Out-of-the-box baryogenesis during relaxation

S. A. Abel, R. S. Gupta, and J. Scholtz

Institute for Particle Physics Phenomenology, Durham University, South Road, Durham DH1 3LE, UK

We show that spontaneous baryogenesis occurs automatically in relaxion models if the reheating temperature is larger than the weak scale, provided the Standard Model fields are charged under the $U(1)$ of which the relaxion is a pseudo-Nambu-Goldstone boson. During the slow roll, the relaxion breaks CPT, biasing the thermal equilibrium in favor of baryons, with sphalerons providing the necessary baryon number violation. We calculate the resulting baryon asymmetry, explore the possible constraints on this scheme and show that there is a swath of parameter space in which the current observations are matched. Successful baryogenesis can be achieved for a range of relaxion masses between 10^{-10} and 10^{-5} eV. The mechanism operates precisely in the region of parameter space where recent work has shown relaxion oscillations to be a dark matter candidate.

I. INTRODUCTION

An interesting scenario has been suggested [1] to explain why the Higgs mass is much smaller than the fundamental scale. Adapting a long-standing idea of Abbott that attempted (unsuccessfully) to explain the smallness of the cosmological constant [2], the "relaxion" mechanism incorporates an interplay of two explicit/anomalous breakings of a Goldstone shift symmetry, to relax the Higgs mass dynamically to values close to the weak scale. The smaller breaking drives the pseudo-Nambu-Goldstone mode (PNGB), the so-called relaxion, which samples over Higgs masses, while the larger breaking comes in the form of a periodic axionlike potential that is proportional to the Higgs vacuum expectation value (VEV): hence the dynamical evolution of the relaxion stops as soon as the Higgs VEV turns on. With a suitable choice of parameters the resultant Higgs mass is of the correct order.

The central achievement of [1] was to relate the Higgs mass to a small technically natural parameter, overcoming the lack of any direct equivalent to the chiral symmetry breaking that protects quark masses. The proposal can be made natural in the colloquial sense as well, by adapting clockworklike scenarios [3].

This paper is motivated by the fact that, being a PNGB, the rolling of the relaxion represents a spontaneous CPT violation. Successful baryogenesis scenarios based on such

dynamical evolution were proposed in [4–10] (also see [11] for a recent review) and were dubbed spontaneous baryogenesis (SBG). One is led to ask if SBG is already present in the relaxion mechanism.

We will demonstrate that SBG is virtually generic. It requires only the most minimal augmentation of the relaxion mechanism, namely the addition of a single operator:

$$\mathcal{O}_1 = \frac{1}{f} \partial_\mu \phi J^\mu, \qquad (1)$$

where ϕ is the relaxion and J^μ is a current of matter fields that has a component orthogonal to both electromagnetic charge Q and $B - L$. We will see that the term in Eq. (1) can be generated simply by charging the matter fields under the $U(1)$ corresponding to the relaxion shift symmetry [12].

Although the term in Eq. (1) is invariant, a time derivative of ϕ yields a CPT violating chemical potential for baryon number [11]. This biases sphaleron transitions for temperatures around the electroweak scale where they are active and dominant [4–8].

Thus remarkably all the required ingredients for SBG are *already present* in the relaxion mechanism, provided the operator in Eq. (1) is generated with sufficient strength.

Successful SBG occurs if the cosmological evolution follows the pattern shown in Fig. 1. There is a period of inflation (red) and reheat to temperature T_r (black). After reheat until the temperature drops below $T_{sph} \sim T_{ew} \sim 130$ GeV [13], $B + L$ violating transitions (blue) are active in the plasma, dying away exponentially fast below T_{sph}. During this period the relaxion rolls (green) towards its final value which it reaches at temperature T_c. Thus one has to satisfy the constraint $T_r > T_{sph} > T_c$, which can be done
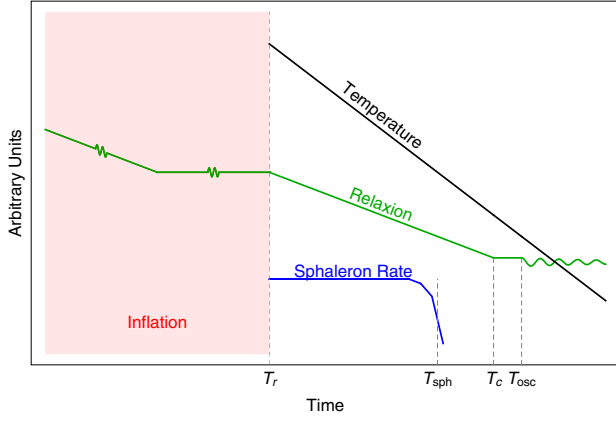
FIG. 1.   The pattern of evolution required for successful baryogenesis during cosmological relaxation.

without violating any bounds. As the temperature drops further the relaxion starts to oscillate about the local minimum and becomes a dark matter candidate [14]. The baryon asymmetry and the dark-matter relic density are correct in roughly the same region of parameter space.

II. DESCRIPTION OF THE MECHANISM

A. The relaxion

The defining feature of relaxion models is that the Higgs mass squared term, μ^2, is dynamical and depends on the classical value of the slowly rolling relaxion, ϕ. We will consider the following typical potential [15],

$$V_{\text{roll}} = (g\phi - M^2 + \cdots)H^{\dagger}H + \lambda(H^{\dagger}H)^2 + gM^2\phi + \cdots,$$
(2)

where M is a cutoff scale. Initially the relaxion field has a value $\phi \gtrsim M^2/g$ such that $\mu^2 > 0$, and electroweak symmetry is unbroken. During inflation, the relaxion field rolls due to the linear term and scans the value of μ^2. As the relaxion crosses $\phi = M^2/g$, μ^2 becomes negative, triggering electroweak symmetry breaking; the Higgs gets a VEV and turns on a crucial ingredient of relaxion models, the backreaction potential:

$$V_{\text{br}} = \Lambda_c^4 \cos\frac{\phi}{f_{\text{w}}} = \tilde{m}^2\langle H\rangle^2 \cos\frac{\phi}{f_{\text{w}}}.$$
(3)

The relaxion halts at ϕ_0, satisfying

$$V'_{\text{roll}}(\phi) + V'_{\text{br}}(\phi) = 0 \Rightarrow gM^2 = \frac{\Lambda_c^4}{f_{\text{w}}}\sin\frac{\phi_0}{f_{\text{w}}}.$$
(4)

At this point, it has a quantum spread across multiple weak-scale vacua. Generally the phase at these minima $\frac{\phi_0}{f_{\text{w}}} \sim 1$, but it can be arbitrarily close to 0 or $\pi/2$ in a small fraction of these vacua [19]. Hence, a large hierarchy between the

Higgs VEV and the cutoff scale can be achieved if g is very small [20].

The backreaction can arise as in the non-QCD model of [1], in which ϕ is the axion of a new strong sector. The axion potential gets a Higgs-dependent contribution because the lightest fermion mass of the strong sector gets a contribution from the Higgs VEV. The radiatively generated, Higgs-independent contributions to the potential are subdominant when

$$f_c \lesssim v, \qquad \Lambda_c^4 \lesssim (16\pi^2)v^4,$$
(5)

where f_c is the analog of the "pion decay constant" in the strong sector. Λ_c depends on f_c but also on the Higgs contribution to the lightest fermion mass, making a separation of scales $\Lambda_c \ll f_c \sim T_c$ technically natural.

Assuming that the reheat temperature after inflation $T_r > v$, one can conclude that $T_r > T_c \sim \sqrt{4\pi}f_c$, where T_c is the critical temperature corresponding to the chiral phase transition of the new strong group. Therefore, the backreaction vanishes after reheating and the relaxion enters a second period of rolling. As the Universe cools and the temperature drops below T_c again, the backreaction reappears. During the second phase of rolling the relaxion obeys $V' \simeq 5H\dot{\phi}$ [24] and the relaxion gets trapped again [14,25] provided

$$m_\phi \lesssim 5H(T_c).$$
(6)

In the second phase of rolling the relaxion is displaced from its original stopping point by a misalignment angle [14],

$$\Delta\theta = \frac{\Delta\phi}{f_{\text{w}}} \simeq \frac{1}{20}\left(\frac{m_\phi}{H(T_c)}\right)^2 \tan\frac{\phi_0}{f_{\text{w}}}.$$
(7)

Once $H(T) < m_\phi/3$, the relaxion starts oscillating about the local minimum. These oscillations give rise to a relic abundance [14], given by

$$\Omega h^2 \simeq 3\Delta\theta^2 \left(\frac{\Lambda_c}{1 \text{ GeV}}\right)^4 \left(\frac{100 \text{ GeV}}{T_{\text{osc}}}\right)^3.$$
(8)

See [26] for an alternative way of explaining dark matter in relaxion models.

B. Spontaneous baryogenesis

We now show that baryogenesis is a generic consequence of the relaxion mechanism. First we note that if SM fermions ψ_i are charged under the relaxion shift symmetry, $\mathcal{O}_1$ is generated (by field redefinitions) with a current of the form

$$J^\mu = \sum_i q_i\bar{\psi}_i\gamma^\mu\psi_i + \text{other spin particles}.$$
(9)

Such field redefinitions also generically result in terms like $\phi F_2 \tilde{F}_2$ and $\phi G \tilde{G}$, where F_2 and G are the SM $SU(2)_L$ and $SU(3)_C$ gauge field strengths. Following the treatment of [7–9] the $\phi F_2 \tilde{F}_2$ term can be removed by a $B + L$ transformation (that will also modify the composition of the current above). We must ensure that a $\phi G \tilde{G}$ term is not generated [otherwise it will lead to an $\mathcal{O}(1)$ strong CP phase once the relaxion stops] by demanding that the relaxion shift symmetry have no triangle anomaly with $SU(3)_C$.

Now we show that the operator $\mathcal{O}_1$ generates a baryon asymmetry. In the presence of the operator $\mathcal{O}_1$, the background value of $\dot{\phi}$ shifts the energy of particles and antiparticles differently (implying a spontaneous breaking of CPT symmetry). This is equivalent to

$$\mu_i = -\bar{\mu}_i = q_i \dot{\phi}/f + (B_i - L_i)\mu_{B-L} + Q_i \mu_Q, \quad (10)$$

where $(\bar{\mu}_i)$ μ_i is the chemical potential for (antiparticles) particles i, Q_i is the electromagnetic charge and $\mu_{Q,B-L}$ are Lagrange multipliers introduced to enforce conservation of Q and $B - L$. Any source of $B + L$ violation generates an asymmetry in the species i of

$$n_i - \bar{n}_i = f(T, \mu) - f(T, \bar{\mu})$$

$$= g_i(q_i \dot{\phi}/f + (B_i - L_i)\mu_{B-L} + Q_i \mu_Q)\frac{T^2}{6}, \quad (11)$$

where $f(T, \mu)$ is the Fermi-Dirac distribution, we have assumed $\mu \ll T$ and the factor g_i incorporates color, as well as 2 spin degrees of freedom for each Weyl fermion, a factor of 2 for complex scalars, and 3 for massive gauge bosons. The chemical potentials μ_Q and μ_{B-L} are determined by solving $n_Q = n_{B-L} = 0$. This gives a baryon number density

$$\eta \equiv \frac{n_B}{s} = g_{SB}\frac{\dot{\phi}}{f}\frac{T^2}{6} \times \frac{45}{2\pi^2 g_* T^3} = \frac{15}{4\pi^2}\frac{g_{SB}}{g_*}\frac{\dot{\phi}}{fT}, \quad (12)$$

where the particular linear combination of currents in J_μ determines the value of $g_{SB} \sim \mathcal{O}(1)$. For instance, in the flavor degenerate example with $n + 1$ Higgs doublets, considered in the Appendix,

$$g_{SB} = \frac{3((33 + 4n)q_{\lambda_e} + 9q_{\lambda_u})}{(111 + 13n)}, \quad (13)$$

where $q_{\lambda_{e,u}}$ are the effective global charges of the Yukawa couplings.

C. Baryogenesis from the relaxion

Having collected the components, we now proceed to see how they fit together in a generic relaxion scenario [1] and to derive constraints. Typically the relaxion stops its slow roll much before inflation ends (we would need extreme fine-tuning in order to guarantee that the relaxion travels far in field space but stops at a time close to the end of inflation). Therefore the first slow roll of the relaxion is not suitable for baryogenesis as any baryon asymmetry generated would be diluted away. As discussed above however, there is a generic scenario in which the reheat temperature T_r at the end of the inflation is high enough to restore the chiral symmetry breaking that is the source of the oscillatory potential for the relaxion. Consequently the relaxion undergoes a second stage of rolling, this time in a radiation dominated phase. It is in this secondary period of rolling that the time derivative of the relaxion can drive baryogenesis. The operative source of baryon number violation is provided by electroweak sphaleron transitions which are faster than the Hubble rate for temperatures $T > T_{sph} \sim 130$ GeV.

Using Eq. (12) and $\dot{\phi} \sim V'(\phi)/(5H)$ (see Sec. II A), the generated baryon number asymmetry is

$$\eta = \frac{15}{4\pi^2}\frac{g_{SB}}{g_*^{3/2}}\frac{m_\phi^2 m_{pl}}{T_{sph}^3}\frac{f_w}{f}, \quad (14)$$

where the experimentally measured value of $\eta = 8.7 \times 10^{-11}$ [27] and the relaxion mass is

$$m_\phi^2 = \Lambda_c^4/f_w^2. \quad (15)$$

From the first of these expressions one can conclude that

$$\frac{f_w}{f} = \eta\left(\frac{15}{4\pi^2}\frac{g_{SB}}{g_*^{3/2}}\frac{m_\phi^2 m_{pl}}{T_{sph}^3}\right)^{-1} \sim 10^9\left(\frac{m_\phi}{10^{-5}\text{ eV}}\right)^{-2}. \quad (16)$$

Therefore successful baryogenesis requires a large-scale separation between f and f_w. This is acceptable because the relaxion dynamics is already driven by the much larger-scale separation, $F/f_w \sim (M/\Lambda_c)^4$ where $F \sim M^2/g$ is the total field excursion. The hierarchy $f \ll f_w \ll F$ can easily be obtained from the clockwork mechanism [3,22,23] with the operator in Eq. (1) arising from the first site of the clockwork, the backreaction potential from an intermediate site and the rolling potential from the last site. In such a setup f would be the only fundamental physical scale, while f_w, F would be fictitious scales.

The prospects for baryogenesis are determined by five unknown parameters $(f_w, f, f_c, \Lambda_c, m_\phi)$ which, due to the two relations in Eqs. (14) and (15), represent three independent variables. Since the sphalerons must decouple before the backreaction potential reemerges at temperature T_c, we need $T_{sph} \gtrsim T_c$, and so we choose $f_c = T_{sph}/\sqrt{4\pi}$ to saturate this bound. As we will see, the bounds for smaller f_c can easily be determined by rescaling. Hence we can display the results in the plane of m_ϕ and f. The other two variables are then related by

$$f_{\rm w} = \left(\frac{\sqrt{8}\pi^2}{9\sqrt{5}}\eta g_*^{3/2}\frac{T_{\rm sph}^3}{m_{\rm pl}}\right)\frac{f}{m_\phi^2}, \qquad (17)$$

$$\Lambda_{\rm c}^4 = \left(\frac{\sqrt{8}\pi^2}{9\sqrt{5}}\eta g_*^{3/2}\frac{T_{\rm sph}^3}{m_{\rm pl}}\right)^2\frac{f^2}{m_\phi^2}. \qquad (18)$$

Now, for a region in the m_ϕ, f-plane to successfully produce the baryon asymmetry, it must satisfy the following constraints:

a. Relaxion constraints: In order to realize the relaxion mechanism successfully, the loop consistency condition in Eq. (5) requires $\Lambda_{\rm c}^4 < 16\pi^2 v^4$, which translates into

$$\frac{\pi^2}{810}\eta^2 g_*^3\frac{T_{\rm sph}^6}{v^4 m_{\rm pl}^2} < \frac{m_\phi^2}{f^2}. \qquad (19)$$

In addition the slow roll requirement in Eq. (6) provides an upper bound on m_ϕ:

$$m_\phi < 5\sqrt{\frac{g_* T_{\rm c}^4}{90 m_{\rm pl}^2}} \sim 3.8\times 10^{-5}~{\rm eV}\left(\frac{T_{\rm c}}{T_{\rm sph}}\right)^2. \qquad (20)$$

One can verify that this is the only bound that depends on $f_c = T_{\rm c}/\sqrt{4\pi}$. Therefore, changing f_c effects only the upper bound on m_ϕ. Finally for a successful clockwork implementation of the relaxion mechanism, it is essential that the explicit/anomalous breaking at the first and last site be much smaller than the decay constant which implies, for the cutoff, $M \lesssim f$. All these constraints are shown in our master plot in Fig. 1.

b. Cosmological constraints: The Universe must reheat to sufficient temperature in order to activate sphalerons, $T_{\rm r} > T_{\rm sph}$. Furthermore, the sphalerons must decouple before the relaxion stops again, $T_{\rm sph} > T_{\rm c}$; otherwise *CPT* is restored while sphalerons are active and they erase any net baryon number. This is satisfied by our aforementioned choice of f_c. Finally, [11] shows that the backreaction of J^μ on $\dot\phi$ is sufficiently small if $f > T_{\rm sph} \sim 130$ GeV.

c. Experimental constraints: The coupling of ϕ to SM fields must not be detectable. There are two types of constraint: the first type arises from the mixing of ϕ with the Higgs boson which results in an emergent fifth force for the relaxion mass range relevant here. This effect does not depend on the choice of J^μ, so it can be treated independently. The mixing angle between the relaxion and the Higgs comes from the backreaction potential in Eq. (3) and is of order $\sin\theta \sim \frac{\Lambda_{\rm c}^4}{f_{\rm w} v m_h^2}$. The constraints on this angle as a function of m_ϕ have been presented in [28] and are reinterpreted in our master plot as the red exclusion area.

The second type of constraint is from the coupling of the relaxion to SM particles via the operator in Eq. (1). In our

region of interest, the most important bounds on the coupling of the relaxion to electrons, photons and nucleons arise from the fact that such a coupling allows for a more efficient cooling of stars and supernovae (see [29]). While most of these bounds can be evaded if J_μ does not contain first generation fermionic currents (recall that a coupling to $G\tilde{G}$ must be absent in any case as discussed in Sec. II B), a coupling to photons, $E\frac{\alpha}{8\pi}\frac{\phi}{f}F_{\mu\nu}\tilde{F}^{\mu\nu}$, would still be induced, where E is the electromagnetic anomaly coefficient for the relaxion shift symmetry and we have ignored terms with a further m_ϕ^2/m_f^2 suppression (m_f being the mass of the fermions in the loop). The best experimental constraint on the operator comes from bounds on the rate of cooling of globular cluster stars, $f/E > 2\times 10^7$ GeV [29]. The constraints from the contribution of ϕ to the number of relativistic degrees of freedom $\Delta N_{\rm eff}$ [30] are always subdominant. In our master plot we show this bound for $E = 1$ as a purple dashed line, which will apply to currents such as $J^\mu = \bar\tau_R\gamma^\mu\tau_R$ or $J^\mu = \bar{t}_R\gamma^\mu t_R - \bar{b}_R\gamma^\mu b_R$ [31].

Clever choices for the global charges can, however, suppress or even lead to a vanishing E. Consider for example the charge assignment where the only nonzero charges are (1) $q_{L_3} = q_{\tau_R} = 1$ where L_3 is the third generation lepton doublet. For this case, $E = 0$ and the bound due to the coupling to photons can be entirely evaded. While we have guaranteed that the charge assignment above has no electromagnetic or color anomaly, it is still not completely free of constraints. First of all flavor mixing effects can reintroduce a coupling to electrons, once $\mathcal{O}_1$ is rotated to the mass basis, but these can be made small by constraining corresponding off-diagonal rotation matrix elements to be small. Moreover, there are very well-motivated approaches to lepton flavor model building [33,34] where the charged lepton mass matrices are, to a very good approximation, diagonal. More important are the renormalization group effects that generate, at low scales, a coupling of the relaxion to electrons and first generation quarks. As we show in the Supplemental Material, even if we couple the relaxion to just third generation leptons, we tend to generate couplings of the relaxion to the first and second generation fermions of order $c \sim 10^{-3}$ through two-loop effects involving electroweak bosons. The stellar energy loss argument can be applied to the relaxion coupling to electrons in globular cluster stars or its coupling to nucleons in the supernova SN1987A. Both of these yield a similar bound, $f/c \gtrsim 10^9$ GeV [29], as shown in Fig. 2. The constraints from $\Delta N_{\rm eff}$ are always subdominant compared to these star cooling constraints [30].

Finally, the star cooling constraints can be entirely evaded if the current in $\mathcal{O}_1$ contains only right-handed neutrinos as proposed in Ref. [35]. In this case the full white region in Fig. 2 would be allowed by constraints.

d. Dark matter: The authors of Ref. [14] point out that the relaxion oscillations can explain the observed dark
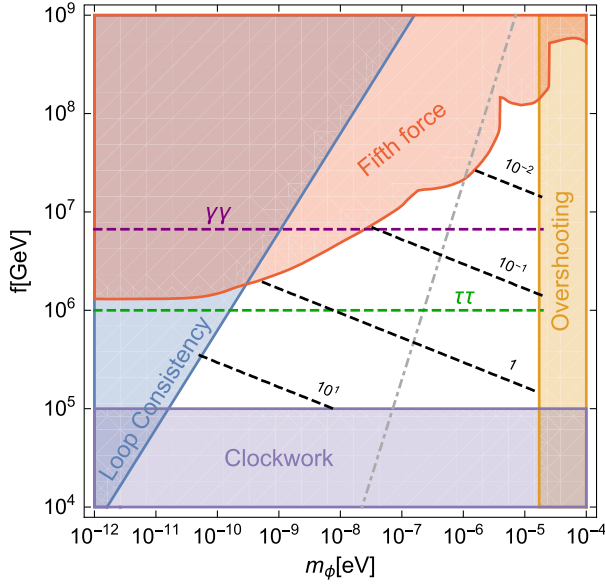
FIG. 2. The parameter space in which spontaneous baryogenesis happens during the second roll of the relaxion. The orange shading shows a region where the relaxion does not get trapped by the backreaction potential. The quantum corrections spoil the backreaction potential in the blue region. The region that leads to rapid star cooling in globular clusters is below the purple dashed line; however, this bound can be relaxed by a suitable choice of current. The region that is ruled out by order 1 coupling between relaxion and tau leptons lies below the green line. The purple region shows the clockwork requirement $M \lesssim f$ for $M \gtrsim 100$ TeV. Finally, the fifth force constraints from mixing with the Higgs particle rule out the region in red. We have added contours of the value of $\tan(\phi_0/f)$ required to obtain the correct relic density as per [14]. Finally, the grey line indicates the region where $f_w = m_{pl}$.

matter abundance. At any point in the plane of Fig. 2, the expected dark matter abundance can be computed up to the square of the factor $\tan(\phi_0/f)$ [see Eqs. (7) and (8)]. We show in Fig. 2 lines indicating the value of $\tan(\phi_0/f)$ required to match the observed dark matter abundance. Values of $\tan(\phi_0/f)$ much larger or smaller than unity are an indication of the level of tuning required to obtain the correct relic density [see the discussion below Eq. (4)]. Remarkably, we find that in the small allowed region obtained after imposing all the theoretical and experimental constraints discussed here, the required tuning to obtain observed relic density ranges from none at all to a maximum of ~100. The relaxion DM should be stable and not decay to photons. We have checked that, even in the example with the least suppressed couplings discussed above, the relaxion decay time to two photons is much longer than the age of the Universe: $\tau \gg H_0^{-1}$.

Finally let us mention how our bounds would change if the $\mathcal{O}(1)$ factor g_{SB} is varied. The clockwork constraint and the constraints arising from the operator $\mathcal{O}_1$ are not directly related to g_{SB}. The effect of g_{SB} on the fifth force or loop consistency bounds can be determined by simply linearly

rescaling f, and the constrained regions move upward (downward) for larger (smaller) values of g_{SB}. Thus a factor of $g_{SB} \sim 10^{-3}$ would be required to close the parameter space entirely.

III. DISCUSSION AND CONCLUSION

We have shown that spontaneous baryogenesis is an integral feature of relaxion models with a high reheat temperature, which induces a second stage of rolling, and that it can generate the SM baryon asymmetry we observe today with almost no adjustment. Indeed the only additional ingredient is a single operator coupling the relaxion to matter currents, $\partial_\mu \phi J^\mu$, which arises automatically if some SM fields are charged under the $U(1)$ of which the relaxion is a pseudo-Nambu-Goldstone boson. Remarkably one encounters (to adapt a phrase from [14]) a *double* "relaxion-miracle," because the baryogenesis mechanism operates most effectively, and with the least tuning, exactly where the relaxion oscillations are a viable dark matter candidate.

ACKNOWLEDGMENTS

APPENDIX A: ORIGIN OF $\mathcal{O}_1$

The simplest explanation for the operator $\mathcal{O}_1$ in Eq. (1) of the main text is that it arises because the SM fermions are charged under the global symmetry for which the relaxion is a PNGB. In this case the Yukawa coupling terms of the SM must carry extra powers of $e^{i\phi/f}$ to be invariant under the relaxion shift symmetry. To explore this possibility we will generalize the discussion in Ref. [9] in order to deal correctly with anomalies. We begin with the Yukawa-induced masses for the fermions during the relaxion evolution, which in the unitary gauge take the form

$$\mathcal{L}_m = -\lambda_u v[u_r^\dagger e^{-i\theta_u} u_l + u_l^\dagger e^{i\theta_u} u_r]$$
$$- \lambda_d v[d_r^\dagger e^{-i\theta_d} d_l + d_l^\dagger e^{i\theta_d} d_r]$$
$$- \lambda_e v[e_r^\dagger e^{-i\theta_e} e_l + e_l^\dagger e^{i\theta_e} e_r], \qquad (A1)$$

where obviously $v = \langle H \rangle$, and a summation over generations is implied. The θ_i are time-dependent phases driven by the relaxion mechanism,

$$\theta_i = q_{\lambda_i} \frac{\phi}{f}, \qquad (A2)$$

where the q_{λ_i} are the difference in global charge between the respective left- and right-handed fermion. The Higgs VEV, $v(t)$, is of course also time dependent, although that has only a minor effect on the baryon asymmetry. For simplicity we will for this illustrative example take the

phases θ_i to be generation independent. This allows us to diagonalize the fermion mass terms as in Eq. (A1).

To determine the physical consequences of the evolving phases in Eq. (A1), one can remove them at the expense of inducing baryon and lepton currents. Broadly speaking this must be equivalent to the operator $\mathcal{O}_1$ with J^μ being some combination of only right-handed currents. We know this because one is free to remove the phases with a rotation on the right-handed fermions only, which avoids topological $SU(2)$ terms.

To see this in more detail, first we remove the phases in Eq. (A1) by making the rotations,

$$q_l \to e^{i\omega} q_l; \qquad u_r \to e^{i(\omega-\theta_u)} u_r; \qquad d_r \to e^{i(\omega-\theta_d)} d_r;$$
$$\ell \to e^{i\bar\omega} \ell; \qquad e_r \to e^{i(\bar\omega-\theta_e)} e_r, \tag{A3}$$

where we are allowed two arbitrary phases, ω and $\bar\omega$ (corresponding to B and L rotations), upon which of course the eventual physics should not depend.

The corresponding change in the classical action is

$$\delta S_0 = -\int \mathrm{d}^4 x [\bar q_l \gamma^\mu q_l \partial_\mu \omega + \bar u_r \gamma^\mu u_r \partial_\mu(\omega-\theta_u)$$
$$+ \bar d_r \gamma^\mu d_r \partial_\mu(\omega-\theta_d) + \bar\ell \gamma^\mu \ell \partial_\mu \bar\omega$$
$$+ \bar e_r \gamma^\mu e_r \partial_\mu(\bar\omega-\theta_e) + \cdots], \tag{A4}$$

where the ellipsis refers to the accompanying shift in the mass terms that removed the phases. At the same time the rotation in Eq. (A3) induces contributions from global anomalies and finite temperature triangle diagrams, which combined (and neglecting all masses except $m_{t,b}$) are

$$\delta S_1 = \int \mathrm{d}^4 x \left[\left(18\omega + 6\bar\omega - \frac{3}{2}\Lambda_t \theta_u - \frac{3}{2}\Lambda_b \theta_d \right) \frac{g_2^2 F_2 \tilde F_2}{32\pi^2} \right.$$
$$- \left(\frac{9\omega}{2} + \frac{3\bar\omega}{2} - \left(4 - \frac{17}{24}\Lambda_t\right)\theta_u \right.$$
$$- \left. \left(1 - \frac{5}{24}\Lambda_b\right)\theta_d - 3\theta_e \right) \frac{g_Y^2 F_Y \tilde F_Y}{32\pi^2}$$
$$\left. + ((3-\Lambda_t)\theta_u + (3-\Lambda_b)\theta_d) \frac{g_3^2 G\tilde G}{32\pi^2} \right], \tag{A5}$$

where the finite temperature pieces are given by

$$\Lambda(m,T) = \sum_{n\in\mathbb{Z}} \frac{8\pi T m^2}{3(m^2 + (2n+1)^2 \pi^2 T^2)^{3/2}}. \tag{A6}$$

At high temperatures, $T \gg m$, these terms can be approximated as

$$\Lambda_{T\gg m} = \frac{14}{3}\zeta(3)\frac{m^2}{\pi^2 T^2}\left(1 - \frac{93\zeta(5)}{56\zeta(3)}\frac{m^2}{\pi^2 T^2} + \cdots\right). \tag{A7}$$

Thus we have implicitly already taken the infinite T limit for all but the top and bottom masses, by setting their Λ's to zero. In the small T limit, one instead finds

$$\Lambda_{T\ll m} = \frac{8}{3}. \tag{A8}$$

Here the requirements for baryogenesis during the relaxion mechanism start to diverge from those of electroweak baryogenesis in Ref. [9]. In particular the anomalous $G\tilde G$ terms for the strong coupling should be zero in the final vacuum such that an $\mathcal{O}(1)$ strong CP phase is not generated once the relaxion stops. This motivates (at least for generation-independent phases) models with the particular physical choice

$$\theta_d = -\theta_u. \tag{A9}$$

Under this assumption we may neglect the effect of strong sphalerons [36,37]. We now follow [9] and remove the $SU(2)$ topological piece in the action as well, by setting

$$\omega = \frac{1}{12}(\Lambda_t - \Lambda_b)\theta_u - \frac{1}{3}\bar\omega \approx -\frac{1}{3}\bar\omega. \tag{A10}$$

This particular choice of ω puts all the important physics in the δS_0 term:

$$\delta S_0 \approx \int \mathrm{d}^4 x [\bar u_r \gamma^\mu u_r \partial_\mu \theta_u - \bar d_r \gamma^\mu d_r \partial_\mu \theta_u$$
$$+ \bar e_r \gamma^\mu e_r \partial_\mu \theta_e + J^\mu_{B-L} \partial_\mu \bar\omega]. \tag{A11}$$

Note that, since we will ultimately set $B - L = 0$, the arbitrary phase $\bar\omega$ can have no effect on the physics. Inserting Eq. (A2), the remaining part can as promised be interpreted as a term of the form of the operator $\mathcal{O}_1$, with

$$J^\mu = q_{\lambda_u}(J^\mu_{u_r} - J^\mu_{d_r}) + q_{\lambda_e} J^\mu_{d_r}. \tag{A12}$$

It is straightforward to generalize the above discussion to generation-dependent charges if we keep in mind some crucial distinctions. First of all, for general charges it is not possible to remove the phases from the Yukawa terms by redefinitions of the singlet quarks alone. The redefinition of the doublet quarks will then generate a $\phi F_2 \tilde F_2$ term that can be removed by a $B + L$ rotation. Finally the condition in Eq. (A9) for not generating a strong CP phase is generalized to the condition that the global symmetry have no triangle anomaly with QCD (see for example Ref. [38]).

APPENDIX B: CALCULATING THE BARYON ASYMMETRY FOR A GIVEN CURRENT

Now we show how the baryon asymmetry can be computed for a given current in Eq. (A12). Starting from Eq. (11) in the main text we solve for μ_{B-L} and μ_{em} by solving $n_{B-L} = n_Q = 0$. For the case at hand, treating $\dot\theta_i = q_{\lambda_i}\dot\phi$ as classical homogeneous background fields, with the q_i's taken from Eq. (A12), we find

$$\mu_{B-L} = -\dot{\bar{\omega}} - \frac{3((7+n)\dot{\theta}_e + 12\dot{\theta}_u)}{111 + 13n},$$

$$\mu_{em} = \frac{3}{2}\frac{(5\dot{\theta}_e - 39\dot{\theta}_u)}{111 + 13n}, \tag{B1}$$

where in order to keep contact with [7–9] we have allowed for n charged Higgs scalars with charge ± 1.

Finally substituting into Eq. (11) of the main text gives

$$n_B = n_L = \frac{3}{(111 + 13n)}((33 + 4n)\dot{\theta}_e + 9\dot{\theta}_u)\frac{T^2}{6},$$

$$= g_{SB}\frac{\dot{\phi}}{F}\frac{T^2}{6}, \tag{B2}$$

where

$$g_{SB} = \frac{3((33 + 4n)q_{\lambda_e} + 9q_{\lambda_u})}{(111 + 13n)} \tag{B3}$$

is a constant of order 1.

Note the physical interpretation which is clear from Eq. (11) of the main text: there is a bias for J^μ production in the plasma, but J^μ does not preserve $B - L$ or Q. Therefore any production of J^μ must be accompanied by a compensating production of J^μ_{B-L} and J^μ_Q. As a consequence even $J^\mu = \bar{e}_r\gamma^\mu e_r$, with no baryon content at all, leads to a baryon asymmetry. (One should be mindful of the background assumption that left-handed and right-handed fields remain in chemical equilibrium throughout, via the Yukawa couplings.)

As expected, there is no dependence of the answer on $\bar{\omega}$, so the physics does not care about how the phase absorption is arbitrarily divided between left- and right-handed fields. As a second check of these expressions, taking $\theta_u = -\theta_d = -\theta_e = -2\bar{\omega}$ gives a current in Eq. (A11) proportional to the hypercharge, as it should because in this case the Yukawas have relaxion charges proportional to the hypercharges of the Higgs bosons appearing in the couplings: then Eq. (B3) yields the answer of [7,8], relevant for Higgs-driven electroweak baryogenesis. (The relaxion charges here are of course completely general.)

APPENDIX C: CALCULATING OF RADIATIVELY GENERATED COUPLINGS TO LEPTONS

As described in [39], anomalous currents can renormalize through nonzero anomalous dimensions. As a result, even if we couple the relaxion to just third generation at some high scale, we will generate a small coupling to other leptons and quarks at lower scales. The anomalous dimension has been calculated in [40]:

$$\frac{d\log Z}{d\log\mu} = \gamma_A = \frac{3n_F C_F}{8}\left(\frac{\alpha_i}{\pi}\right)^2 + \cdots, \tag{C1}$$

where C_F is the Casimir operator of the fundamental representation and n_F is the number of fermionic fields in the fundamental representation. When the leading coupling between the relaxion and the SM is in the leptonic sector, the largest contribution to the anomalous dimension comes from the loop of electroweak (EW) bosons. For our purposes, the anomalous dimension has the form

$$\gamma_{A,EW} = \frac{9n_g}{8}\left(\frac{\alpha_2}{\pi}\right)^2 + \mathcal{O}(\alpha_2^3, \alpha_1^2), \tag{C2}$$

where n_g is the number of active generations (there are four fundamentals per generation). Below m_W, we need to integrate out the W's and Z's and the two-loop diagram responsible for γ_A generates a finite contribution proportional to $G_F^2 m_W^4 \sim \alpha_2^2$ and logarithmic contribution suppressed by $G_F^2 m_l^4 \sim 10^{-7}\alpha_2^2$. For this reason, we can neglect the anomalous dimension below m_W. The electromagnetic contribution is proportional to α_{em}^2 and is therefore subleading. To summarize, the largest contribution to the couplings to leptons and quarks comes from the running induced by electroweak loops from the scale f down to m_W. Consider the charge assignment we consider in the main body of our paper:

$$\mathcal{L} = \frac{\partial^\mu\phi}{f}\left(\sum_Q c_Q\bar{Q}\gamma_\mu Q + \sum_L c_L L^+\gamma_\mu L^-\right), \tag{C3}$$

where $c_Q(f) = c_L(f) = 0$, except for $c_\tau(f) = 1$. As shown in [39], the contributions to other couplings are

$$c_i(m_Z) = c_i(f) + \left(\frac{Z(m_Z)}{Z(f)} - 1\right)\frac{c_\tau}{4n_g}. \tag{C4}$$

Integrating Eq. (C2) we arrive at

$$\frac{Z(m_Z)}{Z(f)} - 1 \sim \frac{9n_g}{8\pi^2}\int_f^{m_Z}\alpha_2^2(\mu)d\log\mu. \tag{C5}$$

Since α_2 only runs by at most 30% between m_Z and 10^9 GeV, it is conservative to use $\alpha_2(\mu) = \alpha_2(m_Z)$. This assumption makes the integral trivial and the generated quark and lepton couplings will bound by

$$c_i = \frac{9n_g}{32\pi^2}c_\tau(f)\alpha_2^2(m_Z)\log f/m_Z$$

$$= 8\times 10^{-4}\log f/10^6 \text{ GeV}. \tag{C6}$$

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