Physics Letters B 732 (2014) 85–90

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Radiative lepton model and dark matter with global $U(1)'$ symmetry

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ARTICLE INFO

Article history:
Received 4 February 2014
Accepted 12 March 2014
Available online 19 March 2014
Editor: J. Hisano

ABSTRACT

We propose a radiative lepton model, in which the charged lepton masses are generated at one-loop level, and the neutrino masses are induced at two-loop level. On the other hand, tau mass is derived at tree level since it is too heavy to generate radiatively. Then we discuss muon anomalous magnetic moment together with the constraint of lepton flavor violation. A large muon magnetic moment is derived due to the vector like charged fermions which are newly added to the standard model. In addition, considering a scalar dark matter in our model, a strong gamma-ray signal is produced by dark matter annihilation via internal bremsstrahlung. We can also obtain the effective neutrino number by the dark radiation of the Goldstone boson coming from the imposed global $U(1)'$ symmetry.

1. Introduction

Even though 26.8% of energy density of our Universe is occupied by a non-baryonic dark matter (DM) [1,2], several current experiments are still under investigation of its nature from various points of view such as direct and indirect searches. As for the direct detection search, for example, XENON100 [3] and LUX [4] provide the most severe constraint on spin independent elastic cross section with nuclei; that is, the cross sections is less than around 10^{-46} cm^2 at 100 GeV scale of DM mass. As for the indirect searches, AMS-02 has recently shown the positron excess with smooth curve in the cosmic ray, and reached the energy up to 350 GeV [5]. This result has a good statistics and supports the previous experiment by PAMELA [6]. On the other hand, the recent analysis of gamma-ray observed by Fermi-LAT tells us that there may be some peak near 130 GeV [7,8]. As for the neutrinos, their small masses and mixing pattern call for new physics beyond the standard model (SM). Plank, WMAP9 and ground-based data recently reported a possible deviation in the effective neutrino number, $\Delta N_{\rm eff} = 0.36 \pm 0.34$ at 68% confidential level [2,9–11]. Compensating this deviation theoretically might come into one of the important issues. In this sense, radiative seesaw models which support a strong correlation between DM and neutrinos come into an elegant motivation. Many authors have proposed such kind of models in, *e.g.*, Refs. [12–46].[1]

In our paper, we propose a model that neutrino masses as well as charged lepton (muon and electron) masses are generated by radiative correction. We obtain a large contribution to muon anomalous magnetic moment from the charged lepton sector as can be seen later. At the same time, one should mind constraints from lepton flavor violations (LFVs) like $\mu \to e\gamma$ since it is closely correlated with anomalous magnetic moment. Since neutrino masses are generated at two-loop level, they are therefore naturally suppressed. As a result, unlike the TeV scale canonical seesaw mechanism, extremely small parameters are not required to lead the observed neutrino mass scale. Moreover the particles running in the loop can be DM candidates. Our scalar DM interacts with vector like charged fermions, which are added to the SM, and the other interaction should be suppressed to satisfy the direct search constraint. Due to the interaction with the vector like charged fermions, a strong gamma-ray signal is emitted by the DM annihilation via internal bremsstrahlung preserving consistency with the thermal relic density of DM [52,53]. In particular it is possible to adapt with the gamma-ray anomaly found in the Fermi data at around 130 GeV. The neutrino effective number is also led without conflicting with the other parts of DM physics.

This paper is organized as follows. In Section 2, we show our model building for the lepton sector, and discuss Higgs sector,

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[1] Radiative models of the lepton mass are sometimes discussed with non-Abelian discrete symmetries due to their selection rules. See for example such kind of models: [47–51].

http://dx.doi.org/10.1016/j.physletb.2014.03.021

Table 1
The particle contents and the charges for fermions. The i, j are generation indices: $i = 1, 2, 3$, $j = 1, 2$.

Particle	L_i	e_j^c	e_3^c	e_i'	$e_i'^c$	n_j'	$n_j'^c$	N^c
$(SU(2)_L, U(1)_Y)$	$(\mathbf{2}, -1/2)$	$(\mathbf{1}, 1)$	$(\mathbf{1}, 1)$	$(\mathbf{1}, -1)$	$(\mathbf{1}, 1)$	$(\mathbf{1}, 0)$	$(\mathbf{1}, 0)$	$(\mathbf{1}, 0)$
$U(1)' \times Z_2$	$(\ell, -)$	$(0, -)$	$(-\ell, -)$	$(\ell, +)$	$(-\ell, +)$	$(\ell, +)$	$(-\ell, +)$	$(0, -)$

Table 2
The particle contents and the charges for bosons.

Particle	Φ	η	χ	Σ
$(SU(2)_L, U(1)_Y)$	$(\mathbf{2}, 1/2)$	$(\mathbf{2}, 1/2)$	$(\mathbf{1}, 0)$	$(\mathbf{1}, 0)$
$U(1)' \times Z_2$	$(0, +)$	$(0, -)$	$(-\ell, -)$	$(\ell, +)$

muon anomalous magnetic moment, and LFV. In Section 3, DM phenomenology such as relic density, strong gamma-ray signal and the neutrino effective number is discussed. We summarize and conclude in Section 4.

2. The model

2.1. Model setup

We construct a radiative lepton model with global $U(1)'$ symmetry, in which charged lepton sector is obtained through one-loop level, and two-loop level for neutrino sector. In the model, only tau mass is generated at tree level, but electron and muon masses are generated at one-loop level. This is because tau mass is too heavy to generate radiatively. The particle contents are shown in Tables 1 and 2. The quantum number $\ell\,(\neq 0)$ in the tables is arbitrary. Here L_i and e_i^c $(i = 1, 2, 3)$ are the SM left-handed and right-handed lepton fields. For right-handed charged leptons e_i^c $(i = 1, 2, 3)$, different charges of $U(1)'$ are assigned to the first, second generation and the third generation in order to distinguish the mass generation mechanism. We add three generations of $SU(2)_L$ singlet vector like charged fermions e_i' and $e_i'^c$ $(i = 1, 2, 3)$, two generations of vector like neutral fermions n_j' and $n_j'^c$ $(j = 1, 2)$, a singlet Majorana fermion N^c.[2] For new bosons, we introduce $SU(2)_L$ doublet scalar η and singlet scalars χ and Σ in addition to the SM Higgs doublet Φ. The SM Higgs Φ should be neutral under $U(1)'$ not to couple quarks to Goldstone boson through chiral anomaly to be consistent with the axion particle search.[3] We assume that only the SM Higgs doublet Φ and the SM singlet Σ have vacuum expectation values. Otherwise the $\mathbb{Z}_2$ symmetry which guarantees DM stability is spontaneously broken.

The renormalizable Lagrangian for Yukawa sector and scalar potential are given by

$$\begin{aligned}
\mathcal{L}_Y &= y_n^\eta n'^c L\eta + y_n^\chi N^c n'\chi + \frac{M_N}{2} N^c N^c + M_{n'} n'^c n' + \text{h.c.} \\
&\quad + y_\tau^\Phi \Phi^\dagger e_3^c L + y_\ell^\eta \eta^\dagger e'^c L + y_\ell^\chi e_{1,2}^c e'\chi \\
&\quad + M_{e'} e' e'^c + \text{h.c.}, \qquad (2.1) \\
\mathcal{V} &= m_1^2 \Phi^\dagger\Phi + m_2^2 \eta^\dagger\eta + m_3^2 \Sigma^\dagger\Sigma + m_4^2 \chi^\dagger\chi \\
&\quad + \lambda_1 (\Phi^\dagger\Phi)^2 + \lambda_2 (\eta^\dagger\eta)^2 + \lambda_3 (\Phi^\dagger\Phi)(\eta^\dagger\eta) \\
&\quad + \lambda_4 (\Phi^\dagger\eta)(\eta^\dagger\Phi) + \lambda_5 [(\Phi^\dagger\eta)^2 + \text{h.c.}] \\
&\quad + \lambda_5' [(\Sigma^\dagger\chi)^2 + \text{h.c.}] + \lambda_5'' [(\Sigma\chi)^2 + \text{h.c.}] \\
&\quad + \lambda_6 (\Sigma^\dagger\Sigma)^2 + \lambda_6' (\chi^\dagger\chi)^2 + \lambda_6'' (\Sigma^\dagger\Sigma)(\chi^\dagger\chi) \\
&\quad + \lambda_7 (\Sigma^\dagger\Sigma)(\Phi^\dagger\Phi) + \lambda_7' (\chi^\dagger\chi)(\Phi^\dagger\Phi) \\
&\quad + \lambda_8 (\Sigma^\dagger\Sigma)(\eta^\dagger\eta) + \lambda_8' (\chi^\dagger\chi)(\eta^\dagger\eta) \\
&\quad + [a(\eta^\dagger\Phi)(\Sigma\chi) + \text{h.c.}] + [a'(\Phi^\dagger\eta)(\Sigma\chi) + \text{h.c.}], \qquad (2.2)
\end{aligned}$$

where λ_5, λ_5', λ_5'', and one of a and a' can be chosen to be real without any loss of generality by absorbing the phases to scalar bosons. The $\Phi^\dagger e_i'^c L$ term which might generate mixing between $e_i'^c$ and e_3^c is not allowed by the $\mathbb{Z}_2$ symmetry. The Yukawa interaction $\Phi^\dagger e_{1,2}^c L$ which gives the tree level masses of electron and muon is forbidden by $U(1)'$ symmetry. The term $N^c L\eta$ which induces one-loop neutrino masses [12] is also excluded by $U(1)'$ symmetry. The couplings λ_1, λ_2, λ_6 and λ_6' have to be positive to stabilize the Higgs potential. Insert the tadpole conditions; $m_1^2 = -\lambda_1 v^2 - \lambda_7 v'^2/2$ and $m_3^2 = -\lambda_6 v'^2 - \lambda_7 v^2/2$, the resulting mass matrix of the neutral component of Φ and Σ defined as

$$\Phi^0 = \frac{v + \phi^0(x)}{\sqrt{2}}, \qquad \Sigma = \frac{v' + \sigma(x)}{\sqrt{2}} e^{iG(x)/v'}, \qquad (2.3)$$

is given by

$$\begin{aligned}
m^2(\phi^0, \sigma) &= \begin{pmatrix} 2\lambda_1 v^2 & \lambda_7 v v' \\ \lambda_7 v v' & 2\lambda_6 v'^2 \end{pmatrix} \\
&= \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} m_h^2 & 0 \\ 0 & m_H^2 \end{pmatrix} \\
&\quad \times \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \qquad (2.4)
\end{aligned}$$

where h implies SM-like Higgs with the mass of 125 GeV and H is an additional CP-even Higgs mass eigenstate. The mixing angle α is given by

$$\tan 2\alpha = \frac{\lambda_7 v v'}{\lambda_6 v'^2 - \lambda_1 v^2}. \qquad (2.5)$$

The Higgs bosons ϕ^0 and σ are rewritten in terms of the mass eigenstates h and H as

$$\begin{aligned}
\phi^0 &= h\cos\alpha + H\sin\alpha, \\
\sigma &= -h\sin\alpha + H\cos\alpha. \qquad (2.6)
\end{aligned}$$

A Goldstone boson G appears due to the spontaneous symmetry breaking of the global $U(1)'$ symmetry. This massless particle would be dark radiation contributing to the effective neutrino number we will discuss later [54].

The resulting mass matrix of the neutral component of η and χ defined as

$$\eta^0 = \frac{\eta_R + i\eta_I}{\sqrt{2}}, \qquad \chi = \frac{\chi_R + i\chi_I}{\sqrt{2}}, \qquad (2.7)$$

is given by

[2] Multi-component vector like fermions are required to produce the observed charged lepton masses and neutrino oscillation data. There are other patterns of particle content to derive proper lepton masses.

[3] If Φ is charged under $U(1)'$, its breaking scale should be very large ($\gtrsim 10^{12}$ GeV), which is inconsistent with the observed value ~ 246 GeV.

$$m^2(\eta_R, \chi_R) = \begin{pmatrix} m^2_{\eta_R} & m^2_{\eta_R\chi_R} \\ m^2_{\eta_R\chi_R} & m^2_{\chi_R} \end{pmatrix}$$
$$= \begin{pmatrix} \cos\beta_R & \sin\beta_R \\ -\sin\beta_R & \cos\beta_R \end{pmatrix} \begin{pmatrix} m^2_{h'_R} & 0 \\ 0 & m^2_{H'_R} \end{pmatrix}$$
$$\times \begin{pmatrix} \cos\beta_R & -\sin\beta_R \\ \sin\beta_R & \cos\beta_R \end{pmatrix}, \tag{2.8}$$

for CP even mass eigenstates where h'_R and H'_R are mass eigenstates of inert Higgses. The imaginary part of these inert Higgses (CP odd states) is defined by replacing the index R into I, hereafter. The mixing angle β_R is given by

$$\tan 2\beta_R = \frac{2m^2_{\eta_R\chi_R}}{m^2_{\chi_R} - m^2_{\eta_R}}. \tag{2.9}$$

The η_R and χ_R are rewritten in terms of the mass eigenstates h'_R and H'_R as

$$\eta_R = h'_R \cos\beta_R + H'_R \sin\beta_R,$$
$$\chi_R = -h'_R \sin\beta_R + H'_R \cos\beta_R. \tag{2.10}$$

Each mass component is defined as

$$m^2_\eta \equiv m^2(\eta^\pm) = m_2^2 + \frac{1}{2}\lambda_3 v^2 + \frac{1}{2}\lambda_8 v'^2, \tag{2.11}$$

$$m^2_{\eta_R} \equiv m^2(\eta_R) = m_2^2 + \frac{1}{2}\lambda_8 v'^2 + \frac{1}{2}(\lambda_3 + \lambda_4 + 2\lambda_5)v^2, \tag{2.12}$$

$$m^2_{\eta_I} \equiv m^2(\eta_I) = m_2^2 + \frac{1}{2}\lambda_8 v'^2 + \frac{1}{2}(\lambda_3 + \lambda_4 - 2\lambda_5)v^2, \tag{2.13}$$

$$m^2_{\chi_R} \equiv m^2(\chi_R)$$
$$= m_3^2 + \frac{1}{2}\left(\frac{1}{2}\lambda''_6 v'^2 + \frac{1}{2}\lambda'_7 v^2 + \lambda'_5 v'^2 + \lambda''_5 v'^2\right), \tag{2.14}$$

$$m^2_{\chi_I} \equiv m^2(\chi_I)$$
$$= m_3^2 + \frac{1}{2}\left(\frac{1}{2}\lambda''_6 v'^2 + \frac{1}{2}\lambda'_7 v^2 - \lambda'_5 v'^2 - \lambda''_5 v'^2\right), \tag{2.15}$$

$$m^2_{\eta_R\chi_R} = \frac{1}{4}vv'(a + a'), \qquad m^2_{\eta_I\chi_I} = \frac{1}{4}vv'(a - a'). \tag{2.16}$$

We note that we need mass splitting between $\eta_R(\chi_R)$ and $\eta_I(\chi_I)$ which is required to generate the non-zero lepton masses. The tadpole conditions for η and χ, which are given by $\partial\mathcal{V}/\partial\eta|_{\rm VEV} = 0$, $\partial\mathcal{V}/\partial\chi|_{\rm VEV} = 0$, $0 < \partial^2\mathcal{V}/\partial\eta^2|_{\rm VEV}$ and $0 < \partial^2\mathcal{V}/\partial\chi^2|_{\rm VEV}$ tell us that

$$0 < m_2^2 + \frac{v^2}{2}(\lambda_3 + \lambda_4 + 2\lambda_5) + \frac{v'^2}{2}\lambda_8,$$
$$0 < m_4^2 + \frac{v^2}{2}\lambda'_7 + \frac{v'^2}{2}(\lambda'_5 + \lambda''_5 + \lambda''_6), \tag{2.17}$$

to satisfy the condition $\langle\eta\rangle = 0$ and $\langle\chi\rangle = 0$ at tree level, respectively. In order to avoid that $\langle\Phi\rangle = \langle\Sigma\rangle = 0$ be a local minimum, we require the following condition:

$$\lambda_7 - \frac{2}{3}\sqrt{\lambda_1\lambda_6} < 0. \tag{2.18}$$

To achieve the global minimum at $\langle\eta\rangle = \langle\chi\rangle = 0$, we find the following condition

$$0 < \lambda'_8 - \frac{2}{3}\sqrt{\lambda_2\lambda'_6}. \tag{2.19}$$

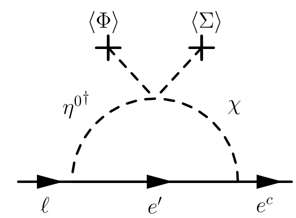

Fig. 1. Radiative generation of charged lepton masses.

Finally, if the following conditions

$$0 < \lambda_3 + \frac{2}{3}\sqrt{\lambda_1\lambda_2}, \qquad 0 < \lambda_7 + \frac{2}{3}\sqrt{\lambda_1\lambda_6},$$
$$0 < \lambda'_7 + \frac{2}{3}\sqrt{\lambda_1\lambda'_6}, \qquad 0 < \lambda_8 + \frac{2}{3}\sqrt{\lambda_2\lambda_6},$$
$$0 < \lambda'_8 + \frac{2}{3}\sqrt{\lambda_2\lambda'_6}, \qquad 0 < \lambda''_6 + \frac{2}{3}\sqrt{\lambda_6\lambda'_6}, \tag{2.20}$$

are satisfied, the Higgs potential Eq. (2.2) is bounded from below.

2.2. Charged lepton and neutrino mass matrix

The tau mass is given at tree level, after the spontaneous symmetry breaking as $m_\tau = y^\Phi_\tau v/\sqrt{2}$. On the other hand, the electron and muon masses are generated at one-loop, as can be seen in Fig. 1 as follows:

$$(m_\ell)_{\alpha\beta} = \sum_i \frac{(y^\eta_\ell)_{\alpha i}(y^\chi_\ell)_{i\beta} M_{e'i} \sin 2\beta_R}{4(4\pi)^2}\left[F\left(\frac{m^2_{h'_R}}{M^2_{e'i}}\right) - F\left(\frac{m^2_{H'_R}}{M^2_{e'i}}\right)\right]$$
$$+ (R \to I), \tag{2.21}$$

where $F(x) = x\log x/(1-x)$. The total mass matrix is diagonalized by bi-unitary matrix. From the mass formula, for example, the Yukawa coupling $(y^\eta_\ell y^\chi_\ell) \sim 1$ is required for muon mass and $(y^\eta_\ell y^\chi_\ell) \sim 0.01$ for electron mass when $M_{e'} \sim 500$ GeV, $\sin 2\beta_{R(I)} \sim 0.1$ and $\mathcal{O}(1)$ of the loop function. The Yukawa coupling y^χ_ℓ should be $\mathcal{O}(1)$ to obtain the observed DM relic density as we will see in Section 3.

The Dirac neutrino mass matrix at one-loop level as depicted in the left hand side of Fig. 2 is given by

$$(m_D)_{i\beta} = \sum_i \frac{(y^\chi_n)_i (y^\eta_n)_{i\beta} M_{n'i} \sin 2\beta_R}{4(4\pi)^2}\left[F\left(\frac{m^2_{h'_R}}{M^2_{n'i}}\right) - F\left(\frac{m^2_{H'_R}}{M^2_{n'i}}\right)\right]$$
$$- (R \to I). \tag{2.22}$$

With the Dirac neutrino mass matrix, the active neutrino mass matrix is obtained by canonical seesaw mechanism as

$$(m_{\nu 1})_{\alpha\beta} = -\frac{1}{M_N}\left(m_D^T m_D\right)_{\alpha\beta}. \tag{2.23}$$

In addition, there is another contribution to the neutrino masses coming from the right hand side of Fig. 2. The mass matrix is expressed as [34]

$$(m_{\nu 2})_{\alpha\beta} = \sum_i \sum_k \frac{(y^\eta_n)_{i\alpha}(y^\chi_n)_i (y^\chi_n)_k (y^\eta_n)_{k\beta} M_{n'i} M_N}{16(4\pi)^4 M_{n'k}} F^{\rm loop}_{ik}, \tag{2.24}$$

where the loop function $F^{\rm loop}_{ik}$ is given by

$$F^{\rm loop}_{ik} = \int d^3x \frac{\delta(x+y+z-1)}{y(y-1)}$$
$$\times \left[\left\{\sin^2 2\beta_R \left(G\left(\frac{M^2_{ih'_R}}{M^2_{n'k}}, \frac{m^2_{h'_R}}{M^2_{n'k}}\right) - G\left(\frac{M^2_{ih'_R}}{M^2_{n'k}}, \frac{m^2_{H'_R}}{M^2_{n'k}}\right)\right)\right.\right.$$

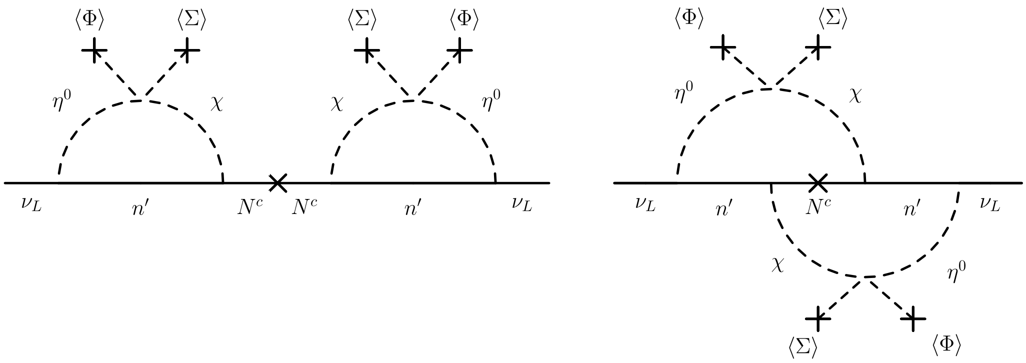

Fig. 2. Radiative generation of neutrino masses.

$$+\left(h'_R \leftrightarrow H'_R\right)\Bigg\}$$
$$-\left\{\sin 2\beta_R \sin 2\beta_I \left(G\left(\frac{M^2_{ih'_R}}{M^2_{n'k}}, \frac{m^2_{h'_I}}{M^2_{n'k}}\right)\right.\right.$$
$$\left.\left.-G\left(\frac{M^2_{ih'_R}}{M^2_{n'k}}, \frac{m^2_{H'_I}}{M^2_{n'k}}\right)\right) - \left(h'_R \leftrightarrow H'_R\right)\right\}$$
$$+ (R \leftrightarrow I)\Bigg], \tag{2.25}$$

with

$$G(x, y) = \frac{-x(1-y)\log x + y(1-x)\log y}{(1-x)(1-y)(x-y)}, \tag{2.26}$$

and

$$M^2_{ia} \equiv \frac{xm^2_{n'i} + yM^2_N + zm^2_a}{y(y-1)} \tag{2.27}$$

where $a = h'_R, H'_R, h'_I, H'_I$. Whole neutrino mass matrix is sum of the two contributions as $m_\nu = m_{\nu 1} + m_{\nu 2}$. From the neutrino mass formula, $(y^\chi_n y^\eta_n) \sim 0.01$ is needed to obtain the proper neutrino mass scale by assuming $M_{n'} \sim 500$ GeV, $M_N \sim 1$ TeV, $\mathcal{O}(0.1)$ of the loop functions.

2.3. The muon anomalous magnetic moment and lepton flavor violation

The muon anomalous magnetic moment, $(g-2)_\mu$, has been measured at Brookhaven National Laboratory. The current average of the experimental results [55] is given by

$$a^{\rm exp}_\mu = 11659208.0(6.3) \times 10^{-10},$$

which has a discrepancy from the SM prediction with 3.2σ [56] to 4.1σ [57] as

$$\Delta a_\mu = a^{\rm exp}_\mu - a^{\rm SM}_\mu = (29.0 \pm 9.0 \text{ to } 33.5 \pm 8.2) \times 10^{-10}.$$

In our model, there are several contributions to the (transition) magnetic moment $\mu_{\alpha\beta}$ which is coefficient of the operator $\mu_{\alpha\beta}\overline{\ell_\alpha}\sigma^{\mu\nu}\ell_\beta F_{\mu\nu}$. The muon anomalous magnetic moment is identified as $\Delta a_\mu = \mu_{\mu\mu}$. The largest contribution comes from photon attaching to vector like charged fermions since it is proportional to $m_\alpha/M_{e'}$ where m_α is charged lepton mass. On the other hand, the other contributions are proportional to $m^2_\alpha/M^2_{e'}$. The contributions coming from the loop of η^+ and n' in neutrino sector are also proportional to $m^2_\alpha/M^2_{n'}$. Thus these are neglected in our calculation, and the (transition) magnetic moment is calculated as

$$\mu_{\alpha\beta} \simeq \sum_{i=1}^{2} \frac{\sin 2\beta_R}{2(4\pi)^2}\frac{m_\alpha}{M_{e'i}}\left(\left(y^\eta_\ell\right)^{\alpha i}\left(y^\chi_\ell\right)^{i\beta}\right.$$
$$\left. + \left(y^{\eta *}_\ell\right)^{\beta i}\left(y^{\chi *}_\ell\right)^{i\alpha}\right)\left[-H\left(\frac{m^2_{h'_R}}{M^2_{e'i}}\right) + H\left(\frac{m^2_{H'_R}}{M^2_{e'i}}\right)\right] + (\mathrm{R} \to \mathrm{I})$$
$$\text{with } H(x) = \frac{1 - 4x + 3x^2 - 2x^2 \ln x}{2(1-x)^3}. \tag{2.28}$$

More preciously, the unitary matrices which diagonalize the charged lepton mass matrix should be multiplied from left and right. It is understood by replacing Yukawa couplings y^η_ℓ, y^χ_ℓ to $y^{\eta'}_\ell$, $y^{\chi'}_\ell$. This expression of the (transition) magnetic moment is closely related with radiative induced charged lepton masses Eq. (2.21). To reproduce the muon mass, for example, $\sin 2\theta_{R(I)}$ and $M_{e'i}$ are taken to be $\mathcal{O}(10^{-2})$ and $\mathcal{O}(1)$ TeV, respectively. Thus we obtain $\Delta a_\mu = \mathcal{O}(10^{-9})$, when $(y^\eta_\ell)(y^\chi_\ell)[H(m^2_{h'_R}/M^2_{e'i}) - H(m^2_{H'_R}/M^2_{e'i})]$ is roughly 0.1.

It is the common fact that muon $g-2$ and lepton flavor violation tend to conflict each other. In LFV processes, $\mu \to e\gamma$ especially gives the most stringent bound. The upper limit of the branching ratio is given by $\mathrm{Br}(\mu \to e\gamma) \leqslant 5.7 \times 10^{-13}$ at 95% confidence level from the MEG experiment [58]. In our model, the diagonal Yukawa matrices y^η_ℓ and y^χ_ℓ are required not to conflict with lepton flavor violating processes such as $\mu \to e\gamma$. Nevertheless, the contribution to $\mu \to e\gamma$ still comes from the neutrino sector, and it is calculated as

$$\mathrm{Br}(\mu \to e\gamma) = \frac{3\alpha_{\rm em}}{64\pi G^2_F m^4_\eta}\left|\sum_i \left(y^\eta_n\right)_{i\mu}\left(y^\eta_n\right)^*_{ie} F_2\left(\frac{M^2_{n'i}}{m^2_\eta}\right)\right|^2, \tag{2.29}$$

where $\alpha_{\rm em} = 1/137$ is the fine structure constant, G_F is the Fermi constant and $F_2(x)$ is the loop function defined in Ref. [59]. From Eq. (2.29), we obtain a rough estimation for the Yukawa coupling $y^\eta_n \lesssim 0.05$ by setting $m_\eta = M_{n'} \sim 500$ GeV. This estimation does not contradict with the discussion of neutrino masses.

3. Dark matter

We have two DM candidates: vector like fermion n', the lightest eigenstate of η^0 and χ (one of h'_R, H'_R, h'_I, H'_I). One may think the scalar DM candidate decays into the SM particles since the SM leptons also have odd charge under the imposed $\mathbb{Z}_2$ symmetry in our model. However, the decay of the DM candidate is forbidden by Lorentz invariance. Namely, this means that the scalar DM candidate can decay into only even number of fermions, however such a decay process is not allowed in the model.

We identify h'_R is DM here since it has interesting DM phenomenology. The mixing angle $\sin\beta_R$ is needed to be small enough since tiny neutrino masses are proportional to the mixing angle. Note that in the limit of $\sin\beta_R \to 0$, there is no contribution from h'_R and H'_R to the charged lepton and neutrino masses as one can see from the previous section. However we still have the contribution of h'_I and H'_I. The neutrino masses are generated from h'_I and H'_I. The parameter relation $a \approx -a'$ is required to construct such a situation as one can see in Eq. (2.16). In this case, the DM candidate h'_R corresponds to just χ_R. Thus we regard χ_R as DM hereafter. The couplings λ'_5, λ''_5, λ''_6 and λ'_7 in the scalar potential also should be suppressed not to have large elastic cross section with nuclei. Otherwise elastic scattering occurs via Higgs exchange and it is excluded by direct detection experiments of DM such as XENON [3] or LUX [4]. The spin independent elastic cross section with proton in the limit of $\sin\beta_R \to 0$ is given by

$$\sigma_p = \frac{C\mu_\chi^2 m_p^2}{\pi m_{\chi_R}^2 v^2}\left(\frac{\mu_{\chi\chi h}\cos\alpha}{m_h^2} + \frac{\mu_{\chi\chi H}\sin\alpha}{m_H^2}\right)^2, \tag{3.1}$$

where μ_χ is reduced mass defined as $\mu_\chi = (m_{\chi_R} + m_p^{-1})^{-1}$, $m_p = 938$ MeV is the proton mass and $C \approx 0.079$. The couplings $\mu_{\chi\chi h}$ and $\mu_{\chi\chi H}$ are given by

$$\mu_{\chi\chi h} = -\left(\lambda'_5 + \lambda''_5 + \frac{\lambda''_6}{2}\right)v'\sin\alpha + \frac{\lambda'_7}{2}v\cos\alpha, \tag{3.2}$$

$$\mu_{\chi\chi H} = \left(\lambda'_5 + \lambda''_5 + \frac{\lambda''_6}{2}\right)v'\cos\alpha + \frac{\lambda'_7}{2}v\sin\alpha. \tag{3.3}$$

The elastic cross section is strongly constrained by LUX as $\sigma_p \lesssim 7.6 \times 10^{-46}$ cm^2 at $m_{\chi_R} \approx 33$ GeV. Thus the couplings λ'_5, λ''_5, λ''_6 and λ'_7 are required to be $\mathcal{O}(0.001)$ in order to satisfy the constraint when $v' \sim 1$ TeV and $\sin\alpha \sim 1$.

Due to the strong constraint from direct detection of DM, the annihilation cross section for the process $\chi_R\chi_R \to f\bar{f}$ via Higgs s-channel is extremely suppressed. The cross section is calculated as

$$\sigma v_{\rm rel} = \frac{y_f^2}{2\pi}\left(1 - \frac{4m_f^2}{s}\right)^{3/2}\left|\frac{\mu_{\chi\chi h}\cos\alpha}{s - m_h^2 + im_h\Gamma_h} + \frac{\mu_{\chi\chi H}\sin\alpha}{s - m_H^2 + im_H\Gamma_H}\right|^2, \tag{3.4}$$

where $s \approx 4m_{\chi_R}^2(1 + v_{\rm rel}^2/4)$, Γ_h and Γ_H are the decay widths of h and H. With the above constraint from direct detection, the typical value of the annihilation cross section is roughly $\sigma v_{\rm rel} \sim 10^{-32}$ cm^3/s which is too small to obtain the observed DM relic density $\Omega h^2 \approx 0.12$ [2].

However there is the Yukawa interaction $y_\ell^\chi e^c e' \chi$. The DM annihilation $\chi_R\chi_R \to \ell\bar{\ell}$ is possible via the Yukawa interaction. When one expands the cross section by the DM relative velocity $v_{\rm rel}$, the s-wave and p-wave of the process are helicity suppressed. Thus this process becomes d-wave dominant in the chiral limit of the final state particles as have been studied in Refs. [52,53]. The annihilation cross section is written as

$$\sigma v_{\rm rel} = \left|\sum_i \frac{(y_\ell^{\chi\dagger} y_\ell^\chi)_{ii}}{(1+\mu_i)^2}\right|^2 \frac{v_{\rm rel}^4}{60\pi m_{\chi_R}^2}, \tag{3.5}$$

where $\mu_i = m_{e'i}^2/m_{\chi_R}^2 > 1$. The Yukawa couplings should be $\mathcal{O}(1)$ to achieve the correct relic density of the DM. As a result of the d-wave suppression of the 2-body cross section, internal bremsstrahlung process $\chi_R\chi_R \to \ell\bar{\ell}\gamma$ which generates sharp gamma ray spectrum around $E_\gamma \sim m_\chi$ becomes strong as can be compared with the experiments such as Fermi-LAT [60] or future project CTA [61] without conflicting with the thermal relic density of DM. The predicted spectrum is stronger than that in case of p-wave dominant Majorana DM [7]. When μ_i is far from 1, the gamma ray spectrum becomes broader. Thus roughly $\mu_i \lesssim 2$ is needed to produce a sharp gamma ray spectrum.

Finally, we mention about the discrepancy of the effective number of neutrino species $\Delta N_{\rm eff}$. This has been reported by several experiments such as Planck [2], WMAP9 polarization [9], and ground-based data [10,11], which tell us $\Delta N_{\rm eff} = 0.36 \pm 0.34$ at the 68% confidence level. Such a deviation $\Delta N_{\rm eff} \approx 0.39$ is achieved, if we take the extra neutral boson H to be light as well as 500 MeV and small mixing angle $\sin\alpha \ll 1$ [54,62,44,63]. Such a light mass is needed to determine the appropriate decoupling era of the extra neutral boson in the early Universe. The mixing angle also should be small enough to suppress the invisible decay of the SM Higgs $h \to HH$. When such a light extra Higgs H is taken into account, smaller scalar couplings λ'_5, λ''_5, λ'_6 are required to be consistent with the constraint on elastic cross section with proton Eq. (3.1). However it does not matter with the estimation of the thermal relic density and the strong gamma-ray signal discussed above because these are induced via the Yukawa coupling y_ℓ^χ. Hence we can derive the neutrino effective number $\Delta N_{\rm eff}$ without any contradiction with the other DM phenomenology.

4. Conclusions

We have constructed a model where the neutrino and charged lepton masses are generated radiatively. The electron and muon masses are obtained from one-loop diagram while the neutrino masses arise through two-loop diagrams. The tau mass is rather heavy to generate radiatively, and is given by the tree level Yukawa interaction. Thus their measured mass hierarchies are naturally explained. Then we have obtained the large muon anomalous magnetic moment ($(g-2)_\mu$) as same as the observed value from the charged lepton sector. Such a large magnetic moment tends to conflict with LFV processes. To avoid this, an appropriate parameter condition has been considered to be consistent with LFV.

The same symmetries that explain charged lepton and neutrino masses also allow some DM candidates. We have shown that our scalar DM can emit a strong gamma-ray by internal bremsstrahlung process which is possible to compare with the experiment such as Fermi-LAT. In addition, the thermal relic density of DM can be consistently derived unlike internal bremsstrahlung of Majorana DM. Simultaneously, when H is light ($m_H \sim 500$ MeV) and the mixing angle $\sin\alpha$ is small enough, the Goldstone boson can play the role of dark radiation and we can also induce a sizable discrepancy in the effective neutrino number $\Delta N_{\rm eff} \approx 0.39$.

Acknowledgements

This work is partly supported by NRF Research Grant 2012R1A2A1A01006053 (S.B.). T.T. acknowledges support from the European ITN project (FP7-PEOPLE-2011-ITN, PITN-GA-2011-289442-INVISIBLES).

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