Dynamic characterization method of accelerometers based on the Hopkinson bar calibration system

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1. Introduction

Higher requirements for impact acceleration measurement put forward the accuracy of accelerometer measurement research [1]. The dynamic characterization can be deemed as the critical theme for the dynamic error estimation and the premise of error compensation on account of systematic characteristics contribution. The actual characterization methods are confined to the dynamic modelling, which could indicate frequency information in the discrete frequency domain [2]. The feasible characteristic index of frequency information is the frequency band of small distortion (FBSD), which is within ±5% or ±10% amplitude deviation due to the extreme conditions of dynamic measurement [3]. In [4-7], the assumed second-order model is adopted to depict the nonlinear error characteristics of accelerometer. Also, in [8-9], the state-space transfer function assumes the accelerometer model to be an ordinary second-order difference function. The aforementioned works make sense in general cases because the ordinary accelerometers can be considered as the system with damp and inertia segments. Actually, in extreme conditions, the accelerometers always exhibit a high-order system feature due to the complicated process. So the order of the accelerometer model should be judged thoughtfully from a physical situation rather than from empirical assumption. In [10], a high-order parameterized accelerometer model is calculated, which has a significant realization of high order system feature of accelerometers. Nevertheless, there is a strong coupling effect for estimating model coefficients and the order, which have not been paid enough attention apparently.

In this paper, a novel dynamic modelling approach is developed. The model order identification error induced from the inaccurate test data or algorithm deviation would influence the coefficient identification directly. So an iteration estimation of coefficients and the order is performed, which could compensate and correct the estimation error with the processing steps. In addition, the dynamic compensation model (DCM) has been introduced to improve the dynamic performance of the model by the frequency coverage extension.

The accurate estimation of the excitation and response signals is the prerequisite for dynamic modelling. Accordingly, the dynamic calibration technology needs to be introduced [11]. The dynamic calibration is required to excite the impact accelerometer with the significant conditions, which is discussed in section 3. The typical dynamic calibration systems for the accelerometer include the Hopkinson bar system, gas gun system, Machete hammer system, impact pendulum system and drop hammer system [12-13], among which the Hopkinson bar could generate a narrower pulse width of the excitation signal. Therefore, the Hopkinson bar system is adopted in this paper to guarantee a better calibration performance.

2. Dynamic calibration theory

2.1 Dynamic modelling

With the feature information extracted from the excitation signal and the response signal, the accelerometer dynamic model could be obtained, which can be depicted as a differential equation for the feature of temporal correlation [15]. The equation described in the discrete time domain is as follows:

$$\sum_{i=0}^{n} a_i T_m(k-i) = \sum_{i=0}^{n} b_i T_g(k-i) + \varepsilon(k)\,,\ (k = 0,\ 1,\ \ldots\ ,\ N) \tag{1}$$

where $T_m(k)$ is the response signal, $T_g(k)$ is the excitation signal, n is the order of the model, a_i and b_i are coefficients of the model, $i = 0, 1, \ldots, n$, $a_0 = 1$, $\varepsilon(k)$ is the error existing in both excitation and response signals, N is the data length.

The dynamic modelling theory focuses on how to estimate the unbiased coefficients and the model order. If we define $A(d^{-1}) = a_i d^{-i}$, $B(d^{-1}) = b_i d^{-i}$, d is the backshift operator. Then Eq. (1) can be written as:

$$A(d^{-1})T_m(k) - B(d^{-1})T_g(k) = \varepsilon(k) \tag{2}$$

The feature information matrix is defined as:

$$\mathbf{D} = [T_g(j), -T_m(j), \ldots, T_g(j+v), -T_m(j+v)]\,,\ \ (j = 1, \ldots, k, v \ge \hat{n}) \tag{3}$$

where $k = N - v$, $\hat{n}$ is the model order estimate, v is a positive integer.

The coefficients array of the dynamic model can be defined as follows:

$$\boldsymbol{\theta} = [b_{\hat{n}}\quad a_{\hat{n}}\quad \ldots\quad b_1\quad a_1\quad b_0\quad 1]^T \tag{4}$$

And also the measurement error array:

$$\mathbf{E} = [\varepsilon(1)\quad \varepsilon(2)\quad \ldots\quad \varepsilon(\hat{n})]^T \tag{5}$$

For the matrix calculation requirement for the column and row number of **D**, the row number of $\boldsymbol{\theta}$ and **E** need to be expanded with zeroes. The expansion results are expressed as:

$$\boldsymbol{\theta}_E = [\boldsymbol{\theta}^T\quad 0\quad \ldots\quad 0]^T\,,\ \ \mathbf{E}_E = [\mathbf{E}^T\quad 0\quad \ldots\quad 0]^T \tag{6}$$

where $\boldsymbol{\theta}_E$ and $\mathbf{E}_E$ are the real number matrices of $k\times 1$-order.

Then Eq. (1) can be written as a concise form:

$$\mathbf{E}_E = \mathbf{D}\cdot\boldsymbol{\theta}_E \tag{7}$$

In order to calculate the coefficients, an indicator function needs to be introduced. Based on the principle of least squares, the indicator function is defined as:

$$J(\boldsymbol{\theta}) = \|\mathbf{E}_E - \mathbf{D}\cdot\boldsymbol{\theta}_E\|_2^2 = (\mathbf{E}_E - \mathbf{D}\cdot\boldsymbol{\theta}_E)^T(\mathbf{E}_E - \mathbf{D}\cdot\boldsymbol{\theta}_E) \tag{8}$$

The information matrix **D** is the real number matrix of $k\times\hat{n}$-order, and there is an orthogonal transformation matrix **H** of $k \times k$-order that could upper triangulate matrix **D**. The information matrix **D** could be divided as follows:

$$\mathbf{H}\cdot\mathbf{D} = \begin{bmatrix} R \\ 0 \end{bmatrix},\ \ \mathbf{H}\cdot\mathbf{E}_E = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \tag{9}$$

where $g_1 \in R^{\hat{n}\times 1}$, $g_2 \in R^{(k-\hat{n})\times 1}$.

Due to the features of the positive defined matrix and Eq. (8), $J(\boldsymbol{\theta})$ could be obtained:

$$J(\boldsymbol{\theta}) = \left\| \begin{matrix} R\boldsymbol{\theta} - g_1 \\ g_2 \end{matrix} \right\|_2^2 = \|R\boldsymbol{\theta} - g_1\|_2^2 + \|g_2\|_2^2 \tag{10}$$

Therefore, the least squares solution is:

$$\hat{\boldsymbol{\theta}} = R^{-1}g_1\,,\ \ J(\hat{\boldsymbol{\theta}}) = \|g_2\|_2^2 \tag{11}$$

From Eq. (11), we know that the variation trend of $J(\boldsymbol{\theta})$ can be indicated by $\|g_2\|_2^2$. If the indicator changes little as n increases, it means that the higher system order contribution to the system performance can be neglected. Then the dynamic model order can be acquired.

However, the algorithm above is based on a hypothetical condition of unbiased estimation. If the error $\varepsilon(k)$ is a coloured noise, then the least squares estimation is biased, and the model needs to be optimized. The signal transfer flow diagram of the dynamic system modelling is shown in Fig.1.

In Fig. 1, the remnant error is $e(k)$, which can be expressed as:

$$e(k) = 1 / A(d^{-1}) \cdot \varepsilon(k) \tag{12}$$

The dynamic model of the accelerometer with unbiased coefficients can be established based on the autocorrelation reduction effect of Eq. (12).

2.2 Design of DCM

In some cases, the vibration frequency of the measured signal is beyond the coverage of the accelerometer frequency band, then the dynamic error and time lag would be generated. Unlike the static error, the dynamic error cannot be corrected with a constant parameter. So the dynamic error and lagging characteristics can only be improved by a dynamic compensation procedure, hence the DCM is introduced.

As the dynamic characteristics of the accelerometer are determined by the pole point of the dynamic model, the DCM can be designed with a zero point equivalent to the pole point of the dynamic model. And the dynamic characteristics of the accelerometer can be offset by a cascaded connection of DCM.

The DCM model should be designed as a discretized transfer function, the pole point of which could be a real number or a plural number. If it is a real number, the transfer feature could be represented by a first order system, whose transfer function could be expressed in z- area with z- transformation:

$$G(z) = \frac{1}{\tau}\frac{z}{z - e^{-T/\tau}} = \frac{1}{\tau}\frac{z}{z - p} \tag{13}$$

where T is the sampling interval, z is the variable symbol in z- area, p is the real pole, $p = e^{-T/\tau}$.

If the compensated system response time is required to be T_r. According to the first-order system response feature, the time constant of DCM should be:

$$\tau_e = T_r / 3 \tag{14}$$

Then, the real pole of the DCM transfer function could be upgraded as:

$$p_e = e^{-T/\tau_e} = e^{-3T/T_r} \tag{15}$$

If the pole point is a plural number, it could be represented by the second-order dynamic model, the transfer function could be expressed in z-area with z- transformation:

$$G(z) = \frac{z \cdot \omega_n \sin(\omega_n \cdot T\sqrt{1-\zeta^2})}{e^{\zeta\omega_n T}\sqrt{1-\zeta^2}(z-p_1)(z-p_2)} \tag{16}$$

where ζ is the damping coefficient and ω_n is the inherent oscillation frequency of the system, p_1 and p_2 are the conjugate pole points that can be expressed as:

$$p_{1,2} = \alpha \pm j\beta = e^{\omega_n T(-\zeta \pm j\sqrt{1-\zeta^2})} \tag{17}$$

$$\theta = \arctan(\beta / \alpha) = \omega_n T \cdot \sqrt{1-\zeta^2} \tag{18}$$

$$\alpha = e^{-\zeta\omega_n T} \cdot \cos\theta \tag{19}$$

$$\zeta\omega_n = \frac{1}{T}\ln\left(\frac{\cos\theta}{\alpha}\right) \tag{20}$$

According to the second order system response feature, the response time reached 95% of the steady-state value should be:

$$T_{rr} = \frac{3}{\zeta\omega_n} = \frac{3T}{\ln\left(\dfrac{\cos\theta}{\alpha}\right)} \tag{21}$$

According to Eq. (17), the conjugate pole amplitude of DCM could be calculated:

$$|p_e| = e^{-\zeta_e \omega_e T} \tag{22}$$

where ζ_e is the DCM camping coefficient and ω_e is the inherent oscillation frequency of DCM.

Based on Eq. (21), the following relationship would be obtained:

$$\omega_e = \frac{3}{\zeta_e \cdot T_{rr}} \tag{23}$$

Then the real part and imaginary part of the DCM pole point are expressed respectively as follows:

$$\alpha_e = |p_e|\cos\theta_e = e^{-3T/T_{rr}} \cos(\omega_e \cdot T\sqrt{1-\zeta_e^{\ 2}}) \tag{24}$$

$$\beta_e = |p_e|\sin\theta_e = e^{-3T/T_{rr}} \sin(\omega_e \cdot T\sqrt{1-\zeta_e^{\ 2}}) \tag{25}$$

The pole point designation depends on the compensation effect requirements. But if the further effect of compensation is needed, an additional pair of pole points could be adopted.

3. Principle and composition of the dynamic calibration system

The dynamic calibration system is engineering realization of the dynamic characterization, which is required to exhibit all or major modes of the accelerometer. Accordingly, the primary issue of dynamic calibration is to generate the excitation signal whose frequency band could fully cover the modes of the accelerometer. The broadband frequency coverage corresponds to a narrow pulse width in the time domain. Hence the dynamic calibration priority is the generation of excitation signal with a narrow pulse width. Meanwhile, the excitation signal needs to be accurately evaluated in the time domain. Therefore, the high precision traceable ability of excitation signal is also needed for the dynamic calibration system. Specific sensor dynamic calibration corresponds to a specific calibration system. The Hopkinson bar is a kind of specific device for impact accelerometer calibration. The Hopkinson bar system consists of a titanium alloy bar, a differential laser Doppler interferometer, an air compressor, a vacuum pump, the signal processing system, and the calibrated object, as shown in Fig. 2.

3.1 Generation of the excitation acceleration signal

The excitation signal is generated by the impact of a projectile on the titanium alloy bar, which excites the accelerometer installed at the end of the bar. As shown in Fig. 2, when the valve opens, the projectile is driven by the force of the compressed gas and vertically hits the aluminium spacer installed at the incident end of the bar. Then, a right-travelling elastic stress wave is generated. At the opposite end of the bar, a raster base is installed and a raster is pasted on its surface along the axis direction. In order to ensure the installation tightness of the raster base, the vacuum fixture is adopted to provide the adherence force. And the raster base material is the same as the material of the bar to ensure the minimal influence on the stress propagation at the fitting surface. The other end of the raster base is the accelerometer linked with the screw thread. The raster base and accelerometer are considered as the calibrated object. The accelerometer could perceive the acceleration when the incident stress wave spreads to the fitting surface of the accelerometer. When the incident stress wave and the reflected stress wave are offset at the fitting face of the calibrated object, the accelerated process terminates and the calibrated object flies out with the involved momentum. The stress wave propagation is shown in Fig. 3.

Since the stress waveform could be considered as harmonic components superposition of different frequencies, and the different components will spread independently according to their own phase velocity, so the waveform can no longer keep prototype and must disperse in the propagation process. In order to research the stress changes at the fitting surface of the calibrated object, we assume that the stress wave is a standard semi-sinusoidal waveform. The incident stress wave amplitude can be expressed as:

$$\sigma_i = \begin{cases} \sin(\pi \cdot t / t_0), & 0 \le t < t_0 \\ 0, & t < 0, or, t \ge t_0 \end{cases} \tag{26}$$

where t_0 is the moment when the acceleration propagation is terminated.

The reflected stress wave amplitude at the same point can be expressed as:

$$\sigma_r = \begin{cases} -\sin\left[\pi \cdot \left(t - \frac{2L}{C}\right) / t_0\right], & \frac{2L}{C} \le t \le t_0 + \frac{2L}{C} \\ 0, & t < \frac{2L}{C}, or, t > t_0 + \frac{2L}{C} \end{cases} \tag{27}$$

where L is the length of the raster base, C is the velocity of elastic stress wave in titanium.

From Eq. (26) and Eq. (27), the sum of the stress wave amplitudes can be obtained:

$$\sigma = \sigma_i + \sigma_r = \begin{cases} \sin(\pi \cdot t / t_0), & 0 \le t < \frac{2L}{C} \\ \sin(\pi \cdot t / t_0) - \sin\left[\pi \cdot \left(t - \frac{2L}{C}\right) / t_0\right], & \frac{2L}{C} \le t < t_0 \\ -\sin\left[\pi \cdot \left(t - \frac{2L}{C}\right) / t_0\right], & t_0 \le t < t_0 + \frac{2L}{C} \\ 0, & 0 < t, or, t \ge t_0 + \frac{2L}{C} \end{cases} \tag{28}$$

The sum of the stress wave amplitudes can resort to zero on the condition that:

$$\sin(\pi \cdot t / t_0) = \sin\left[\pi \cdot \left(t - \frac{2L}{C}\right) / t_0\right], \quad \frac{2L}{C} \le t < t_0 \tag{29}$$

The pulse width of the acceleration signal t_a can be deduced from Eq. (29):

$$t_a = t - \frac{L}{C} = \frac{t_0}{2} \tag{30}$$

From Eq. (30), we can recognize that the pulse width of the acceleration signal is equivalent to the leading edge duration of the stress wave. With this consequence, we could know that the stress loading process, and the geometric dimension of the bar, etc. have an important influence on the acceleration signal pulse width. The relationship between the pulse width and geometric dimension of the bar has been discussed in Ref. 15. This paper focuses on the impact loading process, as other factors can be easily realized with the engineering design.

In order to narrow the acceleration pulse width, the stress loading process of the projectile on the bar needs to be minimized. So that different types of projectile shape are designed and shown in Fig. 4, among which projectiles No. 1 and No. 2 are designed based on the experimental experience, and projectile No. 3 is designed based on stress mechanism analysis. Projectiles No. 1 and No. 2 are made of C45 steel with a different shaped tip, and projectile No. 3 is made of a C45 steel cushion block with a nylon plastic tail. For projectile No. 3, the wave impedance of steel is much higher than that of nylon, most of the left-travelling stress waves are reflected and unloaded at the interface of different materials, and the impact process ends when the reflected wave reaches the projectile impact surface. Therefore, the excitation acceleration pulse width is proportional to the 45 steel cushion thickness. And when No. 3 projectile is used in the experiment, the aluminium spacer originally installed at the incident end of the bar can be omitted to ensure the minimum acceleration pulse width. Due to the geometric dimension influence on the pulse width, the actual acceleration pulse width comparison experiment is conducted after the geometric dimension of the Hopkinson bar is given in section 4.

3.2 Quantification of the excitation acceleration signal

In order to establish a quantitative relationship between the accelerometer inherent frequency and the pulse width of the excitation signal, we can assume that the excitation signal is a standard semi-sinusoidal waveform in the time domain:

$$T_g(t) = a \cdot \sin(\pi / \delta \cdot t) \cdot (U(t) - U(t - \delta)) \tag{31}$$

where $U(t)$ is the standard step signal with unit amplitude, δ is the pulse width of the excitation signal, a is the amplitude of the excitation signal.

The Laplace transformation of Eq. (31) can be depicted as:

$$T_g(s) = \frac{a\pi / \delta}{s^2 + (\pi / \delta)^2}(1 + e^{-\delta s}) \tag{32}$$

And the energy spectrum of the excitation signal in the frequency domain can be deduced:

$$P(\Omega) = [2\frac{a\pi / \delta}{(\pi / \delta)^2 - \Omega^2}\cos(\frac{\Omega\delta}{2})]^2 \tag{33}$$

where $\Omega = 2\pi f$, f is the frequency of the excitation signal.

Then the total energy of the excitation signal can be obtained:

$$E = \int_0^{+\infty} T_g^2(t)dt = a^2\delta / 2 \tag{34}$$

The frequency coverage of the excitation signal can be quantified as the energy spectrum is no less than -3dB:

$$20\log(P(\Omega)) \geq -3dB \tag{35}$$

As the frequency of the excitation signal should cover the inherent frequency of the accelerometer, from Eq. (35), the frequency coverage of the excitation signal can be acquired:

$$\delta \leq 1.06 / f_n \tag{36}$$

where f_n is the inherent frequency of the accelerometer.

Eq. (36) exhibits the specific requirement of the excitation signal pulse width, which is the minimum requirement to excite the accelerometer.

3.3 Traceability of the excitation acceleration signal

As shown in Fig. 2, the excitation signal could be traced by a strain gauge or differential laser Doppler interferometer. For strain gauge applications, the measurement influence factors during the stress propagation need to be considered, especially the length of the strain gauge. However, the derivative operation of the stress signal is required for the acceleration signal calculation, it will lead to the amplification of noise magnitude, which is unfavourable for the dynamic calibration. Therefore, the excitation signal traceability is realized by the differential laser Doppler interferometer in this paper, which could generate continuous Doppler signal and avoid the random phase deviation of the simultaneous Doppler signal. The schematic of the differential laser interferometer is shown in Fig. 5.

The light beam sent by the He–Ne laser device is divided into two parallel light beams through a beam splitter prism. The light beams gather on the raster measurement point through the receiving lens. Diffraction light beams from the raster reach the cathode surface of the photomultiplier (PM) through the reflecting mirror system and the Doppler frequency shift signal is detected. Two rhombic prisms in the interferometer are used to fine-tune the space between the two parallel beams. The receiving lens are set to eliminate aberration. The Doppler frequency shift signal can be expressed as:

$$\Delta f_{\psi,(-\psi)} = \frac{2v}{\lambda}\sin\psi = \frac{v(\varphi - \xi)}{d} \tag{37}$$

where λ is the wavelength, 2ψ is the angle of two incident laser beams, v is raster movement speed, d is the grating space, and φ and ξ represent the diffraction series of the incident light wave.

From Eq. (37), the acceleration can be obtained by derivative calculation of the speed result:

$$a(t) = \frac{d}{dt}[\Delta f_{\psi,(-\psi)}(t)] \cdot d / (\varphi - \xi) \tag{38}$$

Though the demodulated operation of the signal processing system, the excitation acceleration signal can be obtained.

3.4 Encapsulation design for the impact accelerometer

The conventional impact resistance improvement is to encapsulate the accelerometer into a sufficiently strong and rigid shell with a polymer encapsulating material. Unfortunately, the conventional approach could easily find the bubbles in the encapsulating material and easily form stress concentration during solidification, which will severely reduce the impact resistance.

In order to reduce the bubbles and the internal stress of the encapsulating material and the thermal stress caused by the temperature change, a vacuum encapsulating technology is introduced in this paper, which improves the encapsulation strength through two means. Firstly, a vacuum pressure driver is adopted for the encapsulating material flow, which could prevent the formation of bubbles in the encapsulating material and avoid stress concentration at the contact interface of the electronic components. Secondly, temperature and solidification time are controlled accurately. Because temperature has a decisive influence on the final chemical reaction degree after the solidification. Furthermore, prolonging the solidification time can improve the solidification degree by using a thermosetting adhesive. Schematic of the vacuum encapsulating technology is shown in Fig. 6.

In order to verify the mechanical characteristics of the encapsulating material, an epoxy resin cylinder is simulated according to the resin casting compression method. The whole cylinder is encapsulated inside a steel shell. The simulation experiment is carried out with the axisymmetric analysis method. A semi-sinusoidal pulse load with a duration of 200 μs is applied along the vertical direction. The equivalent stress distribution at 100μs is shown in Fig 7.

From Fig. 7, we can see that the stress intensity of the encapsulating material is far less than that of the steel shell. This is because the wave impedance of the encapsulating material is only 0.01 ~ 0.001 times that of steel. In addition, due to the viscoelastic and transverse inertia effect of the encapsulating material, the amplitude attenuation and waveform dispersion will occur during the stress wave propagation, i.e. the encapsulating material can improve the overall impact resistance of the accelerometer.

4. Calibration experiment

In order to verify the characterization method proposed in this paper, the dynamic calibration experiment is carried out with a piezoresistance sensor AYZ-3 (100k), which is used as the calibrated accelerometer. Considering the pulse width requirement and engineering realization feasibility, a Hopkinson bar with a diameter of 16 mm and a length of 1,600 mm is adopted in the experiment. Based on these conditions, the measurement of the excitation signal pulse width generated by the three types of the projectile is performed and the result is shown in Fig. 8. It can be acquired that projectile No. 3 with Δ=3 mm can generate the shortest pulse width which is 38.6μs. And according to Eq. (36), this pulse width could excite the accelerometers whose inherent frequency is below 27.5 kHz, which is an exact fit for AYZ-3 (100k). Therefore, projectile No. 3 with Δ=3 mm is adopted in the calibration experiment.

The excitation acceleration signal calculated from the Doppler signal is shown in Fig. 9.

With the feature information matrix, the model coefficients are directly estimated through the dynamic characterization algorithm described in section 2.1. The trend of $\|g_2\|_2^2$ changes as the order n increases. And when n = 4, $\|g_2\|_2^2$ has a second point of inflexion, and after that, $\|g_2\|_2^2$ remains stable.

In order to establish a high precision dynamic model, $n = 4$ is chosen as the model order. Then, the signals with coloured noise can be optimized by Eq. (12). With an iteration procedure, the dynamic system transfer function can be obtained:

$$H_{OP}(s) = \frac{-2.4941s^4 + 1.9096\times 10s^3 + 2.2756\times 10^6 s^2 - 2.3600\times 10^{12} s + 1.9779\times 10^{17}}{1.2945\times 10s^4 + 5.5784\times 10^6 s^3 + 9.9900\times 10^{11} s^2 + 1.2461\times 10^{17} s + 1.1627\times 10^{22}} \tag{39}$$

The normalized dynamic model simulation results are shown in Fig. 10. The regression curve of the optimized algorithm agrees well with the experimental curve, which indicates that this dynamic model could represent the main accelerometer feature. In order to test the validity and accuracy of the model, more input and output data under the same experimental condition are selected to evaluate the regression effect of the model. By testing multi-group data, the regression effect of the model is proven to be sound. The comparison of the frequency characteristic curve between the experimental result and the model simulation result is shown in Fig.11.

As shown in Fig. 11, the frequency characteristic curve obtained from the experimental result is found to be rough and irregular, whereas the curve obtained from the proposed model is smooth and regular. These differences can be attributed to the measurement noise influence on the discrete sampling values. Therefore, the dynamic characteristic index should be acquired from the dynamic model. From Fig. 11, the FBSD within the ±10% amplitude error is ω_{g1}=9.6 kHz, whereas that within the ±5% amplitude error is ω_{g2}=7.2 kHz.

The dynamic compensation is designed according to the theory of DCM design described in section 2.2. The ratio of the system response time expressed as r_i can be solved and listed in Table 1.

Table 1. Pole analysis of the accelerometer dynamic model

Pole point		T'_{ri}	r_i
p_1	0.9740+j0.1396	185.45	1
p_2	0.9740-j0.1396	185.45	1
P_3	0.8174+j0.0518	15.03	12.34
P_4	0.8174-j0.0518	15.03	12.34

If the compensated ratio of the system response time is r = 12.5, then the nondimensional response time T'_{rr} can be expressed as:

$$T'_{rr} = T'_{r1} / r = 14.836 \tag{40}$$

The damping ratio of the equivalent system is selected as 0.707. Based on Eq. (24) and Eq. (25), the pole points could be obtained as follows:

$$P_{e1,2} = 0.8003 \pm 0.1641i \tag{41}$$

Additional pole point is selected as p_{ea} = 0.4 ± 0.1i, then the discrete transfer function of DCN in z-area could be obtained:

$$G(z) = \frac{15.616z^4 - 55.949z^3 + 75.3252z^2 - 45.1229z + 10.1421}{z^4 - 2.8006z^3 + 2.9581z^2 - 1.3931z + 0.2469} \tag{42}$$

The comparison of the step response signals with and without dynamic compensation can be acquired based on Eq. (42) and is shown in Fig.12. The rise time of the compensated system is obviously shortened, which is 1/9.76 of that without compensation. The comparison of frequency characteristic curves with and without the dynamic compensation is shown in Fig. 13.

As shown in Fig. 13, the FBSD within the ±10% amplitude error is ω_{g1}' = 78.4 kHz, the FBSD within the ±5% amplitude error is ω_{g1}' = 62.2 kHz. Obviously, ω_{g1}'/ω_{g1} = 8.167 and ω_{g2}'/ω_{g2} = 8.639, i.e., the FBSD within the ±10% and ±5% amplitude errors are respectively 8.167 and 8.639 times wider than the result acquired from Fig. 11.

5. Conclusion

In summary, the dynamic characterization reveals the main frequency characteristics of the accelerometer, which is confined as FBSD in this paper. The solution of this paper is divided into two parts: calibration theory analysis and calibration system design. For the part of theory analysis, the proposed modelling method could evaluate the model order according to system characteristics based on the real-time data and overcome the conventional coupling problem of model estimation. The proposed dynamic compensation approach, defined as DCM design in this paper, could compensate the dynamic measurement error based on actual demand in the field and directly correct the real-time data dynamic error under the same condition as the dynamic model.

For the calibration system design, the primary issue is the generation of the excitation signal, which is studied both qualitatively and quantitatively. In terms of qualitative analysis, an important conclusion is drawn, which is the equivalence relation of the excitation acceleration signal. In terms of quantitative analysis, the magnitude requirement of excitation signal pulse width is derived. As for the traceability of the excitation signal, a differential laser Doppler interferometer is introduced, which could quantify the excitation signal pulse width with high precision and can be independent of stress propagation noise interference. And a vacuum encapsulation technique is also developed in this paper, which could improve the overall impact resistance of the accelerometer.

In order to verify the characterization approach in engineering applications, the dynamic calibration experiment has been carried out with the calibrated accelerometer AYZ-3 (100k). The dynamic experiment result shows that the proposed dynamic model could represent the main characteristics of the accelerometer. The dynamic compensation result showed that the FBSDs within the ±5% and ±10% amplitude errors were respectively 8.639 and 8.167 times wider than the results without DCM. In order to verify the effectiveness of the characterization algorithm, this paper adopts the method of verifying the effect of redundant data regression, and the results show that the characterization theory could form a self-consistent system. The dynamic characterization of accelerometer presented in this paper opens opportunities for feature description generalization to the dynamic measurement field.

Author Declaration

All authors have reviewed and approved the final version of the manuscript being submitted. We warrant that the article is the authors' original work, hasn't received prior publication and isn't under consideration for publication elsewhere.

Conflict of interest

This study has been completely carried out by the authors and there is no conflict of interest in the presented article.

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