How does a mastery lesson unfold?

Jeremy Dawson and Yuqian (Linda) Wang discuss how to put planning into practice.

Mastery in mathematics has been promoted in England since 2014. So far, the majority of discussion at school level has attempted to explain what a mastery approach looks like in the classroom; conceptual methods for structuring thinking such as bar modelling and how to question students to increase their depth of understanding. In addition, the National Centre for Excellence in the Teaching of Mathematics (NCETM) has championed the notion of "intelligent practice", which they define as "the type of practice that supports pupils to build conceptual understanding, at the same time developing procedural fluency." (NCETM, 2015) These conversations might help us to recognise a mastery approach when it appears in the classroom. They say less, however, about how to go about bringing such an approach to fruition, or how might teachers achieve a mastery approach? In this article, we argue that this is rooted in the planning stage. We propose a lesson-design structure, the causal connectivity framework, to suggest to teachers how a lesson might be designed from the ground up to embrace a mastery approach in the secondary school classroom.

To get a sense of teachers' concerns about a mastery approach, we analysed articles published in 2016 and 2017, from the primary professional journals in the area of mathematics teaching. These articles suggested that a debate engaging teachers in reflective thinking is beneficial for the development of mathematics mastery in England. The mastery approach is summarised as involving the notion of success for all and the goal of conceptual or deep understanding (NCETM, 2016). However, Ems Lord has argued that these definitions lack any mention of problem-solving (Lord, 2016), one of the three aims of the English national curriculum for mathematics. Given all of this, there appears to have been little advice offered about how to design and structure a lesson to fulfil the three aims in the curriculum through the mastery approach. In MT251, the ATM/MA primary group did add a specific focus on pedagogical elements, for instance proposing a "teach-like-they-do-in-Shanghai" route. More discussions around pedagogical aspects focus on whole-class teaching, mixed-attainment classes and the importance of small steps at the instruction level, although Dietmar Küchemann was critical of aspects of this approach in MT257. In MT259, Anne Watson and colleagues developed the discussion of the core elements of mastery to explore conceptual understanding and fluency, which are two of the three aims in the National Curriculum. Meanwhile, the third aim, problem-solving, is explained as a result or an outcome of mastery. Drury (2015) also points out that problem-solving in unfamiliar situations is nourished by mastery. Causal connectivity, the lesson framework we present here, is designed to bridge this gap in an explicit way.

Turning to lesson design, the NCETM (2016) states:

> Lesson design identifies the new mathematics that is to be taught, the key points, the difficult points and a carefully sequenced learning journey through the lesson. In a typical lesson, the teacher facilitates whole-class interactive discussion, including active debate and argument based around the tasks offered. Through teacher-student and student-student interaction the teacher encourages demonstration, explanation, exploration, analysis and generalisation leading to proof where appropriate.

We use the example of teaching probability to outline the causal connectivity framework below, aiming to show that this framework meets the NCETM's ideals.

Theoretical framework

The causal connectivity framework (see figure 1) we have developed has five interconnected phases:

1. Relevance: Activities are chosen that reflect the experience of students, not necessarily in the field of mathematics. The activity might be based around something that sparks their interest or something that all of them can do. This phase could take a playful character.
2. Analysis and synthesis: This phase extends the activities in the relevance phase towards procedural variation, which is the essence of mastery, by developing the activities to lead to

a mathematical discussion and the key teaching points of the session. The transition from phase 1 to phase 2 involves thinking being directed in a mathematical direction and uses the skills of analysis and synthesis on basic tasks, frequently finding differences and similarities, which generally leads to ...

3. Sorting and ordering.
4. Causal connectivity: Through reflection on the processes that have occurred in phase 1, 2 and 3, students are encouraged to look for potential meaningful connections. Note: The design of a learning sequence hinges upon this critical juncture, because it must be designed in a way that actively reveals the connections that the teacher wishes the students to make.
5. Proof or theorising: Via implication and/or inference, students develop an abstract understanding.

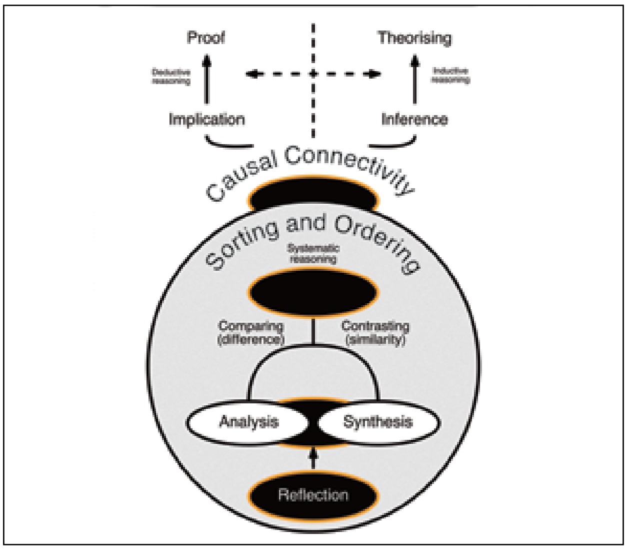


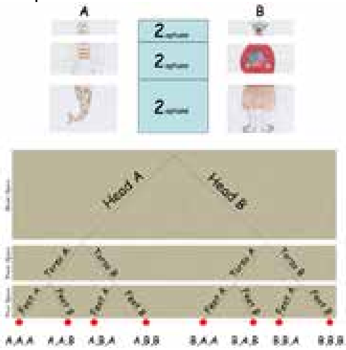
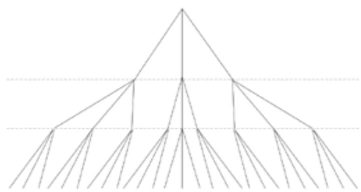
Figure 1: Causal connectivity framework.

We will exemplify the meaning of these phases and how they interconnect with one another through a plan aimed at 11-to 13-year-old students.

The case of probability

The chosen case is a learning sequence that underpins the topic of probability. The table summarises how the causal connectivity framework is linked to the principles of lesson design, to the aims of the curriculum and to the principles of the mastery approach.

Illustration of tasks	Task explanation	Link to the causal connectivity framework	Link to the curriculum	Link to the principles of the mastery approach
Task A1	Fold the paper into three sections to draw a monster: Student A draws the head and then passes the paper to student B to draw the body, who then passes it to student C to draw the legs/tail.			
	This task is based on playful learning and is meant to be interesting and motivational for students.	Phase 1: Relevance		A basis for whole-class interaction.
Task A2	If two monsters are torn into three sections each, how many new monsters (with one head, one body and one leg) can be created?			
	This task is to emphasise or clarify the particular mathematical idea to be focused on. It directs students' learning and encourages thinking mathematically.	Phase 2: Analysis and synthesis	Problem-solving	Implying what new mathematics is to be taught.

<table>
<tr><th>Illustration of tasks</th><th>Task explanation</th><th>Link to the causal connectivity framework</th><th>Link to the curriculum</th><th>Link to the principles of the mastery approach</th></tr>
<tr><td>Task A3</td><td colspan="4">How would you explain your answers mathematically?</td></tr>
<tr><td>Method: Consider the space into which the pieces can be placed.

</td><td>This task is to develop students' ability to analyse the problem through sorting and ordering, with the aim of developing their own way of dealing with a problem. Students might use different strategies: counting by exchanging the parts or presenting the different sections by numbers to list all possibilities.</td><td>Phase 3: Sorting and ordering</td><td>Reasoning</td><td>Active debate and argument based around the tasks at hand.</td></tr>
<tr><td>How could this diagram relate to a similar task?
</td><td>From listening to students justifying their own methods, a generalised method can be developed, linked to an abstract-level understanding.</td><td>Phase 4: Causal connectivity</td><td>Fluency</td><td>Encourage explanation and link with other topics: numbers and powers.</td></tr>
<tr><td>Task A4</td><td colspan="4">How many different creatures could be formed using the three original monsters?</td></tr>
<tr><td></td><td></td><td></td><td>Reasoning and problem-solving</td><td>Encourage exploring.</td></tr>
<tr><td>Task A5</td><td colspan="4">What would happen if instead of cutting our three monsters into three sections, we cut them into four sections?</td></tr>
<tr><td></td><td></td><td></td><td>Reasoning and problem-solving</td><td>Encourage further exploring.</td></tr>
<tr><td>Task A6</td><td colspan="4">Can we generalise our findings for any number of monsters with any number of sections?</td></tr>
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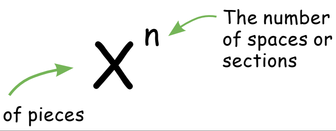
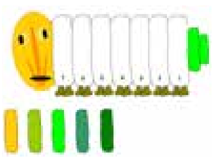
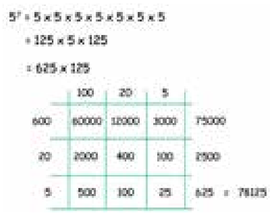
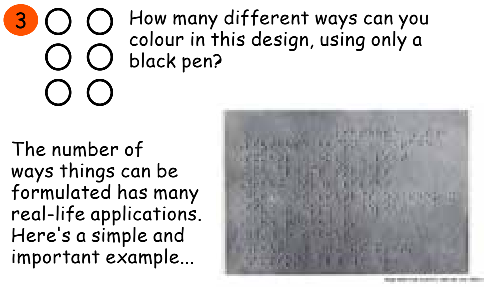

</td><td></td><td>Phase 5: Proof or theorising</td><td>Fluency</td><td>Encourage generalisation and lead to proof.</td></tr>
<tr><td>Task A7</td><td colspan="4">A caterpillar has seven sections to its main body. Each section can be one of 5 colours [shown]. How many distinct and individual caterpillars can there be?</td></tr>
<tr><td>A7:
</td><td>
</td><td></td><td>Problem-solving</td><td>Emphasis on 'what it is' and 'what it is not'.</td></tr>
<tr><td></td><td>How to work out the answer?</td><td></td><td>Reasoning and fluency</td><td>Key facts such as number facts, for example, multiplication is practised.</td></tr>
</table>

Illustration of tasks	Task explanation	Link to the causal connectivity framework	Link to the curriculum	Link to the principles of the mastery approach
Task A8	How many different ways can you colour in this design, using only a black pen?			
			Problem-solving	This adds challenge by applying the content in a new, unfamiliar problem-solving situation.

Final remarks

Our starting point is the assumption that in order to carry out lesson planning using a mastery approach, teachers need guidance with regard to the structuring of lessons and they benefit from a set of principles that determines how to organise the tasks and activities within these lessons. We are currently testing the effectiveness of this framework in a research project, the *Snowflake Bentley project*, to find out if theory translates into practice. However, in this paper, we only describe the framework as guidance for lesson planning in response to the mastery approach.

In these final remarks, we would like to address two issues which arose from the *Snowflake Bentley project:*

Issue 1: What activities might be included in Phase 1 design?

The purpose of Phase 1 is to develop in the students a sense of personal investment in the lesson, a stake in the mathematics that unfolds over its course. This is based on the intention to nurture students' interests in mathematics and their motivation to learn. The activities in Phase 1, ideally, are closely associated with the interests of the students or the class, allowing students an insight into the potential purposes of doing mathematics and how mathematics can be linked to other activities they enjoy. The example above used a playful game in Phase 1 to relate to the topic of probability. Another example might be illustrating unfair versus fair sharing when learning ratios by specifically generating an experience for students so that they experience that emotion of unfairness. For younger learners, storytelling and discussion activities are particularly appropriate. For the *Snowflake Bentley project,* for example, we engaged the children with an illustrated story about William Bentley.

Issue 2: Will adopting the causal connectivity framework make lessons less flexible?

From our experience, we have found that lesson flexibility is not affected. Wang (2015), in her comparative research about students' understanding of mathematics between England and Shanghai, has pointed out that, in the planning stage, teachers in Shanghai tended to be content-focused in order to reveal the structure of the concept and develop depth of understanding while English teachers tended to use various activities in order to arouse students' interest. The mastery approach places its emphasis on the depth of understanding with its core aim to improve students' performance. Under our framework, the underlying logical process during the lesson-planning stage is still to benefit and deepen students' understanding, but we maintain a clear focus on interesting and engaging tasks, which provide clear causal connections that also promote a sense of fun and creativity for both students and teachers.

References

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