Information control in reputational cheap talk

Nejat Anbarci[a], Saptarshi P. Ghosh[1], Jaideep Roy[c,*]

[a]*Department of Economics, Deakin University, 70 Elgar Road, Burwood, Victoria, Australia.*
[b]*Department of Economics, Shiv Nader University, Noida, India.*
[c]*Department of Economics, University of Bath, Bath BA2 7JP, England.*

Abstract

An evaluator is engaged in estimating as precisely as possible the innate talent of a careerist expert by observing the expert's performance in a prediction task, and has the ability to interfere with the expert's private signal by reducing or enhancing its precision. The expert on the other hand knows her talent, observes this interference and can misrepresent private beliefs through strategic predictions to enhance her reputation. We show that when priors are significantly uninformative so that the task is a priori hard, the evaluator reduces the precision of the expert's signal, while when priors are significantly informative, he enhances it. We also find that the evaluator's objectives of maximising the precision of information about talent and maximising the probability of 'truthful expert advice' in the given task are aligned in and only in a priori hard tasks. We discuss implications of these results for market research decisions by a monopolist facing uncertain demand.

Keywords: Reputational cheap talk, variance-minimising evaluation, task difficulty, information control.
JEL: D82, D83.

*Corresponding author

Email addresses: nejat.anbarci@deakin.edu.au (Nejat Anbarci), saptarshi.ghosh@snu.edu.in (Saptarshi P. Ghosh), jaideeproy1234@gmail.com (Jaideep Roy)

1. Introduction

Evaluating talent in order to allocate the 'right person to the right job' is an important and difficult challenge confronted by organisations around the globe.[1] An evaluator of talent is therefore employed to determine as precisely as possible the hidden talent of a careerist expert by observing the expert's performance in a given task. The task can be described by a standard Reputational Cheap Talk (RCT) environment (as studied in Levy 2004 and described in Section 2) where such an expert makes a prediction. There are two possible states of the world, namely the conventional state and the contrarian state (where by definition the common prior probability of the conventional state is higher). For example, the possible binary states considered may be: 'the stock market will go up vs. go down in the next month' or 'the dollar will get stronger vs. weaker against the euro in the next quarter'. The expert knows her talent level privately and makes a prediction about the uncertain state based on common priors and her private signal. The 'un-interfered' precision of her private signal reflects this talent. The evaluator does not know the expert's talent nor does he observe her private signal. He evaluates the expert's talent by using the expert's prediction and the true state of the world that is publicly revealed after the prediction has been made. We use the variance of the evaluator's ex-post beliefs about the expert's talent as a measure of precision.

In this setting the evaluator has the ability to interfere with the expert's information acquisition process and thus the problem faced by the evaluator is to use informational control to minimize the variance of his ex-post beliefs about the expert's talent. There are numerous means by which the evaluator can gain control over the expert's informational quality (see page 722, paragraph 2 in Ivanov (2010) for various applications of information control by the Receiver in a standard cheap-talk setting). For example, the evaluator can strategically obstruct or release evidence, set stringent or easy deadlines, invest less or more in analytical softwares or produce less or more misleading data or disconcerting evidence. In Section 3.2 we provide a concrete application of our framework and results in the case of a monopolist facing uncertain market demand.

The preassigned task is called *a priori hard* if prior information is largely unbiased (that is both the states are almost equally likely), while it is *a priori easy* when this bias towards the 'conventional' state is strong. If a job is a priori easy, then almost everyone is likely to come up with the correct prediction, so the evaluator may think of making the job harder by jamming the signal strength of the expert so that only the high skilled experts can execute it correctly. Alternatively, if the job is a priori hard, almost all the experts may possibly come up with the incorrect prediction, and hence one may be tempted to make the job easier through signal boosting so that only the low skilled experts err. Thus by observing whether a prediction is correct or wrong, the evaluator can make more precise estimation of talent through more informative separation of types. We prove in Section 3 the exact opposite, that is, in order to get more precise information about talent, the evaluator should make 'hard tasks harder and easy tasks easier.'

Though ex-post "correctness" or quality of advice is an important determinant of skill evaluation, it is not the sole criterion to do so. We show that "content," in terms of whether the expert is correct in predicting the conventional or contrarian state plays a nuanced and efficacious role. For a hard task where the prior does not favour a particular alternative very highly, contrarian private information encourages almost all the experts to predict the contrarian alternative as the true state. This they do in hope that if they are proved to be right, their impression in the eyes of the evaluator will be further accentuated compared to the case when they make a conventional prediction. Hence, for a hard task there is excessive contrarianism on the expert's end. To curb this excessive contrarianism, precision of private information must be reduced so that only the extremely smart experts are encouraged to side with their private contrarian signal and make a prediction against the conventional view given by the prior. On the other hand, for an easy task, most experts side with the strong prior and predict the conventional alternative since their primary objective is to be proved right. In this case, making the task easier through signal-boosting curbs this excessive conventionalism, and more experts side with their privately received contrarian signals to make a contrarian prediction. It is important to note that the main result extends to a more general setting where the expert also cares about the correctness of her prediction.

Careerist motivations make experts strategically misrepresent their talent through mimicry of prediction-strategies

[1]There is a large literature on the importance of this match between employees' knowledge, abilities and characteristics of the job they are assigned to (see for example, Edwards (1991), Kristof (1996) and Kristof-Brown et al., (2005)).

used sincerely only by experts with very high talents. If expert advice leads to payoff-relevant actions taken by the evaluator or his employer, minimising the extent of this misrepresentation becomes important as well, particularly if the task at hand is outcome-relevant for the evaluator. We show that the objective of maximising the precision talent evaluation and that of minimising the probability of insincere (i.e., strategic) predictions by careerist experts are perfectly aligned when the task is a priori hard, while they are in conflict if the task is a priori easy. This is good news for organisational design as the real need for experts is felt more acutely for hard tasks when the prior reveals scant information.

1.1. Related literature

We have addressed the question of optimal information control in a reputational cheap talk setting, and to our knowledge this is the first paper to do so. The issue of optimal information control has however been studied in the Crawford and Sobel (1982) (CS) cheap talk setting by Fischer and Stocken (2001) and Ivanov (2010). A key point of difference between the CS framework and the RCT literature (studied in Levy (2004), Levy (2007), Ottaviani and Sorensen (2006a), (2006b)) is that unlike in CS, the receiver in the RCT framework does not necessarily take a payoff-relevant action after receiving a message from the sender while the sender may not care about the action taken, if any, by the receiver; all she cares is about the receiver's beliefs about how informed the sender is. We maintain this point of difference in our RCT model. Even though at the beginning the evaluator in our model chooses the degree of precision of the expert's private signal, the pay-off of the expert is not contingent on any strategic action taken by the evaluator once the expert submits her prediction. Within the CS framework, Fischer and Stocken (2001) consider a discrete set of bias in the players' preferences and focus only on pure strategies, whereas Ivanov (2010) considers a more general CS setting with continuous bias. While in the CS framework it is typically assumed that the sender has full information about the state, both papers show that it can be in the receiver's interest to reduce the sender's information since a less informed sender has weaker incentive compatibility constraints so that for the same preference bias, his messages may carry more credibility. Similarly, Austen-Smith (1994) shows that introducing positive costs of acquiring information in the CS model can expand the range of bias for which informative communication is possible. The issue of information control has also been studied in a variety of other settings. Bergemann and Pesendorfer (2007) analyse an auction framework in which the seller decides on the precision of the valuations for each bidder (without learning their private signals) and chooses the price and bidder to sell to. Lewis and Sappington (1994) study a monopoly market in which a seller can allow buyers to acquire private information about the product. They demonstrate that the optimal policy of the seller is either to let buyers acquire perfect information or have no information at all.

2. The Model

Actors, states, signals and talent: The model comprises two actors, an evaluator and a careerist expert.[2] The evaluator is employed to form a probabilistic belief about the expert's talent as precisely as he can, when the expert faces a preassigned binary-state prediction task with states W_1 and W_2, where $p \in (\frac{1}{2}, 1)$ is the common prior that the true state is W_1, the *conventional state*. The true state is unknown to both actors, and the task will be called *a priori harder* when p is lower as a lower p implies higher entropy of prior information. The expert receives a private signal about the states that can take two values, w_1 and w_2. In the absence of any interference from the evaluator, the precision of these signals is her *talent* t such that the probability of receiving the signal w_i in state W_i is $\mathbb{P}[w_i|W_i;t] = t$, with $t \in [\frac{1}{2}, 1]$. Talent t is *private information* to the expert while the evaluator only knows that t is drawn *uniformly* from the interval $[\frac{1}{2}, 1]$.[3]

Information control and overall precision: Although the evaluator does not know t, he is aware that the *act of signal boosting* is to move the overall precision of the expert's private signal to the right of t to a point in the interval $(t, 1]$, while the *act of signal jamming* is to move the overall precision to the left of t to a point in the interval $[\frac{1}{2}, t)$ for

[2]We follow the baseline model in Levy (2004).

[3]The assumption that t is drawn uniformly is without loss of qualitative generality.

each value of t. Thus irrespective of whether t is known to the evaluator or not, an act of information control is well-defined. Let $Q \in [0,1]$ be the *degree of interference* that determines the choice of this point in the respective intervals. The *overall precision* $\pi(t,Q)$ of these signals $w_i, i = 1,2$ therefore depends on the expert's privately-known talent t and the publicly-observed degree of interference $Q \in [0,1]$ controlled by the evaluator. It follows that with a signal-boosting scheme, we have

$$\pi(t,Q) := \mathbb{P}[w_i|W_i;t,Q] = t + Q(1-t), \tag{1}$$

while with a signal-jamming scheme, we have

$$\pi(t,Q) := \mathbb{P}[w_i|W_i;t,Q] = t - Q(t-1/2). \tag{2}$$

It is important to note that since the overall signal precision π depends on t, for any fixed $Q \neq 1$, the evaluator does not observe π. In contrast, since t, the scheme, and Q are all observed by the expert, she is informed about π. Signal boosting therefore makes the task *interim-easier* while jamming makes it *interim harder*.[4]

Timing of actions and payoffs: Upon receiving her private signal, and after observing the scheme and the associated Q set by the evaluator, the expert makes a prediction about the true state, denoted by P_i to imply that 'the true state is W_i', $i = 1,2$. Not knowing the private signal of the expert, the evaluator chooses the type and degree of information control with the sole target of evaluating the expert's talent t *as accurately as possible* once her prediction and the true state are revealed to him. We use variance of the evaluator's beliefs about t to measure this accuracy and note that our results are qualitatively robust to any other measure of accuracy that is monotonic in variance.

Let $b(t|W_i,P_j)$ be the evaluator's *Bayesian belief* about t based upon the information control chosen by himself, the expert's prediction $P_j \in \{P_1,P_2\}$ and the realisation of the true state $W_i \in \{W_1,W_2\}$.[5] Let $\mathbb{E}(t|W_i,P_j)$ be the consequent mathematical expectation of talent t with the Bayesian belief $b(t|W_i,P_j)$ at the event (W_i,P_j). Let $\mathbb{P}[W_i,P_j|Q,p]$ be the ex-ante probability that the terminal node will be (W_i,P_j) for a given task p and interference level Q. The evaluator's optimisation problem is then to choose a scheme and an interference level $Q \in [0,1]$ in order to *minimise the expected variance* of his eventual belief $b(\cdot)$ given by

$$\mathbb{E}(\mathbb{V}(Q,p)) = \sum_{(W_i,P_j),i,j\in\{1,2\}} \mathbb{P}[W_i,P_j|Q,p] \int_{\frac{1}{2}}^{1} (t-\mathbb{E}(t|W_i,P_j))^2 b(t|W_i,P_i)\mathrm{d}t$$

The careerist expert, on the other hand, makes the prediction in order to maximise the expected talent held by the evaluator based upon $b(t|\cdot,\cdot)$.

3. Main result

The following theorem is our main result. It shows that in order to minimise the expected variance of his ex-post beliefs about expert talent, the evaluator should make an a priori hard task interim-harder and an a priori easy task interim-easier.

Theorem 1. *The optimal information-control strategy for evaluating hidden talent of a careerist expert requires the following interference actions by the evaluator: boost the precision of the expert's private signal if the prediction task is a priori easy (viz. p high) and jam it if the task is a priori hard (viz. p low).*

The intuition of the result follows from two elemental forces at play and it is important to note that they are independent of the exact functional forms of the interference schemes or the assumption of uniform priors used in proving the theorem. First, we note that since a correct prediction is always rewarding in terms of reputation,

[4] An interesting feature of the information control technology is that the marginal impact of precision-boosting is higher for experts with lower talents while for precision-jamming it is higher for experts with higher talents.

[5] For notational ease, we suppress indicating the scheme for which these beliefs are obtained unless the distinction is necessary.

the expert never goes against the conventional state unless her talent exceeds a threshold level and she receives a contrarian private signal. By boosting the overall precision of the expert's private signal, the evaluator can push this threshold down so that experts with lesser talent too start taking actions that are in line with their private signals. On the other hand, by jamming the precision of the expert's private signal, the evaluator can only push this threshold up so that experts with higher talent too turn conservative and go with the conventional state. Ceteris paribus, the evaluator would want to use his informational instruments of signal jamming and signal boosting to keep this threshold 'close to the midpoint' of the interval from where the expert's talent is drawn so that beliefs are balanced in informativeness across different states and predictions pairs. At the same time, the evaluator benefits the most from a contrarian prediction of the expert, as otherwise no talent types can be fully excluded from the admissible set. However, the likelihood of the expert receiving the contrarian signal – the most important precursor for her to predict against the conventional state – itself depends on the prior and the overall precision of the expert's private signal. In particular, a rise in the overall signal precision unambiguously lowers the likelihood of the contrarian signal at a rate that is increasing in the prior bias towards the conventional state. Hence, unless the common prior of the conventional state is rather high, this negative impact of signal boosting on the probability of the expert obtaining a contrarian signal cannot be outweighed by the positive impact of bringing the threshold down. A somewhat reversal of the above reasoning applies to signal jamming. When the prior bias towards the conventional state is low, signal jamming pushes the threshold talent up and towards the midpoint, and at the same time increases the probability of the expert receiving a contrarian signal. Hence the evaluator finds it beneficial to jam.

Remark 1 (Outcome-driven experts)**.** *As clear from the proof of Theorem 1, the conclusion of the theorem holds irrespective of whether the expert is outcome driven (viz. one who wants to make correct predictions) or a careerist. The main reason is that even with a pure outcome driven expert, the equilibrium strategy continues to follow a similar cut-off rule as with careerists: predict the conventional state under a conventional signal and continue to do so under a contrarian signal provided t is not too high in which case predict the contrarian state. Of course, when one injects careerist motivations, the cutoff shifts to the left as more experts are now willing to propose the less likelier outcome to generate a more favourable talent evaluation in case they end up being correct. This follows directly from the fact that outcome driven experts predict according to true posteriors. Thus, increasing the precision of private signal will shift the cutoff to the left while reducing the precision will shift it to the right. Hence, the basic machinery behind Theorem 1 remains robust to outcome motivations of the expert.*

3.1. Information about talent versus state

Suppose alternatively that the sole objective of the evaluator is to reduce the probability of misreporting by the careerist expert. Can information control be used to this end? It turns out that for any a priori task difficulty p, the only way to do this is to adhere to signal-jamming. The intuition is the following: the reputation-driven expert with mediocre talent sometimes predicts in favour of the unconventional state by going against her own posterior probability. This she does in the hope that in case the 'less likelier' prediction turns out to be true, the evaluator will form a more favourable expectation of her talent. However, it is crucial to note that the expert always prefers a correct prediction over an incorrect one, irrespective of whether the prediction is conventional or contrarian. Jamming reduces the reliability of her private signal, and this increases the gain from siding with the conventional alternative that the prior is biased towards by definition. This advantage of inducing conventional behaviour (which is an off-shoot of signal-jamming) partially mitigates the inherent problem of excessive non-conformism embedded in the model with careerist experts. In other words, the expert is encouraged to shed her excessive non-conformism through signal-jamming, and made to predict more in line with her privately formed posterior probability. Thus, the strategy of reducing the signal precision works universally well in this regard. However, we have seen that for tasks that are a priori easy (p high enough), boosting reduces the variance of the evaluator's information about talent. Put together, we arrive at the following interesting corollary, which, of course, holds only for careerist experts as with purely outcome driven ones, there is no scope for manipulative predictions.

Corollary 1. *With careerist experts, the two objectives of* Minimising Noise in Talent-estimation *and* Maximising Truthful Predictions *go hand-in-hand when the prediction task is a priori hard (i.e., p is low enough) but they are in conflict when the task is easy (i.e., p is high enough).*

Theorem 1 and Corollary 1 have interesting implications for market research in the following economic application of a monopolist facing uncertain demand.

3.2. Monopolist with uncertain demand: an application

A monopolist faces uncertain demand that can be high or low and appoints a manager to make a prediction. However, the monopolist is unaware of the manager's innate skill in understanding the market and has resources that he can let the manager use for additional market research. Suppose the monopolist has long term objectives of assigning the manager to future tasks requiring appropriate match between skill and task difficulty, so that all he cares about is to know the talent of the manager as precisely as possible. Theorem 1 (along with Remark 1) shows that irrespective of whether the manager is career driven or outcome driven, the monopolist should spend on additional market research if prior uncertainty about demand is small but reduce spending if prior uncertainty is large. On the other hand, if the monopolist cares about short term profits so that he values information about the true demand contained in the manager's prediction in the present task, then Corollary 1 shows the following for the case when the manager is purely career-driven. If prior uncertainty is large, not allowing the manager to spend on market research increases both the monopolist's information about the manager's talent as well as the informational content in the manager's recommendation regarding the true demand. However, if prior uncertainty is small, the monopolist must give up information in one dimension to gain more in the other.

4. Conclusion

The paper studies an RCT model where the goal of the evaluator is to obtain variance-minimising information about a careerist expert's predictive talent. The evaluator can interfere with the expert's signal through jamming or boosting its precision. We established a simple but powerful point in such an exercise: when the prior of a preassigned prediction task is relatively unbiased (making the prediction task a priori hard), the evaluator will jam the signal of the expert while as the prior bias becomes stronger (making the task easier) the evaluator will boost the precision. We have also shown that the objective of maximising informational precision about expert talent and that of decreasing the probability of insincere prediction from careerist experts through information control are aligned for a priori hard tasks, while for tasks that are a priori easy, they are in conflict. We also show that this conclusion holds irrespective of whether the expert is outcome driven or a careerist. But what if the expert has a choice between accepting or refusing interference schemes whenever possible? Also, could the evaluator do even better by controlling not only private information but also the a priori difficulty of the task? We leave these questions for future research.

Acknowledgements

We thank the Associate Editor and two anonymous referees for guiding us into transforming the paper into a much improved version. We are grateful to Sumon Bhowmik, Sushil Bikhchandani, Sandro Brusco (particularly for suggesting the example in Section 3.2), Hulya Eraslan, Navin Kartik, Abhimanyu Khan, Semih Koray, Bettina Klaus and Gilat Levy for various comments and suggestions. We also thank seminar participants at the Indian Statistical Institute Calcutta, University of Bath, IGIDR-Mumbai and the ASSET meetings in November 2015 for various comments.

Appendix

Proof of Theorem 1 and Corollary 1

The strategy of the proof is as follows. We will assume first that the expert only cares about making a correct prediction. We will then extend the proof to experts with reputational concerns. So suppose for now that the expert *only* cares about making a correct prediction. Given p, t and w_i, the expert's posterior belief about the true state being $W_i, i = 1,2$, is a Bayesian update of p and is given by $\mathbb{P}[W_1|w_i] = \frac{p\pi}{p\pi+(1-p)(1-\pi)}$ if $i = 1$ and $\mathbb{P}[W_1|w_i] = \frac{p(1-\pi)}{p(1-\pi)+(1-p)\pi}$ if $i = 2$, and $\mathbb{P}[W_2|w_i] = 1 - \mathbb{P}[W_1|w_i]$. Thus, a sincere expert's optimal prediction strategy $P^*(t, w_i)$ is as follows: $P^* = P_1$ if $w_i = w_1$ or $w_i = w_2$ and $\pi \leq p$, while $P^* = P_2$ if $w_i = w_2$ and $\pi > p$. The equilibrium prediction strategy employed by the sincere expert is unique and is characterised by a *cutoff talent* τ that solves $\pi(\tau, Q) = p$: for all $t > \tau$, the expert predicts P_2 whenever she receives the signal w_2 and in all other circumstances all expert types predict P_1.

With signal boosting, $\pi = t + Q(1-t))$, and the cutoff talent $\tau \equiv \tau_B$ is $1/2$ if $Q \geq 2p-1$ and $\frac{p-Q}{1-Q}$ otherwise. Thus, starting from $Q = 0$ when $\tau_B = p$, a rise in Q decreases τ_B, reaching $\tau_B = \frac{1}{2}$ when $Q = 2p-1$. Fix some $Q \in [0,1]$ that determines the cutoff τ_B. For each $\mathcal{E} \in \{(W_1,P_1),(W_1,P_2),(W_2,P_1),(W_2,P_2)\}$, let $\mathbb{P}[\mathcal{E}|Q,p]$ be the ex-ante probability of the event $\mathcal{E}$ when the evaluator chooses an amount Q in the signal-boosting scheme. Then the (ex-ante) expected variance of the evaluator's ex-post beliefs about t is given by

$$\mathbb{E}[\mathbb{V}(Q,p)] = \sum_{\mathcal{E}} \mathbb{P}[\mathcal{E}|Q,p]\, \mathbb{V}(\mathcal{E};Q,p) = \sum_{\mathcal{E}=(W_i,P_j)} \mathbb{P}[W_i,P_j|Q,p] \left(\int_{\frac{1}{2}}^{1} (t - \mathbb{E}(W_i,P_j))^2 b(t|W_i,P_j)\mathrm{d}t \right).$$

Thus, the evaluator's problem amounts to choosing $Q \in [0,1]$ in order to minimise $\mathbb{E}[\mathbb{V}(Q,p)]$. Denote this choice by Q_B.

Observation 1. *There exists a unique $p_B \approx 0.73$ such that $Q_B = 0$ if $p \leq p_B$ and $Q_B > 0$ if $p > p_B$.*

To prove Observation 1, consider the terminal events $\mathcal{E}$ faced by the evaluator. Note that under the signal-boosting scheme, τ_B is always < 1. On the other hand, if, and only if, $Q > 2p-1$ can the mathematical expression for τ_B fall below $\frac{1}{2}$ in which case we consider the value of τ_B to be constant and equal to $\frac{1}{2}$. In what follows we assume that $Q \leq 2p-1$ and then prove that in the optimum it cannot be that $Q > 2p-1$.

For any event (W_i,P_j), $i = 1,2; j = 1,2$, observe that $b(t|W_i,P_j)$ is a conditional p.d.f. Denote $f(X)$ as the (joint) p.d.f of event X. Then

$$b(t|W_i,P_j) = \frac{f(t)f(W_i)f(P_j|W_i)}{f(W_i,P_j)} = \frac{f(t)f(W_i)\left[f(w_1|W_i)f(P_j|w_1,W_i) + f(w_2|W_i).f(P_j|w_2,W_i)\right]}{f(W_i,P_j)}.$$

Using the abbreviated notation $\mathbb{P}[W_i,P_j]$ to denote $\mathbb{P}[W_i,P_j|Q,p]$, we have the ex-ante probability of the event (W_i,P_j) to be

$$\mathbb{P}[W_i,P_j] = \Pr(W_i)\Pr(P_j|W_i) = \Pr(W_i)\left[\Pr(w_1|W_i)\Pr(P_j|w_1,W_i) + \Pr(w_2|W_i)\Pr(P_j|w_2,W_i)\right].$$

Consider the terminal event (W_1,P_2) and note that when the prediction is P_2, it must be that $t > \tau_B$. Hence, the top part of the belief $b(t|W_1,P_2)$ is immediate. So suppose $t > \tau_B$. Then

$$b(t|W_1,P_2;t>\tau_B) = \frac{\left(\frac{1-Q}{1-p}\right)p(1-t)(1-Q)\int_{\tau_B}^{1}\left(\frac{1-Q}{1-p}\right)\mathrm{d}t}{\int_{\tau_B}^{1}\left(\left(\frac{1-Q}{1-p}\right)p(1-t)(1-Q)\int_{\tau_B}^{1}\left(\frac{1-Q}{1-p}\right)\mathrm{d}t\right)\mathrm{d}t} = \frac{2(1-t)(1-Q)^2}{(1-p)^2}.$$

Clearly, $b(t|W_1,P_2;t>\tau_B)$ decreases in t in the zone $[\tau_B,1]$ as expected. Using the notation $\mathbb{V}(W_i,P_j)$ to denote the variance of b in the event (W_i, P_j), $i = 1,2$; $j = 1,2$, given (Q,p), we have

$$\mathbb{V}(W_1, P_2) = \frac{(1-p)^2}{18\,(1-Q)^2}.$$

The evaluator, foreseeing his possible future information, computes the ex-ante probability of this event (W_1, P_2) given by

$$\mathbb{P}\left[W_1, P_2\right] = p\left(\int_{\frac{1}{2}}^{1} \pi f(t)\mathrm{d}t.0 + \int_{\frac{1}{2}}^{1} (1-\pi) f(t)\mathrm{d}t \int_{\tau_B}^{1} f(t)\mathrm{d}t\right) = p\left(\int_{\frac{1}{2}}^{1} 2\,(1-t)\,(1-Q)\,\mathrm{d}t\left(\int_{\tau_B}^{1} 2\mathrm{d}t\right)\right) = \frac{p\,(1-p)}{2}.$$

By similar steps and calculations (that we omit reporting in details), one can establish the following:

- For terminal event (W_2, P_2),

$$b(t|W_2, P_2) = \begin{cases} 0 & \text{if } t \leq \tau_B \\ \frac{2(1-Q)(Q+t(1-Q))}{(1+p)(1-p)} & \text{if } t > \tau_B \end{cases},$$

$$\mathbb{V}(W_2, P_2) = \frac{(1-p)^2\left(p^2+4p+1\right)}{18\,(1-Q)^2\,(1+p)^2},$$

  and

$$\mathbb{P}\left[W_2, P_2\right] = \frac{(Q+3)\,(1-p)^2}{2\,(1-Q)}.$$

- For terminal event (W_i, P_1),

$$b(t|W_i, P_1) = \begin{cases} \frac{8\left(p+t(1-p)-\frac{1}{2}\right)}{1+p} & \text{if } W_i = W_1 \\ \frac{4-Q(12-8p)-8t(1-p)(1-Q)}{(3p-1)-Q(3-p)} & \text{if } W_i = W_2 \end{cases},$$

$$\mathbb{V}(W_i, P_1) = \begin{cases} \frac{p^2+4p+1}{72(p+1)^2} & \text{if } W_i = W_1 \\ \frac{Q^2p^2-8Q^2p+13Q^2+10Qp^2-32Qp+10Q+13p^2-8p+1}{72(3Q-3p-Qp+1)^2} & \text{if } W_i = W_2 \end{cases},$$

  and

$$\mathbb{P}\left[W_i, P_1\right] = \begin{cases} \frac{p(p+1)}{2} & \text{if } W_i = W_1 \\ \frac{(1-p)(p(3+Q)-(1+3Q))}{2(1-Q)} & \text{if } W_i = W_2. \end{cases}$$

We now insert these expressions for the probabilities and the corresponding variances to obtain the final expression for the ex-ante variance $\mathbb{E}[\mathbb{V}(Q,p)]$. Some algebraic simplifications yield

$$\frac{\partial \mathbb{E}[\mathbb{V}(Q,p)]}{\partial Q}|_{Q=0} = \frac{(1-p)^2(36p^6+57p^5-318p^4+273p^3-48p^2-16p+4)}{18(1+p)^2(3p-1)^2}.$$

It is routine to check that there exists $p_B \approx 0.7245$ such that for all $p < p_B$, the following holds: $\frac{\partial \mathbb{E}[\mathbb{V}(Q,p)]}{\partial Q}|_{Q=0} > 0$, $\frac{\partial \mathbb{E}[\mathbb{V}(Q,p)]}{\partial Q}|_{Q=2p-1} = \frac{p(p^2+3p+3)}{72(1+p)^2} > 0$, and $\frac{\partial \mathbb{E}[\mathbb{V}(Q,p)]}{\partial Q} = 0$ has no solution. From these it follows that $Q_B = 0$ when $p < p_B$. Also note that for all $p > p_B$, $\frac{\partial \mathbb{E}[\mathbb{V}(Q,p)]}{\partial Q}|_{Q=0} < 0$. From this it follows that when $p > p_B$ then $0 < Q_B \leq 2p-1$. The proof of the observation is completed by noting that Q_B cannot be higher than $2p-1$. Suppose on the contrary, $Q > 2p-1$. Then $\tau_B = \frac{1}{2}$ and the corresponding ex-post beliefs and variances of the evaluator along with the probabilities of the events $\mathcal{E}$ are as follows:

$$b(t|W_1,P_1)=\frac{8Q+8t-8Qt}{Q+3};\mathbb{V}(W_1,P_1)=\frac{Q^2+10Q+13}{72\,(Q+3)^2};\mathbb{P}\,[W_1,P_1]=\frac{p(Q+3)}{4}$$

$$b(t|W_1,P_2)=8-8t;\mathbb{V}(W_1,P_2)=\frac{1}{72};\mathbb{P}\,[W_1,P_2]=\frac{p(1-Q)}{4};$$

$$b(t|W_2,P_1)=8-8t;\mathbb{V}(W_2,P_1)=\frac{1}{72};\mathbb{P}\,[W_2,P_1]=\frac{(1-p)(1-Q)}{4};$$

and

$$b(t|W_2,P_2)=\frac{8Q+8t-8Qt}{Q+3};\mathbb{V}(W_2,P_2)=\frac{Q^2+10Q+13}{72\,(Q+3)^2};\mathbb{P}\,[W_1,P_1]=\frac{(1-p)(Q+3)}{4}.$$

Inserting these expressions in $\mathbb{E}[\mathbb{V}(Q,p|Q>2p-1)]$ we see that

$$\frac{\partial\mathbb{E}[\mathbb{V}(Q,p|Q>2p-1)]}{\partial Q}=\frac{1}{36(3+Q)^2}>0.$$

Therefore, Q must be set at the minimum possible level in the domain $Q\geq 2p-1$ in order to minimise expected variance. From this and the observation that $\frac{\partial\mathbb{E}[\mathbb{V}(Q,p)]}{\partial Q}|_{Q=2p-1}>0$, it follows that at the optimum, the condition $Q_B<2p-1$ must hold. This completes the proof of Observation 1.

Consider next the signal-jamming scheme $\pi=t-Q(t-\frac{1}{2})$. The cutoff talent $\tau\equiv\tau_J$ is such that $\tau_J=1$ if $Q\geq 2(1-p)$ and $\tau_J=\frac{2p-Q}{2(1-Q)}$ otherwise. Thus, starting from $Q=0$ when $\tau_J=p$, a rise in Q increases τ_J reaching $\tau_J=1$ when $Q=2-2p$. Denote by Q_J the ex-ante variance-minimising choice of Q under the signal-jamming scheme.

Observation 2. *There exists a unique prior $p_J\approx 0.62$ such that $Q_J=0$ if $p>p_J$ and $Q_J>0$ when $p<p_J$.*

The strategy of the proof of Observation 2 is similar to that of Observation 1 and can be found in the Online Appendix.

Given Observations 1 and 2, we are now in a position to incorporate reputational cheap talk. We note that if in addition to the desire of predicting correctly, the expert had an incentive to enhance her reputation in the eyes of the evaluator, her payoff would increase in the expectation held by the evaluator about t based on the evaluator's ex-post information and beliefs. Suppose the careerist's prediction strategy is as follows: $P^*=P_1$ if $w_i=w_1$ or $w_i=w_2$ and $\pi\leq\tau^c$, while $P^*=P_2$ if $w_i=w_2$ and $\pi>\tau^c$, where the cutoff talent level τ^c is solved from the following indifference equation of the careerist expert:

$$\mathbb{P}[W_1|w_2]\mathbb{E}(W_1,P_1,\tau^c)+(1-\mathbb{P}[W_1|w_2])\mathbb{E}(W_2,P_1,\tau^c)=\mathbb{P}[W_1|w_2]\mathbb{E}(W_1,P_2,\tau^c)+(1-\mathbb{P}[W_1|w_2])\mathbb{E}(W_2,P_2,\tau^c),\quad(3)$$

where $\mathbb{E}(W_i,P_j,\tau^c)=\mathbb{E}[t|W_i,P_j,Q,p,\tau^c]$. From Proposition 1 in Levy (2004), the following lemma is immediate.

Lemma 1. *For each p, Q and M, $\tau^c<\tau$ where $\pi(\tau,Q)=p$.*

Lemma 2. *For signal-boosting, $\partial\tau^c/\partial Q<0$, while for signal-jamming, $\partial\tau^c/\partial Q>0$.*

To prove Lemma 2, note that Lemma 2 of Levy (2004) implies that

$$\mathbb{E}[W_2,P_2,\tau^c]>\mathbb{E}[W_1,P_1,\tau^c]>\mathbb{E}[W_1,P_2,\tau^c]>\mathbb{E}[W_2,P_1,\tau^c].$$

Now suppose for the signal-boosting case that on the contrary, $\partial\tau^c/\partial Q>0$ holds. In particular, suppose the cutoff rises from τ^c to τ'^c. Now, for all $t\in(\tau^c,\tau'^c)$, LHS of (3) is greater than RHS. But this is impossible since

$\partial\pi(t, Q)/\partial Q > 0$. The proof is analogous for the signal jamming case, and completes the proof of the above lemma. The proof of the Theorem is completed by noting that

$$\partial\tau_B/\partial Q = -\frac{1-p}{(1-Q)^2} < 0 \text{ and } \partial\tau_J/\partial Q = \frac{2p-1}{2(1-Q)^2} > 0,$$

so that the direction of impact of Q on τ and τ^c are identical.

Corollary 1 now follows automatically by noting (i) when $Q = 0$, then $\tau^c < p$ as shown in Proposition 1 in Levy (2004), and (ii) Lemma (2) that is proved above.

□

References

[1] Austen-Smith, D. (1994), "Strategic transmission of costly information", *Econometrica*, 62, 955-963.

[2] Bergemann, D. and Pesendorfer, M. (2007), "Information Structures in Optimal Auctions", *Journal of Economic Theory*, 137, 580-609.

[3] Crawford, V. and Sobel, J. (1982), "Strategic information transmission", *Econometrica*, 50, 1431-1452.

[4] Edwards, J.R. (1991), "Person-job fit: a conceptual integration, literature review, and methodological critique", in Cooper, C. (Ed.), International Review of Industrial and Organizational Psychology, Vol. 6, Wiley, Chichester, 283-357.

[5] Fischer, P. and Stocken, P. (2001) "Imperfect information and credible communication", *Journal of Accounting Research*, 39, 119-134.

[6] Ivanov, M. (2010), "Informational control and organisational design", *Journal of Economic Theory*, 145, 721- 751.

[7] Kristof, A.L. (1996), "Person-organization fit: an integrative review of its conceptualizations, measurement, and implications", *Personnel Psychology*, 49(1), 1-49.

[8] Kristof-Brown, A.L., Zimmerman, R.D. and Johnson, E.C. (2005), "Consequences of individuals? fit at work: a meta-analysis of person-job, person-organization, person-group, and person-supervisor fit", *Personnel Psychology*, 58(2), 281-342.

[9] Levy, G. (2004), "Anti-herding and Strategic Consultation", *European Economic Review*, 48, 503-525.

[10] Levy, G. (2007), "Decision making in committees: transparency, reputation, and voting rules", *American Economic Review*, 97(1), 150-168.

[11] Lewis, T. and Sappington, D. (1994), "Supplying information to facilitate price discrimination", *International Economic Review*, 35, 309-327.

[12] Ottaviani, M., and Sorensen, P.N. (2006a), "Professional advice", *Journal of Economic Theory*, 126 (1), 120 - 142.

[13] Ottaviani, M., and Sorensen, P.N. (2006b), "Reputational cheap talk", *RAND Journal of Economics*, 37 (1), 155 - 175.

Online Appendix

Proof of Observation 2

Recall $\tau_J = \frac{2p-Q}{2(1-Q)}$. Note that since $p > \frac{1}{2}$, therefore τ_J is always greater than $\frac{1}{2}$. Moreover $\tau_J < 1$ if and only if $Q < 2(1-p)$, and assumes the fixed value of 1 when $Q \geq 2(1-p)$. We assume first that $Q < 2(1-p)$ and then show that in the optimum this is indeed the case. So suppose $Q < 2(1-p)$. Consider the probabilities of obtaining the signals $w_i, i = 1, 2$ given by

$$\mathbb{P}[w_2] = \mathbb{P}[w_2|W_1]\mathbb{P}[W_1] + \mathbb{P}[w_2|W_2]\mathbb{P}[W_2] = p - (t - Q(t - \frac{1}{2}))(2p-1), \tag{4}$$

and

$$\mathbb{P}[w_1] = (1-p) + (t - Q(t - \frac{1}{2}))(2p-1). \tag{5}$$

It is routine to verify that a rise in Q increases the probability of the contrarian signal and reduces that of the conventional one. The following lemmas report the ex-post beliefs, variances and probabilities of events. The proofs are ignored as they are analogous to the signal-boosting case.

Lemma 3. *Under a signal-jamming scheme, in the event (W_1, P_2), the evaluator's posterior beliefs for the expert's talent is given as follows:*

$$b(t|W_1, P_2) = \begin{cases} 0 & \text{if } t \leq \tau_J \\ \frac{4(1-Q)(2(1-t)+Q(2t-1))}{4p^2-8p-Q^2+4} & \text{if } t > \tau_J \end{cases}$$

where $b(t|W_1, P_2)$ is again decreasing in t when $t > \tau_J$. Moreover,

$$\mathbb{V}(W_1, P_2) = \frac{(Q+2p-2)^2 \left(Q^2 - 8Qp + 8Q + 4p^2 - 8p + 4\right)}{72\,(1-Q)^2\,(Q + 2\,(1-p))^2}.$$

and

$$\mathbb{P}(W_1, P_2) = \frac{p\,(1+Q)\,(2\,(1-p) - Q)}{4\,(1-Q)}.$$

Lemma 4. *Under a signal-jamming scheme, in the event (W_2, P_2), the evaluator's posterior beliefs for the expert's talent is given as follows:*

$$b(t|W_2, P_2) = \begin{cases} 0 & \text{if } t \leq \tau_J \\ \frac{4(1-Q)(2t-Q(2t-1))}{4+Q^2-4(p^2+Q)} & \text{if } t > \tau_J \end{cases}$$

where $b(t|W_2, P_2)$ is increasing in t when $t > \tau_J$. Moreover,

$$\mathbb{V}(W_2, P_2) = \frac{\left(Q^2 - 8Qp - 4Q + 4p^2 + 16p + 4\right)(Q+2p-2)^2}{72\,(1-Q)^2\,(2p - Q + 2)^2},$$

and

$$\mathbb{P}[W_2, P_2] = \frac{(3-Q)\,(1-p)\,(2\,(1-p) - Q)}{4\,(1-Q)}.$$

Lemma 5. *Under a signal-jamming scheme, in the event $(W_i, P_1), i = 1, 2$, the evaluator's posterior beliefs for the expert's talent is given as follows:*

$$b(t|W_i, P_1) = \begin{cases} \frac{16(p+t(1+pQ))+8Q(1+tQ)-4(2pQ+6tQ+4tp+2+Q^2)}{Q^2-Q+2(1+p(1+Q))} & \text{if } W_i = W_1 \\ \frac{8(2t+Q^2t+2Q+2pQt)-4(2+Q^2+2pQ+4tp+6tQ)}{Q^2+2(1+pQ)-(6p+Q)} & \text{if } W_i = W_2 \end{cases}$$

with $b(t|W_i, P_1)$ increasing (respectively decreasing) in t when $W_i = W_1$ (respectively $W_i = W_2$). Moreover,

$$\mathbb{V}(W_i, P_1) = \begin{cases} \frac{Q^4+4Q^3p-12Q^3+4Q^2p^2-16Q^2p+37Q^2+16Qp^2-28Qp-24Q+4p^2+16p+4}{72(2p-5Q+2Qp+Q^2+2)^2} & \text{if } W_i = W_1 \\ \frac{\left(Q^4+4Q^3p+4Q^2p^2-16Q^2p+Q^2-32Qp^2+20Qp+52p^2-32p+4\right)}{72(2Qp-6p-Q+Q^2+2)^2} & \text{if } W_i = W_2. \end{cases}$$

while the ex-ante probabilities are

$$\mathbb{P}\left[W_i, P_1\right] = \begin{cases} \frac{p\left(2p-5Q+2Qp+Q^2+2\right)}{4(1-Q)} & \text{if } W_i = W_1 \\ \frac{(1-p)\left(6p+Q-2Qp-Q^2-2\right)}{4(1-Q)} & \text{if } W_i = W_2. \end{cases}$$

We now insert these expressions for the probabilities and the corresponding variances to obtain the final expression for the ex-ante variance $\mathbb{E}[\mathbb{V}(Q,p)]$ given by

$$\mathbb{E}[\mathbb{V}(Q,p)] = \mathbb{P}\left[W_1, P_1\right]\mathbb{V}(W_1, P_1) + \mathbb{P}\left[W_1, P_2\right]\mathbb{V}(W_1, P_2) + \mathbb{P}\left[W_2, P_1\right]\mathbb{V}(W_2, P_1) + \mathbb{P}\left[W_2, P_2\right]\mathbb{V}(W_2, P_2).$$

First we note the following: there exists $p_J \approx 0.62$ such that for $p_J < p$, $\frac{\partial \mathbb{E}[\mathbb{V}(Q,p)]}{\partial Q} > 0$ for all Q. Hence for $p_J < p$, we have $Q_J = 0$. At $p = p_J$, one can check that $\mathbb{E}[\mathbb{V}(Q,p)]|_{Q=0} = \mathbb{E}[\mathbb{V}(Q,p)]|_{Q=\hat{Q}}$ where $0 < \hat{Q} < 2\,(1-p)$ such that $\frac{\partial \mathbb{E}[\mathbb{V}(Q,p)]}{\partial Q}|_{Q=\hat{Q}} = 0$, and $\frac{\partial^2 \mathbb{E}[\mathbb{V}(Q,p)]}{\partial Q^2}|_{Q=\hat{Q}} > 0$. There is only one other value of Q where $\frac{\partial \mathbb{E}[\mathbb{V}(Q,p)]}{\partial Q} = 0$, but for that value $\frac{\partial^2 \mathbb{E}[\mathbb{V}(Q,p)]}{\partial Q^2} < 0$. So to determine the optimal Q (denoted by $Q_J \in \{0, \hat{Q}\}$) for $p < p_J$, we need to show that $\mathbb{E}[\mathbb{V}(Q,p)]|_{Q=0} > \mathbb{E}[\mathbb{V}(Q,p)]|_{Q=\hat{Q}}$. One can show that $\frac{\partial \Delta}{\partial p} < 0$ where $\Delta = \mathbb{E}[\mathbb{V}(0,p)] - \mathbb{E}[\mathbb{V}(\hat{Q},p)]$. Hence $Q_J = \hat{Q}$ for $p < p_J$. The proof of the Observation is completed by noting the following lemma.

Lemma 6. *$Q_J < 2(1-p)$.*

To see this, consider any $Q \geq 2(1-p)$. In this case $\tau_J = 1$, and the ex-ante probability of the events as held by E are given by: $\mathbb{P}\left[W_1, P_1\right] = p$, $\mathbb{P}\left[W_2, P_1\right] = 1-p$, $\mathbb{P}\left[W_1, P_2\right] = \mathbb{P}\left[W_2, P_2\right] = 0$. Furthermore, $b(t|W_1, P_1) = b(t|W_2, P_1) = 2$, and the expected talent is given by $\mathbb{E}(W_1, P_1) = \mathbb{E}(W_2, P_1) = \frac{3}{4}$. Therefore we have $\mathbb{V}(W_1, P_1) = \mathbb{V}(W_2, P_1) = \frac{1}{48}$. Hence, in this case $\mathbb{E}[\mathbb{V}(Q,p)] = \frac{1}{48}$. Suppose we denote $\mathbb{E}[\mathbb{V}(Q,p)]$ for the case $Q < 2(1-p)$ as $f(p,Q)$. Now consider $g(p,Q) = \frac{1}{48} - f(p,Q)$. Note that $g(p,Q)|_{Q=2(1-p)} = 0$. Also note $\frac{\partial g(p,Q)}{\partial Q}|_{Q=2(1-p)} = \frac{4p^2-4p-1}{192(2p-1)}$. Note that since $p \in (\frac{1}{2}, 1)$, the numerator is always negative, and the denominator is always positive. Hence $\frac{\partial g(p,Q)}{\partial Q}|_{Q=2(1-p)} < 0$. Hence, we see that when $Q < 2(1-p)$, the function $g(p,Q) > 0$. This proves that at the optimum, $Q_J < 2(1-p)$ must hold. □

This completes the proof of Observation 2. ■