Theory and Decision (2019) 87:201–232
https://doi.org/10.1007/s11238-019-09712-9

Delayed probabilistic risk attitude: a parametric approach

Jinrui Pan[1] · **Craig S. Webb**[2] · **Horst Zank**[2]

Published online: 21 June 2019

Abstract
Experimental studies suggest that individuals exhibit more risk aversion in choices among prospects when the payment and resolution of uncertainty are immediate relative to when it is delayed. This leads to preference reversals that cannot be attributed to discounting. When data suggest that utility is time-independent, probability weighting functions, such as those used to model prospect theory preferences, can accommodate such reversals. We propose a simple descriptive model with a two-parameter probability weighting function where one of these parameters depends on the time at which a prospect is resolved. The time-dependent parameter is responsible for the curvature of the probability weighting function and is regarded as an index of (in)sensitivity towards changes in probabilities. We provide conditions that characterize increased sensitivity towards more distant probabilities; this can account for the observed relatively less risk aversion towards delayed prospects. In our framework, the discount function is unrestricted, such that the model is compatible with empirical findings of non-constant discounting. In a simple application to bargaining we illustrate when it is advantageous for an individual to advance or delay the bargaining resolution time if an opponent displays increased sensitivity towards probability changes with delay.

Keywords Bargaining · Discounting · Probability weighting · Risk and time preferences

✉ Horst Zank
horst.zank@manchester.ac.uk

Jinrui Pan
jinrui.pan@durham.ac.uk

Craig S. Webb
craig.webb@manchester.ac.uk

1 Business School, Durham University, Durham, UK

2 Department of Economics, University of Manchester, Manchester, UK

1 Introduction

Many decisions taken by individuals concern risky outcomes obtained at various points in time. These range from the purchase of lottery tickets, the ordering of goods that need to be delivered at certain dates, the booking of accommodation for business or holidays, to more complex financial instruments such as options or employment contracts with performance-based pay components. Traditionally, such streams of risky outcomes are evaluated by taking a weighted average of the future expected utility (EU) of each risky option, where the weights are determined using a constant discount rate. The literature has questioned this form of discounting from a descriptive point of view (Loewenstein and Prelec 1992; Frederick et al. 2002) as well as the use of EU for risk (Starmer 2000) and has called for more flexibility in modeling temporal discounting and attitudes towards probabilities.

As recent experimental studies suggest that risk attitude may be affected by the time at which prospects are obtained, we propose a simple theory that combines the domains of choice under risk and over time in a specific way. A connection between the domain of risk and that of pure time preferences has been suggested before (Prelec and Loewenstein 1991; Dasgupta and Maskin 2005) and a specific role has been attributed to the treatment of probabilities (Quiggin and Horowitz 1995; Halevy 2008). Recent contributions suggest that there may be subtle differences across these choice domains and that inverse-S weighting functions, as proposed in rank-dependent models and empirically supported by prospect theory (PT; Kahneman and Tversky and Kahneman 1992; Stott 2006; Wakker 2010), can play an important role for time (Wu 1999; Epstein 2008; Baucells and Heukamp 2012; Epper and Fehr-Duda 2015; Gerber and Rohde 2018). Accordingly, we develop a model in which the empirically founded shape of the probability weighting function may be affected by time delay.

In general, we can expect that time delay inflates or deflates any of the descriptively relevant parameters of a static decision theory model for risk and there are too few empirical studies to be conclusive. Earlier studies have often focused on how discounting is affected by future risk (Stevenson 1992; Shelley 1994), how static choice EU-paradoxes fare with delay (Keren and Roelofsma 1995; Weber and Chapman 2005) and, more recently, on how risk behavior is affected by delay (Noussair and Wu 2006). It is, therefore, not much of a surprise that the empirical basis for our motivation to focus on the probability weighting function is arguably thin. We are mainly motivated by the relatively recent study of Abdellaoui et al. (2011), which finds that, in a setting where discounting, attitudes towards outcomes and attitudes towards probabilities are separated, virtually all of the effect of time delay is captured by probability weighting. The experimental design of Abdellaoui et al. is such that discounting cannot be made responsible for changes in the preference among delayed prospects. Moreover, the elicited data suggest that the utility of outcomes is unaffected by delay. As a result, it can only be the treatment of probabilities that can account for changes in the observed choice behavior. This explains why our focus is on the interaction of probability weighting and time delay.

In our model we want to be more precise about which aspects of probability weighting are foremost responsible for changes in probabilistic risk attitudes. Looking somewhat closer at the distortions in probabilities, the results in Table 5 of Abdellaoui et al. (2011) show that the probability weights attached to the probability value of $p = 1/3$, which has often been reported to be the least distorted probability in static binary choice under risk (Wakker 2010), are not significantly impacted by time delay either. Further, at the aggregate level of all individuals in their study, the low probability ($p = 1/6$) is less overweighted with delay, while moderate and large probabilities ($p \in \{1/2, 2/3, 5/6\}$) are less underweighted when prospects are delayed. Thus, while confirming that delayed probabilities are also treated in accordance with the inverse-S shape of standard PT-weighting functions, albeit that the shape is less pronounced with delay, Abdellaoui et al. were able to qualify the finding of Noussair and Wu (2006) and others, which was termed "more risk tolerance with delay". Since underweighting of probabilities is associated with pessimism (formally defined in Wakker 1994), which in PT induces risk-averse choice behavior, we prefer to use the term *relatively less risk aversion with delay* instead. But the complete picture on the treatment of probabilities has to include the observation that, jointly with reduced pessimism with delay, small probabilities are less overweighted (i.e., less optimism; Wakker 1994), and that the undistorted probability (i.e., $p = 1/3$) apparently remains unaffected by delay. Such a treatment of probabilities is better attributed to a change in the index of *insensitivity* (Wakker 2010), a measure that captures the ability of a decision maker to discriminate among probabilities (Wu 1999).[1] Therefore, our working hypothesis is that delay mainly affects insensitivity.

For a theoretical analysis concerning changes in the index of insensitivity with delay, we invoke the two-parameter *constant relative sensitivity* (CRS) weighting function of Abdellaoui et al. (2010). Other parametric probability weighting functions could, in principle, also be adopted,[2] but the CRS-family has the advantage that it connects optimism (overweighting of small probabilities plus concavity of the weighting function) and pessimism (underweighting of medium and large probabilities plus convexity) in a direct way to its insensitivity index, which is the parameter of our interest. The CRS-function also identifies an *elevation* parameter, which Gonzalez and Wu (1999) interpret as "attractiveness to gambling", because it captures a general propensity of an individual to take risks in a particular choice environment. In accordance with the view that insensitivity and elevation represent logically independent psychological components of behavior (Gonzalez and Wu 1999, p. 139), the CRS-function also achieves a clean separation between these parameters based on behavioral preference foundations.

To better understand the relation between optimism/pessimism and insensitivity/elevation it is useful to present more formal arguments. The CRS-weighting function is a power-function over probabilities that are overweighted and is the (dual of that) power-function over probabilities that are underweighted; both functions have the same exponent, σ, as the index for (in-)sensitivity. Except for the probabilities 0 and 1 the CRS-weighting function may have a further fix-point at an intermediate probability, η, which is the index for elevation. At one extreme with $\eta = 1$ only optimism governs choice behavior (i.e., the weighting function is concave and overweighs

[1] Instead of insensitivity, Gonzalez and Wu (1999) use the term *discriminability* and, related to the degree of concavity/convexity of the inverse-S weighting function, *curvature*.

[2] E.g., parametric families proposed by Goldstein and Einhorn (1987), Prelec (1998), or Diecidue et al. (2009).

all probabilities) while at the other extreme with $\eta = 0$ only pessimism is revealed (the weighting function underweighs all probabilities and is convex). The empirically founded inverse-S probability weighting function will have the elevation parameter bounded away from extreme probabilities (as noted earlier, usually around probability $p = 1/3$; Wakker 2010, p. 205/206). Similarly, for the curvature parameter the empirically relevant parameter range is limited to (0, 1) to generate the empirically supported inverse-S shape. A lower value for σ indicates less sensitivity (or, equivalently, more insensitivity), such that at one extreme ($\sigma = 0$) we have no sensitivity at all as each intermediate probability receives the same weight, while at the other extreme ($\sigma = 1$) we have uniform sensitivity as no probability is distorted, i.e., preferences agree with EU.

Given our objective to pragmatically incorporate some descriptive features in our model, it is worth reviewing in some more detail the few experimental findings related to temporal risk attitudes which we seek to capture. We do that in Sect. 2, however, familiar readers can choose to move ahead to Sect. 3 where the theoretical setup is presented. A foundation for time-dependent CRS-probability weighting is provided in Sect. 4. Subsequently, we provide some comparative analyses, some simple applications to bargaining (Sect. 5), some further discussion (Sect. 6), highlight relevant aspects for time preferences which have not been our focus (Sect. 7), and then conclude (Sect. 8). Proofs are presented in the Appendix.

2 Empirical studies on preference changes with delay

Earlier studies have focused on comparing discounting of risky and riskless outcomes (Stevenson 1992) and on comparing discounting of losses and gains in mixed prospects (Shelley 1994). The latter study finds more discounting for risky losses relative to risky gains. This is in contrast to the findings in Thaler (1981) concerning sure outcomes, where more discounting for sure gains than for sure losses is reported. These opposite findings on discounting for risky versus sure gains and losses can be reconciled by invoking time-dependent probability weighting, suggesting an explanation based on changes in probabilistic risk attitude with delay.

Keren and Roelofsma (1995) focused on the immediacy effect, that is, the empirical observation that a strong preference for ‘a sure outcome now’ relative to ‘a somewhat larger delayed outcome’ is reversed when a common delay is added to both alternatives. In Experiment 1, Keren and Roelofsma change the alternatives by adding risk and observe that the immediacy effect is significantly reduced when the outcomes are likely (that is, when their probability is $p = 0.9$) and the effect is next to non-existent if outcomes are made even more risky ($p = 0.5$). In Experiment 2, they replicate Kahneman and Tversky (1979) common consequence choices in a between-subject design and find that the common consequence effect persists when a common delay of 1 year is added. This suggests a role for models building on PT-weighting functions as these are seen as a prominent class that can accommodate Allais’ (1953) common consequence paradox for EU-preferences.

Weber and Chapman (2005) conduct a study similar to Keren and Roelofsma (1995), and find that the immediacy effect is replicated for sure outcomes and that it persists when those same outcomes are made risky (i.e., the outcomes obtain with probability $p = 0.5$). Like Keren and Roelofsma, they find that the common consequence effect persists when a common delay is added. Weber and Chapman also consider common ratio effect choices, adopting the version introduced by Kahneman and Tversky (1979), and find that adding a common delay does not have any measurable impact on the choice behavior in the common ratio tasks. They do, however, find that elicited certainty equivalents for different prospects are affected by a common delay. Baucells and Heukamp (2010) also study the effect of delay in common ratio choice tasks. They find that a preference for the sure outcome is significantly reduced by delay. These findings suggest that subjects become less risk-averse when prospects are delayed.

Noussair and Wu (2006) use choice lists in which the outcomes of two binary prospects are fixed and probabilities vary (Holt and Laury 2001). In prospect A the outcomes were close (\$10 versus \$8) while prospect B had outcomes more spread (\$19.50 versus \$0.50). Starting with probability 0.1 for the higher outcome (residual probability being given to the lower outcome) and repeatedly shifting probability mass in units of 0.1 from the lower to the higher outcome, an expected value maximizer would choose prospect A for all low probabilities of the higher outcome and change to B at the 50:50 probability distribution (and choose the latter from there on). A more risk-averse decision maker may swap from choosing A to choosing B at probabilities above 0.5 for the better outcome. Since the outcome-stimuli are of a small scale and are kept fixed within the choice lists, it is likely that attitudes towards probabilities, as captured in PT, may be the main driver for risk behavior.[3] When the resolution and payment of the prospects are delayed by 3 months, most subjects remain consistent in their choices or become more inclined to accept risk: they start choosing prospects B at lower probabilities for the larger outcomes relative to the treatment when resolution and payment are immediate. Noussair and Wu interpret this finding as individuals become more risk-tolerant when a received prospect's resolution is delayed.

Abdellaoui et al. (2011) study choice behavior for prospects that are obtained and resolved at a common date. This enables them to separate effects attributed to discounting from the treatment of probabilities and attitudes towards outcomes over time much cleaner than in earlier studies. By adopting a general canonical model and eliciting temporal certainty equivalents for correspondingly timed prospects, they obtain data for utility and probability weighting functions for instant and delayed settings. As highlighted in Sect. 1, they find that the probability weights for $p = 1/3$ are not significantly impacted by time (Abdellaoui et al. 2011, Table 5). Further, at the aggregate level, the low probability ($p = 1/6$) is less overweighted with delay, and that moderate and large probabilities ($p \in \{1/2, 2/3, 5/6\}$) are less underweighted when prospects are delayed. The latter is interpreted as more risk tolerance with delay. At the individual level the picture is more mixed: Abdelaoui et al. report in Table 4 a mixed picture for elevation parameters of the Prelec (1998) type probability weighting function, but a clear tendency for increments in the corresponding sensitivity index.

3 Assuming EU with a slightly concave utility for small-scale outcomes (e.g., a power utility with power parameter of 0.88) is not sufficient to explain a preference for A when the best outcome has a probability larger than 0.5. Indeed, the consequence of a concave utility for small-scale gains under EU is that an unreasonable degree of risk aversion for larger scale gains is implied, which has forcefully been criticized by Rabin (2000).

Their last paragraph in Sect. 6 clarifies that it is the range over which pessimism is observed that seems to explain their findings, that is the range where the sensitivity index is mainly responsible for choice behavior.[4] For utility they find no significant effect of time. Overall, the results of Abdellaoui et al. support the view that subjects are more sensitive to changes in probabilities when delayed prospects are evaluated.

3 Theoretical framework

Initially, we present notation for risky objects in a timeless setting and the general preference model used to evaluate these. Then we discuss the specific probability weighting functions adopted in our temporal model. Subsequently, the more general setting is presented in which profiles of lotteries obtained and resolved at specified dates are defined. Following that, we introduce the temporal model with time-dependent constant relative sensitivity weighting functions.

3.1 Risky outcomes

Let $\mathbb{R}_+$ denote the set of non-negative deterministic *outcomes*. General outcomes are denoted $x, y, z, \ldots$, while in specific cases we use $a, b, c, d, \ldots$. A *prospect* is a finite probability distribution over outcomes and is denoted as $\tilde{x} = (p_1 : x_1, \ldots, p_n : x_n)$ meaning that outcome $x_j \in \mathbb{R}_+$ is obtained with probability p_j, for $j = 1, \ldots, n$. As usual, $p_j \geq 0$ for all $j = 1, \ldots, n$ and $\sum_{i=1}^{n} p_i = 1$ is assumed. Let $\mathcal{L}$ denote the set of all prospects.

For convenience of notation we always write prospects with the outcomes ordered from best to worst, i.e., for $\tilde{x} = (p_1 : x_1, \ldots, p_n : x_n)$ we have $x_1 \geq \cdots \geq x_n$. Further, we identify sure outcomes with the corresponding degenerate prospect; also, for $\tilde{x} = (p_1 : x_1, \ldots, p_n : x_n) \in \mathcal{L}$, the set of prospects with the probabilities $\{p_1, \ldots, p_n\}$ fixed is denoted by $\mathcal{L}_{\{p_1,\ldots,p_n\}}$. By $a_j\tilde{x}$ we denote the lottery $\tilde{x} = (p_1 : x_1, \ldots, p_n : x_n)$ with outcome x_j replaced by $a \in \mathbb{R}_+$ if $p_j > 0$ for $j \in \{1, \ldots, n\}$. Given the ordering of outcomes within prospects, the implicit constraint $x_{j-1} \geq a \geq x_{j+1}$ applies. This ensures that both $a_j\tilde{x}$ and $\tilde{x}$ are lotteries in $\mathcal{L}_{\{p_1,\ldots,p_n\}}$.

3.2 Prospect theory for risky outcomes

In our models below, we adopt *prospect theory*, PT for short (Tversky and Kahneman 1992) with an inverse-S probability weighting function. As we do not treat outcomes as gains or losses relative to a fixed reference point, our model boils down to rank-dependent utility (RDU; Quiggin 1982; Segal 1987; Wakker 1994), though much of the literature refers to this as PT because of the specific form of the probability weighting function; we follow this convention. Under PT, the value of a prospect is $\tilde{x} = (p_1 : x_1, \ldots, p_n : x_n) \in \mathcal{L}$ is given by

[4] For instance, of the six highlighted subjects in Abdellaoui et al. (2011, p. 982)'s Figure 3, one subject displays probability weighting functions that indicate changes in elevation while the other subjects appear to have time delay invariant elevation paired with significant changes in curvature.

$$PT(\tilde{x}) = \sum_{j=1}^{n}[w(p_1 + \cdots + p_j) - w(p_1 + \cdots + p_{j-1})]u(x_j), \tag{1}$$

where ($p_0 := 0$ and) $p_j^* := p_1 + \cdots + p_j$ are cumulated probabilities for $j = 1, \ldots, n$ and w is a *probability weighting function* on the unit interval, i.e., $w : [0, 1] \to [0, 1]$ is strictly increasing and continuous and it satisfies $w(0) = 0$ and $w(1) = 1$. Further, u is a *utility* function, i.e., $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing and continuous. In Eq. (1) the probability weighting function is unique and the utility function is *cardinal*, i.e., unique up to multiplication by a positive constant and addition of a real number. If $w(p) = p$ for all $p \in [0, 1]$, then PT reduces to *expected utility* (EU).

3.3 Constant relative sensitivity weighting

In analogy to how power utility functions are related to constant absolute risk aversion in EU, power probability weighting functions are related to modeling risk attitudes related to probabilities. This was exploited in Abdellaoui et al. (2010) to obtain the *constant relative sensitivity* (CRS) weighting functions, which are defined as follows:

$$w(p) = \begin{cases} \eta^{1-\sigma} p^{\sigma}, & \text{if} \quad p \in [0, \eta], \\ 1 - (1-\eta)^{1-\sigma}(1-p)^{\sigma}, & \text{if} \quad p \in (\eta, 1], \end{cases} \tag{2}$$

where $\eta \in [0, 1]$ is the parameter for *elevation* and $\sigma > 0$ is the (*in*)*sensitivity* parameter. To ensure the empirically documented inverse-S shape for the CRS weighting function, we require $0 < \eta < 1$ and $\sigma < 1$. If $\eta = 1$ then the CRS-weighting function in Eq. (2) is a power function that is concave over the entire probability interval, hence exhibiting *optimism* (Wakker 1994); for $\eta = 0$ we have *pessimism* as the CRS-weighting function is convex. Our restrictions on η ensure that optimism is exhibited for small probabilities ($p < \eta$) of good outcomes and pessimism for larger probabilities of less good outcomes ($p > \eta$).

As shown in Abdellaoui et al. (2010), the index of curvature of the CRS weighting function can be measured using the analogous of the Arrow–Pratt coefficient of relative risk aversion (Arrow 1971; Pratt 1964). For the CRS-weighting function this index is constant and equals $1 - \sigma$, hence it is positive and bounded when the shape of the probability weighting function is inverse-S. For intermediate probabilities away from 0 and 1 and close to η the CRS-function can be approximated by a linear weighting function that has slope σ and fixed point $p = \eta$, and which is discontinuous at 0 and at 1. Such probability weighting functions are referred to as NEO-additive as they induce preferences close to EU at non-extreme outcomes but give extra weight to best and worst outcomes (Chateauneuf et al. 2007; Webb and Zank 2011).[5] The constant σ or the relative curvature index $1 - \sigma$ of the CRS weighting function can therefore be used for comparative interpersonal analyses and, in particular, for intrapersonal

[5] For empirically founded intrapersonal comparisons of insensitivity to choice based probabilities derived using different source of uncertainty, Abdellaoui et al. (2011a) have adopted NEO-additive probability weighting functions. See also Webb (2015, 2017) for continuous extensions that approximate NEO-additive preferences.

comparative statics concerning the changes in sensitivity resulting from a temporal delay of prospects. To this aim we expand our static framework and consider profiles of risky outcomes.

3.4 Risky profiles

In this paper we consider preferences over profiles of risky outcomes. We assume discrete time periods $t = 0, \ldots, T$ for $T \geq 1$. The objects of choice are risky *profiles*, i.e., profiles of prospects that are obtained and resolved at the indicated time period. We use bold-faced letters to denote profiles, that is, $\mathbf{x} = (\tilde{x}_0, \tilde{x}_1, \ldots, \tilde{x}_T)$ is a profile where prospect $\tilde{x}_t = (p_{t,1} : x_{t,1}, \ldots, p_{t,n_t} : x_{t,n_t})$ is obtained and resolved at time t for all $t \in \{0, \ldots, T\}$. The set of all profiles is $\mathcal{P} = \Pi_{t=0}^{T}\mathcal{L}$. Sometimes we write $(\tilde{x}_t)\mathbf{x}$ or simply $\tilde{x}_t\mathbf{x}$ to highlight the prospect obtained at time t.

A preference relation is a binary relation $\succcurlyeq$ defined over $\mathcal{P}$, with $\succ$ denoting strict preference, and $\sim$ denoting indifference; reversed symbols denote corresponding preference, as usual. A real-valued function V *represents* $\succcurlyeq$ on $\mathcal{P}$ if $\mathbf{x} \succcurlyeq \mathbf{y} \Leftrightarrow V(\mathbf{x}) \geq V(\mathbf{y})$ for all profiles $\mathbf{x}, \mathbf{y} \in \mathcal{P}$. Profiles of prospects are evaluated by discounted prospect theory. We present this general model within a formal definition.

Definition 1 General *Discounted Prospect Theory* (DPT) holds if the preference relation $\succcurlyeq$ on $\mathcal{P}$ is represented by

$$\text{DPT}(\mathbf{x}) = \sum_{t=0}^{T} D(t)PT_t(\tilde{x}_t), \tag{3}$$

where the *discount function*, $D(\cdot)$, is positive valued with $D(0) = 1$, and PT_t stands for time-dependent prospect theory, which in general has a time-dependent probability weighting function, w_t, and a corresponding utility function, u_t, for $t = 1, \ldots, T$. The specific normalization of the discount function renders it unique as are the weighting functions, while the utility functions are jointly cardinal (i.e., they can be replaced by $v_t = Au_t + B_t$ for a positive A and real valued B_t, $t = 0, \ldots, T$).

DPT differs from the classical constant discounted expected utility (DEU) model as it allows for a general discounting function. For instance, we have not invoked the standard assumption of impatience [that is, $D(t) > D(s)$ for all $s, t \in \{0, \ldots, T\}$ with $t < s$], which is a natural condition in the context of choice among profiles with sure outcomes. Assuming impatience will not affect any of our results below, hence, we have not explicitly required the property. In applications one can be more restrictive by adopting specific parametric forms like exponential discounting (Samuelson 1937; Koopmans 1960), quasi-hyperbolic discounting (Phelps and Pollak 1968; Laibson 1997), or more general parametric families as discussed in Bleichrodt et al. (2009). A further aspect where DPT departs from classical DEU is the evaluation of prospects, where in DPT, the most successful descriptive model for static choice, PT (Wakker 2010; Barberis 2013), is used. That DPT is quite general also follows from the fact that, without invoking further constraints on behavior, the weighting functions and utility functions at different time points can be unrelated.

The preference relation $\succcurlyeq$ on $\mathcal{P}$ induces a corresponding relation, $\succcurlyeq_t$, over prospects obtained at time $t = 0, \ldots, T$, which in turn induces a preference relation over timed outcomes; we use the same symbol, $\succcurlyeq_t$, for the latter. As the preference considered in Definition 1 is additively separable over time periods, the restrictions of the preference relation to individual time periods are well-defined. While DPT allows for general probability weighting functions to depend on time, the specific version we adopt here has further restrictions as stated in the next assumption.

Assumption 1 Throughout we assume that DPT in Eq. (3) has CRS-probability weighting functions in each time period. That is, we assume that, for each induced preference relation $\succcurlyeq_t$ over prospects obtained at time $t = 0, \ldots, T$, the preference conditions of Abdellaoui et al. (2010) are satisfied.

The preceding assumption allows for each induced preference relation $\succcurlyeq_t, t = 0, \ldots, T$, to be represented by PT with a CRS-probability weighting function. The assumption does not impose further relationships between the CRS parameters across time periods nor for the utility functions which, in general, can depend on the delay time $t = 0, \ldots, T$. We are, however, interested in some form of consistency across time periods. To obtain such a connection we require specific preference conditions, which we present in the next section.

4 Preference properties

This section presents additional preference conditions to further restrict the general class of DPT representations. From Definition 1 one can infer that $\succcurlyeq$ on $\mathcal{P}$ is a complete and transitive preference (which are necessary for the existence of a representing function) that satisfies continuity and monotonicity in outcomes (as the discount function is positive valued, the weighting functions are strictly increasing and the utility functions are strictly increasing and continuous in each period). Further, according with our Assumption 1, the preference also satisfies continuity in probabilities as CRS weighting functions are assumed to be continuous on the probability interval. The next subsection focuses on properties for utility and the elevation parameters. Subsequently, we consider properties that relate to the curvature parameter of the CRS-probability weighting function.

4.1 Time-invariant utility and elevation

As our theory is intended for the study of today's preferences over risky profiles and to understand if delay has an effect on attitudes towards probabilities, we feel that the assumption of a common utility (i.e., today's risky utility) being used in the evaluation of profiles of risky prospects is in order. When studying preferences at different time points one would need to account for tastes that may change over time, which could well be captured by a time-dependent utility (see, e.g., Gerber and Rohde 2018). Here, we model today's preference over (streams of) risky objects. For such choices, the study of Abdellaoui et al. (2011) finds less risk aversion for delayed risky prospects but no evidence for a significant impact of time on utility was documented. Arguably, reference to a single experimental study is only indicative evidence and further empirical tests are required to obtain conclusive evidence that time-invariant utility is an adequate assumption in the context of choice over streams of risky prospects. For instance, less risk aversion for delayed prospects can in principle be explained with a less-concave utility for outcomes relative to the current period's utility; however, it would be unclear how to model less optimism for delayed prospects with that assumption. For this reason, we pursue our theoretical analysis by building on the hypothesis that utility is time-invariant.

The next preference condition proposes a consistency requirement for utility across time periods. It demands that riskless outcome-tradeoffs measured at different time periods are invariant to delay. Similar conditions have been used in the derivation of cardinal utility in static decision models for risk and ambiguity (Köbberling and Wakker 2003).

We say that the preference relation $\succcurlyeq$ on $\mathcal{P}$ satisfies *time-invariance for outcome tradeoffs* if for all time periods $t, s \in \{0, \ldots, T\}$, $s < t$, all outcomes $a, b, c, d \in \mathbb{R}_+$ and all profiles $\mathbf{x}, \mathbf{y} \in \mathcal{P}$ any three of the following indifferences imply the fourth:

$$a_s\mathbf{x} \sim b_s\mathbf{y},\; c_s\mathbf{x} \sim d_s\mathbf{y},$$
$$a_t\mathbf{x} \sim b_t\mathbf{y},\; c_t\mathbf{x} \sim d_t\mathbf{y}.$$

Substitution of DPT into the preceding four indifferences, taking differences of the first pair of resulting equations and similarly of the second pair, and cancelling of common terms, one obtains the following utility differences

$$u_s(a) - u_s(b) = u_s(c) - u_s(d)$$
$$u_t(a) - u_t(b) = u_t(c) - u_t(d),$$

which are supposed to hold for all time periods $t, s \in \{0, \ldots, T\}$, $s < t$, and all outcomes $a, b, c, d \in \mathbb{R}_+$. That is, whenever the first equation holds the second must also hold, and it means that the continuous and strictly increasing utility functions u_s and u_t are proportional and can be taken equal. We obtain the following result.

Proposition 1 *Assume that the preference relation $\succcurlyeq$ on $\mathcal{P}$ is represented by DPT as in Definition 1. Then, $\succcurlyeq$ on $\mathcal{P}$ satisfies time-invariance for outcome tradeoffs if and only if $u := u_0 = u_t$ for all time periods $t \in \{1, \ldots, T\}$.* □

The proof of this proposition follows directly from the results of Köbberling and Wakker (2003); their results are tailored for ambiguity but their arguments apply similarly for our framework with risky profiles. Köbberling and Wakker show (see their Corollaries 29 and 10) that the analogous condition of time-invariance for outcome tradeoffs is a powerful property which, for a representation that is continuous and strictly monotonic in outcomes, implies additive separability across time periods; subsequently, the proportionality results for utility are derived. The advantage of using outcome tradeoffs as a preference condition is that the tradeoff tool leans itself to the measurement of utility and this allows for non-parametric tests that can be used to empirically verify the time independence of utility under DPT (Wakker and Deneffe 1996; Abdellaoui 2000; Abdellaoui et al. 2010).

Our next property requires further consistency of preferences across time periods. It ensures that the elevation parameter of the CRS probability weighting functions is also time-invariant. The elevation parameter is regarded as an index that measures a general propensity of a decision maker to take risks. A higher propensity to take risks, i.e., a larger value of the elevation parameter, graphically shifts the entire CRS weighing function upward. Consequently, the region where the CRS-function exhibits optimism increases at the expense of the region where pessimism governs choice behavior. Behaviorally it means that relatively more optimism is exhibited globally for all probabilities and all choices, which is in contrast to the findings of Abdellaoui et al. (2011).

Like risk attitudes captured in the time-invariant utility function, we impose that elevation is not affected by delay, although both assumptions need a better empirical foundation (we discuss this aspect further in Sect. 6). Accordingly, by adopting the terminology of Gonzalez and Wu (1999), we say that the preference relation $\succcurlyeq$ on $\mathcal{P}$ satisfies *time-invariant propensity to gamble* if for all time periods $t, s \in \{0, \ldots, T\}$, $s < t$, all outcomes $a, b, x, y \in \mathbb{R}_+$ and all profiles $\mathbf{x} \in \mathcal{P}$ the following holds:

$$(\eta_s : a, 1 - \eta_s : x)_s\mathbf{x} \sim (\eta_s : b, 1 - \eta_s : y)_s\mathbf{x}$$
$$\Rightarrow (\eta_t : a, 1 - \eta_t : x)_t\mathbf{x} \sim (\eta_t : b, 1 - \eta_t : y)_t\mathbf{x}.$$

Time-invariant propensity to gamble says that, for binary prospects where the probability of the better outcome is not distorted, hence, locally neither optimism nor pessimism can be inferred, only the tradeoffs among outcomes govern choice behavior. There is some empirical support for probabilities that are not distorted, in particular many studies report that probabilities close to $1/4 - 1/3$ are less subjected to distortion (Tversky and Kahneman 1992; Wu and Gonzalez 1996; Abdellaoui 2000; Bleichrodt and Pinto 2000; Abdellaoui et al. 2005; Etchart-Vincent 2004; see also the discussion in Wakker 2010, Chapter 7). Abdellaoui et al. (2010) find no statistically significant difference between the elevation parameters of the CRS-weighting functions for gains and losses, and Abdellaoui et al. (2011, Tables 5, 6) at the aggregate level find no statistical significant differences between the distorted probability of $p = 1/3$ for delayed prospects. Both findings suggest that behavior captured through the elevation index is relatively stable across choice contexts (gains/losses and instant/delayed), and these findings are compatible with our preference requirement and with the property of the CRS-probability weighting functions that have a fixed point at η_t, i.e., $w_t(\eta_t) = \eta_t$ for all $t = 0, \ldots, T$.[6]

By additionally invoking the time-invariance property for the propensity to gamble in Proposition 1 we obtain the DPT model with time-invariant utility and time-invariant elevation index.

[6] Wakker (2010, pp. 205–206) reports that $w(1/3)$ being approximately 1/3 is the most common finding for probability weighting w under PT.

Proposition 2 *Assume that the preference relation $\succcurlyeq$ on $\mathcal{P}$ satisfies Assumption 1 and that time-invariance for outcome tradeoffs holds. Then, $\succcurlyeq$ on $\mathcal{P}$ satisfies time-invariant propensity to gamble if and only if $\eta := \eta_0 = \eta_t$ for all time periods $t \in \{1, \ldots, T\}$.*

Proposition 2 provides a characterization of a special case of DPT with CRS-probability weighting and constant elevation parameter across time; we label it DPT$^{\eta}$. In contrast to the elevation parameter and the utility of outcomes, we do not restrict the curvature parameter of the probability weighting functions in the DPT$^{\eta}$ model. This allows for intrapersonal comparisons of the changes to curvature across time periods. This means that the discounting function and the curvature parameters are the only means to explain changes in choice behavior when prospects are delayed and, as the discounting function is separable from attitude towards outcomes and attitudes towards probabilities in DPT$^{\eta}$, the whole burden of behavioral change with delay will be carried by the curvature parameter. We present this analysis in the next subsection.

4.2 Insensitivity and delay

This subsection relates changes in the curvature of the probability function when risks are commonly delayed to risk aversion. We proceed by recalling some examples of choices among prospects used in the experimental literature, some of which were briefly touched upon in Sect. 2.

The first example is due to Baucells and Heukamp (2010). They tested the effect of delay on the common ratio paradox to EU of Allais (1953). In one experiment (Baucells and Heukamp, Table 1) the choices were among two pairs of prospects that were obtained at time $t = 0$ (now), $t = 1$ (1 month), $t = 2$ (3 months), as follows (payments in EURO):

$$\begin{array}{ccc} A = (1:9)_t & \text{versus} & B = (0.8:12, 0.2:0)_t \\ & \text{and} & \\ A' = (0.1:9, 0.9:0)_t & \text{versus} & B' = (0.08:12, 0.92:0)_t. \end{array}$$

For $t = 0$ they find that 58% favor A in the first choice while 78% prefer B' in the second choice. For $t = 1$ the corresponding proportion of choices are 55% and 74%, and when $t = 2$ the respective proportions are 43% and 79%. The results suggest that delay induces many individuals to change their preference from $A \succ B$ at time $t = 0$ to $B \succ A$ at time $t = 2$. This shift in preference seems to happen gradually as a delay of 1 month does lead to just minor changes in the proportions, such that an immediacy effect as explanation of the common ratio effect can be excluded here. Moreover, as the proportions of choices for B' in the second pair of prospects are relatively stable, it seems as if the subjects in this experiment are better at discriminating between probabilities 0.8 and 1.0 when the choices are delayed, even though the common ratio effect persists, albeit less pronounced.

The implications of delay for the common consequence effect of Allais (1953) were tested in Weber and Chapman (2005). The choices in their experiment were—using the Kahneman and Tversky (1979) version—among two pairs of prospects that were obtained at time $t = 0$ (now), $t = 1$ (1 year), $t = 2$ (25 years), as follows (payments in US$):

$$C = (1 : 2700)_t \quad \text{versus} \quad D = (0.33 : 3000, 0.66 : 2700, 0.01 : 0)_t$$
$$\text{and}$$
$$C' = (0.66 : 2700, 0.34 : 0)_t \quad \text{versus} \quad D' = (0.33 : 3000, 0.67 : 0)_t.$$

They find that the common consequence effect persists when prospects are delayed although it becomes somewhat weaker when $t = 1$. That is, for $t = 0$ prospect C is chosen by 85% of the subjects while 58% favor D'; for $t = 1$ and $t = 2$ the corresponding percentages are 76% and 60% and, respectively, 82% and 57%. While a time delay of 25 years adds a lot of uncertainty and may lead subjects to adhere to current preferences, the delay of 1 year suggests that individuals have a slight tendency to better account for small probability differences (1.0 and 0.99) of gaining a large sum of money, which indirectly means that they better discriminate between those large probabilities.

The preceding two studies have indicated that changes in preference as a result of delay are most likely to be observed when the choice is between a sure or very likely outcome and a non-degenerate prospect. Accordingly, our third summary example looks at the more recent study of Abdellaoui et al. (2011) who employ such choices. More specifically, they elicited certainty equivalents for binary prospects while varying outcomes and probabilities, and they considered settings with no delay ($t = 0$), 6 months delay ($t = 1$) and a delay of 1 year ($t = 2$).[7] Their initial finding is that certainty equivalents tend to increase with delay, indicating less risk aversion as suggested by Noussair and Wu (2006). Such risk behavior can be the result of better discrimination between probabilities when prospects are delayed.

A further finding of Abdellaoui et al. (2011), based on the assumption that utility is a power function as in Tversky and Kahneman (1992), was that statistically there is no significant difference between the elicited power parameters for utility. This means that the observed reduction in risk aversion can mainly be attributed to the treatment of probabilities. Indeed, Abdellaoui et al. find that the weights for probabilities $p \geq 1/2$ increase with delay but remain relatively unchanged for $p \in \{1/6, 1/3\}$. Both, the findings for utility and for probability weighting are in line with the assumptions underlying the DPT$^{\eta}$ model suggested in Proposition 2. Therefore, to explain the phenomenon of less risk aversion as a result of delay, we have just one more degree of freedom left in our model, namely the curvature parameter σ of our CRS-probability weighting functions.

Next we proceed with intrapersonal analysis on the empirically observed phenomenon of more sensitivity due to delay. We adopt the probability midweight tool of van de Kuilen and Wakker (2011). This tool has recently been adopted by Werner and Zank (2019) to provide preference foundations for PT in a static framework without

7 Abdellaoui et al. (2011) also used a setting with an unspecified or "ambiguous" date of delay between 6 and 12 months. Such ambiguity adds an additional consideration into the decision-making process, not captured by DPT, and seems to lead to more risk aversion than observed for the clearly specified dates of 6 and 12 months.

prior knowledge of the location for the reference point. Werner and Zank indicate that the probability midweight method is suitable for empirically detecting reference point effects beyond it being a non-parametric way of eliciting probability weighting effects and its appeal for comparative analyses.

The midweight method of van de Kuilen and Wakker (2011) requires the prior elicitation of a utility midpoint before proceeding with the elicitation of probabilities. Given the continuity assumptions for utility and probability weighting functions under DPT as in Proposition 2, such midpoints for utility and weighting functions are always feasible and well-defined. Hence, we can fix outcomes $x > y > 0$ such that $u(x) - u(y) = u(y) - u(0)$ and find, for some $t \in \{0, \ldots, T-1\}$, the probability p_t such that

$$(\eta : x, 1-\eta : y)_t\mathbf{z} \sim (p_t : x, 1-p_t : 0)_t\mathbf{z}.$$

Then, adding a delay of one time period leads to reduced insensitivity (equivalently, increased sensitivity) if

$$(\eta : x, 1-\eta : y)_{t+1}\mathbf{z} \sim (p_{t+1} : x, 1-p_{t+1} : 0)_{t+1}\mathbf{z}$$

implies that $p_t > p_{t+1}$. If this implication holds for all $t \in \{0, \ldots, T-1\}$ we say that $\succcurlyeq$ on $\mathcal{P}$ satisfies *increasing sensitivity with delay*.

To further clarify the implications of delay on the sensitivity parameter of the CRS-probability weighting functions, we proceed with some derivations resulting from substitution of DPT$^\eta$ into the first of the preceding indifferences. After canceling the common terms related to $\mathbf{z}$ outside period t, we obtain

$$w_t(\eta)u(x) + [1 - w_t(\eta)]u(y) = w_t(p_t)u(x) + [1 - w_t(p_t)]u(0).$$

Next, exploiting that η is a fix-point of w_t and that y is a utility midpoint between 0 and x, we obtain

$$w_t(p_t) = \frac{1+\eta}{2}, \tag{4}$$

such that p_t is a *midweight* between η and 1 for w_t (i.e., a probability midpoint on the w_t-scale). Similarly, using DPT$^\eta$ in the second indifference, we obtain that p_{t+1} is a midweight between η and 1 for w_{t+1}. We conclude that increasing sensitivity to delay implies that $p_t > p_{t+1}$ and $w_t(p_t) = w_{t+1}(p_{t+1})$. The corresponding implication for the CRS-weighting functions of adjacent time periods can be seen in Fig. 1, where the horizontal axis depicts cumulated probabilities which are weighted by w_t, respectively, w_{t+1} to values on the vertical axis:

Figure 1 indicates, that the requirement for the midweight p_{t+1} to be smaller than p_t implies that w_{t+1} is closer to the 45-degree line [where $w_s(p) = p$ as in EU] than w_t is. This means that w_{t+1} is steeper than w_t, hence, is more sensitive to changes in probabilities. This can also be inferred from substitution of the CRS-probability weighting function in Eq. (4). As $\eta < p_{t+1} < p_t < 1$, one obtains

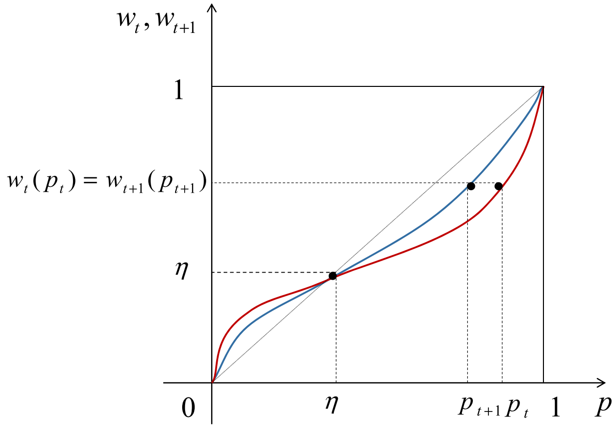

Fig. 1 Effect of increasing sensitivity to delay

$$1 - (1-\eta)(1/2)^{1/\sigma_{t+1}} = p_{t+1} < p_t = 1 - (1-\eta)(1/2)^{1/\sigma_t}$$

from which $\sigma_t < \sigma_{t+1}$ follows as a result of reduced insensitivity with delay.

Next, we discuss the empirically observed phenomenon of less risk aversion with delay. Recall the example of choices in Baucells and Heukamp (2010) discussed above. It appears as if at time 0 individuals have a preference for $A \succ_0 B$ and at a later date $t > 0$ this is reversed to $B \succcurlyeq_t A$. Such behavior suggests that the preference for the sure A today is changing as a result of delay to a preference for the risky B. Thus, an increased willingness to take risk is observed when prospects are obtained later. Formally, we say that $\succcurlyeq$ on $\mathcal{P}$ exhibits *reduced risk aversion with delay* if for all outcomes $x > y > 0$, all profiles $\mathbf{z} \in \mathcal{P}$, all probabilities $\eta < p < 1$ and all time periods $t \in \{0, \ldots, T-1\}$ we have

$$(p : x, 1-p : 0)_t\mathbf{z} \sim (1 : y)_t\mathbf{z} \Rightarrow (p : x, 1-p : 0)_{t+1}\mathbf{z} \succ (1 : y)_{t+1}\mathbf{z}.$$

Reduced risk aversion is formulated for probabilities where the CRS weighting functions display pessimism, i.e., they are convex-shaped (Chew et al. 1987; Chateauneuf and Cohen 1994; Wakker 1994, 2010; Baucells and Heukamp 2006; Ryan 2006; Schmidt and Zank 2008). For positive probabilities smaller than η the CRS-weighting functions exhibit optimism, i.e., risk proneness relative to EU-preferences as revealed through concavity of the probability weighting function. It seems less plausible to demand or detect less risk aversion over a domain of prospects where the typical behavior would be more risk-seeking relative to EU. For this reason, reduced risk aversion due to delay is defined for choices among binary prospects where the typical finding is risk aversion. Substitution of DPT$^\eta$ into the preceding preferences implies that there must be less convexity of the CRS-weighting function as a result of delay. This means that $\sigma_t < \sigma_{t+1}$ for all $t \in \{0, \ldots, T-1\}$. We summarize the analysis of this section in the following theorem.

Theorem 1 *Assume that* $\succcurlyeq$ *on* $\mathcal{P}$ *is represented by DPT*$^{\eta}$ *as in Proposition 2. Then the following statements are equivalent:*

(i) *The preference* $\succcurlyeq$ *on* $\mathcal{P}$ *satisfies increased sensitivity with delay;*
(ii) *The preference* $\succcurlyeq$ *on* $\mathcal{P}$ *exhibits reduced risk aversion with delay;*
(iii) *For all* $t \in \{0, \ldots, T-1\}$ *we have* $0 < \sigma_t < \sigma_{t+1} < 1$.

The data in Abdellaoui et al. (2011) indicate that indeed the subjects in their study exhibit less risk aversion with the delay of prospects. They inferred this through the elicitation of certainty equivalents for binary prospects. Theorem 1 indicates alternative means of detecting reduced risk aversion through the identification of midweights, the non-parametric tool proposed by van de Kuilen and Wakker (2011). Empirically, it is unclear if the monotonic relationship for the sensitivity parameter of the CRS-weighting function is valid for all conceivable time periods. As the example in Weber and Chapman (2005) suggests, a delay of 25 years seems to lead to more insensitivity for moderate probabilities. Such a distant time period may indeed induce subjects not to worry about small changes in probabilities as the outcome of those prospects will affect them only in the very distant future, if at all (for instance, survival considerations may play a role, which are not part of our DPT model).

5 An application to bargaining

In this section, we provide an application of our model to bargaining. In doing so, we demonstrate how reduced aversion to risk due to delay introduces interesting dynamic issues that do not arise under discounted expected utility. A central result of non-cooperative bargaining theory (Rubinstein 1982) is that agreements will be reached immediately. The reason for this prediction is that delay is costly to all parties involved in the bargaining process. Delays may nonetheless occur and a literature has emerged that attempts to explain how delays in agreement can result. Most explanations for bargaining delays have appealed to asymmetric information (Rubinstein 1985; Gul and Sonnenschein 1988; Abreu and Gul 2000). Other explanations have considered alternative assumptions regarding the opportunity costs of delay (Fernandez and Glazer 1991), the possibility of credible threats to reduce the available surplus (Avery and Zemsky 1994), and the way disagreement outcomes are decided (Busch and Wen 1995).

In the standard cooperative setting, assuming DEU, delay has a similar effect to enforcing immediate agreement; the timing of the bargaining and the timing of the outcome are immaterial for reaching a solution. Aiming at immediate agreement may therefore explain why in the cooperative setting time has been invoked as a convenient additional dimension. For the more general preferences discussed in this paper, new possibilities to obtain solutions arise, hence, we need to expand the standard cooperative setting. Suppose that two individuals, labeled A and B, are bargaining over the chance to receive an indivisible rent R obtained at time $\tau \in \{0, \ldots, T\}$. Disagreement results in outcome d for each individual. The actual bargaining takes place at time $t_b \in \{0, \ldots, \tau\}$, a period that we assume endogenously given. The fact that individuals are bargaining at time t_b over an outcome received at time τ introduces a possible bargaining outcome delay, as measured by $\tau - t_b$. We ask the question if this delay is advantageous for any of the individuals in a bargaining context and compare the result with the classical DEU case.

We next assume that both individuals maximize DPT with constant relative sensitivity (i.e., DPT$^\eta$ for individuals A, B with respective discount functions D_A, D_B, utilities u_A, u_B and weighting functions $w_{A,t}$, $w_{B,t}$ with corresponding parameters η_A, η_B and $\sigma_{A,t}$, $\sigma_{B,t}$). After normalizing utilities so that $D_A(\tau)u_A(R) = D_B(\tau)u_B(R) = 1$ and $D_A(\tau)u_A(d) = D_B(\tau)u_B(d) = 0$. The bargaining set in utility space with bargaining outcome time τ and bargaining time t_b is:

$$\mathcal{B}_\tau^{t_b} = \{(w_{A,\tau-t_b}(p_A), w_{B,\tau-t_b}(p_B)) : p_A, p_B \geq 0, p_A + p_B \leq 1\},$$

with the interpretation that either individual A obtains R with probability p_A, or individual B obtains R with probability p_B at time τ [else 0 utility is obtained leading to (0, 0), the disagreement outcome pair].

A few potential solutions to the one-shot bargaining problem could be considered, and we follow Köbberling and Peters (2003) who argued that for PT-preferences—like the one considered here—the Kalai–Smorodinsky (KS) solution is appropriate. Existence is guaranteed, as the Pareto frontier of the bargaining set in utility space is strictly decreasing (Conley and Wilkie 1991). One can then show that, in utility space, the KS-solution is given by:

$$\text{KS}(\mathcal{B}_\tau^{t_b}, (0, 0)) = \{(w, w) : w_{A,\tau-t_b}(p) = w_{B,\tau-t_b}(1 - p) = w, p \in [0, 1]\},$$

which is unique and well-defined.

With bargaining time being t_b and bargaining outcome time being τ various scenarios emerge allowing for comparative analyses. For expositional reasons and to maintain simplicity, we consider the case of two periods, 0 and 1, only, and we compare the results of DPT$^\eta$-preferences with those implied by DEU.

Claim 1 Let discounted expected utility hold. For a given bargaining time, bargaining outcome delays are never beneficial. For a given bargaining outcome time, bargaining delays have no effect.

The following table illustrates the statements in Claim 1. In Table 1, the change from $t_b = 0, \tau = 0$ to $t_b = 0, \tau = 1$ corresponds to a bargaining outcome delay. Since for both individuals the discount function is strictly decreasing under DEU, it follows that $D_i(1) < D_i(0) = 1$, which implies $D_i(1)/2 < 1/2$, for $i = A, B$. Thus, if individuals have a choice, they would be best placed to bargain in period 0 and

Table 1 Bargaining outcomes under DEU

Bargaining/outcome time	Utilities at time $t = 0$
$t_b = 0, \tau = 0$	$(1/2, 1/2)$
$t_b = 0, \tau = 1$	$(D_A(1)/2, D_B(1)/2)$
$t_b = 1, \tau = 1$	$(D_A(1)/2, D_B(1)/2)$

Table 2 Bargaining outcomes under DEU

Bargaining/outcome time	Utilities at time $t = 0$
$t_b = 0, \tau = 0$	(w, w) with $w_{A,0}(p) = w_{B,0}(1 - p) = w$
$t_b = 0, \tau = 1$	$(D_A(1)w, D_B(1)w)$ with $w_{A,1}(p) = w_{B,1}(1 - p) = w$
$t_b = 1, \tau = 1$	$(D_A(1)w, D_B(1)w)$ with $w_{A,0}(p) = w_{B,0}(1 - p) = w$

reach immediate agreement with outcomes paid out at the same time. To see that, for a given outcome time, bargaining time delays have no effect, we compare the cases in the last two columns of Table 1. We observe that for both scenarios ($t_b = 0$, $\tau = 1$ and $t_b = 1$, $\tau = 1$) the utilities are identical: $(D_A(1)/2, D_B(1)/2)$. For DPT$^\eta$-preferences the results are different, as we state in our next claim.

Claim 2 Let DPT hold with reduced risk aversion. Then, bargaining outcome delays can be beneficial.

Table 2 illustrates the possible outcomes under the assumptions of Claim 2. To give a concrete example where one individual's preferences deviate from DEU, suppose that individual A maximizes DEU (i.e., DPT$^\eta$ with $\sigma_A = 1$) and individual B is purely pessimistic ($\eta_B = 0$). Let $\sigma_{B,0} = 1/2$ while $\sigma_{B,1} = 1$. That is, individual B is pessimistic today but has EU-preferences over delayed prospects. It then follows that

$$\mathrm{KS}(\mathcal{B}_\tau^{t_b}, (0, 0)) = \{(w, w) : w = 1 - w^{\sigma_\tau}\}.$$

For the case $t_b = 0$, $\tau = 0$ this implies that the KS-solution corresponds to the chances of receiving R shared as $(\frac{3-\sqrt{5}}{2}, \frac{\sqrt{5}-1}{2})$. If $t_b = 1$, $\tau = 1$, because both individuals have EU-preferences in this period, their outcome is similar to the preceding case but discounted.

Now considering the case $t_b = 0$, $\tau = 1$. The KS-solution gives probabilities of obtaining the rent $(1/2, 1/2)$. As the payment is delayed, the corresponding utilities are $(D_A(1)/2, D_B(1)/2)$. Individual A compares utility relative to the bargaining outcome time being $\tau = 0$, thus $\frac{D_A(1)}{2}$ with $\frac{3-\sqrt{5}}{2}$. therefore, individual A would prefer to delay the bargaining outcome if $D_A(1) > 3 - \sqrt{5} \approx 0.76$.

Let us explain the intuition for why delayed bargaining outcomes can be beneficial. In our example, individual A's probability of receiving the rent is determined such that $p = 1 - p^{\sigma_{B,\tau}}$. By implicit differentiation we obtain

$$\frac{\mathrm{d}p_A}{\mathrm{d}\sigma_{B,\tau}} = -\frac{ln(p_A)p^{\sigma_{B,\tau}}}{1 + \sigma_{B,\tau}p^{\sigma_{B,\tau}-1}},$$

which is positive for all $p \in (0, 1)$. This means that individual A's chance of receiving the rent increases with individual B's reduction in risk aversion, provided that $\sigma_{B,t_b} < 1$. The argument used here follows the explanation provided by Köbberling and Peters

(2003): the higher risk aversion of B corresponds to less curvature in the region where pessimism governs behavior, which means that individual B is more difficult to please. Hence, A must offer B a better deal at A's own expense. With reduced risk aversion for the bargaining outcome, B is easier to please, thus A's chances of obtaining the rent improve. Provided that this improvement is large enough to counteract the fact that delayed outcomes are discounted, individual A will prefer the bargaining outcome delay.

The simple example of an application to bargaining of this section illustrates the implication of one individual exhibiting pessimism today, which is reduced by delay of the risky bargaining outcome. At the bargaining time, pessimism for one individual can have a negative effect on both individuals in the bargaining problem. Clearly, optimism of one individual at the bargaining time but not at the outcome time has the opposite effect for the opponent's outcome. As one expects that the probability weight w, which reflects a similar perceived chances to obtain the rent, is likely to exceed the individuals η-value (if the latter are close to 1/3 as the data for static choice suggests), the case for pessimism appears to be the more prominent. Hence our simple analysis was focused on this case only.

6 Further discussion

In this section, we reconsider the psychological interpretation of insensitivity and elevation and speculate if a categorization of a prospects resolution and payout time into "now" versus "anytime later" psychologically can affect choice behavior and if this would call for elevation to also be dependent on time delay. Alternatively, we consider a potential behavioral explanation based on a "source-dependent" argument, where the interpretation of "risk today" being different to "delayed risk" has similarity to the distinction into sources of uncertainty (Abdellaoui et al. 2011a). Finally, we briefly review some empirical studies that view changes in risk attitude with delay as being a feature that needs to be captured by utility.

Recall that Gonzalez and Wu (1999) regard insensitivity as the index that captures the ability of decision makers to discriminate between probabilities and see this as one important psychological dimension that affects the choice behavior of individuals facing risk. It is, therefore, plausible that this ability captures a cognitive aspect concerning the processing of probabilistic information. In turn, such processing of information may be affected by an individual's emotional state (e.g., Kahneman 2011; Evans and Stanovich 2013). A potential (good or bad) outcome that affects the decision maker instantly can intensify emotional feelings, thereby, putting the decision maker in a so-called "hot" state of cognition in which the ability to discriminate among probabilities is reduced because the focus is on the immediate consequences of obtaining the outcome. By contrast, in a choice situation where the resolution of uncertainty and the receipt of the outcome is more distant in the future, the decision maker is in a "cold" state of cognition as emotional aspects are close to neutral. Being in different cognition states will then be revealed in the treatment of the probabilities of corresponding outcomes showing different degrees of insensitivity for the weighting function.[8]

While curvature as index of (in)sensitivity captures one psychological dimension that has an impact on the treatment of probabilities, Gonzalez and Wu (1999) identified a second dimension that is relevant. This dimension refers to the propensity of individuals to take risks and is captured by the *elevation* measure. This inclination to take risks, which Gonzalez and Wu termed "attractiveness to gambling", can be seen as a tradeoff between optimism and pessimism in the sense that a more elevated inverse-S weighting function reduces the range of probabilities over which pessimism is revealed and, hence, increases the range of probabilities where optimism is exhibited. As this means more overweighting of unlikely best outcomes and less underweighting of likely worst outcomes, elevation can be regarded as an index of *relative pessimism* (Abdellaoui et al. 2011a). In some sense elevation reflects the relative confidence level of an individual in taking risks when acting in a particular choice environment. For instance, a person aquainted with financial investment products may appear optimistic when choosing among different home or car insurance policies with deductibles, while in a situation where the choices are among medical drugs with potential side effects the very same person may appear relatively more pessimistic. Such behavior is independent of their cognitive processing of probabilistic information. Thus, elevation as a measure for a propensity to take risk may reflect a personal trait of individuals; it can vary across decision contexts while over time such traits may not change (Frey et al. 2017). This has been our assumption and for our model we have provided the testable preference condition that can serve for an empirical investigation.

For choice behavior where the probabilistic information is generated by different sources of uncertainty (Fox and Tversky 1995; Kilka and Weber 2001), such as ambiguity (unknown probabilities for events) versus risk (known probabilities), descriptively elevation may be particularly important. As shown in the study of Abdellaoui et al. (2011a), across these sources of uncertainty relative pessimism (i.e., elevation) seems to vary considerably. Their results also indicate more insensitivity to ambiguity relative to risk; which is natural as under ambiguity there is little objective information available for processing, hence, making it more difficult for individuals to reach a decision. However, when considering different ambiguity sources (e.g., temperature in Paris or changes in the value of the French Stock Index CAC40 on a particular day), Abdellaoui et al. did not find significant differences for insensitivity. By contrast, their measure of relative pessimism varied significantly across ambiguity sources, e.g., Parisians have a higher relative pessimism index for choices involving uncertainty over the tempera-

[8] Such behavior would also accord with construal-level theory of psychological distance (Trope and Liberman 2010) applied to probabilities. One can think of decisions with immediate resolution as triggering Footnote 8 continued
emotions because the decision maker is instantly affected by the outcome. Fearing the worst outcome or being hopeful to obtain the best outcome of a prospect might be more accentuated and would be reflected in higher probability weights at the expense of those for intermediate outcomes. If uncertainty resolution and outcome receipt are delayed the decision maker's emotions may be supressed because of the absence of immediate affect. The opposite behavior may also show. The desirability of a future potential outcome that can serve to fulfill a planned action (like going on holiday to one's favorite destination next summer) might trigger emotions that show as inflated weights for the probabilities of extreme outcomes in a prospect. We are not aware of experimental studies that show such an effect on the probability weighting functions.

ture in Paris than for choices where the uncertainty is over foreign temperature events (Abdellaoui et al., p. 19, Figure 9). Such findings support the view that elevation captures aspects of familiarity or confidence in the source of uncertainty that generates the ambiguity. As our setting considers a single source of uncertainty, namely risk provided as objective probabilities, we extrapolate from here that elevation may be source-dependent but otherwise it can be regarded as a delay-independent attribute of an individual's probabilistic risk attitude.[9]

While the time-dependence of probability weighting is central in this paper, it should be noted that in our model we assume that the cardinal utility under risk is equal to the utility for intertemporal outcomes. We think that, when considering choices among prospects resolved at the same date, adding a time-delay of 6 months–1 year for resolution and receipt of outcomes may have less impact on the utility and individuals may accurately foresee this. For longer delays the tastes of individuals may change as well as the environment within which they act, so that more ambiguity about the future tastes gradually becomes significant. Then the assumption of a time-invariant utility may no longer be justified. Indeed, assuming a time-invariant utility is not uncontroversial, for instance, Booij and van Praag (2009) show that the degree of risk aversion may be affected by time-preferences. Assuming EU-preferences for risk, Andreoni and Sprenger (2012) and Coble and Lusk (2010) find differences in the utilities for risk and for time.[10]

Accounting for non-EU preferences, Abdellaoui et al. (2013), who empirically measured risky and temporal utility for gains and losses, find that utility curvature and loss aversion are more pronounced for risk relative to time. Comparing risky utility revealed from choices with utility under certainty derived from strength of preference judgments, Abdellaoui et al. (2007) find little difference when accounting for potential biases attributed to the treatment of probabilities under risk. Similarly, Abdellaoui et al. (2011) conclude that probability weighting is accountable for all of the effect on risk attitude of delay and suggested that the reduction in risk aversion relative to immediately resolved risk is attributable to less bias in the perception of probabilities mainly by subjects exhibiting more sensitivity towards moderate and large probabilities, or equivalently, relatively less pessimism. It is precisely this aspect that we isolate as the focus for this paper.

The assumptions of time-invariance for utility and elevation in our theory are falsifiable, although there is some, albeit limited, empirical evidence to support them.

[9] One could argue that delaying the resolution of prospects adds ambiguity about the probabilities and outcomes. Such an argument would be in the spirit of saying that delaying a sure outcome makes the latter equivalent to a risky outcome obtained today (Halevy 2008). For risk, and in the absence of certainty, this would mean that relatively more pessimism would govern choices over those delayed prospects. As reviewed earlier, however, the literature reports on experiments where the opposite behavior of less risk aversion is documented when adding delay to binary choices. It is therefore unclear how to reconcile elevation with less risk aversion for delayed prospects.

[10] Within a discounted EU-framework such differences can be explained by disentangling the time-elasticity of substitution from the degree of risk aversion captured by a standard utility function under risk (e.g., Selden 1978, and for non-EU preferences see Chew and Epstein 1990). Indeed, Miao and Zhong (2015) and Cheung (2015) revisited the hypothesis put forward by Andreoni and Sprenger (2012) and find evidence supporting a separation of time-elasticity of substitution from risk aversion in the utility function. As in our setting such a separation is not sufficient to explain classical EU-paradoxes in risky choice, we abstract away from this issue.

For instance, the study of Abdellaoui et al. (2011) provides direct results on utility curvature and elevation that do not contradict our position. That said, we think that there is scope for further empirical exploration of the impact of delay and the passage of time on the various components of risk attitude. Certainly, in a dynamic context with a long planning horizon, more flexibility is justified as preferences may change over long passages of time and individuals choice behavior will be affected by anticipated changes in tastes as well as weighting functions (Gerber and Rohde 2018); the research on these dynamic aspects of risk and time is not, however, our current aim.

7 Other aspects of behavior not accounted for

This paper has mentioned several experimental studies that support time-dependent probability weighting and, in doing so, has focused on those empirical findings that we seek to accommodate in our model. For a more balanced impression, it is worthwhile to report on a few experimental results that are related to resolution of uncertainty and which also give some support for adopting time-dependent probability weighting functions. That said, we have to be clear that in our framework, where the time of resolution of uncertainty and the time of outcome receipt are the same, a separate analysis of uncertainty resolution independent of the date at which outcomes are received falls outside the scope of our model.

Ahlbrecht and Weber (1997) consider preferences over risky prospects with fixed payment dates but distinct timing of the resolution of some of the uncertainty. They compare choices among gain prospects, where all resolution of uncertainty is early, gradual resolution where some uncertainty is resolved early and some late, or all resolution is late; similarly, they also implement choices among loss prospects. Ahlbrecht and Weber find that a large majority of their subjects have a consistent preference for the timing of resolution and that most subjects prefer early relative to late resolution of uncertainty when these are the only possibilities. Ahlbrecht and Weber go on to compare these consistent subjects with their choices among the gradually resolved prospects and report various inconsistencies that cannot be accommodated by a transformation of risky utility as proposed in the Kreps and Porteus (1978) type models.

A similar argument was presented in the study of Chew and Ho (1994). They observed risk attitudes in accordance with PT for instantly resolved uncertainty while, for temporal prospects, they report that risk attitude and attitude towards resolution of uncertainty do not seem correlated. As the timing of the payment is common in the prospects with gradually resolved uncertainty, discounting cannot explain the findings in these studies either. Indirectly these findings also give support for a model that allows for probability weighting to be time-dependent, as suggested in Wu (1999).

In our framework, we have not disentangled the timing of the resolution from the date at which an outcome of a prospect is obtained. We adopted an additively separable model of time preferences in which we could circumvent the effect of impatience or discounting when discussing the effect of delay on the perception of probabilities. However, as indicated in Epstein (2008) probability weighting functions may be a useful tool to capture behavior that is exclusively related to resolution delay rather than outcome delay. For that a more flexible framework is required than the one adopted for the purposes of this paper.

8 Conclusions

Motivated by empirical observations that a joint delay in resolution and receipt of risky outcomes seems to generate less risk aversion, a framework is proposed where the primitives are choices among profiles of timed prospects. Our setting deviates from classical discounted utility only in that it replaces profiles of sure outcomes by profiles of risky prospects, while it is sufficiently rich to incorporate settings adopted in previous empirical studies on the interaction of time on risk attitude. To analyze this interaction, we hypothesize that only attitudes towards probabilities can vary with delayed prospects. Hence, we propose a simple descriptive discounted utility model that separates temporal discounting from the evaluation of risky outcomes. We do allow the discounting function to be quite general, and thereby obtain flexibility to incorporate descriptive features related to the time discounting dimension. When it comes to the evaluation of prospects, however, we adopt a PT-specification that is arguably restrictive.

To achieve simplicity, we had to specify the parameters that are allowed to vary with delay and those which are fixed. This calls for a tradeoff, of course, between descriptive generality and tractability of the model. In general, minimal deviations from established models have been regarded as compelling as they provide a pragmatic account of several desiderata (e.g., Gul 1991, whose model of disappointment adds just one parameter over and above EU, thereby achieving descriptive appeal in addition to maintaining many normative features). For our purposes here, we adopted prospect theory, the most successful descriptive theory for risk, within a temporal setting in which, specifically, the CRS-probability weighting function of Abdellaoui et al. (2010) was considered, which identifies a single parameter as index for (in)sensitivity towards changes in probabilities.

To accommodate the empirical findings on less risk aversion with delay we have developed preference conditions that describe such choice behavior. Subsequently, we have demonstrated that, within our framework, there is a one-to-one correspondence between increased sensitivity with delay as a probabilistic risk aspect, and reduced risk aversion with delay as a revealed choice behavior. Our Theorem 1 also provides a relationship of these choice behaviors to the sensitivity parameters of the CRS-weighting functions at different points in time. We think that these results point to testable implications, hence, they are useful to develop new empirical studies on the interaction of risk and time. Our application to cooperative bargaining indicates that delays can be beneficial for individuals bargaining over indivisible objects if opponents display reduced risk aversion. This is a simple application of a model that can be regarded as a minor deviation from discounted expected utility, where "minor" is meant to reflect that empirical regularities have been incorporated yet a tractable version of a potentially much more general model has been obtained.

Acknowledgements Mohammed Abdellaoui and Peter P. Wakker provided us with constructive comments on earlier versions of the paper. The paper builds on Chapter 3 of Jinrui Pan's PhD-thesis "Time and Risk Preferences: Theoretical Models for Individual Decision Making" Manchester, UK: The University of Manchester; 2015.

Appendix: proofs

Proof of Proposition 1 As mentioned in the main text, the proof of this proposition follows from the results of Köbberling and Wakker (2003). Our time invariance of outcome tradeoffs comes down to Köbberling and Wakker's tradeoff consistency property, which they use to obtain a subjective expected utility foundation (see their Theorem 5). We assume that general DPT holds, which, together with the time-invariance of outcome tradeoffs, implies that the conditions in statement (ii) of Köbberling and Wakker's Theorem 5 are satisfied. As indicated in the main text, this implies that utility in DPT is time-independent. Given that our general DPT model satisfies a strong outcome monotonicity property, all periods are essential for the preference; hence, the uniqueness results stated in Köbberling and Wakker's Observation 6(c) apply. Different to the normalization employed for obtaining uniqueness of subjective probabilities in subjective expected utility derivations, the normalization used for the unique weights in DPT each time period is such that $D(0) = 1$ and if, additionally, one requires impatience, then the weights $D(t)$ have to be strictly decreasing; but the result here holds without this assumption. This completes the proof of Proposition 1. □

Proof of Proposition 2 We can assume that Proposition 1 holds. That is, the preference is represented by DPT with common time-independent utility $u(\cdot)$, general discount function $D(\cdot)$, and time-dependent CRS-probability weighting function $w_t(\cdot)$. Next, we show that $\succcurlyeq$ on $\mathcal{P}$ satisfies time-invariant propensity to gamble if and only if $\eta := \eta_0 = \eta_t$ for all time periods $t \in \{1, \dots T\}$. Recall that the preference relation $\succcurlyeq$ on $\mathcal{P}$ satisfies time-invariant propensity to gamble if for all time periods $t, s \in \{0, \dots, T\}$, $s < t$, all outcomes $a, b, x, y \in \mathbb{R}_+$ and all profiles $\mathbf{x} \in \mathcal{P}$ the following holds:

$$(\eta_s : a, 1 - \eta_s : x)_s\mathbf{x} \sim (\eta_s : b, 1 - \eta_s : y)_s\mathbf{x}$$
$$\Rightarrow (\eta_t : a, 1 - \eta_t : x)_t\mathbf{x} \sim (\eta_t : b, 1 - \eta_t : y)_t\mathbf{x}.$$

Obviously, if $\eta := \eta_0 = \eta_t$ for all time periods $t \in \{1, \dots T\}$, then time-invariant propensity to gamble follows immediately.

Next, assume that time-invariant propensity to gamble holds. Set $\eta := \eta_0$ and let $a, b \in \mathbb{R}_+$ be arbitrary distinct outcomes and $\mathbf{x} \in \mathcal{P}$ be an arbitrary profile. By continuity of u there exist distinct outcomes $x, y \in \mathbb{R}_+$ such that $(\eta : a, 1 - \eta :$

$x)_0\mathbf{x} \sim (\eta : b, 1-\eta : y)_0\mathbf{x}$. Substitution of DPT as given by Proposition 1 implies

$$D(0)PT_0(\eta : a, 1-\eta : x) + \sum_{t=1}^{T} D(t)PT_t(\tilde{x}_t) = D(0)PT_0(\eta : b, 1-\eta : y) + \sum_{t=1}^{T} D(t)PT_t(\tilde{x}_t),$$

which, after cancellation of common terms, gives:

$$w_0(\eta)u(a) + [1 - w_0(\eta)]u(x) = w_0(\eta)u(b) + [1 - w_0(\eta)]u(y)$$

or, using the property of the CRS-weighting functions that $w_0(\eta) = \eta$ and our assumption that $0 < \eta < 1$, equivalently

$$\frac{\eta}{1-\eta} = \frac{u(y) - u(x)}{u(a) - u(b)}. \tag{5}$$

Further, time-invariant propensity to gamble implies that $(\eta_s : a, 1-\eta_s : x)_s\mathbf{x} \sim (\eta_s : b, 1-\eta_s : y)_s\mathbf{x}$ holds for all time periods $s \in \{1, \ldots, T\}$. This implies, by a similar argument as above, that

$$\frac{\eta_s}{1-\eta_s} = \frac{u(y) - u(x)}{u(a) - u(b)}.$$

Given Eq. (5) and the fact that $g(\eta_s) = \eta_s/(1-\eta_s)$ is strictly increasing on the interval $(0, 1)$ with image $(0, \infty)$, it follows that there is a unique solution for η_s that satisfies $\eta/(1-\eta) = \eta_s/(1-\eta_s)$, namely that $\eta_s = \eta$ for all $s \in \{1, \ldots, T\}$. Hence, $w_s(\eta) = \eta$ for all time periods $s \in \{0, \ldots, T\}$. This completes the proof of Proposition 2. □

Proof of Theorem 1 We assume that the preference $\succcurlyeq$ on $\mathcal{P}$ is represented by DPT$^{\eta}$ as in Proposition 2. Next we assume Statement (i) and derive Statement (iii) of the theorem. If $\succcurlyeq$ on $\mathcal{P}$ satisfies increasing sensitivity with delay, then for all outcomes $x > y > 0$ such that $u(x) - u(y) = u(y) - u(0)$, and all $\mathbf{z} \in \mathcal{P}$ such that for some time period $t \in \{0, \ldots, T-1\}$ and probabilities p_t, p_{t+1} the two indifferences

$$(\eta : x, 1-\eta : y)_t\mathbf{z} \sim (p_t : x, 1-p_t : 0)_t\mathbf{z}$$

and

$$(\eta : x, 1-\eta : y)_{t+1}\mathbf{z} \sim (p_{t+1} : x, 1-p_{t+1} : 0)_{t+1}\mathbf{z}$$

are satisfied, it follows that

$$p_t > p_{t+1}.$$

As continuity of u ensures that we can always find outcomes $x > y > 0$ such that $u(x) - u(y) = u(y) - u(0)$ and continuity (and strict monotonicity) of the probability weighting functions ensures that there exists (unique) probabilities $p_t, p_{t+1} \in (\eta, 1)$ such that the above indifferences hold, substitution of of DPT$^\eta$ implies that

$$\eta u(x) + (1-\eta)u(y) = w_t(p_t)u(x) + [1 - w_t(p_t)]u(0)$$

and

$$\eta u(x) + (1-\eta)u(y) = w_{t+1}(p_{t+1})u(x) + [1 - w_{t+1}(p_{t+1})]u(0)$$

from which

$$w_t(p_t) = w_{t+1}(p_{t+1})$$

follows for all $t \in \{0, \ldots, T-1\}$. Further, exploiting that $u(x) - u(y) = u(y) - u(0)$ we find that

$$\begin{aligned} \eta u(x) + (1-\eta)u(y) &= w(p_t)u(x) + [1 - w_t(p_t)]u(0) \\ \Leftrightarrow \eta[u(x) - u(y)] + u(y) &= w_t(p_t)[u(x) - u(0)] + u(0) \\ \Leftrightarrow (\eta+1)[u(x) - u(y)] &= 2w_t(p_t)[u(x) - u(y)], \end{aligned}$$

from which we obtain that

$$w_t(p_t) = \frac{\eta+1}{2} \quad \text{for all} \quad t \in \{0, \ldots, T\}.$$

Next we substitute the CRS-weighting function in the preceding equation and obtain

$$\begin{aligned} 1 - (1-\eta)^{1-\sigma_t}(1-p_t)^{\sigma_t} &= (\eta+1)/2 \\ \Leftrightarrow 1 - (\eta+1)/2 &= (1-\eta)^{1-\sigma_t}(1-p_t)^{\sigma_t} \\ \Leftrightarrow 1/2 &= [(1-p_t)/(1-\eta)]^{\sigma_t} \\ \Leftrightarrow (1-\eta)(1/2)^{1/\sigma_t} &= 1 - p_t \\ \Leftrightarrow 1 - (1-\eta)(1/2)^{1/\sigma_t} &= p_t \quad \text{for all} \quad t \in \{0, \ldots, T\}. \end{aligned}$$

Now we use the condition that $p_t > p_{t+1}$ for all $t \in \{0, \ldots, T-1\}$ together with the preceding equation to find that

$$1 - (1-\eta)(1/2)^{1/\sigma_t} = p_t > p_{t+1} = 1 - (1-\eta)(1/2)^{1/\sigma_{t+1}}$$

which implies

$$(1/2)^{1/\sigma_t} < (1/2)^{1/\sigma_{t+1}} \Leftrightarrow 1/\sigma_t > 1/\sigma_{t+1}$$

for all $t \in \{0, \ldots, T-1\}$. As the parameters σ_s are strictly bounded between 0 and 1 for all time periods s, we obtain $\sigma_t < \sigma_{t+1}$ for all $t \in \{0, \ldots, T-1\}$. This completes the derivation of Statement (iii) from Statement (i).

Next we assume Statement (iii) and prove Statement (ii). Assume an arbitrary time period $t \in \{0, \ldots, T-1\}$, outcomes $x > y > 0$, profile $\mathbf{z} \in \mathcal{P}$, and probabilities $\eta < p < 1$ such that

$$(p : x, 1-p : 0)_t\mathbf{z} \sim (1 : y)_t\mathbf{z}.$$

We need to show that

$$(p : x, 1-p : 0)_{t+1}\mathbf{z} \succ (1 : y)_{t+1}\mathbf{z}.$$

Assume to the contrary that

$$(p : x, 1-p : 0)_{t+1}\mathbf{z} \preccurlyeq (1 : y)_{t+1}\mathbf{z}.$$

From the indifference $(p : x, 1-p : 0)_t\mathbf{z} \sim (1 : y)_t\mathbf{z}$ and substitution of DPT$^\eta$ it follows that

$$w_t(p)[u(x) - u(0)] = u(y) - u(0)$$

while the preference $(p : x, 1-p : 0)_{t+1}\mathbf{z} \preccurlyeq (1 : y)_{t+1}\mathbf{z}$ and substitution of DPT$^\eta$ implies

$$w_{t+1}(p)[u(x) - u(0)] \leq u(y) - u(0).$$

Combining the latter inequality with the preceding equation and cancellation of common terms gives

$$w_{t+1}(p) \leq w_t(p)$$

But this contradicts the fact that, as a result of $\sigma_{t+1} > \sigma_t$, the CRS-weighting function w_{t+1} gives higher values to probabilities in $(\eta, 1)$ than w_t does. Hence, $(p : x, 1-p : 0)_{t+1}\mathbf{z} \succ (1 : y)_{t+1}\mathbf{z}$ follows. As $t \in \{0, \ldots, T-1\}$, outcomes $x > y > 0$, profile $\mathbf{z} \in \mathcal{P}$, and probabilities $\eta < p < 1$ were arbitrary chosen, the implication holds for all $t \in \{0, \ldots, T-1\}$, outcomes $x > y > 0$, profile $\mathbf{z} \in \mathcal{P}$, and probabilities $\eta < p < 1$. This means that the preference $\succcurlyeq$ satisfies reduced risk aversion with delay, which completes the derivation of Statement (ii) from Statement (iii).

Next we assume that Statement (ii) holds and derive Statement (i). Suppose that for some arbitrary outcomes $x > y > 0$ such that $u(x) - u(y) = u(y) - u(0)$, profile $\mathbf{z} \in \mathcal{P}$, time period $t \in \{0, \ldots, T-1\}$ and probabilities p_t, p_{t+1} we have the two indifferences

$$(\eta : x, 1-\eta : y)_t\mathbf{z} \sim (p_t : x, 1-p_t : 0)_t\mathbf{z}$$

and

$$(\eta : x, 1-\eta : y)_{t+1}\mathbf{z} \sim (p_{t+1} : x, 1-p_{t+1} : 0)_{t+1}\mathbf{z}.$$

We need to show that $p_t > p_{t+1}$. By continuity and strict monotonicity of the utility u there exists some outcome $0 < z < x$ such that

$$(p_t : x, 1 - p_t : 0)_t\mathbf{z} \sim (1 : z)_t\mathbf{z}.$$

Now reduced risk aversion with delay implies that

$$(p_t : x, 1 - p_t : 0)_{t+1}\mathbf{z} \succ (1 : z)_{t+1}\mathbf{z}.$$

Continuity and strict monotonicity of the CRS-probability weighting function w_{t+1} implies that there exists a positive probability $q < p_t$ such that

$$(q : x, 1 - q : 0)_{t+1}\mathbf{z} \sim (1 : z)_{t+1}\mathbf{z}.$$

Further, by transitivity $(p_t : x, 1 - p_t : 0)_t\mathbf{z} \sim (1 : z)_t\mathbf{z}$ and $(\eta : x, 1 - \eta : y)_t\mathbf{z} \sim (p_t : x, 1 - p_t : 0)_t\mathbf{z}$ imply

$$(\eta : x, 1 - \eta : y)_t\mathbf{z} \sim (1 : z)_t\mathbf{z},$$

such that substitution of DPT$^\eta$ and cancellation of common terms gives

$$\eta u(x) + (1 - \eta)u(y) = u(z).$$

This equation is independent of the time period t, hence, using the fact that η is a common fix point of all CRS-weighting functions, we obtain $w_{t+1}(\eta)u(x) + [1 - w_{t+1}(\eta)]u(y) = u(z)$ from which we derive

$$(\eta : x, 1 - \eta : y)_{t+1}\mathbf{z} \sim (1 : z)_{t+1}\mathbf{z}.$$

Given our assumption that $(\eta : x, 1 - \eta : y)_{t+1}\mathbf{z} \sim (p_{t+1} : x, 1 - p_{t+1} : 0)_{t+1}\mathbf{z}$ and transitivity, we obtain

$$(p_{t+1} : x, 1 - p_{t+1} : 0)_{t+1}\mathbf{z} \sim (1 : z)_{t+1}\mathbf{z},$$

hence, $q = p_{t+1}$. Now reduced risk aversion with delay implies that

$$(p_{t+1} : x, 1 - p_{t+1} : 0)_t\mathbf{z} \prec (1 : z)_t\mathbf{z}.$$

Together with $(p_t : x, 1 - p_t : 0)_t\mathbf{z} \sim (1 : z)_t\mathbf{z}$ and transitivity we obtain

$$(p_{t+1} : x, 1 - p_{t+1} : 0)_t\mathbf{z} \prec (p_t : x, 1 - p_t : 0)_t\mathbf{z},$$

which can only hold if $p_{t+1} < p_t$ (as the CRS-weighting functions are strictly increasing).

This conclusion has been obtained starting with arbitrary outcomes $x > y > 0$ such that $u(x) - u(y) = u(y) - u(0)$, arbitrary profile $\mathbf{z} \in \mathcal{P}$ and time period $t \in \{0, \ldots, T-1\}$ such that the indifferences

$$(\eta : x, 1-\eta : y)_t\mathbf{z} \sim (p_t : x, 1-p_t : 0)_t\mathbf{z}$$

and

$$(\eta : x, 1-\eta : y)_{t+1}\mathbf{z} \sim (p_{t+1} : x, 1-p_{t+1} : 0)_{t+1}\mathbf{z}$$

hold. Hence, the conclusion is valid for all probability midpoints p_t, p_{t+1} and time periods $t \in \{0, \ldots, T-1\}$. This completes the derivation of Statement (i) from Statement (ii).

To summarize: first, we have shown that Statement (i) implies Statement (iii), then we have derived Statement (ii) from Statement (iii) and , finally, we have proven that Statement (ii) implies Statement (i). This completes the proof of the theorem. □

References

Abdellaoui, M. (2000). Parameter-free elicitation of utility and probability weighting functions. *Management Science*, *46*(11), 1497–1512.

Abdellaoui, M., Baillon, A., Placido, L., & Wakker, P. P. (2011a). The rich domain of uncertainty: Source functions and their experimental implementation. *American Economic Review*, *101*(2), 1–31.

Abdellaoui, M., Barrios, C., & Wakker, P. P. (2007). Reconciling introspective utility with revealed preference: Experimental arguments based on prospect theory. *Journal of Econometrics*, *138*, 356–378.

Abdellaoui, M., Bleichrodt, H., l'Haridon, O., & Paraschiv, C. (2013). Is there one unifying concept of utility? An experimental comparison of utility under risk and utility over time. *Management Science*, *59*(9), 2153–2169.

Abdellaoui, M., Diecidue, E., & Öncüler, A. (2011). Risk preferences at different time points: An experimental investigation. *Management Science*, *57*(5), 975–987.

Abdellaoui, M., l'Haridon, O., & Zank, H. (2010). Separating curvature and elevation: A parametric probability weighting function. *Journal of Risk and Uncertainty*, *41*, 39–65.

Abdellaoui, M., Vossmann, F., & Weber, M. (2005). Choice-based elicitation and decomposition of decision weights for gains and losses under uncertainty. *Management Science*, *51*(9), 1384–1399.

Abreu, D., & Gul, F. (2000). Bargaining and reputation. *Econometrica*, *68*, 85–118.

Ahlbrecht, M., & Weber, M. (1997). Preference for gradual resolution of uncertainty. *Theory and Decision*, *43*, 167–185.

Allais, M. (1953). Le comportement de l' homme rationnel devant le risque: Critique des postulats et axiomes de l' ecole américaine. *Econometrica*, *21*(4), 503–546.

Anderhub, V., Gneezy, U., Güth, W., & Sonsino, D. (2001). On the interaction of risk and time preferences: An experimental study. *German Economic Review*, *2*(3), 239–253.

Andersen, S., Harrison, G. W., Lau, M. I., & Rutstrom, E. E. (2008). Lost in state space: Are preferences stable? *International Economic Review*, *49*, 1091–1121.

Anderson, L. R., & Stafford, S. L. (2009). Individual decision-making experiments with risk and intertemporal choice. *Journal of Risk and Uncertainty*, *38*, 51–72.

Andreoni, J., & Sprenger, C. (2012). Risk preferences are not time preferences. *American Economic Review*, *102*(7), 3357–3376.

Arrow, K. J. (1971). *Essays in the theory of risk bearing*. Amsterdam: North-Holland.

Avery, C., & Zemsky, P. B. (1994). Money burning and multiple equilibria in bargaining. *Games and Economic Behavior*, *7*, 154–168.

Barberis, N. (2013). Thirty years of prospect theory in economics: A review and assessment. *Journal of Economic Perspectives*, *27*, 173–195.

Baucells, M., & Heukamp, F. H. (2006). Stochastic dominance and cumulative prospect theory. *Management Science*, *52*(9), 1409–1423.
Baucells, M., & Heukamp, F. H. (2010). Common ratio using delay. *Theory and Decision*, *68*, 149–158.
Baucells, M., & Heukamp, F. H. (2012). Probability and time trade-off. *Management Science*, *58*(4), 831–842.
Bleichrodt, H., & Pinto, J. L. (2000). A parameter-free elicitation of the probability weighting function in medical decision analysis. *Management Science*, *46*(11), 1485–1496.
Bleichrodt, H., Rohde, K. I. M., & Wakker, P. P. (2009). Koopmans' constant discounting for intertemporal choice: A simplification and a generalization. *Journal of Mathematical Psychology*, *52*, 341–347.
Booij, A. S., & van Praag, B. M. S. (2009). A simultaneous approach to the estimation of risk aversion and the subjective discount rate. *Journal of Economic Behavior and Organization*, *70*, 374–388.
Busch, L.-A., & Wen, Q. (1995). Perfect equilibria in a negotiation model. *Econometrica*, *63*(3), 545–565.
Chateauneuf, A., & Cohen, M. (1994). Risk seeking with diminishing marginal utility in a non-expected utility model. *Journal of Risk and Uncertainty*, *9*, 77–91.
Chateauneuf, A., Eichberger, J., & Grant, S. (2007). Choice under uncertainty with the best and worst in mind: NEO-additive capacities. *Journal of Economic Theory*, *137*, 538–567.
Cheung, S. L. (2015). Risk preferences are not time preferences: On the elicitation of time preference under conditions of risk: Comment (#11). *American Economic Review*, *105*(7), 2242–2260.
Chew, S. H., & Epstein, L. G. (1990). Nonexpected utility preferences in a temporal framework with an application to consumption-savings behavior. *Journal of Economic Theory*, *50*, 54–81.
Chew, S. H., & Ho, J. L. (1994). Hope: An empirical study of attitude toward the timing of uncertainty resolution. *Journal of Risk and Uncertainty*, *8*, 267–288.
Chew, S. H., Karni, E., & Safra, Z. (1987). Risk aversion in the theory of expected utility with rank dependent probabilities. *Journal of Economic Theory*, *42*, 370–381.
Coble, K. H., & Lusk, J. L. (2010). At the nexus of risk and time preferences: An experimental investigation. *Journal of Risk and Uncertainty*, *41*, 67–79.
Conley, J. P., & Wilkie, S. (1991). The bargaining problem without convexity: Extending the Egalitarian and Kalai-Smorodinsky solutions. *Economics Letters*, *36*, 365–369.
Dasgupta, P., & Maskin, E. (2005). Uncertainty and hyperbolic discounting. *American Economic Review*, *95*(4), 1290–1299.
Diecidue, E., Schmidt, U., & Zank, H. (2009). Parametric weighting functions. *Journal of Economic Theory*, *144*, 1102–1118.
Epper, T., & Fehr-Duda, H. (2015). Risk preferences are not time preferences: Balancing on a budget line: Comment (#12). *American Economic Review*, *105*(7), 2261–2271.
Epstein, L. G. (2008). Living with risk. *Review of Economic Studies*, *75*, 1121–1141.
Etchart-Vincent, N. (2004). Is probability weighting sensitive to the magnitude of consequences? An experimental investigation on losses. *Journal of Risk and Uncertainty*, *28*, 217–235.
Evans, J. S. B. T., & Stanovich, K. E. (2013). Dual-Process theories of higher cognition: Advancing the debate. *Perspectives on Psychological Science*, *8*(3), 223–241.
Fernandez, R., & Glazer, J. (1991). Striking a bargaining between two completely informed agents. *American Economic Review*, *81*, 240–252.
Fox, C. R., & Tversky, A. (1995). Ambiguity aversion and comparative ignorance. *Quarterly Journal of Economics*, *110*, 585–603.
Frederick, S., Loewenstein, G., & O'Donoghue, T. (2002). Time discounting and time preference: A critical review. *Journal of Economic Literature*, *40*(2), 351–401.
Frey, R., Pedroni, A., Mata, R., Rieskamp, J., & Hertwig, R. (2017). Risk preference shares the psychometric structure of major psychological traits. *Science Advances*, *3*, e1701381.
Gerber, A., & Rohde, K. (2018). Weighted temporal utility. *Economic Theory*, *66*, 187–212.
Goldstein, W. M., & Einhorn, H. J. (1987). Expression theory and the preference reversal phenomena. *Psychological Review*, *94*, 236–254.
Gonzalez, R., & Wu, G. (1999). On the shape of the probability weighting function. *Cognitive Psychology*, *38*, 129–166.
Gul, F. (1991). A theory of disappointment aversion. *Econometrica*, *59*, 667–686.
Gul, F., & Sonnenschein, H. (1988). On delay in bargaining with one-sided uncertainty. *Econometrica*, *56*(3), 601–611.
Halevy, Y. (2008). Strotz meets Allais: Diminishing impatience and the certainty effect. *American Economic Review*, *98*(3), 1145–1162.

Holt, C. A., & Laury, S. K. (2001). Risk aversion and incentive effects. *American Economic Review*, *92*(5), 1644–1655.
Kahneman, D. (2011). *Thinking, fast and slow*. New York: Farrar, Straus and Giroux.
Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, *47*(2), 263–291.
Keren, G., & Roelofsma, P. (1995). Immediacy and certainty in intertemporal choice. *Organizational Behavior and Human Decision Processes*, *63*(3), 287–97.
Kilka, M., & Weber, M. (2001). What determines the shape of the probability weighting function under uncertainty. *Management Science*, *47*, 1712–1726.
Köbberling, V., & Peters, H. (2003). The effect of decision weights in bargaining problems. *Journal of Economic Theory*, *110*, 154–175.
Köbberling, V., & Wakker, P. P. (2003). Preference foundations for nonexpected utility: A generalized and simplified technique. *Mathematics of Operations Research*, *28*, 395–423.
Koopmans, T. C. (1960). Stationary ordinal utility and impatience. *Econometrica*, *28*(2), 287–309.
Kreps, D. M., & Porteus, E. L. (1978). Temporal resolution of uncertainty and dynamic choice theory. *Econometrica*, *46*(1), 185–200.
Laibson, D. I. (1997). Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics*, *112*(2), 443–477.
Loewenstein, G., & Prelec, D. (1992). Anomalies in intertemporal choice: Evidence and an interpretation. *Quarterly Journal of Economics*, *107*(2), 573–597.
Miao, B., & Zhong, S. (2015). Risk preferences are not time preferences: Separating risk and time preference: Comment (#13). *American Economic Review*, *105*(7), 2272–2286.
Noussair, C., & Wu, P. (2006). Risk tolerance in the present and the future: An experimental study. *Managerial Decision Economics*, *27*(6), 401–412.
Phelps, E. S., & Pollak, R. A. (1968). On second-best national saving and game-equilibrium growth. *Review of Economic Studies*, *35*(2), 185–199.
Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica*, *32*(1/2), 122–136.
Prelec, D. (1998). The probability weighting function. *Econometrica*, *66*(3), 497–527.
Prelec, D., & Loewenstein, G. F. (1991). Decision making over time and under uncertainty: A common approach. *Management Science*, *37*(7), 770–786.
Quiggin, J. (1982). A theory of anticipated utility. *Journal of Economic Behavior and Organization*, *3*, 323–343.
Quiggin, J., & Horowitz, J. (1995). Time and risk. *Journal of Risk and Uncertainty*, *10*, 37–55.
Rabin, M. (2000). Risk aversion and expected-utility theory: A calibration theorem. *Econometrica*, *68*(5), 1281–1292.
Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica*, *50*(1), 97–109.
Rubinstein, A. (1985). A bargaining model with incomplete information on time preferences. *Econometrica*, *53*(5), 1151–1172.
Ryan, M. J. (2006). Risk aversion in RDEU. *Journal of Mathematical Economics*, *42*, 675–697.
Samuelson, P. A. (1937). A note on measurement of utility. *Review of Economic Studies*, *4*(2), 155–161.
Schmidt, U., & Zank, H. (2008). Risk aversion in cumulative prospect theory. *Management Science*, *54*(1), 208–216.
Segal, U. (1987). The Ellsberg paradox and risk aversion: An anticipated utility approach. *International Economic Review*, *28*(1), 175–202.
Selden, L. (1978). A new representation of preferences over "Certain x Uncertain" consumption pairs: The "Ordinal Certainty Equivalent" hypothesis. *Econometrica*, *46*(5), 1045–1060.
Shelley, M. K. (1994). Gain/loss asymmetry in risky intertemporal choice. *Organizational Behavior and Human Decision Processes*, *59*, 124–159.
Starmer, C. (2000). Developments in non-expected utility theory: The hunt for a descriptive theory of choice under risk. *Journal of Economic Literature*, *38*, 332–382.
Stevenson, M. K. (1992). The impact of temporal context and risk on the judged value of future outcomes. *Organizational Behavior and Human Decision Processes*, *52*, 455–491.
Stott, H. P. (2006). Cumulative prospect theory's functional menagerie. *Journal of Risk and Uncertainty*, *32*, 101–130.
Thaler, R. (1981). Some empirical evidence on dynamic inconsistency. *Economics Letters*, *8*, 201–207.
Trope, Y., & Liberman, N. (2010). Construal-level theory of psychological distance. *Psychological Review*, *117*, 440–463.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, *5*, 297–323.

van de Kuilen, G., & Wakker, P. P. (2011). The midweight method to measure attitudes toward risk and ambiguity. *Management Science*, *57*(3), 582–598.

Wakker, P. P. (1994). Separating marginal utility and probabilistic risk aversion. *Theory and Decision*, *36*, 1–44.

Wakker, P. P. (2010). *Prospect theory for risk and ambiguity*. Cambridge: Cambridge University Press.

Wakker, P. P., & Deneffe, D. (1996). Eliciting von Neumann–Morgenstern utilities when probabilities are distorted or unknown. *Management Science*, *42*(8), 1131–1150.

Webb, C. S. (2015). Piecewise additivity for non-expected utility. *Economic Theory*, *60*, 371–392.

Webb, C. S. (2017). Piecewise linear rank-dependent utility. *Theory and Decision*, *82*, 403–414.

Webb, C. S., & Zank, H. (2011). Accounting for optimism and pessimism in expected utility. *Journal of Mathematical Economics*, *47*, 706–717.

Weber, B. J., & Chapman, G. B. (2005). The combined effects of risk and time on choice: Does uncertainty eliminate the immediacy effect? Does delay eliminate the certainty effect? *Organizational Behavior and Human Decision Processes*, *96*(2), 104–118.

Werner, K. M., & Zank, H. (2019). A revealed reference point for prospect theory. *Economic Theory, 67*(4), 731–773.

Wu, G. (1999). Anxiety and decision making with delayed resolution of uncertainty. *Theory and Decision*, *46*, 159–198.

Wu, G., & Gonzalez, R. (1996). Curvature of the probability weighting function. *Management Science*, *42*(12), 1676–1690.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.