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Highlights

- We generalize a theory of Biot for a porous solid based on nonlinear elasticity theory to incorporate temperature effects.
- Acceleration waves are studied in detail in the fully nonlinear theory.
- The wavespeeds are found explicitly and the amplitudes are then determined.
- The possibility of shock formation is discussed.

Thermal effects on nonlinear acceleration waves in the Biot theory of porous media

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Abstract

We generalize a theory of Biot for a porous solid based on nonlinear elasticity theory to incorporate temperature effects. Acceleration waves are studied in detail in the fully nonlinear theory. The wavespeeds are found explicitly and the amplitudes are then determined. The possibility of shock formation is discussed.

Keywords: acceleration waves, porous media, nonlinear deformations, thermal effects

1. Introduction

The topic of wave propagation in porous and otherwise compressible media is one of great interest in the current research literature, see e.g. Biot [1], Brunnhuber and Jordan [2], Christov [3], Christov and Jordan [4], Christov et al. [5], Ciarletta and Straughan [6–8], Jordan [9–14], Jordan and Puri [15], Jordan and Saccomandi [16], Jordan et al. [17, 18], Paoletti [19], Rossmanith and Puri [20, 21], Wei and Jordan [22].

In a recent paper, Ciarletta et al. [23], we developed a fully nonlinear acceleration wave analysis for an isothermal theory of porous media which incorporates finite deformation in nonlinear elasticity, this theory having been proposed by Biot [24]. This work extends previous work by Jordan [10] and by Ciarletta and Straughan [6] who studied nonlinear acoustic waves in a porous medium when the elastic skeleton is rigid. The aim of the present article is to incorporate temperature effects into the Biot model and then extend the analysis of Ciarletta et al. [23] to the non-isothermal situation. We emphasize that this is not a trivial extension since we find the elastic wave, the pressure wave, and the temperature wave are intrinsically coupled. Temperature effects on sound absorption and propagation in porous metals are a topic of current research interest, see e.g. Wang et al. [25], Zhang [26], Pi et al. [27]. Thus, this work is timely.

When the skeleton in a porous body is allowed to deform it is not trivial to analyse nonlinear wave motion. One way to achieve this has employed a theory of a mixture of a fluid and an elastic solid, see e.g. De Boer and Liu [28]. A second way is to include a distribution of voids in an elastic body, see e.g. Iesan [29], Ciarletta and Straughan [7, 8]. A third way is via the Biot pressure function theory, see Biot [24], Ciarletta et al. [23]. A comparison of wave motion in the last two mentioned theories is given in chapter 4 of Straughan [30]. It is worth pointing out that Biot [24] is critical of employing a mixture theory approach, and Chen [31] is likewise critical of using acceleration waves in mixture theories. Tong et al. [32] and Legland et al. [33] studied nonlinear waves in an isotropic Biot (cf. Biot [34]) theory for acoustic wave propagation, but no thermal effects were included.

2. Nonlinear thermoelastic theory of porous media

We use standard indicial notation in conjunction with the Einstein summation convention throughout. We denote points in the reference configuration by X_A and these are transformed into the current configuration by the mapping

$$x_i = x_i(X_A, t). \tag{1}$$

The deformation gradient F_{iA} and displacement vector u_i are defined by

$$F_{iA} = \frac{\partial x_i}{\partial X_A}, \tag{2}$$

$$u_i = x_i - X_i. \tag{3}$$

We commence with the momentum equation, cf. Biot [24], Ciarletta et al. [23],

$$\rho \ddot{x}_i = \frac{\partial \Pi_{Ai}}{\partial X_A} + \rho f_i, \tag{4}$$

where ρ is the density (in the reference configuration), Π_{Ai} is the Piola-Kirchhoff stress tensor, and f_i denotes the body force. A superposed dot denotes differentiation with respect to time.

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For a thermoelastic body the balance of energy equation may be written as, see e.g. Straughan [30], p.53,

$$\rho\theta\dot{\eta} = -\frac{\partial q_A}{\partial X_A} + \rho r, \tag{5}$$

where $\theta(\mathbf{X}, t)$ is the temperature, η is the specific entropy, q_A is the heat flux, and r denotes an externally supplied heat source. In terms of the Helmholtz free energy function ψ the entropy is given by

$$\eta = -\frac{\partial \psi}{\partial \theta}. \tag{6}$$

The Biot [24] theory involves the pressure, $p(\mathbf{X}, t)$, in the pores in the elastic body. This theory employs a conservation law for the pressure function which may be written as

$$\frac{\partial m}{\partial t} = \frac{\partial J_A}{\partial X_A}. \tag{7}$$

In equation (7) J_A is a flux term and both J_A and m depend on F_{iA} and upon a function ϕ which is a nonlinear function of the pressure p.

In this work we propose a thermoelastic theory for porous media based upon equations (4), (5) and (7) and we propose that ψ and m depend on the variables

$$\begin{aligned} \psi &= \psi(F_{iA}, p, \theta, X_A) \\ m &= m(F_{iA}, p, \theta, X_A) \end{aligned} \tag{8}$$

while the fluxes J_A and q_A have the functional dependence

$$\begin{aligned} q_A &= q_A(F_{iB}, p, p_{,B}, \theta, \theta_{,B}, X_B) \\ J_A &= J_A(F_{iB}, p, p_{,B}, \theta, \theta_{,B}, X_B). \end{aligned} \tag{9}$$

Without loss of generality we now set the body force f_i and heat supply r to be zero.

3. Nonlinear acceleration waves

The governing system of equations is (4), (5) and (7) and we define an acceleration wave for a solution to this system to be a singular surface $\mathscr{S}$ across which $\ddot{x}_i$, $\dot{x}_{i,A}$, $x_{i,AB}$, $\ddot{p}$, $\dot{p}_{,A}$, $p_{,AB}$, $\ddot{\theta}$, $\dot{\theta}_{,A}$ and $\theta_{,AB}$ and their higher derivatives suffer a finite discontinuity, but x_i, p, θ are continuously differentiable throughout $\mathbb{R}^3$ for $t \in [0, \mathcal{T}]$ for some time interval.

Nonlinear acceleration wave analysis is well known, see e.g. Chen [35], and so we give minimal details of the calculations. One employs the constitutive theory (8) and (9) in equations (4), (5) and (7) and then we take the jumps of the resulting equations. We find that

$$[\ddot{x}_i] = \frac{\partial^2 \psi}{\partial F_{jB} \partial F_{iA}} [x_{j,AB}], \tag{10}$$

where we have used the fact that $\Pi_{Ai} = \rho \partial\psi/\partial F_{iA}$, and we further obtain

$$\frac{\partial m}{\partial F_{iA}} [\dot{F}_{iA}] = \frac{\partial J_A}{\partial F_{jB}} [F_{jB,A}] + \frac{\partial J_A}{\partial p_{,B}} [p_{,BA}] + \frac{\partial J_A}{\partial \theta_{,B}} [\theta_{,BA}], \tag{11}$$

$$\rho\theta \frac{\partial \eta}{\partial F_{iA}} [\dot{F}_{iA}] = -\frac{\partial q_A}{\partial F_{iK}} [F_{iK,A}] - \frac{\partial q_A}{\partial \theta_{,K}} [\theta_{,KA}] - \frac{\partial q_A}{\partial p_{,K}} [p_{,KA}], \tag{12}$$

where $[\,\cdot\,]$ denotes the jump of a quantity, eg. $[f] = f^- - f^+$.

Define now the amplitudes a_i, b and c by

$$a_i(t) = [\ddot{u}_i], \qquad b(t) = [\ddot{p}], \qquad c(t) = [\ddot{\theta}]. \tag{13}$$

Using the Hadamard and compatibility conditions, see e.g. Chen [35], Truesdell and Toupin [36], equations (10), (11) and (12) may be rearranged as

$$(\rho U_N^2 \delta_{ij} - Q_{ij}) a_j = 0, \tag{14}$$

$$-\left(U_N N_A \frac{\partial m}{\partial F_{iA}} + \frac{\partial J_A}{\partial F_{iB}} N_A N_B \right) a_i = = \frac{\partial J_A}{\partial p_{,B}} N_A N_B b + \frac{\partial J_A}{\partial \theta_{,B}} N_A N_B c, \tag{15}$$

$$\left(\rho\theta \frac{\partial \eta}{\partial F_{iA}} U_N N_A - \frac{\partial q_A}{\partial F_{iA}} N_A N_B \right) a_i = = \frac{\partial q_A}{\partial \theta_{,B}} N_A N_B c + \frac{\partial q_A}{\partial p_{,B}} N_A N_B b, \tag{16}$$

where Q_{ij} is the acoustic tensor, namely

$$Q_{ij} = \rho \frac{\partial^2 \psi}{\partial F_{jB} \partial F_{iA}} N_A N_B, \tag{17}$$

U_N is the speed of $\mathscr{S}$ at the point $\mathbf{X}$, and N_A is the unit normal to $\mathscr{S}$ at $\mathbf{X}$ in the reference configuration.

To discuss propagation conditions from (14) we use the relation

$$N_A = F_{iA} n_i \frac{|\nabla_{\mathbf{x}} \mathfrak{s}|}{\nabla_{\mathbf{X}} \mathscr{S}} \tag{18}$$

see Truesdell and Toupin [36], eq. (182.8), where $\mathfrak{s}$ and n_i correspond to $\mathscr{S}$ and N_A, but in the current configuration. One now rewrites Q_{ij} in (17) as a function $Q_{ij}(\mathbf{n}, U_N)$ in the current configuration to deduce an acceleration wave may propagate provided a_i is an eigenvector of Q_{ij}, see Truesdell and Noll [37], p.271. Existence results for longitudinal and transverse waves are discussed at length in Truesdell [38] and in Chen [35], pp. 316–322, and the arguments given there hold also for the case in hand. Thus the wavespeed follows from

$$\rho U_N^2 = |\mathbf{Q}(\mathbf{N})\mathbf{n}|.$$

Once the amplitude a_i is determined equations (15) and (16) become a system of two simultaneous linear equations which yield the pressure and thermal amplitudes b and c.

4. Amplitude calculation

We calculate the amplitudes in the case of a one-dimensional wave. It is possible to calculate the amplitudes for a three-dimensional wave but the differential geometry involved is technical and the one-dimensional case yields much of the associated physics.

The one dimensional equivalents of equations (4), (5) and (7) may be written

$$\ddot{u} = \frac{\partial \psi_F}{\partial X}, \qquad \dot{m} = \frac{\partial J}{\partial X}, \qquad \rho\theta\dot{\eta} = -\frac{\partial q}{\partial X}, \tag{19}$$

and the associated constitutive theory is

$$\begin{aligned} \psi &= \psi(F, p, \theta), & m &= m(F, p, \theta), \\ J &= J(F, p, p_X, \theta, \theta_X), & q &= q(F, p, p_X, \theta, \theta_X). \end{aligned} \tag{20}$$

We suppose the wave is moving into a region in which u_X, p and θ are constants, so that u_X^+, p^+ and θ^+ are constants, where the jump notation $[\ddot{u}] = \ddot{u}^- - \ddot{u}^+$ is used. The idea is to differentiate equation (19), and take the jumps, and then employ the one-dimensional equivalents of equations (14), (15) and (16) together with the Hadamard relation and the equation for the jump of a product. Since the calculations are now well known we simply state the final result.

Denote by

$$a = [\ddot{u}], \qquad b = [\ddot{p}], \qquad c = [\ddot{\theta}],$$

and then one may show

$$\frac{\delta a}{\delta t} + ka^2 - \gamma a = 0, \tag{21}$$

where $\delta/\delta t$ is the rate of change seen by an observer on the wave, and the coefficients k and γ have form

$$k = \frac{\psi_{FFF}}{2U_N^3}, \qquad \gamma = \frac{\alpha}{2U_N}\psi_{Fp} - \frac{\beta}{2U_N}\psi_{F\theta}, \tag{22}$$

where

$$\begin{aligned} \alpha &= \frac{1}{D}\{q_{\theta X}(J_F + U_N m_F) + J_{\theta X}(-q_F + \rho\theta\eta_F U_N)\}, \\ \beta &= \frac{1}{D}\{q_{pX}(J_F + U_N m_F) + J_{pX}(-q_F + \rho\theta\eta_F U_N)\}, \\ D &= J_{pX} q_{\theta X} - J_{\theta X} q_{pX}. \end{aligned} \tag{23}$$

The solution to (21) is

$$a(t) = \frac{a(0)}{e^{-\gamma t} + (ka(0)/\gamma)[1 - e^{-\gamma t}]}. \tag{24}$$

When $a(0) < 0$ the wave amplitude $a(t)$ blows-up in a finite time

$$T = \frac{1}{\gamma}\log\left[\frac{ka(0) - \gamma}{ka(0)}\right]. \tag{25}$$

The amplitudes b and c follow from (24) and use of relations

$$[p_{XX}] = -\frac{\alpha}{U_N^2}a, \qquad [\theta_{XX}] = \frac{\beta}{U_N^2}a.$$

It is worth comparing (24) and (25) to the equivalent expressions in the isothermal case with zero pores, cf. Straughan [39], p.304, and the isothermal case with pores, cf. Ciarletta et al. [23]. For all three cases k has the same value. However, γ is not present in the isothermal, zero pore case, whereas $\gamma = (m_F U_N + J_F)\psi_{Fp}/2U_N J_{pX}$ for the isothermal case with pores, see Ciarletta et al. [23]. The effect of the inclusion of the thermal terms is clearly seen in (22), (23) and (25).

It is difficult to give precise values to γ since as Tong et al. [32] point out there are only a few reports on nonlinear effects and the estimation of the required nonlinear parameters, such as the results of Legland et al. [33]. One, therefore, requires a specific constitutive model fitting to the functions ψ, m, q_A and J_A for each specific material. A typical form for ψ in a rigid non-porous solid is

$$\psi = c(\theta - \theta\ln\theta),$$

cf. Straughan [40], p. 110, and a typical form without temperature or porous effects is

$$\psi = \alpha_1 u_X^2 + \beta_1 u_X^3 + \gamma_1 u_X^4$$

cf. Straughan [39], p. 303. One might try, therefore, a free energy of form

$$\begin{aligned} \psi = c(\theta - \theta\ln\theta) + \alpha_1 u_X^2 + \beta_1 u_X^3 + \gamma_1 u_X^4 + \\ \zeta_1 u_x\theta + \delta\theta^2 + \zeta_2 u_X p + \epsilon p^2. \end{aligned} \tag{26}$$

This leads to an entropy of form

$$\eta = c\ln\theta - \zeta_1 u_X - 2\delta\theta.$$

It is key to retain the ζ_1 and ζ_2 terms in (26) because without them γ in (21) is zero. It is clear that experimental values of coefficients are required, as noted by Tong et al. [32].

To make a direct analysis of the effect of γ we may take $a(0) = -1$ in (24), (25). Then

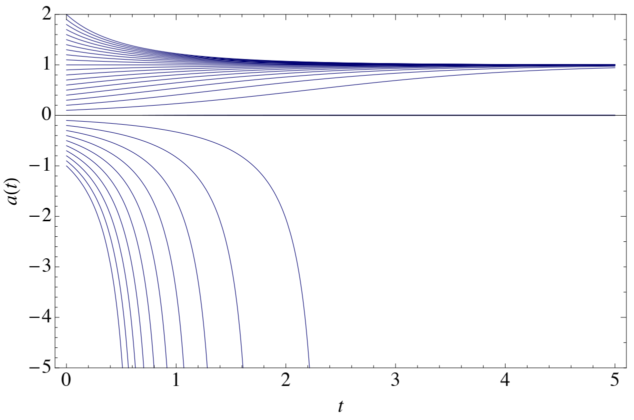
$$a(t) = \frac{-1}{\exp(-\gamma t) - (k/\gamma)(1 - \exp(-\gamma t))}, \qquad T = \frac{1}{\gamma}\log\left(\frac{k+\gamma}{k}\right)$$

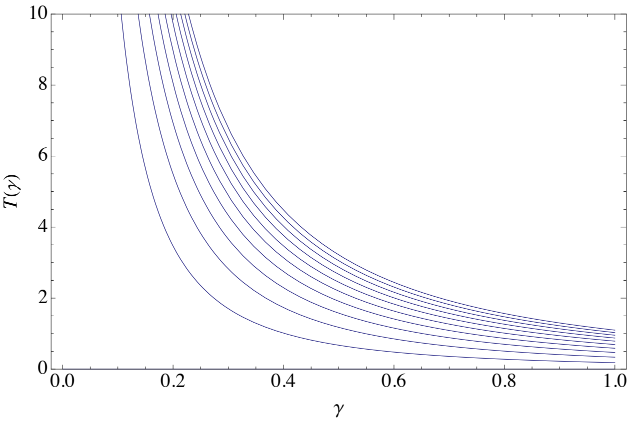
whereas when $\gamma = 0$ the analogous expressions are

$$a(t) = \frac{-1}{1 - kt}, \qquad T = \frac{1}{k}.$$

For the $\gamma > 0$ case we may write, for γ small,

$$a(t) = \frac{-1}{1 - kt - \gamma t + O(\gamma^2)},$$





and so we see that the γ term is causing the blow-up to be more rapid. For example, when $k = 1$, $T(\gamma = 0) = 1$, $T(\gamma = 0.1) = 0.953$, $T(\gamma = 1) = 0.693$, and $T(\gamma = 2) = 0.549$.

We provide two graphical representations of the model. In Fig.1 we show the function $a(t)$ as defined by eq. (24) with $k = 1$, $\gamma = 1$ and for several values of $a(0)$, from -1.0 to 2.0, in steps of 0.1. We can see that for $a(0) < 0$ there is the aforesaid blow-up, while for $a(0) > 0$ the function tends to the constant γ/k.

The second graphics, Fig.2, shows the behaviour of the blow-up time as a function of γ for $a(0) = -1$ and for different values of k from 0.1 to 2.0 in steps of 0.2. It can be seen that the blow-up decreases with γ, while increases with k.

5. Conclusions

We have presented a theory for the evolutionary behaviour of a thermoelastic body which contains pores. The theory involves three variables, namely, the displacement x_i, the temperature θ, and the pressure in the pores p. This theory originates from the original development of Biot [24]. We are not aware of another fully nonlinear thermoelastic development which contains pores in the spirit of this work. There is a continuum thermodynamic development of the theory of an elastic body containing a distribution of voids. The structure of the equations there are very different from those here and a detailed comparison in the linear case is given in chapter 4 of Straughan [30].

A fully nonlinear acceleration wave analysis is performed for the theory given here. The wavespeed is calculated for an acceleration wave in the three-dimensional case and this has the same form as that of classical nonlinear thermoelasticity. The wave amplitudes are determined for a one-dimensional acceleration wave moving into an equilibrium region. In this case $[\ddot{u}] \equiv \ddot{u}^-$ where the minus indicates the value at the left of the wave and if $\ddot{u}^-(\mathbf{X}, 0) < 0$ it is found that the amplitude may blow-up in a finite time T. Since $\ddot{u}^- = U_N^2 u_{XX}^-$ this means $u_{XX}^- \to -\infty$ in a finite time which is suggestive of the formation of a shock wave at time T.

In the purely isothermal elastic case $T = -1/ka(0)$ whereas T involves γ when a porous elastic body is employed. The precise effect of temperature and the porosity is given by equation (25) which involves derivatives of the variables m, η, J and q.

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