^n} \| \mathscr{Y} - \mathscr{H}_c \mathscr{S} \|^2, \tag{19}$$

where $n = 2 \times n_t \times n_r$. We now present the differential STBC and then show how to combine it with full rate full diversity perfect algebraic STBC through the use of the Cayley transform.

## 4 Differential perfect STBC for MU-MIMO with downlink transmission

In this section, the differential encoding and decoding process for downlink transmission in a MU-MIMO system is discussed. In particular, this section demonstrates how to use a high rate space-time coding such as the perfect STBC with differential STBC for MU-MIMO systems. Here, we assume neither the transmitter nor the receiver has prior knowledge of the CSI. One method of implementing differential STBC with multiple antennas and a high data rate is to encode the transmitted data differentially using a CD transform and to decode differentially without any knowledge of the CSI. The proposed work combines the perfect algebraic STBC with the CD transform that constructs full rate and full diversity differential STBC. In the following, we first review the differential STBC and the CD transform and then utilise the CD transform with perfect algebraic STBC in a MU-MIMO framework.

### *4.1 Differential STBC for MU-MIMO system*

In differential STBC, the communications are done in blocks of $n_t$ transmissions, which implies that the transmitted signal for any user $k$ is an $n_t \times n_t$ matrix. The received despread signal block in (6) for the $k$th user at the $\tau$-th block, $\tau = 0, \ldots, N$, can be re-expressed as

$$\hat{\boldsymbol{Y}}_\tau = \boldsymbol{H}\boldsymbol{X}_\tau + \hat{\boldsymbol{Z}}_\tau, \tag{20}$$

where $\hat{\boldsymbol{Y}}_\tau$, $\boldsymbol{X}_\tau$, and $\hat{\boldsymbol{Z}}_\tau$ are the despread received signal matrix, the transmitted perfect algebraic STBC matrix, and the despread noise matrix for the $k$th user at the $\tau$th block, respectively. The transmitted perfect algebraic STBC codeword matrix is encoded differentially as follows [21, 22]:

$$\boldsymbol{X}_\tau = \boldsymbol{X}_{\tau-1}\boldsymbol{U}_{z_\tau}, \tag{21}$$

where $\boldsymbol{U}_{z_\tau}$ is a unitary data matrix utilised by the Cayley transform (we specify $\boldsymbol{U}_{z_\tau}$in the next subsection), $z_\tau \in \{0, \ldots, L-1\}$ is the transmitted data, and $\boldsymbol{X}_{\tau-1}$ is the transmitted matrix of the previous block. The transmitted matrix for the initial block of each user $k$ is set to be identity, i.e. $\boldsymbol{X}_0 = \boldsymbol{I}_{n_t}$.

If we assume that the channels stay constant for two consecutive blocks, i.e. $\boldsymbol{H}_\tau = \boldsymbol{H}_{\tau-1} = \boldsymbol{H}$, then (20) can be written as

$$\begin{aligned}\hat{\boldsymbol{Y}}_\tau &= \boldsymbol{H}\boldsymbol{X}_{\tau-1}\boldsymbol{U}_{z_\tau} + \hat{\boldsymbol{Z}}_\tau \\ &= \hat{\boldsymbol{Y}}_{\tau-1}\boldsymbol{U}_{z_\tau} + \hat{\boldsymbol{Z}}_\tau - \hat{\boldsymbol{Z}}_{\tau-1}\boldsymbol{U}_{z_\tau}.\end{aligned}$$

Therefore, the fundamental differential system equation for the $k$th user is given as

$$\hat{\boldsymbol{Y}}_\tau = \hat{\boldsymbol{Y}}_{\tau-1}\boldsymbol{U}_{z_\tau} + \hat{\boldsymbol{Z}}_\tau', \tag{22}$$

where

$$\hat{\boldsymbol{Z}}_\tau' = \hat{\boldsymbol{Z}}_\tau - \hat{\boldsymbol{Z}}_{\tau-1}\boldsymbol{U}_{z_\tau}. \tag{23}$$

Since $\boldsymbol{U}_{z_\tau}$ is a unitary matrix, the entries of the additive noise term $\hat{\boldsymbol{Z}}_\tau'$ are i.i.d. $\mathscr{CN}(0, 2)$. Thus, $\hat{\boldsymbol{Z}}_\tau'$ is statically independent of $\boldsymbol{U}_{z_\tau}$ and has twice the power. Therefore, for the $k$th user, the ML decoder of the differential STBC is

$$\hat{z}_\tau = \arg \max_{n = 0, \ldots, L-1} \| \hat{\boldsymbol{Y}}_\tau - \hat{\boldsymbol{Y}}_{\tau-1}\boldsymbol{U}_n \|_F^2, \tag{24}$$

thus, the receiver does not need CSI to perform the decoding process. The PEP of transmitting $\boldsymbol{U}_n$ and mistakenly decoding $\boldsymbol{U}_{n'}$ has the following upper bound [21, 22]:

$$P_e(\boldsymbol{U}_n \to \boldsymbol{U}_{n'}) \le \frac{1}{2}\prod_{i=1}^{r}\left[1 + \frac{\rho^2}{4(1+2\rho)}\sigma_i^2(\boldsymbol{U}_n - \boldsymbol{U}_{n'})\right]^{-n_r}, \tag{25}$$

where $\sigma_i(\cdot)$ denotes the $i$th singular value of the codeword difference matrix. At high SNRs, we can neglect the one in (25) and write the following upper bound based on the non-zero singular values as

$$P_e(\boldsymbol{U}_n \to \boldsymbol{U}_{n'}) \lesssim 8^{rn_r} \cdot \frac{\rho^{-rn_r}}{|\det(\boldsymbol{U}_n - \boldsymbol{U}_{n'})|^{2n_r}}. \tag{26}$$

We define the diversity gain to be $G_d$ and the coding gain to be $G_c$. The diversity gain is given by

$$G_d = rn_r. \tag{27}$$

The differential STBC also achieves full diversity order of $n_t n_r$ if the unitary matrix is fully diverse. Using (27) and the right-hand side of (26), we have

$$\begin{aligned}&P_e(\boldsymbol{U}_n \to \boldsymbol{U}_{n'}) \\ &\quad\lesssim 8^{G_d}|\det(\boldsymbol{U}_n - \boldsymbol{U}_{n'})|^{-2n_r}\rho^{-G_d} \\ &\quad\lesssim \left[\left(8^{G_d}|\det(\boldsymbol{U}_n - \boldsymbol{U}_{n'})|^{-2n_r}\right)^{-G_d^{-1}}\right]^{-G_d} \cdot \rho^{-G_d} \\ &\quad\lesssim (G_c \cdot \rho)^{-G_d},\end{aligned} \tag{28}$$

where

$$G_c = \left(8^{G_d}|\det(\boldsymbol{U}_n - \boldsymbol{U}_{n'})|^{-2n_r}\right)^{-G_d^{-1}}. \tag{29}$$

By using (27) in (29), we have

$$G_c \simeq |\det(\boldsymbol{U}_n - \boldsymbol{U}_{n'})|^{2/r}. \tag{30}$$

The PEP will be lower in the case that we receive multiple replicas of the signal using diversity. In other words, diversity is the slope of the error probability curve in terms of the received SNR in a log–log scale. In this case, taking the log for both sides of (28) implies that

$$\log(P_e) = -G_d[\log(G_c) + \log(\rho)], \tag{31}$$

or more explicitly

$$\log(G_c) = \frac{\log(P_e)}{-G_d} - \log(\rho). \tag{32}$$

This coding gain ratio is a measure of the worst case separation between encoded symbols. It, therefore, determines the worst case for PEP, and hence the block error rate (BLER). The differential STBC also achieves full diversity order of $n_t n_r$ if the unitary matrix is fully diverse, i.e. $r = n_t$. Therefore, to minimise the PEP, the following conditions should be satisfied [3, 11]:

- To maximise the diversity gain, the rank criterion $r$ of $(\boldsymbol{U}_n - \boldsymbol{U}_{n'})$ should be maximised.
- To maximise the coding gain $G_c$, the minimum determinant of $(\boldsymbol{U}_n - \boldsymbol{U}_{n'})$ should be maximised.
- Non-vanishing minimum determinant on the coding gain.

### *4.2 Differential perfect algebraic STBC for a MU-MIMO system*

For MU-MIMO differential transmission schemes, the information must first be encoded in a unitary matrix to ensure the same transmit power for different blocks. This can be achieved by applying the Cayley transform as [16]

$$U_{z_\tau} = \frac{I_{n_t} - jA_\tau}{I_{n_t} + jA_\tau} = (I_{n_t} - jA_\tau)(I_{n_t} + jA_\tau)^{-1}, \tag{33}$$

where $j = \sqrt{-1}$ and $A_\tau$ is $n_t \times n_t$ Hermitian matrix at block time $\tau$ (we drop the subscript on $A$ from now on for simplicity). As proposed in [16], the output of the Caylay transform is unitary if, and only if, $A$ is a Hermitian matrix. Therefore, we must ensure the Hermitian property for the transmitted perfect STBC signals. Furthermore, according to [10, 16], the Hermitian constraints require real constellations and real rotation matrices to maintain the Hermitian property for matrix $A$.

For differential perfect algebraic STBC, the input symbol vectors, $s_1, \dots, s_{n_r}$, are first rotated by the real rotation matrix $M \in \mathbb{R}^{n_t \times n_t}$ and then threaded differentially into different layers $l$, where $l = 1, \dots, n_t$. Let $l_i$, $1 \le i \le n_t$ denote the $i$th layer. Hence, a layer can be formed such that [10]

$$l_i = \{((n_t - i - t)_{n_t}, t) : 0 \le t < n_t\}. \tag{34}$$

Therefore, the placement of the real rotated symbols into the code block in differential threading is very similar to the case for coherent encoding, but reversed. Then, the differential perfect algebraic STBC codeword can be expressed as

$$X = \sum_{l=1}^{n_r} \mathrm{diag}(Ms_l) \cdot a \cdot (e^{l-1})^{\mathrm{T}}, \tag{35}$$

where $e$ is defined in (13) but with differential case, we use $\gamma = 1$

$$a = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 1 & 0 & 0 & 0 \\ 1 & \cdots & 0 & 0 & 0 \end{bmatrix}, \tag{36}$$

and $M \in \mathbb{R}^{n_t \times n_t}$ is a real rotation matrix as in [10, 18, 19]. The code block generated in the differential perfect algebraic STBC case must be Hermitian. Then the Hermitian conversion for the matrix $A$ in differential perfect STBC can be written as [10]

$$\begin{aligned} A \triangleq \frac{1}{\sqrt{2}}\Big(&\big(j \cdot \mathrm{triu}(X) + (\mathrm{tril}(X))^{\mathrm{H}}\big) \\ &+ \big((-j \cdot \mathrm{triu}(X))^{\mathrm{H}} + (\mathrm{tril}(X))\big)\Big) + \mathrm{diag}(X), \end{aligned} \tag{37}$$

where $\mathrm{triu}(\cdot)$ denotes the $n_t \times n_t$ matrix that contains only the above diagonal elements of $X$, and $\mathrm{tril}(\cdot)$ denotes the $n_t \times n_t$ matrix that contains only the below diagonal elements of $X$. Therefore, if the input matrix $A$ is Hermitian, the Cayley transformed matrix in (33) will be unitary. Furthermore, since the differential perfect algebraic STBC requires multiplying the new code block by the previous block, the resulting new transmitted output in (21) remains unitary.

*Example 2:* For $n_t = 4$, $n_r = 4$, and $K = 1$, we have $l = 4$ layers. Then, the Hermitian matrix $A$ is in the form of

$$A = \begin{bmatrix} x_{14} & \frac{x_{23} + jx_{13}}{\sqrt{2}} & \frac{x_{32} + jx_{12}}{\sqrt{2}} & \frac{x_{41} + jx_{11}}{\sqrt{2}} \\ \frac{x_{23} - jx_{13}}{\sqrt{2}} & x_{22} & \frac{x_{31} + jx_{21}}{\sqrt{2}} & \frac{x_{44} + jx_{24}}{\sqrt{2}} \\ \frac{x_{32} - jx_{12}}{\sqrt{2}} & \frac{x_{31} - jx_{21}}{\sqrt{2}} & x_{34} & \frac{x_{43} + jx_{33}}{\sqrt{2}} \\ \frac{x_{41} - jx_{11}}{\sqrt{2}} & \frac{x_{44} - jx_{24}}{\sqrt{2}} & \frac{x_{43} - jx_{33}}{\sqrt{2}} & x_{42} \end{bmatrix}.$$

Therefore, with this formulation, given the invertible equivalent channel matrix and the transmitted codeword block $X$, it is easy to determine the input symbols, by using the Hermitian matrix $A$ as a road map. The matrix $A$ points out the elements of the $X$ matrix that include each symbol, and they can be scaled and summed to form the best estimate of the input symbol. For example, from position (4,1) and (1,4) in matrix $A$, we have

$$a_{41} = \frac{x_{41} - jx_{11}}{\sqrt{2}}, \quad a_{14} = \frac{x_{41} + jx_{11}}{\sqrt{2}},$$

and these are the only symbols involved in these positions from matrix $X$. Accordingly, the original input symbols from the transmitted codeword block $X$ are as follows:

$$x_{41} = \frac{1}{\sqrt{2}}\{a_{41} + a_{14}\}, \quad x_{11} = \frac{1}{\sqrt{2}}\Im\{a_{14} - a_{41}\}.$$

### *4.3 Decoding the differential perfect algebraic STBC for a MU-MIMO system*

For the MU-MIMO downlink system, the differential transmissions are implemented in blocks, in which each user $k$ receives the sum of all the transmit waveforms of other users; then the received signal blocks for each user must be detected independently. Thus, if $G$ denotes the matrix having all $N + 1$ received signal blocks for the $k$th user, i.e.

$$G = [\hat{Y}_0 \ \hat{Y}_{\tau-1} \ \hat{Y}_\tau \ \cdots \ \hat{Y}_N]. \tag{38}$$

When encoding using (21), the decoding process for $X_\tau$ for the $k$th user would be according to the last two blocks of $G$ as in the following notation:

$$G = [\underbrace{\hat{Y}_0 \hat{Y}_1} \cdots \underbrace{\hat{Y}_{\tau-1} \hat{Y}_\tau} \cdots \underbrace{\hat{Y}_{N-1} \hat{Y}_N}]. \tag{39}$$

Then the combined information between the unitary matrix $U_{z_\tau}$ and the received signal blocks $(\hat{Y}_{\tau-1}, \hat{Y}_\tau)$ in the differential scheme at the $k$th user can be expressed as

$$\begin{bmatrix} \hat{Y}_{\tau-1} \\ \hat{Y}_\tau \end{bmatrix} = H \begin{bmatrix} X_{\tau-1} \\ X_{\tau-1} U_{z_\tau} \end{bmatrix} + \begin{bmatrix} \hat{Z}_{\tau-1} \\ \hat{Z}_\tau \end{bmatrix}. \tag{40}$$

For differential perfect algebraic STBC encoding, it is assumed that for any user $k$ the channel matrix $H$ changes slowly (channel coherence time is large enough) and extends over several matrix transmission periods. In such a case, the BS transmission starts with a reference matrix $X_0$, followed by several information matrices. The Hermitian matrix $A$ is used to form an equivalent channel model for differential decoding. An easier way to represent this model is to rewrite the differential receiver equation using the Cayley transform [16]

$$\begin{aligned} \hat{Y}_\tau &= \hat{Y}_{\tau-1} U_{z_\tau} + \hat{Z}_\tau - \hat{Z}_{\tau-1} U_{z_\tau} \\ &= \hat{Y}_{\tau-1}(I_{n_t} - jA)(I_{n_t} + jA)^{-1} + \hat{Z}_\tau \\ &\quad - \hat{Z}_{\tau-1}(I_{n_t} + jA)^{-1}(I_{n_t} - jA). \end{aligned}$$

By multiplying both sides by $(I_{n_t} + jA)$, we have

$$\begin{aligned} \hat{Y}_\tau(I_{n_t} + jA) = \hat{Y}_{\tau-1}(I_{n_t} - jA) + \hat{Z}_\tau(I_{n_t} + jA) \\ - \hat{Z}_{\tau-1}(I_{n_t} - jA), \end{aligned}$$

which can be simplified as

$$\begin{aligned} \hat{Y}_\tau - \hat{Y}_{\tau-1} = -j(\hat{Y}_\tau + \hat{Y}_{\tau-1})A + \hat{Z}_\tau(I_{n_t} + jA) \\ - \hat{Z}_{\tau-1}(I_{n_t} - jA). \end{aligned} \tag{41}$$

Note that due to the differential detection with matrix $\boldsymbol{A}$ as a unitary Cayley transform, the additive noise in (41) has the covariance

$$2(\boldsymbol{I}_{n_t} + j\boldsymbol{A})(\boldsymbol{I}_{n_t} - j\boldsymbol{A}) = 2(\boldsymbol{I}_{n_t} + \boldsymbol{A}^2), \tag{42}$$

which results in some performance degradation. Then, the ML decoder can be given as [16]

$$\hat{\boldsymbol{s}} = \arg \min_{s_1 \cdots s_{n_r}} \| \left(\hat{\boldsymbol{Y}}_\tau - \hat{\boldsymbol{Y}}_{\tau-1}\right) - \left(\frac{1}{j}(\hat{\boldsymbol{Y}}_\tau + \hat{\boldsymbol{Y}}_{\tau-1})\boldsymbol{A}\right) \|^2. \tag{43}$$

To find the ML solution vectors without an exhaustive search, the sphere decoding method is used, as it considers only a small set of vectors rather than all possible transmitted signal vectors. We obtain the SD representation by constructing an equivalent channel model for the differential system equation in (41). Let $\boldsymbol{C} = \hat{\boldsymbol{Y}}_\tau - \hat{\boldsymbol{Y}}_{\tau-1}$ and $\boldsymbol{B} = -j(\hat{\boldsymbol{Y}}_\tau + \hat{\boldsymbol{Y}}_{\tau-1})$, then the differential equivalent channel is

$$\boldsymbol{C} = \boldsymbol{B}\boldsymbol{A} + \hat{\boldsymbol{Z}}_d, \tag{44}$$

where $\hat{\boldsymbol{Z}}_d = \hat{\boldsymbol{Z}}_\tau(\boldsymbol{I}_{n_t} + j\boldsymbol{A}) - \hat{\boldsymbol{Z}}_{\tau-1}(\boldsymbol{I}_{n_t} - j\boldsymbol{A})$ is the additive Gaussian noise with zero mean and covariance $2(\boldsymbol{I}_{n_t} + \boldsymbol{A}^2)$. Now, the received spread signal for the $k$th user is vectorised as

$$\begin{aligned} \operatorname{vec}(\boldsymbol{C}^{\mathrm{T}}) &= \operatorname{vec}\left((\boldsymbol{B}\boldsymbol{A})^{\mathrm{T}}\right) + \operatorname{vec}(\hat{\boldsymbol{Z}}_d^{\mathrm{T}}) \\ &= \mathscr{B}_{\mathrm{d}} \operatorname{vec}(\boldsymbol{S}) + \operatorname{vec}(\hat{\boldsymbol{Z}}_d^{\mathrm{T}}), \end{aligned} \tag{45}$$

where $\mathscr{B}_{\mathrm{d}}$ is the new $n_t n_r \times n_t n_r$ effective channel matrix of the differential perfect STBC, which is given by

$$\mathscr{B}_{\mathrm{d}} = \tilde{\boldsymbol{B}}\tilde{\boldsymbol{A}} \cdot (\boldsymbol{I}_{n_r} \otimes \boldsymbol{M}), \tag{46}$$

where $n_t n_r \times n_t n_r$ matrix $\tilde{\boldsymbol{B}}$ is given by

$$\tilde{\boldsymbol{B}} = \begin{bmatrix} (\operatorname{diag}(\boldsymbol{b}_1)\boldsymbol{a})^{\mathrm{T}} & \cdots & (\operatorname{diag}(\boldsymbol{b}_1)\boldsymbol{a} \cdot \boldsymbol{e}^{n_r - 1})^{\mathrm{T}} \\ \vdots & \ddots & \vdots \\ (\operatorname{diag}(\boldsymbol{b}_{n_r})\boldsymbol{a})^{\mathrm{T}} & \cdots & (\operatorname{diag}(\boldsymbol{b}_{n_r})\boldsymbol{a} \cdot \boldsymbol{e}^{n_r - 1})^{\mathrm{T}} \end{bmatrix}. \tag{47}$$

The $n_t n_r \times n_t n_r$ block diagonal matrix $\tilde{\boldsymbol{A}}$ is in the form of

$$\tilde{\boldsymbol{A}} = \frac{1}{\sqrt{2}} \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_2 & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{A}_{n_r} \end{bmatrix}, \tag{48}$$

where $\boldsymbol{A}_1, \boldsymbol{A}_2, \ldots, \boldsymbol{A}_{n_r}$ are the scaled sub-matrices of the original Hermitian matrix $\boldsymbol{A}$, and each is of size $n_t \times n_t$. To define the block diagonal matrix $\tilde{\boldsymbol{A}}$ completely, we give an example.

*Example 3:* By using the same entities as in Example 2. Then, the sub-matrices of the $16 \times 16$ block diagonal matrix $\tilde{\boldsymbol{A}}$ are in the form

$$\boldsymbol{A}_1 = \begin{bmatrix} j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \\ 0 & -j & 1 & 0 \\ -j & 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} j & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -j & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{A}_3 = \begin{bmatrix} j & 1 & 0 & 0 \\ -j & 1 & 0 & 0 \\ 0 & 0 & j & 1 \\ 0 & 0 & -j & 1 \end{bmatrix}, \quad \boldsymbol{A}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & j & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -j & 0 & 1 \end{bmatrix}.$$

Now, we convert the complex received vector in (45) to its equivalent real and imaginary parts, i.e.

$$\mathscr{C} = \mathscr{H}_{\mathrm{d}}\mathscr{S} + \mathscr{Z}_{\mathrm{d}}, \tag{49}$$

where

$$\mathscr{C} = \left[\Re(\operatorname{vec}(\boldsymbol{C}^{\mathrm{T}})) \quad \Im(\operatorname{vec}(\boldsymbol{C}^{\mathrm{T}}))\right]^{\mathrm{T}},$$

$$\mathscr{S} = \left[\Re(\operatorname{vec}(\boldsymbol{S})) \quad \Im(\operatorname{vec}(\boldsymbol{S}))\right]^{\mathrm{T}},$$

$$\mathscr{Z}_{\mathrm{d}} = \left[\Re(\operatorname{vec}(\hat{\boldsymbol{Z}}_{\mathrm{d}}^{\mathrm{T}})) \quad \Im(\operatorname{vec}(\hat{\boldsymbol{Z}}_{\mathrm{d}}^{\mathrm{T}}))\right]^{\mathrm{T}},$$

and

$$\mathscr{H}_{\mathrm{d}} = \begin{bmatrix} \Re(\mathscr{B}_{\mathrm{d}}) & -\Im(\mathscr{B}_{\mathrm{d}}) \\ \Im(\mathscr{B}_{\mathrm{d}}) & \Re(\mathscr{B}_{\mathrm{d}}) \end{bmatrix}.$$

The SD can be implemented to decode the $k$th user symbols $\hat{\mathscr{S}}$ such that

$$\hat{\mathscr{S}} = \arg \min_{\boldsymbol{S} \in \mathbb{Z}^n} \| \mathscr{C} - \mathscr{H}_{\mathrm{d}}\mathscr{S} \|^2. \tag{50}$$

## 5 Computational complexity and rate analysis

The matrix $\mathscr{H}_{\mathrm{d}}$ in (49) has the size of $2n_t n_r \times 2n_t n_r$, thus we have $2n_t n_r$ equations and $2n_t n_r$ unknowns. The SD usually benefits from having more equations and fewer unknowns because the computational complexity is polynomial, yet goes exponential when the difference between the number of equations and unknowns grows large. To allow for a low-complexity decoder and to have at least as many equations as unknowns when $n_t \geq n_r$, the number of threads $l$ per block per user is constrained by [9]

$$l \leq \min(n_t, n_r), \tag{51}$$

and since the number of symbols per block per user is $q = n_t l$. Hence, in this study, we impose the following constraint:

$$q \leq \min(n_t^2, n_t n_r). \tag{52}$$

In this case, the maximum rate of the code essentially depends on the number of threads $l$ per block per user, the total number of $q$ symbols per block per user sent in that threads, the cardinality of constellation $L$, and the orthogonal spreading code period per user. Since the channel is used $n_t$ times, the system transmission rate per channel per user is

$$R = \frac{q}{K \cdot n_t} \cdot \log_2(L) \quad \text{bits/s/Hz}. \tag{53}$$

There are $K$ users in the system, each transmitting $q$ symbols per block. Therefore, the total bit rate per system is

$$R = \frac{q}{n_t} \cdot \log_2(L) \quad \text{bits/s/Hz}. \tag{54}$$

Note that the rate is independent of the number of users. Through a wise choice of the number of threads per block $l \leq \min(n_t, n_r)$, systems that achieve this transmission rate will have full rate and full diversity [7].

### 5.1 Rate analysis

As discussed earlier, the differential perfect algebraic scheme achieves full diversity full rate over a MU-MIMO channel where at different time slots and different antennas, different symbols are transmitted. Table 1 briefly summarises and compares the rate parameters of differential perfect algebraic STBC with other

**Table 1** The rate and diversity parameters of classic STBC representatives

| STBC scheme | Parameters | | | | |
|---|---|---|---|---|---|
| Algebraic perfect | $n_t > 1$ | $n_r \geq 1$ | $G_d = n_t n_r$ | $q = n_t n_r$ | $R = (q/n_t) \cdot \log_2(L)$ |
| G2-STBC [17] | $n_t = 2$ | $n_r \geq 1$ | $G_d = n_t n_r$ | $q = 2$ | $R = \log_2(L)$ |
| QO-STBC [4] | $n_t = 4$ | $n_r \geq 1$ | $G_d = n_t n_r$ | $q = 4$ | $R = \log_2(L)$ |

**Table 2** Computational complexity of coherent perfect algebraic STBC

| Steps | Operation | Flops | Case $(2,2,2) \times 6$ |
|---|---|---|---|
| 1 | $\sum_{l=1}^{n_r} \mathrm{diag}(\boldsymbol{M}\boldsymbol{s}_l) \cdot \boldsymbol{e}^{l-1}$ | $\mathcal{O}(Kn_r(16n_t^3 - 2n_t^2))$ | 20,304 |
| 2 | $\boldsymbol{HXV}$ | $\mathcal{O}(K(16Kn_t^2 n_r - 2\,\mathrm{K}n_t n_r))$ | 10,152 |
| 3 | $\boldsymbol{YV}^{\mathrm{H}}$ | $\mathcal{O}(K(8Kn_t^2 n_r - 2n_t n_r))$ | 5112 |
| 4 | $\boldsymbol{I}_{n_r} \otimes \boldsymbol{M}$ | $\mathcal{O}(K(n_t^2 n_r^2))$ | 432 |
| 5 | $\tilde{\boldsymbol{H}} \cdot (\boldsymbol{I}_{n_r} \otimes \boldsymbol{M})$ | $\mathcal{O}K(8n_t^3 n_r^3 - 2n_t^2 n_r^2)$ | 40,608 |
| | | | total = 76,608 |

**Table 3** Computational complexity of the proposed differential perfect STBC

| Steps | Operation | Flops | Case $(2,2,2) \times 6$ |
|---|---|---|---|
| 1 | $\sum_{l=1}^{n_r} \mathrm{diag}(\boldsymbol{M}\boldsymbol{s}_l) \cdot \boldsymbol{a} \cdot \left(\boldsymbol{e}^{l-1}\right)^{\mathrm{T}}$ | $\mathcal{O}(2\,\mathrm{K}n_r(24n_t^3 - 2n_t^2))$ | 61,344 |
| 2 | $\boldsymbol{HXUV}$ | $\mathcal{O}(K(24Kn_t^3 - 2\,\mathrm{K}n_t^2))$ | 46,008 |
| 3 | $\boldsymbol{YV}^{\mathrm{H}}$ | $\mathcal{O}(K(8Kn_t^2 n_r - 2n_t n_r))$ | 5112 |
| 4 | $\boldsymbol{I}_{n_r} \otimes \boldsymbol{M}$ | $\mathcal{O}(K(n_t^2 n_r^2))$ | 432 |
| 5 | $\tilde{\boldsymbol{B}}\tilde{\boldsymbol{A}} \cdot (\boldsymbol{I}_{n_r} \otimes \boldsymbol{M})$ | $\mathcal{O}K(16n_t^3 n_r^3 - 2n_t^2 n_r^2)$ | 82,080 |
| | | | total = 194,976 |

practical STBC schemes that offer reasonable data rates and diversity such as differential Alamouti code (G2-STBC) [17], and differential quasi-orthogonal code (QO-STBC) [4]. In terms of the MIMO's diversity feature shown in Table 1, the three MIMO schemes of Algebraic, Alamouti, and quasi-orthogonal STBCs are capable of attaining the full diversity order of $n_t n_r$, which minimises the PEP of (25) according to its rank criterion. With regard to the transmission rate as seen in Table 1, differential perfect algebraic STBC introduced in this paper is capable of achieving the full MIMO transmission rate, provided that the parameters satisfy $q = n_t n_r$, which results in a maximised rate gain of $R = (q/n_t) \cdot \log_2(L)$. In the other STBCs shown in Table 1, every element of a codeword matrix is a linear combination of the input symbols and limited by a fixed number of transmit antennas, i.e. $n_t = 2$ or $n_t = 4$. The number of symbols is selected such that an orthogonal STBC is feasible. Such a limit on the number of symbols is not necessary if the orthogonality condition of the STBC is relaxed as in differential perfect algebraic STBC. For example, with $n_t = 2$, $n_r = 2$, and 8-PAM; rates for differential perfect algebraic STBC and G2-STBC, are 6 and 3, respectively. Similarly, with $n_t = 4$, $n_r = 4$, and 8-PAM; rates for differential perfect algebraic STBC and QO-STBC, are 12 and 3, respectively. That shows the difference in the rate.

### *5.2 Complexity analysis*

To provide more insight into the computational complexity, the notion of flops is introduced in this section, where flops denote the floating point operation (FLOPs). We use the total number of FLOPs to measure the computational complexity of different schemes. We summarise the total FLOPs needed for the matrix operations below [23, 24]:

- Multiplication of $m \times n$ and $n \times p$ complex matrices: $8mnp - 2mp$.
- QR decomposition of an $m \times n$ $(m \leq n)$ complex matrix: $16(n^2 m - nm^2 + (1/3)m^3)$.
- Singular value decomposition [SVD] of an $m \times n$ $(m \leq n)$ complex matrix where only $\boldsymbol{\Sigma}$ and $\boldsymbol{V}$ are obtained: $32(nm^2 + 2m^3)$.
- SVD of an $m \times n$ $(m \leq n)$ complex matrix where $\boldsymbol{U}$, $\boldsymbol{\Sigma}$ and $\boldsymbol{V}$ are obtained: $8(4n^2 m + 8nm^2 + 9m^3)$.
- Inversion of an $m \times m$ real matrix using Gauss–Jordan elimination: $4m^3/3$.

For the cases shown in Tables 2–4, we show the operations and the required FLOPs for the algorithms of the coherent perfect algebraic STBC, the differential perfect algebraic STBC, and the differential G2-STBC in [17], respectively. For illustration, we consider a system with $K = 3$ users, each user with $n_r = 2$ receive antennas, and $n_t = 6$ transmit antennas; this scenario is denoted as $(2,2,2) \times 6$. For simplicity, and without loss of generality, we also assume that all users have the same number of receive antennas. Note that, in Table 4, $\bar{n}_r = \sum_{j=1, j \neq k}^{K} n_r^j$. Clearly, the proposed differential perfect algebraic STBC scheme requires the highest complexity.

Furthermore, for the cases shown in Figs. 2 and 3, we show the computational complexity of the system dimensions. In Fig. 2, we first set the number of receive antennas for each user to be $n_r = 2$ and increase the number of users K. Similarly, in Fig. 3, the number of users is fixed to be $K = 4$ while the number of receive antennas for each user is increased gradually. From both figures, the computational complexity of the proposed system is higher and increases exponentially. The reason is that the differential perfect algebraic STBC scheme requires higher rate and as a result, the number of antennas increases exponentially and thus the size of unknown variables for the equivalent channel matrix equation also increases exponentially. We also observe that varying the number of receive antennas has a much higher impact on the complexity than varying the number of users. Therefore, for high-rate systems that support different types of terminals, it is better to keep the number of receive antennas for each terminal as low as possible.

As shown above, it is worth noting that the perfect STBC combined with differential STBC scheme proposed in this study relaxes the orthogonality conditions of the standard orthogonal

**Table 4** Computational complexity of differential Alamouti STBC in [17]

| Steps | Operation | Flops | Case $(2,2,2) \times 6$ |
|---|---|---|---|
| 1 | $\bar{\boldsymbol{H}}^{\dagger}$ | $\mathcal{O}(K(\frac{4}{3}\bar{n}_r^3 + 16\bar{n}_r^2 n_t - 2n_t\bar{n}_r))$ | 4720 |
| 2 | $(\boldsymbol{I} - \boldsymbol{H}^{\dagger}\bar{\boldsymbol{H}})\boldsymbol{\Phi}$ | $\mathcal{O}(K(8n_t^2\bar{n}_r + 14n_t^2 - 4n_t))$ | 4896 |
| 3 | $\mathbf{QR}((\boldsymbol{I} - \bar{\boldsymbol{H}}^{\dagger}\bar{\boldsymbol{H}}))$ | $\mathcal{O}(K(\frac{16}{3}n_t^3))$ | 3456 |
| 4 | $\mathbf{SVD}(\boldsymbol{H}(\boldsymbol{I} - \bar{\boldsymbol{H}}^{\dagger}\bar{\boldsymbol{H}}))$ | $\mathcal{O}(K(64n_r^3 + 8n_t^2 n_r + 32n_t n_r^2 - 2n_t n_r))$ | 5496 |
| 5 | $\boldsymbol{HFX}$ | $\mathcal{O}(K(16n_t n_r + 24n_r))$ | 720 |
| | | | total = 19,288 |

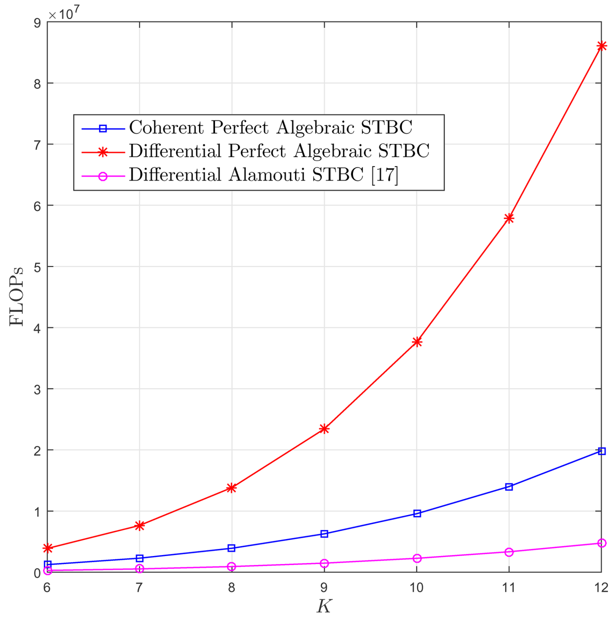


**Fig. 2** *Comparison of the computational complexity for differential perfect algebraic, coherent perfect algebraic, and differential Alamouti with $n_r = 2$, and $n_t = K \times n_r$*

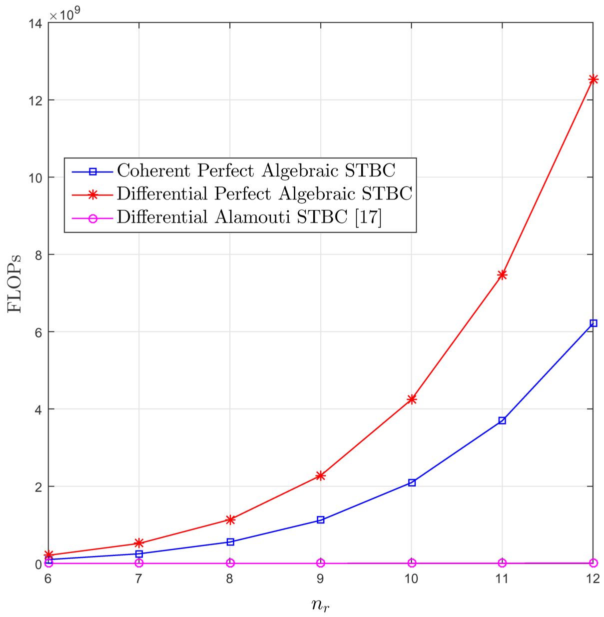


**Fig. 3** *Comparison of the computational complexity for differential perfect algebraic, coherent perfect algebraic and differential Alamouti with $K = 4$, and $n_t = K \times n_r$*

STBC codes such as G2-STBC and QO-STBC codes. Therefore, in this study, the number of transmit symbols per block in downlink is much higher and that will result in increasing the overall rate of the system. Thus, we can transmit and receive in high rate without needing the CSI.

## 6 Simulation results and discussion

In this section, the performance of the differential perfect algebraic space-time modulation scheme for the MU-MIMO downlink transmission is examined. In this section, we assume the channel is modelled as quasi-static, where the fading block matrix between the transmitter and receiver is constant (but unknown) between two successive channel uses. The SNR per user is defined as SNR= $n_r\rho$. The Monte Carlo simulation is used to evaluate the performance in terms of the BLER and bit error rate (BER).

*The differential perfect algebraic STBC with multiple users*: In Fig. 4, the BLER performance curve is first simulated and plotted for one, two, three, and four user systems. Each user has two receive antennas and 4-PAM symbols are used. The BS has four transmit antennas. In this case, each STBC block has $q = 8$ symbols per user with $R = 4$. The transmitted codeword $X$ for each user consists of two layers $4 \times 4$ perfect algebraic STBC, i.e. $l = n_r = 2$. The $4 \times 4$ rotation matrix $\boldsymbol{M}$ is given in [18, 19]. Hadamard matrices are used as the orthogonal spreading matrices to cancel the CCI. It is shown that the MU-MIMO system for all cases (e.g. in the case of one, two, three, and four users) achieves the same performance as a single-user MIMO system; i.e. on multiple users, the orthogonal spreading codes allowed to eliminate CCI. This results in every user being processed as if it was a single-user case so that the results for every user are identical and the CCI is completely eliminated and full rate full diversity is achieved with the differential perfect algebraic STBC.

*Diversity gain, coding gain, and rates :* In Fig. 4, the slope of the BLER curves for high SNR converges to the slope determined by the diversity. As shown in (27), this slope is $-rn_r$ (in a log–log scale) where $r$ is the rank of the codeword difference matrix. For differential algebraic STBC codes, the number of threads $l$ determines the rank $r$, and this is limited by the number of receive antennas, $n_r$, if $n_r \leq n_t$. Hence the diversity slope is $-n_r^2$. Furthermore, $-rn_r$ is also related to the number of symbols encoded in each codeword block over symbol time periods. Thus, the number of symbols per block is $n_t n_r$. For the case of Fig. 4, we have $l = 2$ threads and four symbol time periods in the $4 \times 4$ algebraic codeword. Each thread has four symbols and only two symbols are encoded and transmitted in any one symbol time period. Clearly, when two threads are populated, eight entries of the $4 \times 4$ code block are populated and the other eight are filled with zeros. Therefore, the rank of the codeword difference matrix $r = 2$ for this approach of coding. Then we have $G_d = rn_r = 4$. In Fig. 4, we plot the diversity line based on the actual BLER curves using the estimated lower bounds formula in (32) of the coding gain $G_c$. The BLER curves appear to approach a slope of $-4$ asymptotically and the diversity gain is $G_d = 4.0257$, i.e. the full diversity for this case is achieved with the rate of $R = 4$. Using the method above, the estimated value for a lower bound for coding gain is $G_c = 0.154$ (as a linear ratio).

*Differential algebraic versus orthogonal differential Alamouti:* In Fig. 5, the BLER performance is plotted and compared between the differential algebraic STBC and the differential Alamouti code [17]. We use the differential Alamouti code as a benchmark scheme. First, we examine the performance for both schemes at the same rate $R = 2$ with $n_t = 2$ and $n_r = 2$. To get $R = 2$ for both schemes, we use 4-PAM constellation size for the differential Alamouti and 2-PAM for differential algebraic by using (54). The

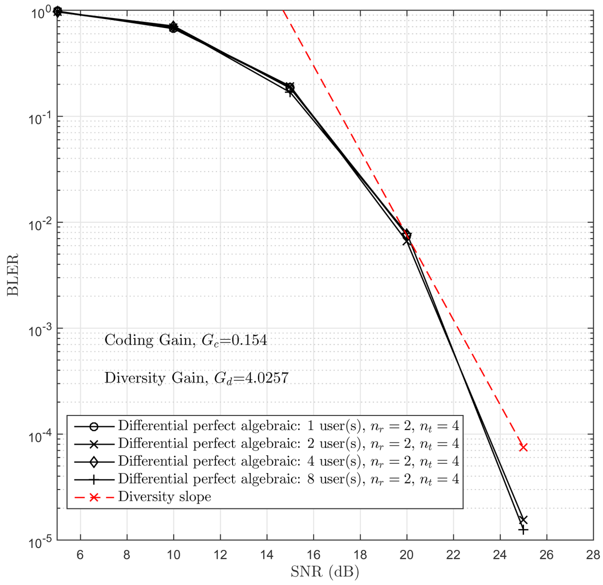


**Fig. 4** *BLER performance of the proposed MU-MIMO STBC downlink transmission with differential algebraic STBC with a one-, two-, three-, and four-user system model, $R = 4$, $n_t = 4$, $n_r = 2$, $l = 2$, and 4-PAM*

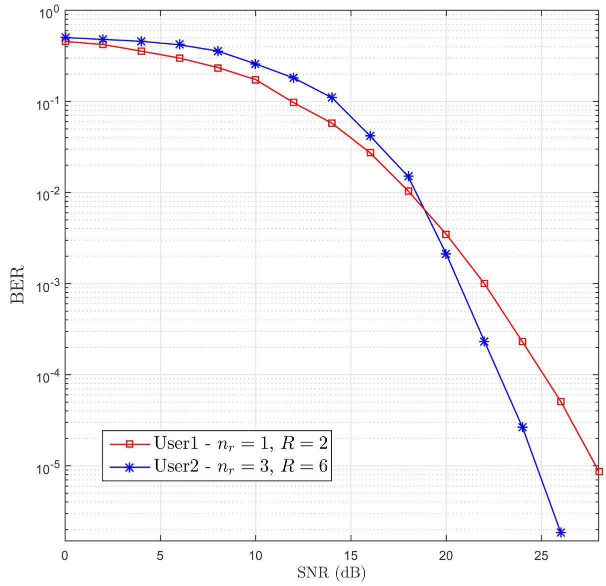


**Fig. 6** *BER performance of the proposed MU-MIMO STBC downlink transmission with differential algebraic STBC with a two-user system model with two different rates and layers, $n_t = 4$, and 4-PAM*

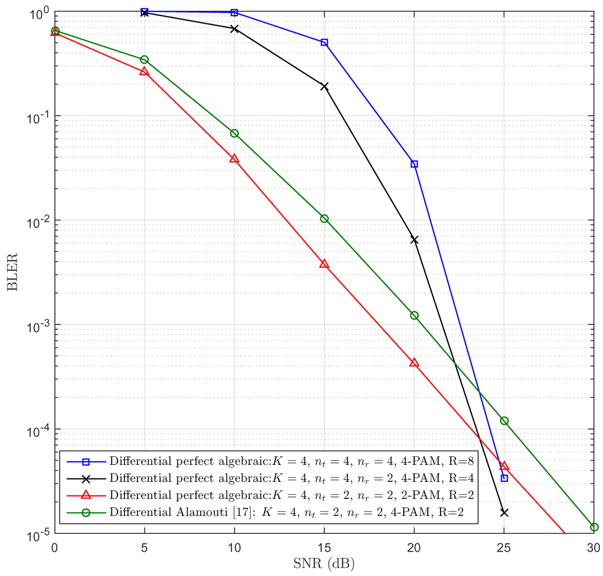


**Fig. 5** *BLER performance of the proposed MU-MIMO STBC downlink transmission for differential algebraic and the orthogonal Alamouti Code [17] for different rates*

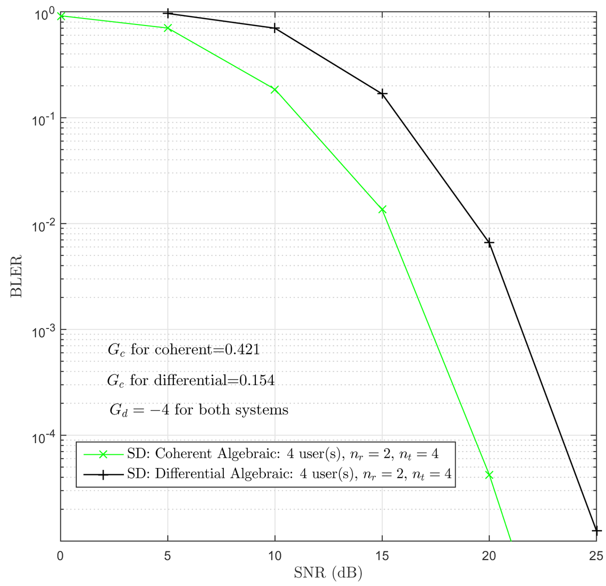


**Fig. 7** *BLER performance of the proposed MU-MIMO STBC downlink transmission for differential algebraic and coherent algebraic STBC with a four-user system, $R = 4$, $n_t = 4$, $n_r = 2$, $l = 2$, and 4-PAM*

figure shows that the performance of the proposed scheme outperforms the differential Alamouti for the same rate. Second, we increase the rate of the proposed scheme from $R = 2$ to $R = 4$ and $R = 8$, then we compare it to the differential Alamouti. The differential Alamouti with $R = 2$ is initially better at low SNR. However, the differential algebraic curves converge and outperform at high SNR, even if their rates are two or four times the rate of the differential Alamouti scheme thanks to their steeper diversity slope, i.e. $G_d = 4$ and $G_d = 16$.

*The impact of multiple receive antenna diversity:* In Fig. 6, we assume a two-user system; user 1 is equipped with one receive antenna, and user 2 has three receive antennas, and a 4-PAM constellation is used. The BS has four transmit antennas. The rate for user 1 is 2 bits/s/Hz with one layer and for user 2 is 6 bits/s/Hz with three layers. The BER performance of the system is shown in Fig. 6. We see that the performance of user 2 outperforms that of user 1 at high SNR, even though its rate is three times the rate of user 1, because of its receive antenna diversity.

*Coherent algebraic versus differential algebraic STBC:* In Fig. 7, the BLER performance is plotted and compared between the coherent algebraic and the differential algebraic STBC. For a fair comparison of the two schemes, we consider the same setup, namely four users, $n_t = 4$, each user has $n_r = 2$, $l = 2$ threads, $R = 4$, 4-PAM, SD, and with unitary Cayley matrix. Furthermore, to quantify this performance loss in both schemes, the receiver's SNR is calculated for the same unit to transmit power. For the differential algebraic detection scheme, the power of noise at the receiver is approximately two times the power of the noise for the coherent detection as shown in (42). Therefore, the received SNR of the differential detection scheme is approximately half of that of the coherent detection scheme for the same transmission power. This results in about a 4 dB difference in the performance at high SNRs as expected. The coding gain for both schemes is calculated, and it is $G_c = 0.421$ for the coherent scheme and $G_c = 0.154$ for the differential. The diversity for both schemes is $G_c \simeq -4$.

*The impact of multiple access interference (MAI):* Fig. 8 illustrates the results of repeating the same experiment for a two-user system each user has two receive antennas but with higher rate $R = 6$ and 8-PAM. The performance of the system in Fig. 8 underperforms that of the system in Fig. 4 because of its higher

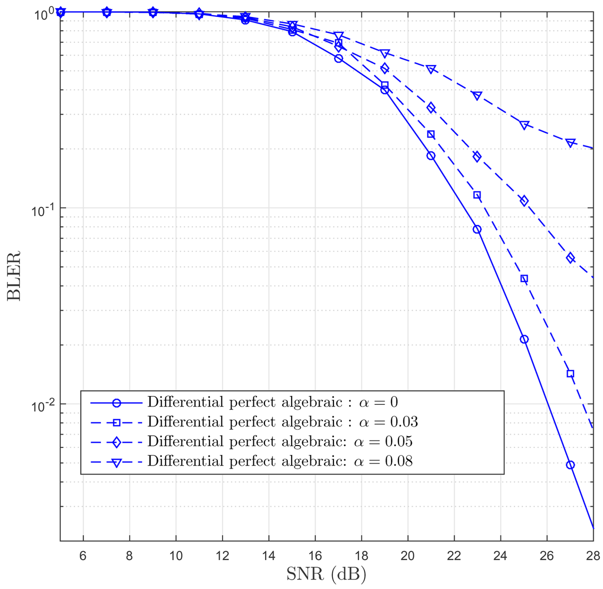


**Fig. 8** *BLER performance of the proposed MU-MIMO STBC downlink transmission with differential algebraic STBC using a two-user system model, $R = 6$, $n_t = 4$, $n_r = 2$, and 8-PAM*

rate. Furthermore, in this figure, we examine the effect of error in the spreading matrices $\boldsymbol{V}$. Increasing the number of users in the system, the high mobility, and multipath propagation may result in MAI in orthogonal spreading matrices, which destroy the orthogonality of the transmitted signals for multiple users. For the two-user system, let the error spreading matrix for user 1 be $\bar{\boldsymbol{V}}_1 = \boldsymbol{V}_1 + \alpha \boldsymbol{V}_2$, where $\alpha$ is the error coefficient. Therefore, the conditions for the orthogonality of the spreading matrix for user 1 and user 2 are as follows:

$$\bar{\boldsymbol{V}}_1 \bar{\boldsymbol{V}}_1^{\mathrm{H}} = \boldsymbol{I}_{n_t} + \alpha^2 \boldsymbol{I}_{n_t}, \tag{55}$$

$$\boldsymbol{V}_2 \bar{\boldsymbol{V}}_1^{\mathrm{H}} = \alpha \boldsymbol{I}_{n_t}. \tag{56}$$

The values of $\alpha$ are chosen to be 0.03, 0.05, and 0.08. It is shown that the error in the orthogonality of the spreading matrix $\boldsymbol{V}$ occurs among users when $\alpha > 0$.

## 7 Conclusion

In this study, a differential perfect algebraic STBC scheme for MU-MIMO with downlink transmission has been proposed. The Caylay differential STBC that we have introduced does not require channel knowledge, either at the transmitter or receiver. To simplify the receivers' equipment in the MU-MIMO system, the impact of the receiver channel estimation process and/or overhead problem can potentially be solved and avoided by using the Caylay differential STBC. Furthermore, we show how to use the differential STBC combined with perfect algebraic STBC to achieve a full rate and full diversity differential system. Due to the multiple users, there is a need for the separation of the data streams and this is achieved by the use of orthogonal spreading matrices. For this system, to limit the number of possible codewords, a near-optimal SD is performed to decode the signals at the receiver. The proposed schemes yield low complexity transceivers while also providing high rate with good performance. However, the system in this study has higher computational complexity because of its higher rate. Monte Carlo simulation results demonstrate the effectiveness of the proposed schemes.

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