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Key Points:
- The expressions of SINR and MI are derived to characterize the target detection and parameter estimation performance, respectively
- The problem of LPI-based adaptive jamming waveform design for distributed multiple-radar systems is investigated
- Both jamming waveform design schemes are solved analytically, and the method of Lagrange multipliers is exploited to solve those problems

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Adaptive Jamming Waveform Design for Distributed Multiple-Radar Architectures Based on Low Probability of Intercept

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Abstract This paper investigates the problem of low probability of intercept (LPI)-based adaptive jamming waveform design for distributed multiple-radar architectures. Such a smart jammer system adopts a multibeam working mode, where multiple simultaneous jamming beams are synthesized to interfere with multiple radars independently. The primary objective of the smart jammer is to minimize the total jamming power by optimizing the transmitted jamming waveform while the achieved signal-to-interference-plus-noise ratio (SINR) and mutual information (MI) between the received echoes from the target at each radar receiver and the target impulse responses are enforced to be below specified thresholds. First, the expressions of SINR and MI are derived to characterize target detection and characterization performance, respectively. Then, two different LPI-based jamming waveform design strategies are proposed to minimize the total noise jamming power by optimizing the jamming waveform while the achieved SINR/MI is enforced to be below a certain threshold. The resulting optimization problems are solved analytically by employing the technique of Lagrange multipliers. With the aid of some numerical examples, it is illustrated that the two schemes result in different jamming waveform design results, which is useful to guide jamming power allocation for various jamming tasks. It is also shown that the LPI performance of the smart jammer can be efficiently improved by exploiting the proposed jamming waveform design criteria.

1. Introduction

1.1. Background and Motivation

Recently, the electromagnetic environment of modern warfare is becoming much more complex, which has brought up the problem of investigating the interaction between radar and target (L. L. Wang et al., 2013, 2014). In general, the open literature is now rich with a number of studies, which concentrate on waveform design to improve the performance of radar system. Due to the fact that the performance of target detection and parameter estimation relies on the output signal-to-interference-plus-noise ratio (SINR) and mutual information (MI), much effort has been devoted to the problem of radar waveform design, which takes the issue of SINR and MI maximization into account (Cheng, Liao, et al.,2018). In Leshem et al. (2007), the authors employ an information theoretic method to design radar waveforms for multiple extended targets, which can be accomplished utilizing a linear sum of MI between target radar signatures and the related received beams. Romero, Bae, and Goodman optimize the matched illumination waveforms for both deterministic and stochastic extended targets (Romero et al., 2011), and the connections between signal-to-noise ratio (SNR) and MI is discussed. Cheng, He, et al.(2018) study the problem of joint design of transmit waveform and receive filter for MIMO radar in signal-dependent interference with a peak-to-average-power ratio constraint and a waveform similarity constraint. The resulting optimization problem is solved by the sequential optimization algorithm based on the semidefinite relaxation with randomization approach. In Zhu et al. (2018), the problem of radar waveform design under the small waveform power conditions is studied for detecting a doubly spread target in colored noise, where the superiority of the proposed algorithm over the linear frequency-modulated waveform is validated. Moreover, Shi et al. propose three power minimization-based robust orthogonal frequency division multiplexing radar waveform design strategies for radar and communication systems in coexistence (Shi et al., 2018), which differ in the way the communication signals scattered off the target are considered as useful energy, as interference, or ignored altogether. It is also shown that the radar-transmitted power can be significantly reduced by employing the communication signals scattered off the target at the radar receiver.

Nevertheless, all the algorithms mentioned above prefer to investigate the competition between a smart radar and a dumb target, in which the radar system can effectively detect and extract target information , while the target is unable to interfere with the radar (Song, Willett, Zhou, & Lu 2012). With the development of direct digital synthesis and digital radio frequency memory (L. L. Wang et al., 2014), many noncooperative targets which are equipped with countermeasure systems can intelligently confuse radar systems for self-protection (Y. X. Wang et al., 2018). As introduced in (Song, Willett, and Zhou 2012), radar jamming refers different techniques of spoofing a radar system with barrage noise of false target information, which can be either mechanical or electronic. The mechanical jamming approach exploits the designed hardware such as corner reflectors and decoys to produce false target echoes, while the electronic jamming method actively emits interfering waveforms toward the radar system. Thus, many scientific publications have studied the antijamming techniques and the interaction between a smart radar and a smart jammer (Song, Willett, Zhou, and Lu 2012; L. L. Wang et al., 2014).

Technically speaking, the antijamming performance of modern radar system can be enhanced by adopting well-designed waveforms, coherent sidelobe cancellers, and joint optimization of transmit waveform and receiver filter (Zhang et al., 2016). For instance, in Song, Willett, and Zhou (2012), it is shown that a distributed MIMO radar system can counter the deceptive jamming owing to the spatial diversity, where the antijamming capability is a feature of MIMO radar. Li et al. (2014a) investigate the problem of jammers suppression in colocated MIMO radar, which is solved by utilizing the reduced dimension beamspace designs with adaptiveness to suppress jammers. Further, the robust beamforming for MIMO radar in the presence of powerful jamming signals is studied in Li et al. (2014b), and two minimum variance distortionless response type beamformers are proposed. It is illustrated that the capability of efficient jammers suppression employing these optimizations is unique in MIMO radar systems. In addition, Zhang et al. (2016) propose a novel method to optimize the transmitting array in order to suppress the active jamming for colocated MIMO radar, which can obtain enhanced performance for conventional or adaptive beamforming performance degradation cases.

On the other hand, Song, Willett, Zhou, and Lu (2012) investigate the interaction between a smart MIMO radar and a smart target from a game theory perspective, which is modeled as a two-person zero-sum game, and various equilibria solutions are analytically derived. In Gao et al. (2015), the authors extend the results in Song, Willett, Zhou, and Lu (2012) and model the adversarial competition between a statistical MIMO radar and an intelligent jammer as a Bayesian game, where the radar system and the jammer have incomplete information. Furthermore, reference Deligiannis et al. (2016) adopts game theory to tackle the problem of competitive power allocation for MIMO radar in the presence of multiple jammers, the main objective of the radar system is to guarantee a desired target detection performance utilizing the minimum transmission power, while the intelligent jammers optimize the jamming power to maximize the interference to the radars. In Wang et al. (2013), the SINR- and MI-based jamming design algorithms are presented to protect the target from detection and estimation, respectively. However, the precise characteristics of radar waveform is impossible to estimate in practice. To this end, Wang et al. design the minimax robust jamming methods based on SINR and MI criteria (L. L. Wang et al., 2014), and the worst-case jamming performance is optimized, where the radar waveform spectrum is assumed to lie in an uncertainty set confined by known upper and lower bounds. Later, the authors investigate the optimal and robust jamming power allocation for distributed MIMO radar based on the MI and the minimum mean square error (L. L. Wang et al., 2017). The obtained results are useful for the implementation of cognitive jammers. In Y. X. Wang et al. (2018), the waveform design algorithm for radar and the extended target in electronic warfare environment is proposed, and different countermeasure models are presented.

Nowadays, the concept of low probability of intercept (LPI) should be taken into account in designing radar systems (Pace, 2009; Schleher, 2006; Shi et al., 2018; Shi, Zhou, & Wang, 2017; Zhang et al., 2015). Precisely, large revisit interval, low transmission power, short dwell time, ultralow sidelobe antenna, and waveform optimization can result in better LPI performance of radar system. Similarly, the LPI-based jamming technique is also of critical importance to modern jammer systems, whereas more achievements are seldom published due to the inherent sensitivities associated with military radar and electronic warfare systems (Zhang et al., 2016). In our previous work (Shi et al., 2015), the LPI-based radar jamming waveform design methods are proposed, whose aim is to minimize the transmitted jamming power, while the achievable radar performance outage probability is enforced to be greater than a given confidence level. In Shi, Wang, et al. (2017), the LPI-based orthogonal frequency division multiplexing radar jamming power allocation is addressed for a joint radar and wireless communication system, where various schemes are proposed to minimize the noise jamming power by optimizing the multicarrier jamming power allocation subject to a predetermined MI threshold. Numerical results confirm that the LPI performance of the jammer is remarkably enhanced by the proposed strategies.

Overall speaking, the reported works show that adaptive jamming waveform design is an important method to improve the LPI performance of jammer systems. Although the existing approaches provide us a guidance to tackle the problem of LPI-based jamming waveform optimization, they are all concentrated on the monostatic radars. However, for the case of distributed multiple-radar architectures, those algorithms are no longer applicable, owing to the fact that the jamming performance for multiple-radar systems not only relies on the radar-transmitted waveforms but also relies on the target impulse responses and signal-dependent clutters. Above all, to the best of our knowledge, the problem of LPI-based adaptive jamming waveform design for distributed multiple-radar systems, which is the focus of this paper, has not been well investigated until now.

1.2. Our Contributions

Motivated by the aforementioned facts, this study investigates the problem of LPI-based adaptive jamming waveform design for distributed multiple-radar architectures. To be specific, the primary objective is to minimize the transmitted jamming power of a smart jammer by optimizing the jamming waveform while the achieved SINR/MI is enforced to be below a certain threshold. The designed jamming waveform results are sent back to the system controller to form the next jamming scheme. Therefore, the smart jammer system can be viewed as a reaction to the electromagnetic environment, based on which it optimally designs the jamming waveforms.

To sum up, the major contributions of this work are summarized in the below:

1.) *The analytical expressions of SINR and MI between the received echoes from the target at the radar receiver and the target impulse responses are derived to evaluate the target detection and characterization performance respectively, which incorporate the target spectra, the power spectral densities (PSDs) of signal-dependent clutters, and each radar transmission waveforms.* In this work, the adaptive jamming waveform design criteria are formulated based on the target detection and parameter estimation performance.
2.) *In order to improve the LPI performance of smart jammer, the problem of LPI-based adaptive jamming waveform design for distributed multiple-radar architectures is investigated.* It is assumed that the precise characteristics of the target spectra, the PSDs of the signal-dependent clutters, and the radar-transmitted waveforms are perfectly estimated by the jammer (Shi, Wang, et al., 2017; L. L. Wang et al., 2014). Mathematically speaking, the LPI-based jamming waveform design strategies are built to improve the LPI performance of a smart jammer. Thus, it is actually an optimization problem of minimizing an objective function about the total transmitted jamming power, while meeting a predefined jamming performance requirement. Specifically, with the derived expressions of SINR and MI, two associated LPI-based jamming waveform design criteria are proposed, which minimize the total jamming power by optimizing the transmitted jamming waveform while the achieved SINR/MI is enforced to be below a certain threshold.
3.) *Both LPI-based adaptive jamming waveform design criteria are formulated and solved analytically, where the technique of Lagrange multipliers is utilized to solve the jamming waveform design problems.* In addition, the bisection search approach is adopted to find the optimal solutions for the resulting problems.
4.) *Various numerical examples are presented to verify the effectiveness of the proposed LPI-based jamming waveform design schemes via Monte Carlo simulations.* Moreover, we also reveal the relationship between the jamming power allocation and different radar transmission waveforms. It is worth to mention that similar jamming waveform design results can be expected in the scenarios where the bandwidth is extremely narrow or extremely wide.

1.3. Organization of the Paper and Notations

The reminder of the paper is organized as follows: Section 2 introduces the considered system model and the underlying assumptions needed in this paper. Section 3 describes the optimization problems to be solved. In section 3.1, the basis of the LPI-based jamming waveform design for distributed multiple-radar architectures

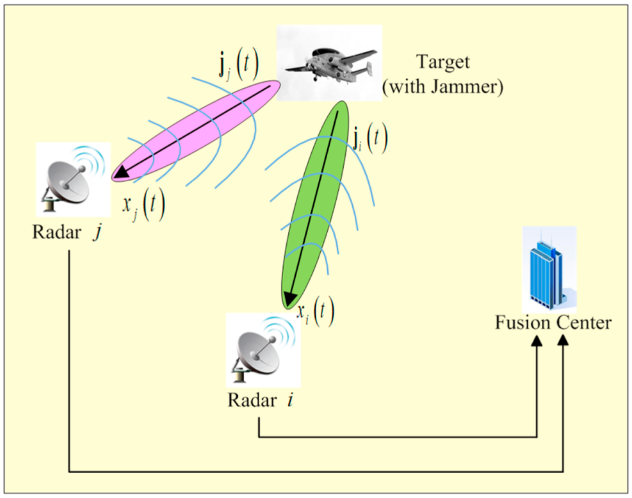

Figure 1. Illustration of the system model.

is provided. The LPI-based jamming waveform design schemes with SINR and MI criteria are proposed in sections 3.2 and 3.3, respectively, where the established problems of adaptive jamming waveform design are formulated and solved analytically. Various numerical examples are provided in section 4 to verify the effectiveness of the proposed jamming waveform design schemes. Finally, conclusions are drawn and directions for future work are provided in Section 5.

Notations: Throughout this paper, the following notations are adopted. The continuous time domain signal is represented by $x(t)$; the Fourier transform of $x(t)$ is $X(f)$. The symbol $*$ denotes the convolution operator. $\mathbb{E}\{\cdot\}$ denotes the expectation operator. $\max\{m, n\}$ denotes the larger value between m and n; the superscript $(\cdot)^*$ indicates the optimality, and the superscript $(\cdot)^H$ indicates the complex conjugation.

2. System and Signal Models

2.1. Problem Scenario

In the overall paper, we consider a scenario, where a distributed multiple-radar system consisting of multiple radar nodes aiming at detecting/tracking an extended target, as illustrated in Figure 1. There are M_R radars networking together in a self-organizing fashion (Xu, 2011). The distributed multiple-radar system has a common precise knowledge of time and space. The radars transmit orthogonal waveforms and receive and process all these echoes scattered off the target, which send the estimates of the target to the fusion center for information fusion with data link. Each radar works with an antenna directed to the target to receive the scattered echoes. For simplicity, it is assumed that the radar antennas are directional and steered toward the target; thus, the radar-transmitted signals do not arrive at the other radar receivers through direct paths.

The target is equipped with a smart jammer system, which can intelligently interfere with the multiple-radar configuration for self-protection. Thus, we will focus on the self-protection jamming under the assumption that the exact knowledge of the target spectra, the PSDs of the signal-dependent clutters, and the radar-transmitted waveforms are perfectly estimated by the jammer. It is worth mentioning that the system model and the mathematical derivations can straightforwardly be extended to the stand-off jamming scenario.

The primary objective of the smart jammer is to minimize the total jamming power by optimizing the transmitted jamming waveform while the achieved SINR/MI performance is enforced to be below a certain threshold. The ith radar receives the echoes scattered off the target and the signal-dependent clutter due to its transmission waveform $x_i(t)$, whose finite duration is T_i. As mentioned before, the phase coded waveforms of radars are orthogonal to each other, thus the interference from one waveform to the another can be minimized or even removed (Xu, 2011). Due to the unique structure of the phased antenna array, various desired beam patterns can be generated by the transmitters (Yan et al., 2015). Thus, the smart jammer can launch multiple directional beams simultaneously to interfere with multiple radars in a multibeam mode. The transmitted jamming waveform for radar i is denoted by $\mathbf{j}_i(t)$, which can only be received by radar i.

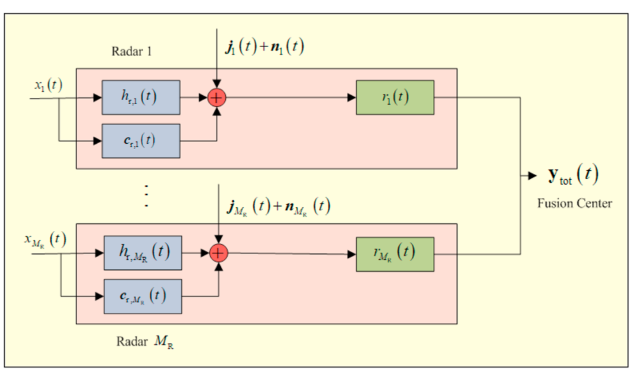

Figure 2. Known target signal model.

2.2. Signal Model

Figure 2 shows the known target signal model in the presence of a smart jammer. The complex-valued baseband target impulse response with respect to radar i is denoted by $h_{r,i}(t)$ with finite duration T_{h_i}. Let $X_i(f)$ and $H_{r,i}(f)$ represent the Fourier transforms of $x_i(t)$ and $h_{r,i}(t)$. Let $J_i(f)$ denote PSD of the transmitted jamming waveform $\mathbf{j}_i(t)$. Let the signal-dependent clutter with respect to the ith radar $\mathbf{c}_{r,i}(t)$ be a complex-valued, zero-mean Gaussian random process with PSD $S_{ccr,i}(f)$. $\mathbf{n}_{r,i}(t)$ denotes the additive white Gaussian noise (AWGN) of radar i, which is a complex-valued, zero-mean process with PSD $S_{nnr,i}(f)$. $r_i(t)$ denotes the ith complex-valued receive filter impulse response, whose Fourier transform is $R_i(f)$.

The overall output signal of the radar system is $\mathbf{y}_{\text{tot}}(t)$. As a consequence, the output signal in radar i $\mathbf{y}_i(t)$ can be expressed as follows (Shi et al., 2016; Xu & Liang, 2010):

$$\mathbf{y}_i(t) = \underbrace{h_{r,i}(t) * x_i(t) * r_i(t)}_{\text{Target return}} + \underbrace{\left(\mathbf{c}_{r,i}(t) * x_i(t) + \mathbf{j}_i(t) + \mathbf{n}_{r,i}(t)\right) * r_i(t)}_{\text{Signal-dependent clutter plus interference return}}. \tag{1}$$

In the following, we describe the optimization problems in order to improve the LPI performance of a smart jammer. Precisely, we propose two different jamming waveform design strategies to be addressed depending on SINR and MI criteria, respectively.

3. Problem Formulation

3.1. Basis of the Technique

Mathematically speaking, the LPI-based adaptive jamming waveform design for distributed multiple-radar systems can be formulated as a problem of optimizing transmitted jamming waveform to minimize the total jamming power of smart jammer, while the achieved SINR/MI is enforced to be below a specified threshold. First, the expressions of SINR and MI are derived to evaluate the target detection and parameter estimation performance, respectively. We are then in a position to design the jamming waveform in order to minimize the transmitted jamming power of smart jammer. The resulting optimization problems are solved analytically by utilizing the method of Lagrange multipliers. The general LPI-based jamming waveform design criteria are detailed as follows.

3.2. LPI-Based Jamming Waveform Design With SINR Criterion

As previously mentioned, it is supposed that the precise characteristics of the target spectra, the PSDs of the signal-dependent clutters, and the radar-transmitted waveforms are perfectly estimated by the smart jammer. In the light of the definitions in (Y. X. Wang et al., 2018), the SINR can be utilized as a metric to evaluate target detection performance for radar systems. Let $y_{r,i}(t)$ and $y_{\mathbf{n},i}(t)$ be the signal and interference components of the output signal $\mathbf{y}_i(t)$, respectively, which can be given by

$$\begin{cases} y_{r,i}(t) = h_{r,i}(t) * x_i(t) * r_i(t), \\ y_{\mathbf{n},i}(t) = \left(\mathbf{c}_{r,i}(t) * x_i(t) + \mathbf{j}_i(t) + \mathbf{n}_{r,i}(t)\right) * r_i(t). \end{cases} \tag{2}$$

Then, the output SINR at time t_0 can be expressed by

$$\begin{aligned} (\text{SINR})_{t_0} &\triangleq \sum_{i=1}^{M_R} \frac{|y_{r,i}(t_0)|^2}{\mathbb{E}\{|y_{\mathbf{n},i}(t_0)|^2\}} \\ &= \sum_{i=1}^{M_R} \frac{|\int_{-\infty}^{+\infty} R_i(f)H_{r,i}(f)X_i(f)\exp(j2\pi f t_0)\mathrm{d}f|^2}{\int_{-\infty}^{+\infty} |R_i(f)|^2 (S_{\text{ccr},i}(f)|X_i(f)|^2 + J_i(f) + S_{\text{nnr},i}(f))\mathrm{d}f} \\ &= \sum_{i=1}^{M_R} \frac{\left|\int_{-\infty}^{+\infty} R_i(f)\sqrt{J_i(f)+V_i(f)}\frac{H_{r,i}(f)X_i(f)}{\sqrt{J_i(f)+V_i(f)}}\exp(j2\pi f t_0)\mathrm{d}f\right|^2}{\int_{-\infty}^{+\infty} |R_i(f)|^2 (J_i(f)+V_i(f))\mathrm{d}f} \\ &\leq \sum_{i=1}^{M_R} \frac{\int_{-\infty}^{+\infty} |R_i(f)|^2 (J_i(f)+V_i(f))\mathrm{d}f \int_{-\infty}^{+\infty} \frac{|H_{r,i}(f)X_i(f)|^2}{J_i(f)+V_i(f)}\mathrm{d}f}{\int_{-\infty}^{+\infty} |R_i(f)|^2 (J_i(f)+V_i(f))\mathrm{d}f}, \end{aligned} \tag{3}$$

where $|X_i(f)|^2$ denotes the energy spectral density of $x_i(t)$, and $V_i(f) = S_{\text{ccr},i}(f)|X_i(f)|^2 + S_{\text{nnr},i}(f)$. Applying Schwarz's inequality (Papoulis & Pillai, 2002), the overall SINR can achieve its maximum value

$$(\text{SINR})_{t_0} = \sum_{i=1}^{M_R} \int_{-\infty}^{+\infty} \frac{|H_{r,i}(f)X_i(f)|^2}{J_i(f)+V_i(f)}\mathrm{d}f \tag{4}$$

if and only if the matched filter of each radar is of the form

$$R_i(f) = \frac{[gH_{r,i}(f)X_i(f)\exp(j2\pi f t_0)]^H}{J_i(f)+V_i(f)}, \tag{5}$$

where g is an arbitrary constant. It is assumed that the matched filter of each radar can be optimally designed in the form of equation (5), and the jamming bandwidth is adjusted to be the same with the radar transmission waveforms in order to improve the jamming efficiency. Therefore, the expression of the overall output SINR can be expressed by

$$\begin{aligned}(\mathrm{SINR})_{\mathrm{tot}} &\simeq \sum_{i=1}^{M_{\mathrm{R}}} \int_{\mathcal{BW}} \frac{|H_{\mathrm{r},i}(f)|^2 |X_i(f)|^2}{J_i(f) + V_i(f)} \mathrm{d}f \\ &= \Delta f \sum_{i=1}^{M_{\mathrm{R}}} \sum_{k=1}^{K} \frac{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2}{J_i(f_k) + V_i(f_k)},\end{aligned} \tag{6}$$

where $\mathcal{BW}$ denotes the bandwidth of radar and jammer waveforms, Δf denotes subband bandwidth, and K denotes the total number of subbands in frequency. It is noteworthy that the target detection performance relies on the output SINR. In the environment of modern electronic warfare, the target is equipped with a smart countermeasure system. In order to minimize the output SINR and prevent the distributed multiple-radar systems from detecting the target, the jammer system will optimize its transmitted jamming waveform according to the prior knowledge of the target spectra, the PSDs of the signal-dependent clutters, and the radar-transmitted waveforms. We can find from (6) that the output SINR depends on the target spectra $H_{\mathrm{r},i}(f)$, the PSDs of the signal-dependent clutters $S_{\mathrm{ccr},i}(f)$, the PSDs of the AWGN $S_{\mathrm{nnr},i}(f)$, and the radar-transmitted waveforms $X_i(f)$. Obviously, it is seen that, for fixed parameters of $H_{\mathrm{r},i}(f)$, $S_{\mathrm{ccr},i}(f)$, and $X_i(f)$, the SINR performance can be improved by allocating more jamming power over the frequency band $\mathcal{BW}$, which in turn leads to poorer LPI performance for the jammer system. Thus, the SINR of the distributed multiple-radar systems should be below a desired level that is determined according to the requirement for the jamming task. In other words, we obtain

$$\Delta f \sum_{i=1}^{M_{\mathrm{R}}} \sum_{k=1}^{K} \frac{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2}{J_i(f_k) + V_i(f_k)} \leq \beta_{\mathrm{SINR}}, \tag{7}$$

where β_{SINR} is a specified SINR threshold for the jamming task.

Consequently, with the optimization criterion of minimizing the total jamming power over the whole frequency band, the problem of LPI-based adaptive jamming waveform design under the SINR constraint can be mathematically formulated as follows:

$$\textit{Problem 1}: \min_{J_i(f_k)>0,\forall i} \Delta f \sum_{i=1}^{M_{\mathrm{R}}} \sum_{k=1}^{K} J_i(f_k), \tag{8a}$$

$$\text{s.t.}: \quad \Delta f \sum_{i=1}^{M_{\mathrm{R}}} \sum_{k=1}^{K} \frac{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2}{J_i(f_k) + V_i(f_k)} \leq \beta_{\mathrm{SINR}}. \tag{8b}$$

It can be noticed that since the integration kernel of (8b) is convex, the optimal jamming waveform can be obtained by employing the approach of Lagrange multipliers. We invoke the technique of Lagrange multipliers yielding the following objective function:

$$\mathcal{L}(J_i(f_k), \xi) = \Delta f \sum_{i=1}^{M_{\mathrm{R}}} \sum_{k=1}^{K} J_i(f_k) + \xi \left[\beta_{\mathrm{SINR}} - \Delta f \sum_{i=1}^{M_{\mathrm{R}}} \sum_{k=1}^{K} \frac{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2}{J_i(f_k) + V_i(f_k)} \right], \tag{9}$$

where $\xi < 0$ is the Lagrange multiplier for the constraint (8b).

Taking the derivative of $\mathcal{L}(J_i(f_k), \xi)$ with respect to $J_i(f_k)$ and setting it to zero yields the $J_i(f_k)$ that minimizes $\Delta f \sum_{i=1}^{M_{\mathrm{R}}} \sum_{k=1}^{K} J_i(f_k)$, where $J_i(f_k)$ can be given by:

$$J_i^*(f_k) = -V_i(f_k) + \sqrt{-\xi^* \cdot |H_{\mathrm{r},i}(f_k)|^2 \cdot |X_i(f_k)|^2}. \tag{10}$$

Let $\overline{A^*} = \sqrt{-\xi^*}$, rearranging terms, and ensuring $J_i^*(f_k)$ to be positive, the $J_i^*(f_k)$ that minimizes the total jamming power under SINR criterion can be expressed by

$$J_i^*(f_k) = \max\left\{0, \overline{B_i}(f_k) \left[\overline{A^*} - \overline{D_i}(f_k)\right]\right\}, \tag{11}$$

where $\overline{B_i}(f_k)$ and $\overline{D_i}(f_k)$ are given by

$$\begin{cases} \overline{B_i}(f_k) = |H_{\mathrm{r},i}(f_k)||X_i(f_k)|, \\ \overline{D_i}(f_k) = \frac{V_i(f_k)}{|H_{\mathrm{r},i}(f_k)||X_i(f_k)|}, \end{cases} \tag{12}$$

and $\overline{A^*}$ is a constant which can be calculated according to the SINR constraint:

$$\Delta f \sum_{i=1}^{M_\mathrm{R}} \sum_{k=1}^{K} \frac{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2}{V_i(f_k) + \max\left\{0, \overline{B_i}(f_k)[\overline{A^*} - \overline{D_i}(f_k)]\right\}} \leq \beta_{\mathrm{SINR}}. \tag{13}$$

Remark 1: The complex-valued jamming waveform in (11) that minimizes the total transmitted jamming power has a frequency spectrum obtained by performing the water-filling operation on the function $\overline{B_i}(f_k)\left[\overline{A^*} - \overline{D_i}(f_k)\right]$. To be specific, the optimal jamming waveform $J_i^*(f_k)$ depends not only on the target spectra and the PSDs of the signal-dependent clutters but also on the radar-transmitted waveforms. It should be noticed the importance of restricting $H_{\mathrm{r},i}(f_k)$ and $X_i(f_k)$ to be positive at each frequency sample within the bandwidth $\mathcal{BW}$, which is due to the fact that it ensures that the $\overline{D_i}(f_k)$ in (12) is always defined. In addition, the numerical search necessary to find the optimal solution to *Problem 1* is one-dimensional search over the parameter $\overline{A^*}$.

According to the above derivations, our presented iterative algorithm, which can design the LPI-based optimal jamming waveform for the distributed multiple-radar architectures, is summarized in Algorithm 1. We can see from (11) that the jamming waveform is a strictly monotonic increasing function of $\overline{A}$ (Yan et al., 2018). This monotonic property guarantees that, between the predetermined lower and upper bounds $\overline{A_{\min}}$ and $\overline{A_{\max}}$, there does exist a unique solution $\overline{A^*}$ that satisfies (13). Therefore, we utilize the bisection search method to solve it (see Algorithm 2 for details). After that, the LPI-based optimal jamming waveform design solution to Problem 1 can be obtained through (11).

Algorithm 1: LPI-Based Jamming Waveform Design for *Problem 1*

Input: Set β_{SINR}, Δf, M_R, K, iterative index $n = 1$ and the convergence parameter $\epsilon > 0$
Output: $J_i^*(f_k)$

```
while SINR^(n) − β_SINR ≤ ε do
    for i = 1, ···, M_R do
        Calculate J_i^(n)(f_k) by solving (11);
        Calculate SINR^(n) ← Δf Σ_{i=1}^{M_R} Σ_{k=1}^{K} |H_{r,i}(f_k)|^2 |X_i(f_k)|^2 / (J_i^(n)(f_k) + V_i(f_k));
        Obtain A^(n+1) via bisection search in Algorithm 2;
    end
end
Output the final solution;
```

Overall speaking, by employing the approach of Lagrange multipliers, we transform the constrained convex problem into a linear equation solving problem, and then adopt the bisection search method to tackle it (Shi et al., 2018). Therefore, our solution procedure has a much lower computational complexity in the order of $\mathcal{O}(M_\mathrm{R}\log_2(\overline{A_{\max}} - \overline{A_{\min}})/\epsilon)$, when compared with the conventional exhaustive search method. In this case, the optimal solution to *Problem 1* can be obtained through a more efficient way.

3.3. LPI-Based Jamming Waveform Design With MI Criterion

The MI between the received echoes from the target at each radar receiver and the target impulse responses is usually employed as the criterion to characterize the target parameter estimation performance in radar systems. Thus, as the opponent of a distributed multiple-radar system, the key objective of the smart jammer is to minimize the achievable MI by exploiting barrage noise jamming technique. Then, the overall MI can

Algorithm 2: Detailed Steps of the Bisection Search Approach

Input: $\overline{A^{(n)}}$, $\overline{A_{\max}}$, $\overline{A_{\min}}$, the tolerance $\epsilon > 0$
Output: $\overline{A^{(n+1)}}$

while $SINR^{(n)} - \beta_{SINR} \leq \epsilon$ **do**
  for $i = 1, \cdots, M_R$ **do**
    $\overline{A^{(n)}} \leftarrow \frac{\overline{A_{\max}}+\overline{A_{\min}}}{2}$;
    Calculate $J_i^{(n)}(f_k)$ from (11) and update SINR$^{(n)}$;
    if $SINR^{(n)} < \beta_{SINR}$ **then**
      $\overline{A_{\max}} \leftarrow \overline{A^{(n)}}$;
    else
      $\overline{A_{\min}} \leftarrow \overline{A^{(n)}}$;
    end
    $\overline{A^{(n)}} \leftarrow \frac{\overline{A_{\max}}+\overline{A_{\min}}}{2}$;
    Set $n \leftarrow n + 1$;
  end
end
Output the final solution;

be written as follows (Shi et al., 2016; Xu & Liang, 2010):

$$
\begin{aligned}
(\mathrm{MI})_{\mathrm{tot}} &\simeq \sum_{i=1}^{M_{\mathrm{R}}} T_{y_i} \int_{\mathcal{BW}} \log \left\{ 1 + \frac{|H_{\mathrm{r},i}(f)|^2 |X_i(f)|^2}{T_{y_i}[J_i(f) + V_i(f)]} \right\} \mathrm{d}f \\
&= \Delta f \sum_{i=1}^{M_{\mathrm{R}}} T_{y_i} \sum_{k=1}^{K} \log \left\{ 1 + \frac{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2}{T_{y_i}[J_i(f_k) + V_i(f_k)]} \right\},
\end{aligned} \tag{14}
$$

where $T_{y_i} = T_{h_i} + T_i$ denotes the duration of the convolution output. It is important to remark that the target estimation accuracy depends on the overall MI. As it can be observed from (14), the achievable MI is related to the target spectra $H_{\mathrm{r},i}(f)$, the PSDs of the signal-dependent clutters $S_{\mathrm{ccr},i}(f)$, the PSDs of the AWGN $S_{\mathrm{nnr},i}(f)$, and the radar-transmitted waveforms $X_i(f)$. As described in previous sections, it is evident that the minimization of the achievable MI indicates that the echoes contains as little information of the target a possible, leading to poor target estimation performance, which may be in contradiction with the LPI performance for the jammer system. Hence, the overall MI should be below a specified threshold, that is,

$$
\Delta f \sum_{i=1}^{M_{\mathrm{R}}} T_{y_i} \sum_{k=1}^{K} \log \left\{ 1 + \frac{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2}{T_{y_i}[J_i(f_k) + V_i(f_k)]} \right\} \leq \beta_{\mathrm{MI}}, \tag{15}
$$

where β_{MI} is a specified MI threshold for the jamming task.

Similarly to *Problem 1*, the LPI-based jamming waveform design strategy with MI criterion can be described as follows:

$$
\textit{Problem 2}: \quad \min_{J_i(f_k)>0,\forall i} \ \Delta f \sum_{i=1}^{M_{\mathrm{R}}} \sum_{k=1}^{K} J_i(f_k), \tag{16a}
$$

$$
\text{s.t.}: \quad \Delta f \sum_{i=1}^{M_{\mathrm{R}}} T_{y_i} \sum_{k=1}^{K} \log \left\{ 1 + \frac{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2}{T_{y_i}[J_i(f_k) + V_i(f_k)]} \right\} \leq \beta_{\mathrm{MI}}. \tag{16b}
$$

It is easy to confirm that the function within the integration kernel of (16b) is convex, and thus, the method of Lagrange multipliers can be invoked, which results in the objective function as

$$
\begin{aligned}
\mathcal{K}(J_i(f_k), \xi) &= \Delta f \sum_{i=1}^{M_{\mathrm{R}}} \sum_{k=1}^{K} J_i(f_k) \\
&+ \xi \left(\beta_{\mathrm{MI}} - \Delta f \sum_{i=1}^{M_{\mathrm{R}}} T_{y_i} \sum_{k=1}^{K} \log \left\{ 1 + \frac{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2}{T_{y_i}[J_i(f_k) + V_i(f_k)]} \right\} \right),
\end{aligned} \tag{17}
$$

where $\xi < 0$ is the Lagrange multiplier for the constraint (16b).

Table 1
Key Parameters of the System

Parameter	Value
$\mathcal{BW}$	512 MHz
$T_{y,i}(\forall i)$	0.01 s
β_{SINR}	−2.5 dB
Δf	4 MHz
$S_{\mathrm{nn},i}(f_k)$	1.66×10^{-14} W/Hz
β_{MI}	0.466 nats

Similarly, taking the derivative of $\mathcal{K}(J_i(f_k), \xi)$ with respect to $J_i(f_k)$ and setting it to 0 yields the $J_i(f_k)$ that minimizes $\Delta f \sum_{i=1}^{M_{\mathrm{R}}} \sum_{k=1}^{K} J_i(f_k)$, where $J_i(f_k)$ can be given by

$$J_i^*(f_k) = \max\left\{0, -R_i(f_k) + \sqrt{R_i^2(f_k) + S_i(f_k)\left[\bar{\bar{A^*}} - \bar{\bar{D}}_i(f_k)\right]}\right\}, \tag{18}$$

where $R_i(f_k)$, $S_i(f_k)$, and $\bar{\bar{D}}_i(f_k)$ are given by

$$\begin{cases} R_i(f_k) = V_i(f_k) + \frac{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2}{T_{y_i}}, \\ S_i(f_k) = \frac{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2}{T_{y_i}}, \\ \bar{\bar{D}}_i(f_k) = \frac{V_i^2(f_k) + |H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2 V_i(f_k)/T_{y_i}}{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2 / T_{y_i}}, \end{cases} \tag{19}$$

and $\bar{\bar{A^*}}$ is a constant that determined by the MI constraint:

$$\Delta f \sum_{i=1}^{M_{\mathrm{R}}} T_{y_i} \sum_{k=1}^{K} \log\left\{1 + \frac{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2}{T_{y_i}[J_i^*(f_k) + V_i(f_k)]}\right\} \le \beta_{\mathrm{MI}}. \tag{20}$$

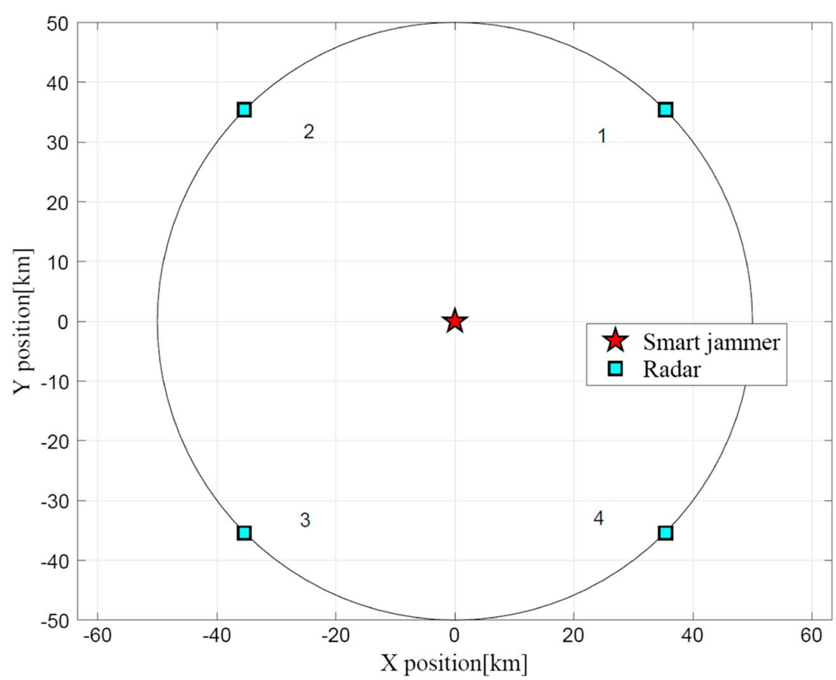

Figure 3. Deployment of the distributed multiple-radar systems with respect to the smart jammer.

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Algorithm 3: LPI-Based Jamming Waveform Design for *Problem 2*

Input: Set β_{MI}, Δf, M_{R}, K, iterative index $n = 1$ and the convergence parameter $\epsilon > 0$

Output: $J_i^*(f_k)$

while $MI^{(n)} - \beta_{MI} \le \epsilon$ **do**

  for $i = 1, \cdots, M_{\mathrm{R}}$ **do**

    Calculate $J_i^{(n)}(f_k)$ by solving (24);

    Calculate $\mathrm{MI}^{(n)} \leftarrow \Delta f \sum_{i=1}^{M_{\mathrm{R}}} T_{y_i} \sum_{k=1}^{K} \log \left\{ 1 + \frac{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2}{T_{y_i}[J_i^{(n)}(f_k)+V_i(f_k)]} \right\}$;

    Obtain $\overline{\overline{A^{(n+1)}}}$ via bisection search in *Algorithm 2*;

  end

end

Output the final solution;

To obtain further intuition, we apply a first-order Taylor approximation to

$$Q_i(f_k) = -R_i(f_k) + \sqrt{R_i^2(f_k) + S_i(f_k)\left[\bar{\bar{A}}^* - \bar{\bar{D}}_i(f_k)\right]}, \tag{21}$$

and the approximation yields:

$$Q_i(f_k) \simeq \bar{\bar{B}}_i(f_k)\left[\bar{\bar{A}}^* - \bar{\bar{D}}_i(f_k)\right], \tag{22}$$

where $\bar{\bar{B}}_i(f_k)$ is expressed by

$$\bar{\bar{B}}_i(f_k) = \frac{|H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2 / T_{y_i}}{2V_i(f_k) + |H_{\mathrm{r},i}(f_k)|^2 |X_i(f_k)|^2 / T_{y_i}}. \tag{23}$$

Therefore, the $J_i^*(f_k)$ that minimizes the total jamming power under MI criterion is given by

$$J_i^*(f_k) \simeq \max\left\{0, \bar{\bar{B}}_i(f_k)\left[\bar{\bar{A}}^* - \bar{\bar{D}}_i(f_k)\right]\right\}. \tag{24}$$

Remark 2: Remarkably, it is worth to mention that the complex-valued jamming waveform in (24) that minimizes the total transmitted jamming power also performs a water-filling operation. The LPI-based jamming waveform design approach under MI criterion is depicted in Algorithm 3.

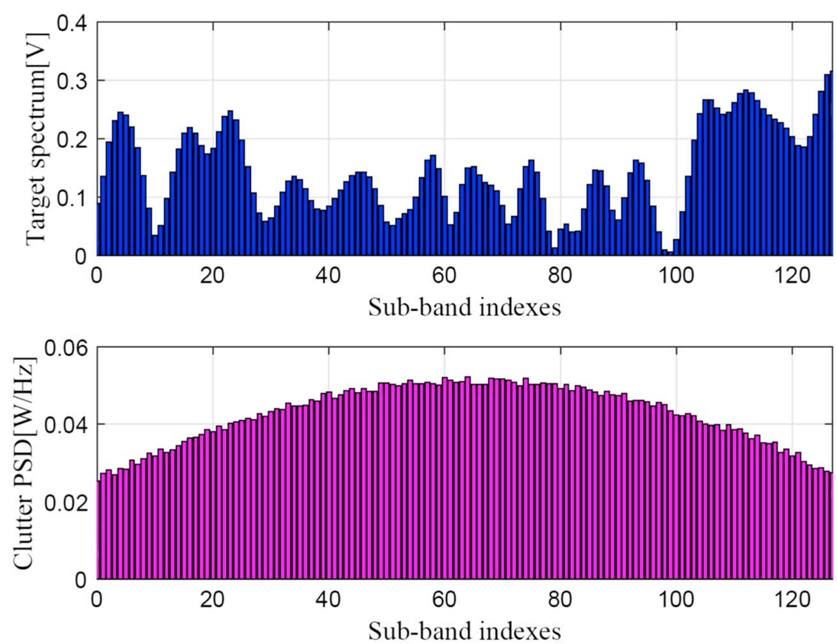

Figure 4. Target and signal-dependent clutter spectra with respect to Radar 1.

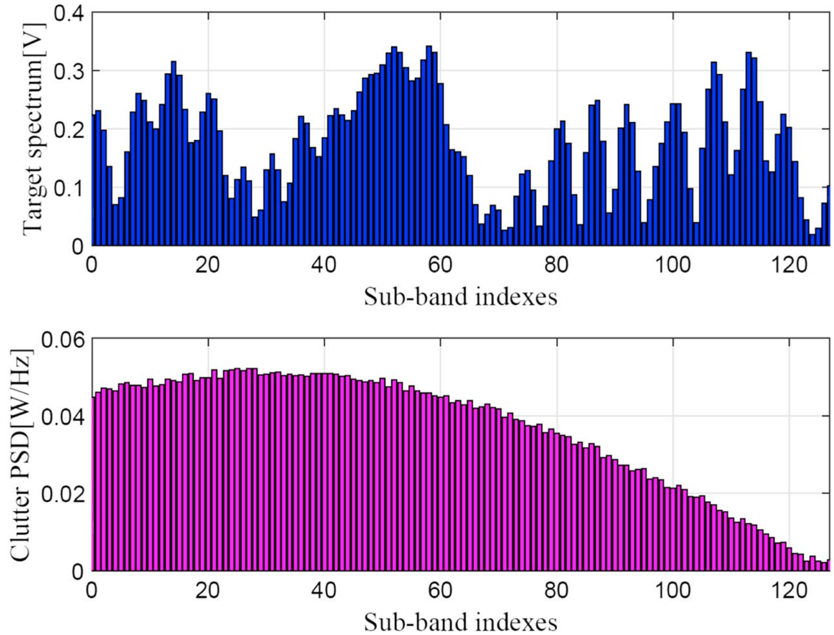

Figure 5. Target and signal-dependent clutter spectra with respect to Radar 2.

4. Performance Evaluation Results and Discussion

In this section, various numerical simulations are provided to validate the accuracy of the theoretical derivations as well as show the improvement of LPI performance for smart jammer brought by our presented jamming waveform design criteria.

In all simulations, we consider a distributed multiple-radar architecture with $M_{\mathrm{R}} = 4$ spatially diverse radars. Unless otherwise specified, the default values for the system parameters are utilized as provided in Table 1. Without loss of generality, we utilize the distributed multiple-radar system in Figure 3 as an example. It is noteworthy that the antenna beam of the smart jammer is very narrow so that it can respond independently to each other. Throughout the simulations, the carrier frequency of the multiple-radar system is

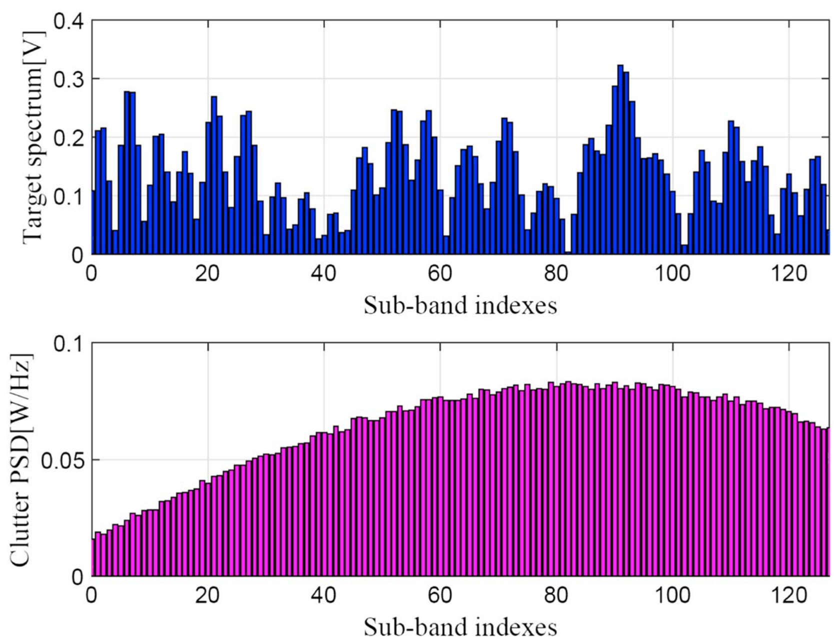

Figure 6. Target and signal-dependent clutter spectra with respect to Radar 3.

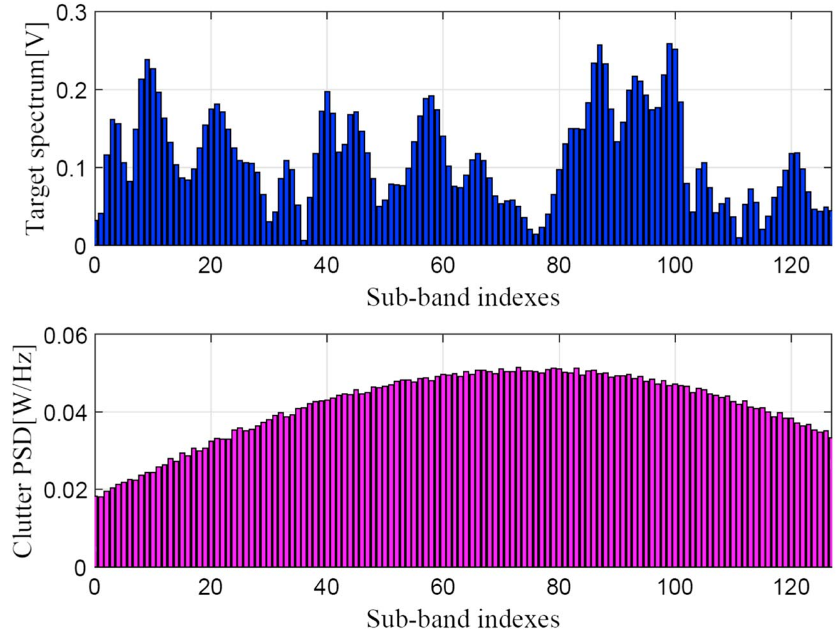

Figure 7. Target and signal-dependent clutter spectra with respect to Radar 4.

3 GHz. It should be pointed out that the predefined target detection constraint β_{SINR} is set to be −2.5 dB, which is approximately equivalent to the values of $\beta_{MI} = 0.466$ nats for a desired target parameter estimation performance (Romero et al., 2011; Shi et al., 2018). In order to solve the problems of LPI-based jamming waveform design *Problem 1* and *Problem 2*, it is also assumed that the precise knowledge of the target spectra, the PSDs of the signal-dependent clutters, and the radar-transmitted waveforms are perfectly estimated by the smart jammer (Shi, Zhou, & Wang, 2017; L. L. Wang et al., 2014).

4.1. Jamming Waveform Design Results

As mentioned before, it is assumed that the exact knowledge of the target spectra, the PSDs of the signal-dependent clutters, and the radar-transmitted waveforms are perfectly estimated by the jammer

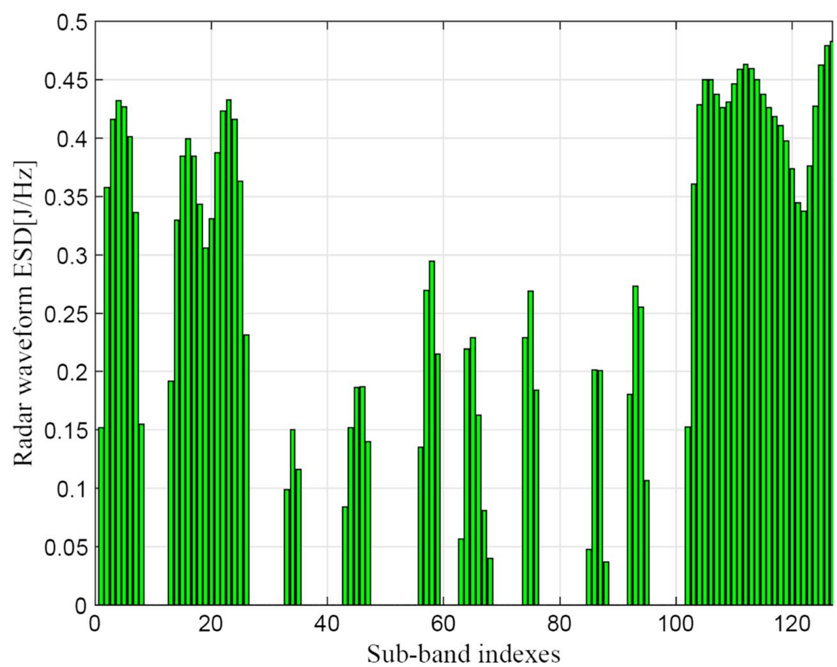

Figure 8. Transmitted waveform of Radar 1.

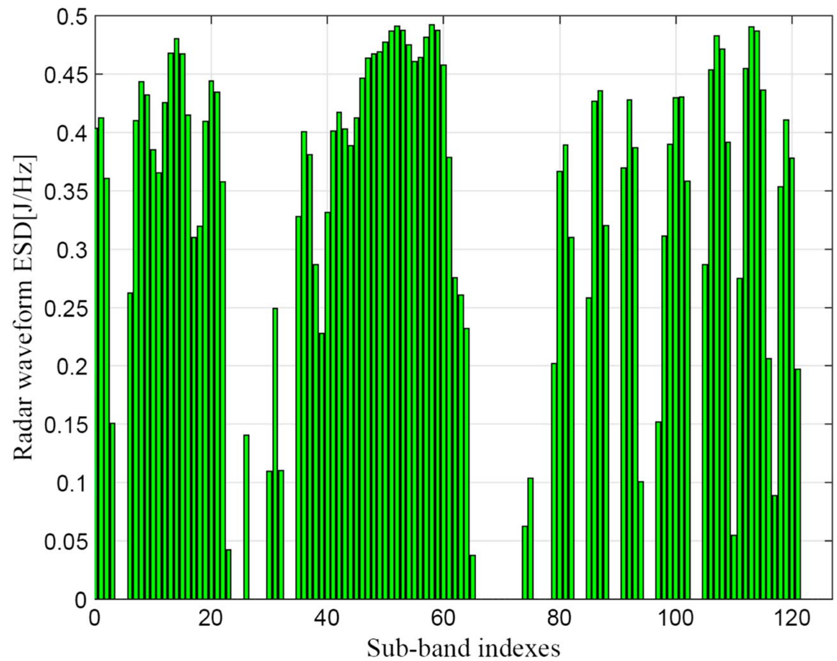

Figure 9. Transmitted waveform of Radar 2.

system. The target spectra and PSDs of signal-dependent clutters with respect to different radars are illustrated in Figures 4–7, respectively. The transmitted waveforms of different radars are shown in Figures 8–11, respectively, which can be intercepted and estimated by the jammer through previous received signals. Without loss of generality, the transmitted waveform of each radar is designed to maximize the target information. The jamming waveform design results of the aforementioned two strategies are provided in Figures 12–15, respectively, which give insight about the jamming power allocation for the LPI performance of smart jammer. It is evident that for both strategies proposed in this paper, the two LPI-based jamming waveform design schemes lead to different jamming power allocation results. The SINR criterion degrades the target detection

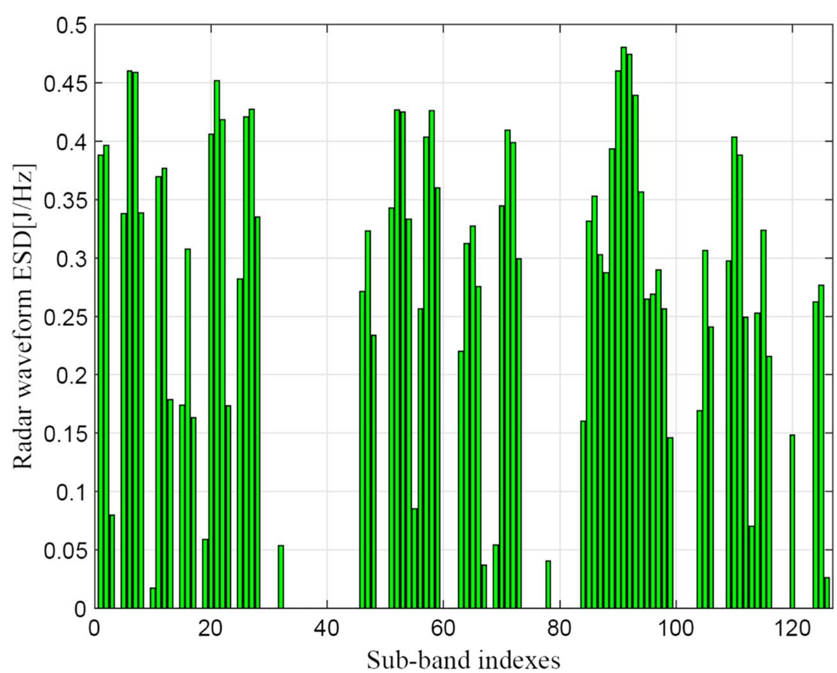

Figure 10. Transmitted waveform of Radar 3.

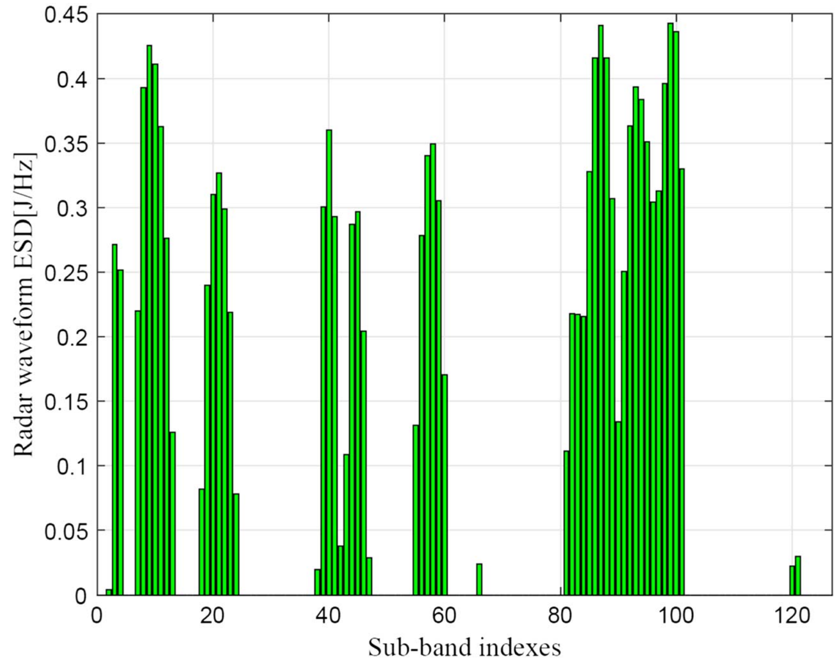

Figure 11. Transmitted waveform of Radar 4.

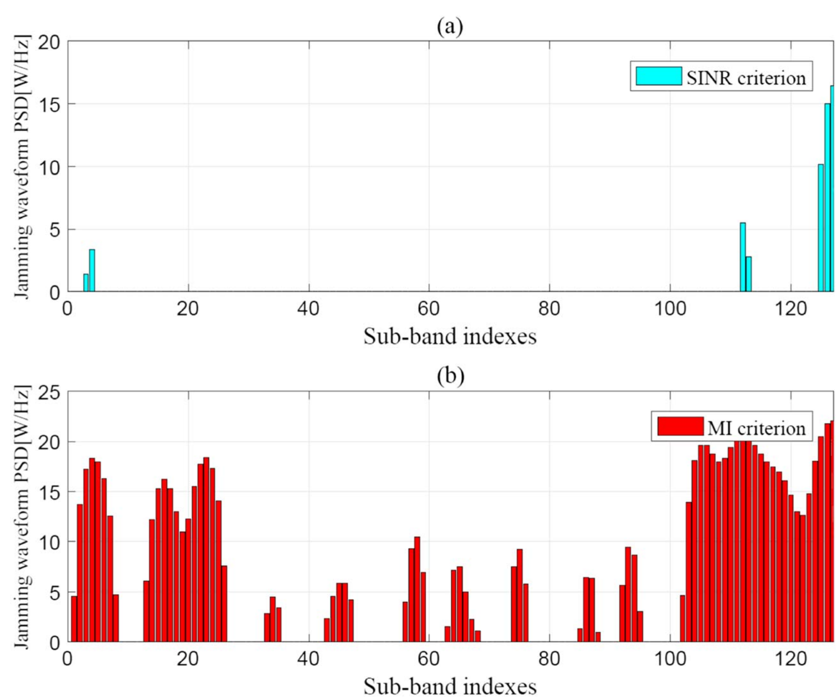

Figure 12. Low probability of intercept-based jamming waveform design results for Radar 1: (a) Signal-to-interference-plus-noise ratio (SINR) criterion; (b) mutual information (MI) criterion. PSD = power spectral density.

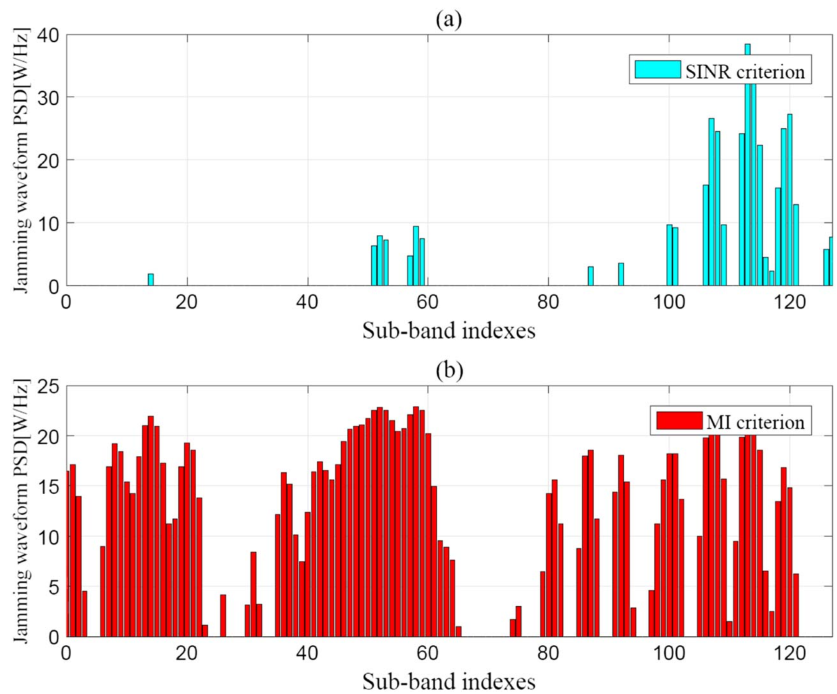

Figure 13. Low probability of intercept-based jamming waveform design results for Radar 2: (a) Signal-to-interference-plus-noise ratio (SINR) criterion; (b) mutual information (MI) criterion. PSD = power spectral density.

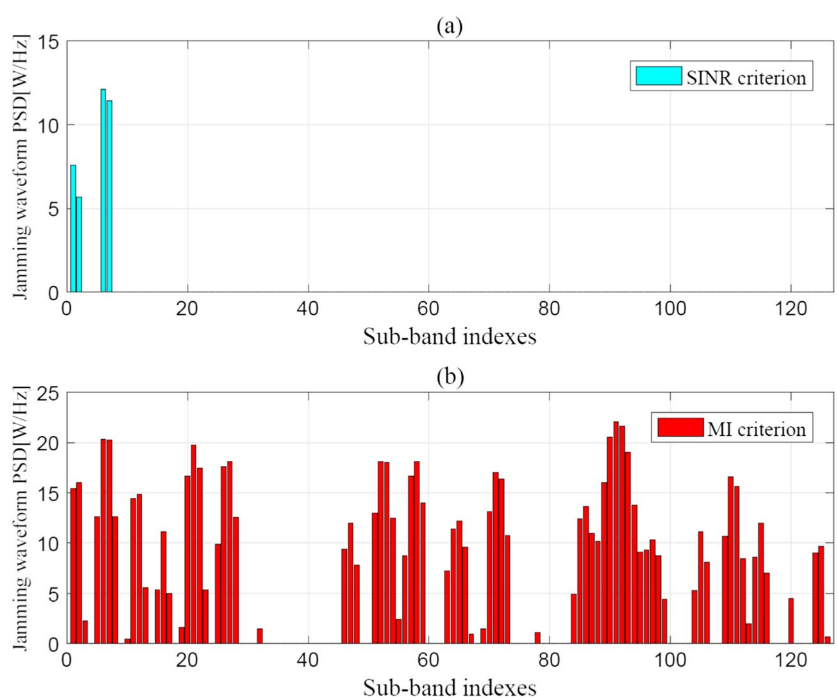

Figure 14. Low probability of intercept-based jamming waveform design results for Radar 3: (a) Signal-to-interference-plus-noise ratio (SINR) criterion; (b) mutual information (MI) criterion. PSD = power spectral density.

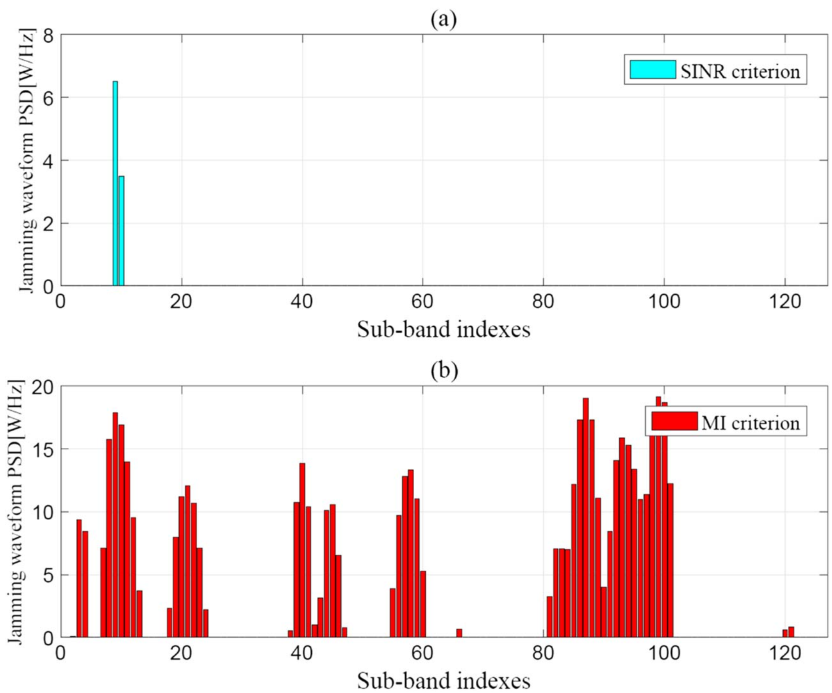

Figure 15. Low probability of intercept-based jamming waveform design results for Radar 4: (a) Signal-to-interference-plus-noise ratio (SINR) criterion; (b) mutual information (MI) criterion. PSD = power spectral density.

performance to a predetermined SINR level, which is able to prevent the target being detected. Similarly, the MI criterion degrades the target parameter estimation performance to a specified MI level. Physically speaking, the LPI-based jamming waveform design strategies are determined by the target spectra, the PSDs of the signal-dependent clutters, the PSDs of the AWGN, and the transmitted waveform of each radar. Specifically, for the situation where the radar-transmitted waveform $|X_i(f)|^2$ is large, we should concentrate more jamming power for the frequency subbands of jamming waveform. Whereas for the situation where $|X_i(f)|^2$ is weak, we should allocate less power for the corresponding subbands of jamming waveform. From the above figures, it can be clearly observed that the LPI-based jamming waveform design scheme with SINR criterion is formed by the water-filling action to minimize the total transmitted jamming power for a specified SINR requirement, which only allocates the minimum jamming power into the single frequency subband with the largest value of $|X_i(f)|^2$. However, the LPI-based jamming waveform design scheme with MI criterion tends to place the jamming power into multiple frequency subbands. This is because the logarithm calculation lowers the values of these large coefficients, which allows for less dominant frequency subbands in the MI spectral density to be significant.

Figure 16 shows the jamming power ratio results employing the LPI-based jamming waveform design strategies with SINR and MI criteria, respectively, which give insight about jamming power allocation for the LPI performance of a smart jammer. In Figure 16, different colors denote the ratios of the transmitted jamming power for different radars. Here the jamming power ratio is defined as $\eta_i(f_k) = \frac{\sum_{k=1}^{K} J_i(f_k)}{\sum_{i=1}^{M_R} \sum_{k=1}^{K} J_i(f_k)}$. Note that similar jamming waveform design results can be obtained in the cases where the bandwidth is ultra narrow or ultra wide. Whether the bandwidth is narrow or wide depends on different applications. For example, the conventional radars are ultra narrow wideband systems, where the targets are modeled as point targets, whereas the ultra wideband radar can be utilized for target recognition, target imaging, etc. Moreover, it should be pointed out that these two schemes are easy to realize and will undoubtedly achieve better LPI performance for the smart jammer system.

4.2. LPI Performance of the Smart Jammer

For the sake of completeness, let us express the superiority of the proposed LPI-based jamming waveform design strategies on the LPI performance of a smart jammer. Figure 17 depicts the comparisons of total jamming power for different algorithms, where Monte Carlo simulations with 5×10^4 independent trials

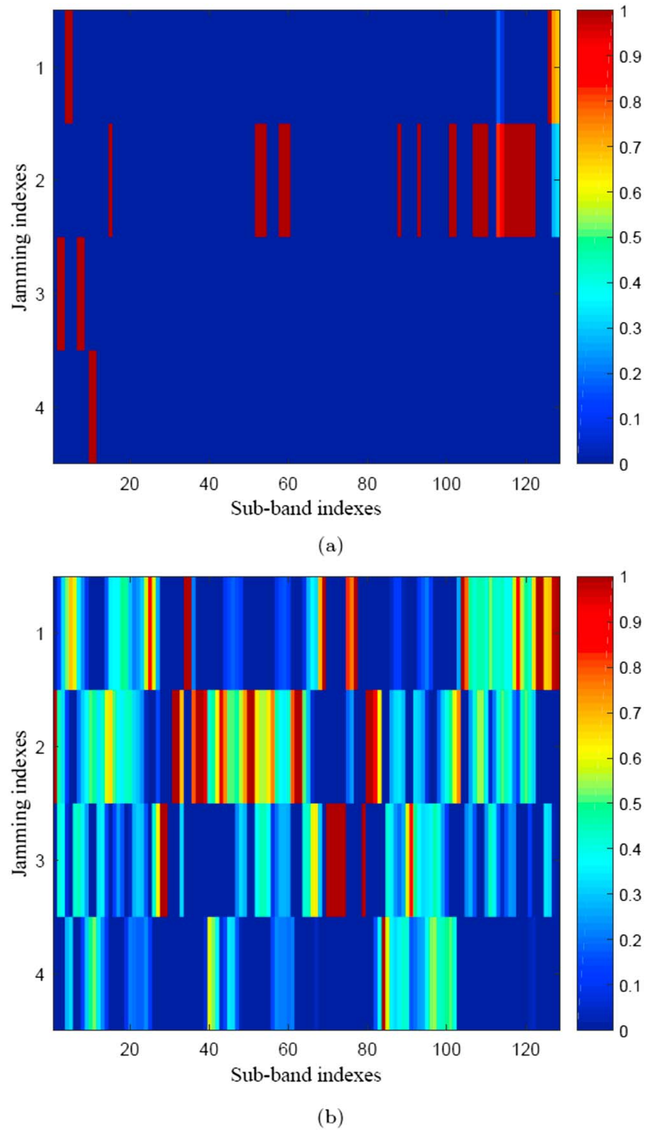

Figure 16. The transmit power ratio results of smart jammer: (a) LPI-based jamming waveform design strategy with signal-to-interference-plus-noise ratio criterion; (b) LPI-based jamming waveform design strategy with mutual information criterion. LPI = low probability of intercept.

are conducted to achieve an average performance. The aim of this comparison is to show how the presented strategies are able to improve the LPI performance of jammer system. Analyzing the results in Figure 17, we can observe that a better LPI performance of jammer system can be obtained when employing the LPI-based optimal jamming waveforms, which are optimized supposing that the precise target spectra, the PSDs of the signal-dependent clutters, and the radar-transmitted waveforms are known. It shows that the smart jammer transmits the minimum power to interfere with multiple radars for a predetermined SINR/MI threshold.

Besides, the total transmitted jamming power of the uniform jamming waveform in both criteria is also presented in Figure 17, where the uniform jamming waveform design methods are considered as the corresponding benchmarks. As introduced in Shi, Zhou, and Wang (2017) and L. L. Wang et al. (2014), the uniform jamming waveform spreads the barrage noise jamming power averagely in the whole frequency band without any prior information about the target, clutter, and radar-transmitted waveforms. As predicted in previous sections, for the same SINR/MI threshold value, the LPI performance improvement is quite apparent compared with the uniform jamming waveform design schemes which are usually adopted in traditional barrage noise jamming because of not employing the prior knowledge. Also, as the specified

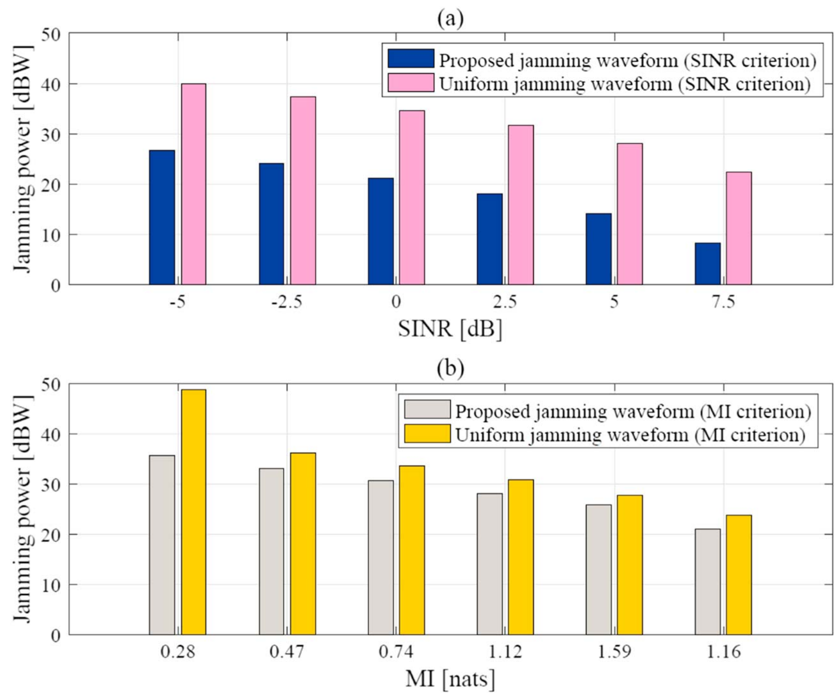

Figure 17. Comparisons of total jamming power for different criteria: (a) LPI-based jamming waveform design strategy with signal-to-interference-plus-noise ratio (SINR) criterion; (b) LPI-based jamming waveform design strategy with mutual information (MI) criterion. LPI = low probability of intercept.

SINR/MI threshold increases, less jamming power is transmitted to impair the target detection/estimation performance for the distributed multiple-radar architectures, and vice versa.

On the other hand, one can see from Figure 17a together with Figure 17b that more jamming power are saved by exploiting the LPI-based jamming waveform design strategy with SINR criterion, which indicates that the SINR criterion only distributes a minimum noise jamming power into a single frequency subband with the largest gain of $|X_i(f)|^2$.

4.3. Discussion

Combining the previous results, it can be concluded that the proposed LPI-based jamming waveform design strategies are particularly attractive for practical implementations for the following reasons (Yan et al., 2018). First, the presented jamming waveform design schemes can significantly reduce the total transmitted jamming power of a smart jammer system when compared with the uniform design scheme. Second, according to the discussions in L. L. Wang et al. (2014), the proposed jamming waveform design strategies are robust and can be used in practical scenario, where the precise characteristics of radar waveforms are impossible to capture. Third, the proposed LPI-based jamming waveform design schemes give insight about jamming power allocation for the LPI performance of a smart jammer and result in different jamming waveform design results, which are useful to guide jamming power allocation for various jamming tasks.

5. Conclusion and Future Work

In this paper, we have studied the problem of LPI-based jamming waveform design for distributed multiple-radar architectures. The mathematical expressions of SINR and MI are derived to characterize the target detection and parameter estimation performance respectively, and two different LPI-based jamming waveform design criteria are presented to minimize the total jamming power of the smart jammer by optimizing the transmit jamming waveform while the achieved SINR/MI is enforced to be below a predetermined threshold. The resulting problems are formulated and solved analytically. Numerical simulations have been provided to demonstrate that the LPI performance of the jammer is remarkably enhanced by employing the proposed LPI-based jamming waveform design criteria. To generalize the proposed jamming waveform design strategies, our future work is to investigate the problem of robust jamming waveform design for distributed multiple-radar systems in the presence of the spectral uncertainty of radar waveforms.

Acknowledgments
We note that there are no data sharing issues since all of the numerical information is provided in the figures, which are realized by MATLAB software. The MATLAB programs and numerical data are available as supporting information. This research is supported by the National Natural Science Foundation of China (Grants 61801212, 61371170, and 61671239), the Natural Science Foundation of Jiangsu Province (Grant BK20180423), the National Aerospace Science Foundation of China (Grants 20172752019 and 2017ZC52036), the Priority Academic Program Development of Jiangsu Higher Education Institutions (PADA), and Key Laboratory of Radar Imaging and Microwave Photonics (Nanjing University Aeronautics and Astronautics), Ministry of Education, Nanjing University of Aeronautics and Astronautics, Nanjing, 210016, China. In particular, the author Chenguang Shi would like to highlight the unwavering and invaluable support of his wife, Ying Hu.

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