The A_4, S_4 and A_5 flavour symmetries in light of data on neutrino mixing*

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We consider the A_4, S_4 and A_5 discrete lepton flavour symmetries broken down to non-trivial residual symmetries in the charged lepton and neutrino sectors in such a way that at least one of them is a Z_2. Such symmetry breaking patterns lead to predictions for some of the three neutrino mixing angles and/or the Dirac CP violation phase δ of the neutrino mixing matrix. First, we perform a statistical analysis of these predictions, which uses as input the latest global data on the neutrino mixing parameters. We find 14 phenomenologically viable cases. Further, we assess the viability of these cases taking into account the prospective uncertainties in the determination of the mixing angles, planned to be achieved in current and future neutrino oscillation experiments. We find that only six cases would be compatible with the assumed prospective data. We show that this number will be further reduced by a precision measurement of δ.

Keywords: Discrete flavour symmetries; neutrino mixing; CP violation; sum rules.

1. Introduction

In spite of the tremendous success of the Standard Theory, we still do not know why the number of fermion generations is three, what determines the patterns of quark and lepton masses, and what the origins of quark and neutrino mixing are. In the attempts to understand the origins of flavour, a variety of *flavour symmetries* have been proposed and explored in last decades. Symmetries described by both continuous and discrete groups have been considered. Discrete non-Abelian symmetries (see, e.g., Refs. 1–4 for reviews) allow for rotations in the flavour space by fixed (large) angles, which is particularly attractive in view of the fact that two of the three neutrino mixing angles are large [5,6].

In the framework of discrete flavour symmetry approach to 3-neutrino mixing, on which we will concentrate in the present article, it is assumed that at some high-energy scale there exists a (lepton) flavour symmetry described by a non-Abelian

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discrete (finite) group G_f. The lepton doublets of the three fermion generations are usually assigned to an irreducible 3-dimensional representation of this group, because one aims to unify the three lepton flavours, and this is the case we will consider in the present article. At low energies the flavour symmetry has necessarily to be broken, because the electron, muon and tauon charged leptons and the three massive neutrinos are distinct. Generally, G_f is broken in such a way that the charged lepton and neutrino mass matrices, M_e and M_ν, or more precisely, the product $M_e M_e^\dagger$ and M_ν ($M_\nu^\dagger M_\nu$) in the Majorana (Dirac) neutrino case, are left invariant under the action of its Abelian subgroups G_e and G_ν, respectively. These *residual symmetries* constrain the forms of the unitary matrices U_e and U_ν diagonalising $M_e M_e^\dagger$ and M_ν ($M_\nu^\dagger M_\nu$), and thus of the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) neutrino mixing matrix $U_{\mathrm{PMNS}} = U_e^\dagger U_\nu$.

If $G_e = Z_k$, $k > 2$ or $Z_m \times Z_n$, $m, n \geq 2$, and $G_\nu = Z_2 \times Z_2$ ($G_\nu = Z_k$, $k > 2$ or $Z_m \times Z_n$, $m, n \geq 2$) for Majorana (Dirac) neutrinos, the matrices U_e and U_ν are fixed (up to permutations of columns and diagonal phase matrix on the right). This leads to certain fixed values of the solar, atmospheric and reactor neutrino mixing angles θ_{12}, θ_{23} and θ_{13} of the standard parametrisation of U_{PMNS} (see, e.g.,[7]) . Tri-bimaximal (TBM) mixing[8,9] (see also[10]), characterised by $\theta_{12} = \arcsin(1/\sqrt{3}) \approx 35^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$, is a well-known example of a symmetry form arising from a specific breaking pattern. Namely, it arises naturally by breaking $G_f = S_4$ to $G_e = Z_3$ and $G_\nu = Z_2 \times Z_2$[11]. This highly symmetric mixing pattern was ruled out, however, once θ_{13} was found to have a non-zero value, $\theta_{13} \cong 0.15$. The relatively large value of θ_{13} opened up a possibility of establishing the status of Dirac CP violation (CPV) in the lepton sector by measuring the Dirac phase δ present in U_{PMNS}. At the same time, it implied that the TBM and other symmetry forms of U_{PMNS} predicting $\theta_{13} = 0$ (see, e.g.,[3,4]) have to be "perturbed", so that the symmetry values of θ_{13}, as well as of θ_{12} and θ_{23} get corrected to values compatible with the data. This gave a boost to investigations of the possible forms of corrections (see, e.g.,[12–14]) and of alternative flavour symmetries and symmetry breaking patterns (see, e.g.,[15–19]). The most distinctive feature of the discussed approach to neutrino mixing based on non-Abelian discrete flavour symmetries is the predictions of the values of some of the neutrino mixing angles and leptonic CPV phases, or of existence of correlations between the values of at least some the neutrino mixing angles and/or between the values of the neutrino mixing angles and the Dirac CPV phase in U_{PMNS} (see, e.g.,[3,12,14,16–19]).

In[18] all symmetry breaking patterns, i.e., all possible combinations of residual symmetries, which could lead to correlations between some of the three neutrino mixing angles and/or between the neutrino mixing angles and the Dirac CPV phase δ, were considered. Namely, (A) $G_e = Z_2$ and $G_\nu = Z_k$, $k > 2$ or $Z_m \times Z_n$, $m, n \geq 2$; (B) $G_e = Z_k$, $k > 2$ or $Z_m \times Z_n$, $m, n \geq 2$ and $G_\nu = Z_2$; (C) $G_e = Z_2$ and $G_\nu = Z_2$; (D) G_e is fully broken and $G_\nu = Z_k$, $k > 2$ or $Z_m \times Z_n$, $m, n \geq 2$; and (E) $G_e = Z_k$, $k > 2$ or $Z_m \times Z_n$, $m, n \geq 2$ and G_ν is fully broken. For each pattern, *sum rules*, i.e., relations between the neutrino mixing angles and/or between the neutrino mixing angles and the Dirac CPV phase δ, when present, were derived.

In the present article based on Ref. 20, we concentrate on patterns A, B and C, which involve non-trivial residual symmetries in both sectors in such a way that at least one of them is a Z_2. We assume $G_f = A_4$, S_4 and A_5. When choosing these flavour symmetries, we are guided by minimality: A_4, S_4 and A_5 are among the smallest (in terms of the number of elements) discrete groups admitting a 3-dimensional irreducible representation. We perform a statistical analysis of the sum rule predictions derived in [18], taking into account (i) the latest global data on the neutrino mixing parameters [21], and (ii) the prospective uncertainties in the determination of the neutrino mixing angles, which are planned to be achieved in the next generation of neutrino oscillation experiments.

2. Residual Symmetry Patterns and Sum Rules

In this Section, we summarise the results for patterns A, B and C obtained in [18]. They are used later to perform statistical analyses.

Pattern A: $G_e = Z_2$ and $G_\nu = Z_k$, $k > 2$ or $Z_m \times Z_n$, $m,n \geq 2$. The Z_2 residual symmetry in the charged lepton sector fixes the matrix U_e up to a $U(2)$ transformation in the i-j plane. This transformation can be parametrised in terms of a matrix containing one angle and three phases. Two of the three phases can be removed by a redefinition of the charged lepton fields. Therefore the three neutrino mixing angles and the Dirac phase are expressed in terms of the remaining two free parameters. As a result, correlations between the observables arise. Namely, the considered type of residual symmetries leads to sum rules for $\sin^2\theta_{23}$ and $\cos\delta$, except in one case (case A3, see further).

Depending on the plane in which the $U(2)$ transformation is performed, one has three cases. The first one, which we denote as A1, corresponds to the transformation in the 1-2 plane and leads to the following sum rules:

$$\sin^2\theta_{23} = 1 - \frac{\cos^2\theta^\circ_{13}\cos^2\theta^\circ_{23}}{1-\sin^2\theta_{13}}\,, \tag{1}$$

$$\cos\delta = \frac{\cos^2\theta_{13}(\sin^2\theta^\circ_{23} - \cos^2\theta_{12}) + \cos^2\theta^\circ_{13}\cos^2\theta^\circ_{23}(\cos^2\theta_{12} - \sin^2\theta_{12}\sin^2\theta_{13})}{\sin 2\theta_{12}\sin\theta_{13}|\cos\theta^\circ_{13}\cos\theta^\circ_{23}|(\cos^2\theta_{13} - \cos^2\theta^\circ_{13}\cos^2\theta^\circ_{23})^{\frac{1}{2}}}\,, \tag{2}$$

where the angles θ°_{13} and θ°_{23} are fixed once the flavour symmetry group G_f and the residual symmetry subgroups G_e and G_ν are specified. In the second case, A2, which corresponds to the free $U(2)$ transformation in the 1-3 plane, one has different relations:

$$\sin^2\theta_{23} = \frac{\sin^2\theta^\circ_{23}}{1-\sin^2\theta_{13}}\,, \tag{3}$$

$$\cos\delta = -\frac{\cos^2\theta_{13}(\cos^2\theta^\circ_{12}\cos^2\theta^\circ_{23} - \cos^2\theta_{12}) + \sin^2\theta^\circ_{23}(\cos^2\theta_{12} - \sin^2\theta_{12}\sin^2\theta_{13})}{\sin 2\theta_{12}\sin\theta_{13}|\sin\theta^\circ_{23}|(\cos^2\theta_{13} - \sin^2\theta^\circ_{23})^{\frac{1}{2}}}\,, \tag{4}$$

where also the angle θ°_{12} is fixed once G_f, G_e and G_ν are specified. Finally, case A3 corresponding to the $U(2)$ transformation in the 2-3 plane predicts $\sin^2\theta_{13} = \sin^2\theta^\circ_{13}$ and $\sin^2\theta_{12} = \sin^2\theta^\circ_{12}$, while $\cos\delta$ remains unconstrained.

Pattern B: $G_e = Z_k$, $k > 2$ or $Z_m \times Z_n$, $m, n \geq 2$ and $G_\nu = Z_2$. The residual Z_2 symmetry determines the matrix U_ν up to a $U(2)$ transformation in the i-j plane. For Dirac neutrinos, two of the three phases parametrising this transformation can be removed by a re-phasing of the neutrino fields. For Majorana neutrinos, these two phases will contribute to the Majorana phases in the PMNS matrix. In either case, they will not enter into the expressions for the mixing angles and the Dirac phase, which depend on the remaining two free parameters (an angle and a phase). Pattern B leads to sum rules for $\sin^2\theta_{12}$ and $\cos\delta$, again except in one case (case B3, see further).

Again, depending on the plain of the $U(2)$ transformation, we have three cases. Case B1 corresponding to $(ij) = (13)$ yields

$$\sin^2\theta_{12} = \frac{\sin^2\theta^\circ_{12}}{1 - \sin^2\theta_{13}}\,, \tag{5}$$

$$\cos\delta = -\frac{\cos^2\theta_{13}(\cos^2\theta^\circ_{12}\cos^2\theta^\circ_{23} - \cos^2\theta_{23}) + \sin^2\theta^\circ_{12}(\cos^2\theta_{23} - \sin^2\theta_{13}\sin^2\theta_{23})}{\sin 2\theta_{23}\sin\theta_{13}|\sin\theta^\circ_{12}|(\cos^2\theta_{13} - \sin^2\theta^\circ_{12})^{\frac{1}{2}}}\,, \tag{6}$$

where θ°_{12} and θ°_{23} are fixed once the symmetries are specified. In case B2, $(ij) = (23)$, the sum rules of interest read:

$$\sin^2\theta_{12} = 1 - \frac{\cos^2\theta^\circ_{12}\cos^2\theta^\circ_{13}}{1 - \sin^2\theta_{13}}\,, \tag{7}$$

$$\cos\delta = \frac{\cos^2\theta_{13}(\sin^2\theta^\circ_{12} - \cos^2\theta_{23}) + \cos^2\theta^\circ_{12}\cos^2\theta^\circ_{13}(\cos^2\theta_{23} - \sin^2\theta_{13}\sin^2\theta_{23})}{\sin 2\theta_{23}\sin\theta_{13}|\cos\theta^\circ_{12}\cos\theta^\circ_{13}|(\cos^2\theta_{13} - \cos^2\theta^\circ_{12}\cos^2\theta^\circ_{13})^{\frac{1}{2}}}\,. \tag{8}$$

At last, case B3, $(ij) = (12)$, leads to $\sin^2\theta_{13} = \sin^2\theta^\circ_{13}$ and $\sin^2\theta_{23} = \sin^2\theta^\circ_{23}$, and no sum rule for $\cos\delta$.

Pattern C: $G_e = Z_2$ and $G_\nu = Z_2$. In this case, both U_e and U_ν are determined up to $U(2)$ transformations in the i-j and k-l planes, respectively. Thus, we have four free parameters (two angles and two phases) in terms of which θ_{ij} and δ are expressed. However, as shown in [18], this number is reduced to three after an appropriate rearrangement of these parameters. As a consequence, a sum rule for either $\cos\delta$ or one of the three $\sin^2\theta_{ij}$ arises.

Depending on the planes in which the free $U(2)$ transformations are performed, we have nine possibilities. We number them as in [18], i.e., cases C1–C9. Four of them lead to sum rules for $\cos\delta$, which we summarise bellow.

$$\text{C1, } (ij,kl) = (12,13): \quad \cos\delta = \frac{\sin^2\theta^\circ_{23} - \cos^2\theta_{12}\sin^2\theta_{23} - \cos^2\theta_{23}\sin^2\theta_{12}\sin^2\theta_{13}}{\sin\theta_{13}\sin 2\theta_{23}\sin\theta_{12}\cos\theta_{12}}\,, \tag{9}$$

$$\text{C3, } (ij,kl) = (12,23): \quad \cos\delta = \frac{\sin^2\theta_{12}\sin^2\theta_{23} - \sin^2\theta^\circ_{13} + \cos^2\theta_{12}\cos^2\theta_{23}\sin^2\theta_{13}}{\sin\theta_{13}\sin 2\theta_{23}\sin\theta_{12}\cos\theta_{12}}\,, \tag{10}$$

$$\text{C4, } (ij,kl) = (13,23): \quad \cos\delta = \frac{\sin^2\theta^\circ_{12} - \cos^2\theta_{23}\sin^2\theta_{12} - \cos^2\theta_{12}\sin^2\theta_{13}\sin^2\theta_{23}}{\sin\theta_{13}\sin 2\theta_{23}\sin\theta_{12}\cos\theta_{12}}\,, \tag{11}$$

$$\text{C8, } (ij,kl) = (13,13): \quad \cos\delta = \frac{\cos^2\theta_{12}\cos^2\theta_{23} - \cos^2\theta^\circ_{23} + \sin^2\theta_{12}\sin^2\theta_{23}\sin^2\theta_{13}}{\sin\theta_{13}\sin 2\theta_{23}\sin\theta_{12}\cos\theta_{12}}\,. \tag{12}$$

The neutrino mixing angles in these cases can be treated as free parameters. Other two cases, C5 and C9, yield correlations between $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$. Namely,

$$\text{C5, } (ij,kl) = (23,13): \quad \sin^2\theta_{12} = \frac{\sin^2\theta^\circ_{12}}{1-\sin^2\theta_{13}}\,, \tag{13}$$

$$\text{C9, } (ij,kl) = (23,23): \quad \sin^2\theta_{12} = \frac{\sin^2\theta^\circ_{12} - \sin^2\theta_{13}}{1-\sin^2\theta_{13}}\,. \tag{14}$$

In cases C2 and C7, instead, there are correlations between $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$:

$$\text{C2, } (ij,kl) = (13,12): \quad \sin^2\theta_{23} = \frac{\sin^2\theta^\circ_{23}}{1-\sin^2\theta_{13}}\,, \tag{15}$$

$$\text{C7, } (ij,kl) = (12,12): \quad \sin^2\theta_{23} = \frac{\sin^2\theta^\circ_{23} - \sin^2\theta_{13}}{1-\sin^2\theta_{13}}\,. \tag{16}$$

Finally, in case C6, $(ij,kl) = (23,12)$, $\sin^2\theta_{13}$ is predicted to be equal to $\sin^2\theta^\circ_{13}$. In cases C2, C5, C6, C7 and C9, $\cos\delta$ remains unconstrained.

3. Predictions for the Mixing Angles and the Dirac CPV Phase

A brief description of the discrete groups A_4, S_4 and A_5 we consider as flavour symmetries can be found in Sec. 3 of Ref. 20. In the case of $G_f = A_4$, we find *only one* phenomenologically viable case[18]. Namely, this is case B1 with $(G_e, G_\nu) = (Z_3, Z_2)$, which yields $(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (1/3, 1/2)$ and corresponds to the TBM mixing matrix corrected from the right by a $U(2)$ transformation in the 1-3 plane. Making use of Eqs. (5) and (6) and the current best fit values of $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ for NO [henceforth NO (IO) stands for normal (inverted) ordering of neutrino masses] from[21], we find $\sin^2\theta_{12} = 0.341$ and $\cos\delta = -0.353$. For $G_f = S_4$, there are 6 more

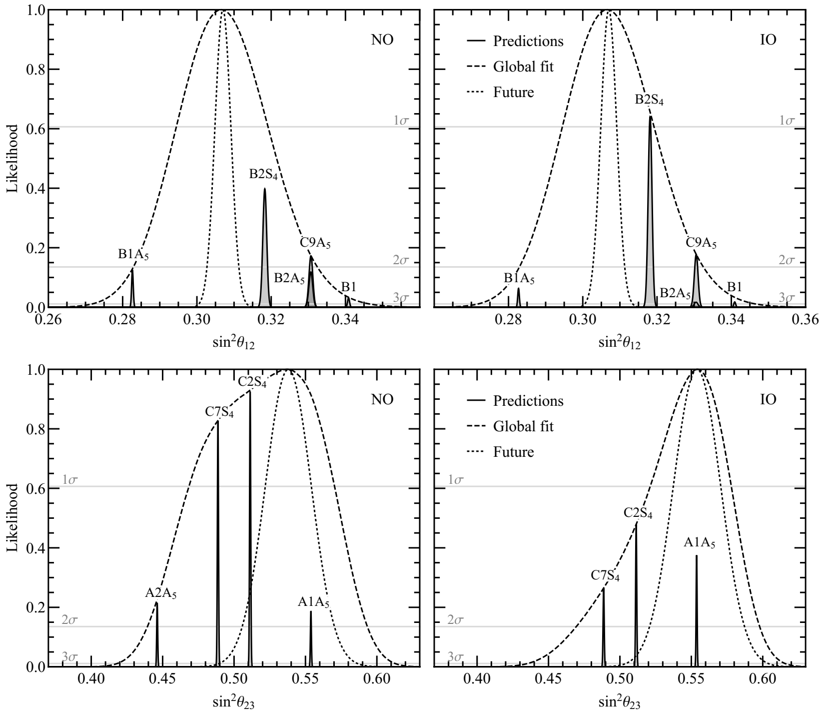

Fig. 1. Upper [lower] panel: predictions for $\sin^2\theta_{12}$ [$\sin^2\theta_{23}$] obtained using the current global data on the neutrino mixing parameters. "Future" refers to the scenario with $\sin^2\theta_{12}^{\rm bf} = 0.307$ [$\sin^2\theta_{23}^{\rm bf} = 0.538$ (0.554) for NO (IO)] (current best fit values) and the relative 1σ uncertainty of 0.7% [3%] expected from JUNO [DUNE and T2HK]. See text for further details. (From Ref. 20.)

viable cases. They are summarised in Table 3 of Ref. 20. The A_5 flavour symmetry leads to 7 more phenomenologically viable cases, see Table 4 in [20]. Thus, the total number of phenomenologically viable cases for the considered groups is 14.

Further, we construct an approximate global likelihood function [13,22] for the observable of interest ($\sin^2\theta_{12}$, $\sin^2\theta_{23}$ or $\cos\delta$). This likelihood uses as input the one-dimensional χ^2 projections for $\sin^2\theta_{ij}$ and δ from [21] and takes into account the correlations between the mixing parameters (the sum rules).

In Fig. 1, we present the likelihood functions for $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$, obtained for NO and IO spectra in all the cases compatible at 3σ with the current global data [21]. The corresponding likelihood profiles are very narrow because their widths are determined by the small experimental uncertainty on $\sin^2\theta_{13}$. In the upper (lower) panel, the dashed line corresponds to the likelihood for $\sin^2\theta_{12}$ ($\sin^2\theta_{23}$) extracted from the global analysis. The dotted line represents the prospective precision on $\sin^2\theta_{12}$ ($\sin^2\theta_{23}$) of 0.7% (3%), which is planned to be achieved by JUNO [23] (DUNE [24,25] and T2HK [26,27]). It is obtained under the assumption that the best fit value(s) of $\sin^2\theta_{12}$ ($\sin^2\theta_{23}$) will not change in the future. If it is indeed the case, then, as is clear from Fig. 1, all five models leading to the predictions for $\sin^2\theta_{12}$ will be ruled out by the JUNO measurement of this parameter.

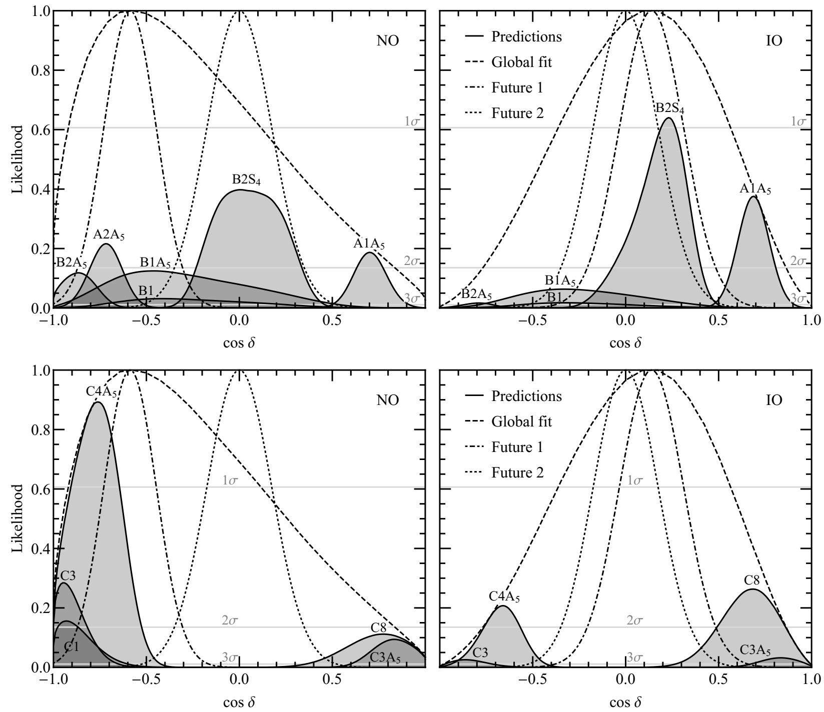

Fig. 2. Predictions for $\cos\delta$ obtained using the current global data on the neutrino mixing parameters. "Future 1" refers to the scenario with $\delta^{\rm bf} = 234°$ (278°) for NO (IO) spectrum (current best fit values) and the 1σ uncertainty on δ of 10°. "Future 2" corresponds to $\delta^{\rm bf} = 270°$ and the 1σ uncertainty on δ of 10°. See text for further details. (From Ref. 20.)

As has been discussed in Sec. 2, cases A and B of interest lead not only to predictions for $\sin^2\theta_{23}$ and $\sin^2\theta_{12}$, respectively, but also to predictions for $\cos\delta$. At the same time, cases C1, C3, C4 and C8 lead only to predictions for $\cos\delta$. We summarise the results of statistical analysis of these predictions in Fig. 2. We find that the predictions for $\cos\delta$ in cases B are very sensitive to the value of θ_{23}, which is determined with a larger uncertainty than θ_{12} and θ_{13}. This results in quite broad likelihood profiles. For cases A, the uncertainty in predicting $\cos\delta$ is driven by the uncertainty on $\sin^2\theta_{12}$, since $\sin^2\theta_{23}$ is almost fixed in these cases. Thus, the resulting likelihood profiles are not so broad in cases A1A$_5$ and A2A$_5$. The dashed line stands for the likelihood extracted from the global analysis. At present, all (almost all) values of $\cos\delta$ are allowed at 3σ for NO (IO) spectrum. We also show the dash-dotted and dotted lines which represent two benchmark cases. The first case, marked as "Future 1", corresponds to the current best fit NO (IO) value $\delta^{\rm bf} = 234°$ (278°) and the prospective 1σ uncertainty on δ of 10°. The second case, "Future 2", corresponds to the potential best fit value $\delta^{\rm bf} = 270°$ (for both NO and IO cases) and the same 10° error on δ. The likelihoods in cases C peak at values of $|\cos\delta| \sim 0.5 - 1$. Looking at the dotted line, we see that if in the future the best fit value of δ shifted to 270° and the next generation of long-baseline experiments

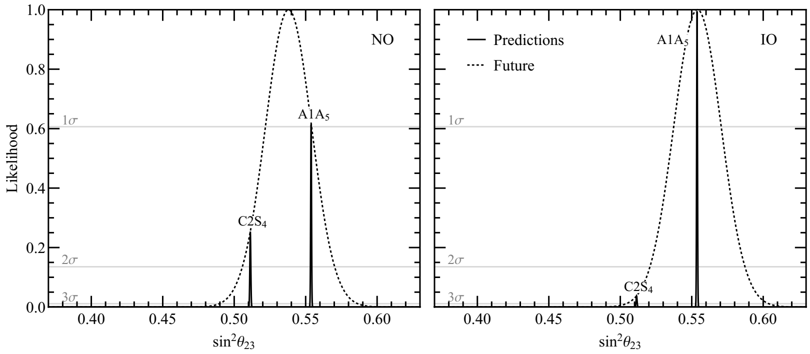

Fig. 3. Predictions for $\sin^2\theta_{23}$ obtained using the current best fit values and the prospective uncertainties in the determination of the neutrino mixing angles. "Future" refers to the scenario with $\sin^2\theta_{23}^{\rm bf} = 0.538$ (0.554) for NO (IO) spectrum (current best fit values) and the relative 1σ uncertainty of 3% expected from DUNE and T2HK. See text for further details. (From Ref. 20.)

managed to achieve the 1σ uncertainty on δ of 10°, all cases C viable at the moment would be disfavoured at around 3σ only by the measurement of δ.

Next we will assess in greater detail the impact of the future precision measurements of the neutrino mixing angles on the predictions discussed above. To this aim, we perform a statistical analysis of these predictions assuming that (i) the current best fit values of the mixing angles will not change in the future, and (ii) the prospective relative 1σ uncertainties on $\sin^2\theta_{12}$, $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ will amount to 0.7% (JUNO), 3% (DUNE and T2HK) and 3% (Daya Bay[28]), respectively. It is worth noting that in this analysis, we do not assume any experimental information on δ. The results presented below should be considered only as indicative.

It follows from Fig. 1 that JUNO will be able to rule out all the cases predicting $\sin^2\theta_{12}$, if the best fit value of this parameter does not shift in the future (see the dotted line). The results for the cases predicting $\sin^2\theta_{23}$ are shown in Fig. 3. As could be expected, two cases out of the four survive the assumed prospective constraints on $\sin^2\theta_{23}$. We emphasise that the conclusion about the excluded cases should be revised if the best fit value of $\sin^2\theta_{23}$ shifts, e.g., to 0.5.

Finally, we perform a statistical analysis of the predictions for $\cos\delta$ in case $\mathrm{A1A_5}$ (all cases B as well as case $\mathrm{A2A_5}$ would be ruled out by the assumed prospective data) and cases C. We show the obtained results in Fig. 4. The width of the likelihood profiles in case $\mathrm{A1A_5}$ is much smaller than that of the corresponding profiles in Fig. 2. This makes even more evident the fact that improving the precision on the mixing angles leads to sharper predictions for $\cos\delta$, which can and should be considered as an additional motivation of measuring the mixing angles with a high precision[12,13]. What concerns cases C, we find that under the assumptions made case C1 would be ruled out. Thus, we would be left with four cases, two of which (C3 and $\mathrm{C3A_5}$) lead to predictions lying in the corners of the parameter space for $\cos\delta$. A high precision measurement of δ is crucial to firmly establish the status of the considered cases.

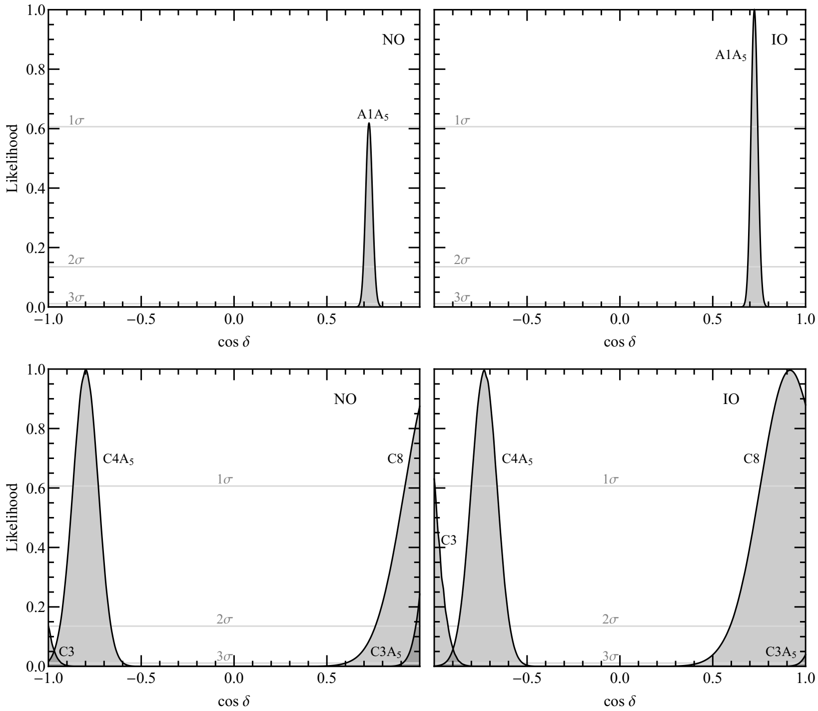

Fig. 4. Predictions for $\cos\delta$ obtained using the current best fit values and the prospective uncertainties in the determination of the neutrino mixing angles. See text for further details. (From Ref. 20.)

4. Summary and Conclusions

We have considered the A_4, S_4 and A_5 discrete flavour symmetry groups broken to non-trivial residual symmetry subgroups G_e and G_ν in the charged lepton and neutrino sectors in the following ways: (A) $G_e = Z_2$ and $G_\nu = Z_k$, $k > 2$ or $Z_m \times Z_n$, $m, n \geq 2$; (B) $G_e = Z_k$, $k > 2$ or $Z_m \times Z_n$, $m, n \geq 2$ and $G_\nu = Z_2$; and (C) $G_e = Z_2$ and $G_\nu = Z_2$. In the cases corresponding to pattern A (B) sum rules for $\sin^2\theta_{23}$ ($\sin^2\theta_{12}$) and $\cos\delta$ arise, while pattern C leads to sum rules for either $\sin^2\theta_{12}$ or $\sin^2\theta_{23}$ or $\cos\delta$ [18].

First, we have performed a statistical analysis of the sum rule predictions using as input the latest global neutrino oscillation data [21]. We have found 14 cases compatible with these data at 3σ. Five of them lead to very sharp predictions for $\sin^2\theta_{12}$, and four others to similarly sharp predictions for $\sin^2\theta_{23}$. Phenomenologically viable cases A and B, which are six in total, lead as well to predictions for $\cos\delta$. Five out of eight viable C cases also lead to predictions for $\cos\delta$. The corresponding likelihoods peak at values of $|\cos\delta| \sim 0.5 - 1$. As we have shown, the number of these cases could be further reduced by a sufficiently precise measurement of δ.

Further, we have performed a statistical analysis of the predictions discussed above assuming that (i) the current best fit values of the mixing angles will not change in the future, and (ii) the prospective relative 1σ uncertainties on $\sin^2\theta_{12}$, $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ will amount to 0.7%, 3% and 3%, respectively. Such uncertainties are planned to be achieved by the JUNO, T2HK/DUNE and Daya Bay experiments, respectively. Under the assumptions made, all the cases predicting $\sin^2\theta_{12}$ get ruled out. In what concerns the cases predicting $\sin^2\theta_{23}$, two out of the four would "survive" this test. We have found that only one case among six cases A and B viable at present would be compatible with the prospective data on the neutrino mixing angles. Four out of five cases C predicting $\cos\delta$ satisfy the expected constraints on the mixing angles. Thus, in total, six cases out of 14 viable at present are compatible with the assumed prospective data on the neutrino mixing angles. Five of these cases will be further critically tested by sufficiently precise data on the Dirac phase δ, e.g., if δ is measured with 1σ uncertainty of $10°$.

The results obtained in the present study show that the high precision data on neutrino mixing, planned to be obtained in the next generation of neutrino oscillation experiments, will be crucial for testing the ideas of existence of new fundamental discrete flavour symmetry in the lepton sector of particle interactions.

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