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Covid-19 Control and the Economy: Test, Test, Test*

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Abstract

Hard lockdowns have left policymakers to face the ethical dilemma of choosing between saving lives and saving the economy. However, massive testing could have helped to respond more effectively to Covid-19 crisis. In this paper, we study the trade-off between infection control, lockdown and testing. The aim is to understand how these policies can be effectively combined to contain Covid-19 without damaging the economy. An extended SIR epidemic model is developed to identify the set of testing and lockdown levels that lead to a reproduction number below one, thus to infection control and saving lives. Depending on whether the testing policy is static or dynamic, the model suggests that testing 4% to 7% of the population is the way to safely reopen the economy and the society.

I. Introduction

Covid-19 has created an unprecedented global health and economic crisis, which caused thousands of deaths, threatened business and wiped out millions of jobs. Almost every country in the world had to take extreme measures to control the spread of the virus and save lives. One of these measures is lockdown that halts all but essential businesses. IMF, however, predicted that hard lockdowns are likely to cause the worst recession in a century. Another measure, namely testing, was introduced by officials to help control the infection, but the majority of the countries used it at a small scale. The World Health Organization (WHO) believes that large-scale testing is necessary to stop the virus, and its Director-General once said '*We have a simple message for all countries: test, test, test.*' In this paper, we develop an extension of SIR epidemic model to study the trade-off between infection control, lockdown and testing. The objective is to understand how these measures can be effectively combined to contain Covid-19 without damaging the economy too much.

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The lockdown policy has contributed significantly to reducing the transmission of Covid-19 in all affected countries, which has saved thousands of lives. However, this was at the cost of bringing the economy to a standstill. To minimize Covid-19's economic impact, countries of the world were invited to find ways out of their lockdowns without causing another epidemic outbreak. But without a vaccine, most countries were struggling to provide an exit roadmap that balances between reopening the economy and controlling the infection. Testing is another measure that has been used by countries to reduce the infection. The WHO views large-scale testing as an effective way to stop the virus, and called on countries around the world for urgent action to join forces to identify, isolate and contact trace people with Covid-19. Moreover, we know for a fact that the countries – including Iceland, China, Germany and South Korea – that built a high-testing capacity (they tested a greater proportion of their population than the rest of the world) fought the pandemic in an effective way by finding out those who have Covid-19 and quickly isolate or/and treat them. These countries that have managed to keep their case counts and deaths tolls low have also reopened their economies earlier than most other countries in the world.

Given the high importance of the debate on how countries can contain the virus and save their economies, in this paper we are interested in studying the trade-off between Covid-19 infection control and the two policies discussed previously. Our main motivation is simply to understand how to effectively combine these policies so that the infection remains under control and the lockdown is lifted at least partially. To achieve our objectives, we use a key measure of the contagiousness of an infectious disease, namely the reproduction number R, which tells us how many people will contract the disease from a person with this disease. This number is extremely valuable for scientists and policymakers as it can tell if a disease, like Covid-19, can or not cause a proper epidemic outbreak. Our analysis can help determine the levels of lockdown and testing that are needed to push R below 1 – the level at which the disease starts to decline – without risking too much the economy, which should guide countries in need of defining a roadmap for easing the lockdown.

To obtain our results, we first develop a simple extension of SIR epidemic model that links the reproduction number to lockdown and testing policies. We then examine two versions of the extended model, one in which the testing policy is static and the other one in which the testing policy is dynamic. Using the model with the static testing policy (constant testing capacity), we identify the set of testing levels (fractions of the population tested each day) and lockdown levels (fractions of the economic activity shut down during the lockdown) that lead to a reproduction number below one, thus to infection control and saving lives. We show that this set is not empty as long as the false negative rate of the used test is strictly below one and people who contract the virus are self-isolated. A numerical illustration of this model indicates that when a soft lockdown is imposed, say 10% shut down of the economic activity, then high testing capacity of as much as testing 4% of the population each day is needed to have the reproduction number below 1. When the lockdown is slightly higher, say 30% shut down of the economic activity, then testing about 1.5% of the population daily will be enough to get the infection under control. We provide the sets of all levels of lockdown and testing capacity that produce different levels of R.

Next, we consider a SIR model with a testing policy that mimics the dynamic of infection, which is characterized by an expansion, peak and decline phases. Using this dynamic model, we provide a simple formula that expresses the minimum reproduction number – reached at a given time horizon and for a given lockdown level – as a function of the parameters that govern the testing policy. In addition, we identify the set of lockdown levels and values of the parameters of the testing policy that lead to a reproduction number below 1 at a given targeted time horizon. A numerical illustration of this model shows that for 3-month time horizon, the minimum R will be below one when the maximum daily testing capacity is as high as testing 7% of the population daily, and this is even when the used test is not highly reliable. However, for a maximum daily testing capacity of as bad as 0.1%, the infection will not be controlled, except if a hard lockdown of at least 65% level is imposed, which will significantly damage the economy.

We finally provide a general discussion of the economic costs of lockdown and testing policies and how comparing these costs – in light of our findings – can guide countries in terms of setting out roadmaps for easing their lockdowns. Using the results we obtained previously, it seems that a massive testing policy would cost much less than a hard lockdown. For the United States for example, it is estimated that the cost of Covid-19 lockdown is between \$4.2 and \$5.4 trillions a year, whereas the cost of a massive testing (7% of Americans daily) is estimated at maximum \$550.55 billions a year. According to our findings, in terms of controlling the infection and saving lives, a massive testing policy with a soft lockdown would lead to better results than a small-scale testing policy with a hard lockdown. Hence, it seems obvious that roadmaps for lifting lockdowns safely should seriously consider testing populations massively.

Before moving on to the rest of the paper, here is a summary of some economic research that has been initiated so far on Covid-19 pandemic and that can be linked to our paper. Acemoglu *et al*. (2021) examine optimal lockdown policies by developing a multi-risk SIR model that vary between age groups. Solving a planning problem, they find that optimal policies differentially targeting age groups (oldest group) outperform optimal uniform policies. Alvarez, Argente and Lippi (2021) are interested in studying optimal lockdown policies for a planner who wants to control the fatalities of a pandemic while minimizing the output costs. Solving the problem of a planner who uses the lockdown of the citizens as a single instrument to deal with the epidemic, they find that the optimal policy depends on the fraction of infected and susceptible in the population. Both Acemoglu *et al*. (2021) and Alvarez *et al*. (2021) look at lockdown policies. In comparison with their models, our approach is based on a simple extended SIR model where both lockdown and testing policies are endogenous. We can also allow for an endogenous time horizon at which Covid-19 infection is minimized. Berger, Herkenhoff and Mongey (2020) extend the baseline SIR model to study the potential role of broad testing in ameliorating quarantine measures. By calibrating their model, they find that randomly testing asymptomatic individuals can stand-in for costly quarantine measures. Compared to Berger *et al*. (2020), our approach is based on simple theory (with few parameters) that identifies the set of all levels of lockdown and testing capacities that lead to infection control. We show that this set is not empty as long as the false negative rate of the test is below one and the

isolation period is not zero. Without using simulations or calibration, but simple numerical calculations, we are able to estimate the level of testing capacity (4% to 7% of the population daily) that is needed to control the infection and safely reopen the economy and the society. Finally, Bandyopadhyay *et al*. (2021) consider a model of a disease with unknown virulence to illustrate the trade-off between learning and habit formation (e.g. social distancing, developing hygienic habit) that have opposing impacts on the timing of a lockdown. On the one hand, their model shows that an early lockdown is beneficial not only to slow down the spread of the disease, but also creates beneficial habit formation that persists even after the lockdown is lifted. On the other hand, against this benefit of an early lockdown, their model shows that there is a cost from loss of information about the virulence and spread of the disease in the population in addition to a direct cost to the economy. Moreover, based on the prior probability of the disease being virulent, Bandyopadhyay *et al*. (2021) characterize the timing, intensity and duration of a lockdown with the above-mentioned trade-offs.

The rest of the paper is organized as follows. Section II defines the traditional SIR model and develops an extension that depends on lockdown and testing measures. Section III establishes the results of studying the trade-off between the reproduction number, lockdown and testing in the context of SIR models with static and dynamic testing policies. Section IV provides numerical illustrations of the trade-off between the reproduction number, lockdown and testing using static and dynamic testing policies. Section V contains a general discussion of the economic costs of lockdown and testing policies. Section VI concludes and all mathematical proofs are deferred to the Appendix.

II. Theoretical model

We begin by recalling the Susceptible, Infectious and Recovered (SIR) epidemic model that was formulated by Kermack and McKendrick (1927). To derive their model, Kermack and McKendrick (1927) assume that the population can be divided into a set of three distinct groups that are defined with respect to disease status: (i) group of individuals who are uninfected and susceptible (S) of catching the disease, (ii) group of individuals who are infected (I) by the concerned pathogen and (iii) group of individuals recovered (R) from the disease. They also assume that encounters ('matching') between infected and susceptible individuals occur at a rate proportional to their respective numbers in the population. The rate of new infections can thus be defined as βSI, where β represents the contact rate or what is know as infectivity parameter. Infected individuals are assumed to recover with a constant probability at any time, which translates into a constant per capita recovery rate that we denote by γ, and thus an overall rate of recovery γI. Formally, the SIR model is represented by three differential equations, and the key one defines the dynamic of infection:

$$dI_t = \left(\beta \frac{S_t I_t}{N} - \gamma I_t\right) dt,$$

where S_t is the size of susceptible population at time t, I_t is the size of infected population at time t, and N is the total population which also includes the recovered

population at time t, say R_t. Now, if we discretize the previous continuous-time differential equation, we obtain

$$\Delta I_t = \left(\frac{\beta}{\gamma}\frac{S_t}{N} - 1\right)\gamma I_t.$$

At the beginning of an infection, the previous equation becomes:

$$\Delta I|_{t=0} = \left(\frac{\beta}{\gamma} - 1\right)\gamma I_0, \quad \text{since } S_0 \simeq N. \tag{1}$$

The ratio $\frac{\beta}{\gamma}$, known as R-naught [hereafter $R_0 = \frac{\beta}{\gamma}$], represents the natural or basic reproduction number of infectious disease at the time of its apparition, where no vaccine is yet available and no health and economic measures were taken to control the spread of the infection. It measures the number of people who will contract the contagious disease from one person with that disease. This number is very important as it can tell if a disease, like Covid-19, can or not cause a proper epidemic outbreak. Using equation (1), we see that if $R_0 > 1$, then $\Delta I|_{t=0} > 0$, that is, each existing infection causes more than one new infection, thus there will be a proper epidemic outbreak with an increase in the number of the infectious that can reach a high fraction of the population. However, if $R_0 < 1$, then $\Delta I|_{t=0} < 0$, that is, each existing infection causes less than one new infection, thus the disease will decline and eventually die out.

Like we said before, R-naught is not informative about the measures taken to control the spread of the infection. Thus, it cannot be used to evaluate the effectiveness of the health and economic measures implemented to contain the virus and save lives. In the following, we develop a simple extension of SIR model which we use to assess the impact of these measures on the reproduction number R. We are particularly interested in studying the trade-off between R and two policies that have been introduced by governments to control the spread of Covid-19, namely Lockdown and Testing. To study the effect of these policies on R, we extend the SIR model as follows:

$$\Delta I_t = \left[\left((1-\phi_{t-1})(1-L)^2\frac{\beta}{\gamma}\right)\frac{S_t}{N} - 1\right]\gamma I_t, \tag{2}$$

where the dynamic of infection depends on two additional factors: (i) L that represents the level of lockdown or the fraction of the economic activity shut down during the lockdown; and (ii) ϕ_{t-1} that represents the fraction of infected in isolation at time $t-1$, which – as we will see later – depends on the daily testing capacity or the fraction of the population tested each day. For example, $L = 0.35$ can be interpreted as 35% of the total economic activity is shut down during the lockdown, and $\phi_{t-1} = 0.01$ means that there is 1% of infected people in isolation at time $t - 1$. In the extended model, the term $(1 - L)$ is squared because of its double effect on reducing susceptible and infected population simultaneously once the lockdown is imposed. In addition, this quadratic form ensures that R is convex and a decreasing function of L (higher the level of lockdown lower the R).

Alvarez *et al*. (2021) and Acemoglu *et al*. (2021) have also considered SIR models that depend on lockdowns in a quadratic form. As pointed out by Acemoglu *et al*. (2021), the 'matching' form captured by $(1-L)^2$ is similar to the quadratic matching term in Diamond (1982) coconut model, where the number of matches between two groups (here susceptible and infected groups) is the product of their size.

We now model the fraction of infected in isolation ϕ_{t-1} as follows:

$$\phi_{t-1} = T_{t-1}(1-n)l, \tag{3}$$

where T_{t-1} represents the fraction of the population tested at time $t-1$, l is the number of days in isolation, n is the false negative rate of the used test, and $T_{t-1}(1-n)$ is the fraction of infections that enter isolation at time $t-1$. Similar expression of ϕ was considered by Paul Romer during his talk organized by Princeton University in response to Covid-19. Combining equations (2) and (3), the SIR model becomes:

$$\Delta I_t = \left[\left((1-T_{t-1}(1-n)l)(1-L)^2\frac{\beta}{\gamma}\right)\frac{S_t}{N}-1\right]\gamma I_t, \tag{4}$$

where the dynamic of infection depends on the testing policy T_{t-1} and lockdown policy L. As stated by the WHO, testing policy helps quickly identify the infected cases, quickly treat seriously infected people and immediate isolation to stop the spread of the virus. Early testing of the population helps identify anyone who came into contact with infected people so they too can be quickly isolated and/or treated. The director general of WHO, Dr. Tedros Adhanom Ghebreyesus once said: '*We cannot stop this pandemic [Covid-19] if we do not know who is infected*.' Daily testing of a fraction of the population is expected to reduce the reproduction number and in turn the spread of the virus.

From the SIR model in equation (4), the reproduction number at time t, say R_t, is given by:

$$R_t = (1-T_{t-1}(1-n)l)(1-L)^2\frac{\beta}{\gamma}. \tag{5}$$

Equation (5) says that the reproduction number depends not only on the traditional public health measures like improving hygiene practices by washing hands for at least 40 seconds – which is captured by the contact rate β, but also on testing a fraction of the population each day and on lockdown. The lockdown policy intends to reduce the reproduction number R_t by forcing people to stay at home (stay-at-home order), which can minimize the level of contact and stop the spread of the infection. We next use equation (5) to study the trade-off between infection control and lockdown and testing policies.

III. Trade-off between R, lockdown and testing

Static testing policy

In this section, we study a simple version of previously extended SIR model with a constant T and two periods 0 and 1. At period 0, the government discovers the

infectious disease (Covid-19), finds its basic reproduction number R_0, and implements lockdown and testing measures to stop its spread. At period 1, the government calculates the reproduction number R_1 and uses it to decide if the infection is under control ($R_1 < 1$) or not. Under this framework and using equation (5), R_1 is given by:

$$R_1 = \frac{\beta}{\gamma}(1 - T(1-n)l)(1-L)^2. \tag{6}$$

In terms of controlling the infection and making it more likely to die out, it suffices to identify the set of values of T and L that lead to an R_1 below 1, which might help minimize the cost and the burden of extreme lockdown and testing policies. To identify this set, we first re-write equation (6) as follows:

$$R_1 = \frac{\beta}{\gamma}(1 - T(1-n)l)\psi_L, \tag{7}$$

where $\psi_L = (1 - L)^2$. As we will see later, this reparametrization of L is needed to apply some results on the identification of sets that can be represented by quadratic forms. We now focus on identifying the set of all values of T and ψ_L such that $R_1 < 1$. Notice that equation (7) can be written in the following quadratic form (see proof of Proposition 1):

$$\rho' A \rho + b'\rho + c = 0, \tag{8}$$

where $\rho = (T, \psi_L)'$, and A, b, and c are defined in Proposition 1 below. The locus of points $(T, \psi_L)'$ that satisfy an equation of the form (8) constitutes a quadric surface; see Dufour and Taamouti (2005) and references therein. These include as special cases various figures such as ellipsoids, paraboloids, hyperboloids and cones. The shape of the set of $(T, L)'$, however, does not necessarily look like the shape of the set of $(T, \psi_L)'$. The following proposition identifies the set of all values of (T, ψ_L) $[(T, L)]$ that satisfy $R_1 < 1$.

Proposition 1. The set of all values of (T, ψ_L), say $S_{(T,\psi_L)}$, where $\psi_L = (1 - L)^2$, such that $R_1 < 1$ – that is, the infection is under control and *more likely to die out* – can be identified as follows:

(i) It is represented by a quadratic form that depends on a symmetric matrix A, a vector b and a scalar c:

$$S_{(T,\psi_L)} = \left\{\rho = (T, \psi_L)' : \rho' A \rho + b'\rho + c < 0\right\},$$

where $A = \begin{pmatrix} 0 & nl - l \\ nl - l & 0 \end{pmatrix}$, $b = (0, 1)'$, and $c = -\frac{\gamma}{\beta}$;

(ii) If the *isolation period is not zero*, $l \neq 0$, and if the false negative rate of the used test is strictly below one, $n < 1$, then the set $S_{(T,\psi_L)}$ is not empty (unbounded).

The set of all levels of testing capacity T and lockdown L such that $R_1 < 1$, say $S_{(T,L)}$, can be deduced from the set $S_{(T,\psi_L)}$ in Proposition 1 using the mapping $L = 1 - (\psi_L)^{1/2}$ and projection techniques; see for example Dufour (1997), Dufour and Kiviet (1998), Dufour and Jasiak (2001), and Dufour and Taamouti (2005). Given the form of the mapping between L and ψ_L, it is straightforward to see that if the set of ψ_L is not empty, then the one of L will not be empty too. Section IV provides a numerical illustration of the set $S_{(T,L)}$ using real values of the parameters $\frac{\beta}{\gamma}$, n, and l.

Dynamic testing policy

We now study a SIR model with multiple periods and time-varying testing policy as stated by equation (4). This model implies a dynamic reproduction number, say R_{t+1}, that depends on the fraction of the population tested at time t (T_t) and the level of lockdown (L). We assume that L is fixed over time until the infection is ended or at least under control, which represents the situation occurred in most countries that imposed lockdowns. Our results, however, can be straightforwardly extended for an L that changes over time. Formally, the dynamic of the reproduction number is given by:

$$R_{t+1} = \frac{\beta}{\gamma}(1 - T_t(1-n)l)(1-L)^2, \tag{9}$$

where the fraction of the population tested at each time t is now changing over time.

We next use equation (9) to examine the trade-off between infection control, testing and lockdown over a time horizon. Our objective is to identify the set of L and the parameters of the process of T_t such that the reproduction number is minimized or below 1 at a certain time horizon h. To achieve this, we first specify the functional form of T_t. Hereafter, we consider a quadratic form for the testing process and we justify our choice. Our results, however, can be extended for any other linear or nonlinear functional form of T_t.

Regarding the quadratic form of T_t, we think it is more rational to have a testing policy that mimics the dynamic of infection, which is characterized by an expansion (acceleration), peak and decline phases. This implies that high fraction of the population should be tested during the expansion and peak phases and low fraction should be tested during the decline phase; which might help save some resources. This testing policy can be modelled using the following deterministic function:

$$T_t = \mu + \xi t + \eta t^2, \tag{10}$$

where μ, ξ and η are parameters that govern the dynamic of testing. Linton and Li (2020) used similar function to predict the number of new cases of Covid-19 infection in many countries. To mimic the quadratic shape of the dynamic of infection, the parameter η has to be negative. Later (see section 'Dynamic testing policy'), we rescale the time to the unit interval [0, 1] to guarantee that T_t takes values between zero and one. At a given horizon h and considering that the infection starts at $t = 0$, combining equations (9) and (10), we obtain:

$$R_h = \frac{\beta}{\gamma}\left[1 - \left(\mu + \xi(h-1) + \eta(h-1)^2\right)(1-n)l\right](1-L)^2. \tag{11}$$

For a given level of lockdown L, the following proposition provides the minimum value of R_h and the time horizon h at which this minimum is reached.

Proposition 2. For known $\frac{\beta}{\gamma}$, n, and l, and for a given level of lockdown L, the minimum value of R_h in equation (11) is given by:

$$R_{\min} = \frac{\beta}{\gamma}\left[1 - \left(\mu - \xi^2/4\eta\right)(1-n)l\right](1-L)^2,$$

which occurs at horizon

$$h_{\min} = -\xi/2\eta + 1, \quad \text{for } \eta < 0.$$

Interestingly, Proposition 2 states that for a fixed L, the minimum $R_{\min}$ and the time horizon $h_{\min}$ at which that minimum is reached depend on the parameters that govern the testing policy (μ, ξ and η). The horizon $h_{\min}$ depends on the ratio $\xi/2\eta$, thus the reproduction number will take a longer time before it reaches its minimum if that ratio is high and vice versa. This analysis should guide policymakers when they set up their testing policy (when they decide about μ, ξ and η).

According to the epidemiologists, however, having R below 1 should be sufficient to get the infection under control and save lives. For a given horizon h, the following proposition identifies the set of all values of μ, ξ, η and L such that R_h is below 1.

Proposition 3. For a given time horizon h, the set of all values of (μ, ξ, η, ψ_L), say $S_{(\mu,\xi,\eta,\psi_L)}$, where $\psi_L = (1-L)^2$, such that $R_h < 1$ – that is, the infection is under control and *more likely to die out* – can be identified as follows:

(i) It is represented by a quadratic form that depends on a symmetric matrix A^{dyn}, a vector b^{dyn} and a scalar c^{dyn}:

$$S_{(\mu,\xi,\eta,\psi_L)} = \left\{\rho^{dyn} = (\mu, \xi, \eta, \psi_L)' : \rho^{dyn\prime} A^{dyn} \rho^{dyn} + b^{dyn\prime} \rho^{dyn} + c^{dyn} < 0\right\},$$

where

$$A^{dyn} = \begin{pmatrix} 0 & 0 & 0 & nl - l \\ 0 & 0 & 0 & -(h-1)(1-n)l \\ 0 & 0 & 0 & -(1-n)l(h-1)^2 \\ nl - l & -(h-1)(1-n)l & -(1-n)l(h-1)^2 & 0 \end{pmatrix},$$

$b^{dyn} = (0, 0, 0, 1)'$, and $c^{dyn} = -\frac{\gamma}{\beta}$;

(ii) The set $S_{(\mu,\xi,\eta,\psi_L)}$ is not empty (unbounded).

The set of all levels of lockdown L and values of the parameters of the testing process (μ, ξ, η) such that $R_h < 1$, say $S_{(\mu,\xi,\eta,L)}$, can be deduced from the set $S_{(\mu,\xi,\eta,\psi_L)}$ in Proposition 3 using the mapping $L = 1 - (\psi_L)^{1/2}$ and projection techniques. Given

the form of the mapping between L and ψ_L, it is straightforward to see that if the set of ψ_L is not empty, then the one of L will not be too.

IV. Numerical illustration

This section provides some numerical illustrations of the trade-off between R, L and T for both static and dynamic testing policies. The parameter values we used for the numerical computations were obtained from the WHO's official reports and published work in epidemiology and medicine.

Static testing policy

To illustrate the trade-off between R, L and T, we first use model (6) in which the testing policy is assumed to be static. We now discuss the choice of the values of the parameters $\frac{\beta}{\gamma}$, n, and l of that model. Regarding $\frac{\beta}{\gamma}$ or what is known as R_0, the values that were reported at the beginning of Covid-19 infection – before countries started lockdown and testing policies – provide good estimates for $\frac{\beta}{\gamma}$. In an early statement of the WHO which was published on the 23 January 2020, we read '*Human-to-human transmission is occurring and a preliminaryR_0estimate of 1.4–2.5 was presented*.' Some early published work in epidemiology and medicine also reported values of $\frac{\beta}{\gamma}$ between 2.5 and 3.28; see for example Liu *et al*. (2020) and Singh *et al*. (2020). For our illustration, we consider different values of $\frac{\beta}{\gamma}$, but for the sake of brevity we only report the results for $\frac{\beta}{\gamma} = 2.5$; and the other results are available upon request.

Concerning the number of days in isolation l, we follow the recommendation of WHO and take $l = 14$ days. For the false negative rate n, online sources and newspapers report different values that depend on the used test. We consider several values that range from 1% to 10%, and even 50% in the next section. Using values $\frac{\beta}{\gamma} = 2.5$, $l = 14$, and $n = 2\%$, the reproduction number in equation (6) can be expressed as follows:

$$R_1 = 2.5(1 - 13.72\,T)(1 - L)^2. \tag{12}$$

On the one hand, Figure 1 illustrates the trade-off between R_1 and T for different levels of L. From this, we first see that whatever the level of lockdown is, R_1 decreases when the fraction of the population tested each day increases. Second, for the range of low levels of lockdown, say $L \in (0, 0.30)$, Covid-19 infection will be under control ($R_1 < 1$) only when the testing capacity is above certain levels that are determined by the points of intersections between the horizontal line at one and the downward trending lines. For example, if a government implements a soft lockdown of 10%, then to control Covid-19 infection at least 4% of the population must be tested daily and those who are infected need to self-isolate for at least 14 days. When the level of lockdown is slightly higher at 30%, then to have the infection under control the government only needs to test at least 1.5% of the population daily and isolate the infected ones. These results are in line with the figures suggested by the '*Roadmap to Pandemic Resilience*' published by Harvard University.1 The roadmap suggests that 2%–6% of the US population need to be

[1]See the link to the *Roadmap to Pandemic Resilience*: https://ethics.harvard.edu/Covid-Roadmap

tested every day in order to control the infection and re-open the economy. Figure 1 also shows that Covid-19 infection will be under control when the level of lockdown is above 38%–40%, and this is whatever the level of testing. However, hard lockdown would significantly damage the economy as recently warned by IMF. In addition, the speed for stopping the virus (getting R_1 close to zero) will depend on the testing capacity as shown in that same figure.

On the other hand, Figure 2 illustrates the trade-off between R_1 and L for different levels of T. This shows that whatever the level of the testing capacity is, R_1 decreases when the level of lockdown increases. Interestingly, comparing the slopes of the curves in Figures 1 and 2, we see that marginal increases in T reduce more R_1 than marginal increases in L, which might suggest that the testing policy is more effective than the lockdown policy. Furthermore, Figure 2 confirms the previously found results. Again we see that when the testing capacity is low, then high levels of lockdown are required in order to control the infection and vice versa. When the testing capacity is between 0% and 4%, Covid-19 infection becomes under control ($R_1 < 1$) only when the level of lockdown is above certain levels that are determined by the points of intersections

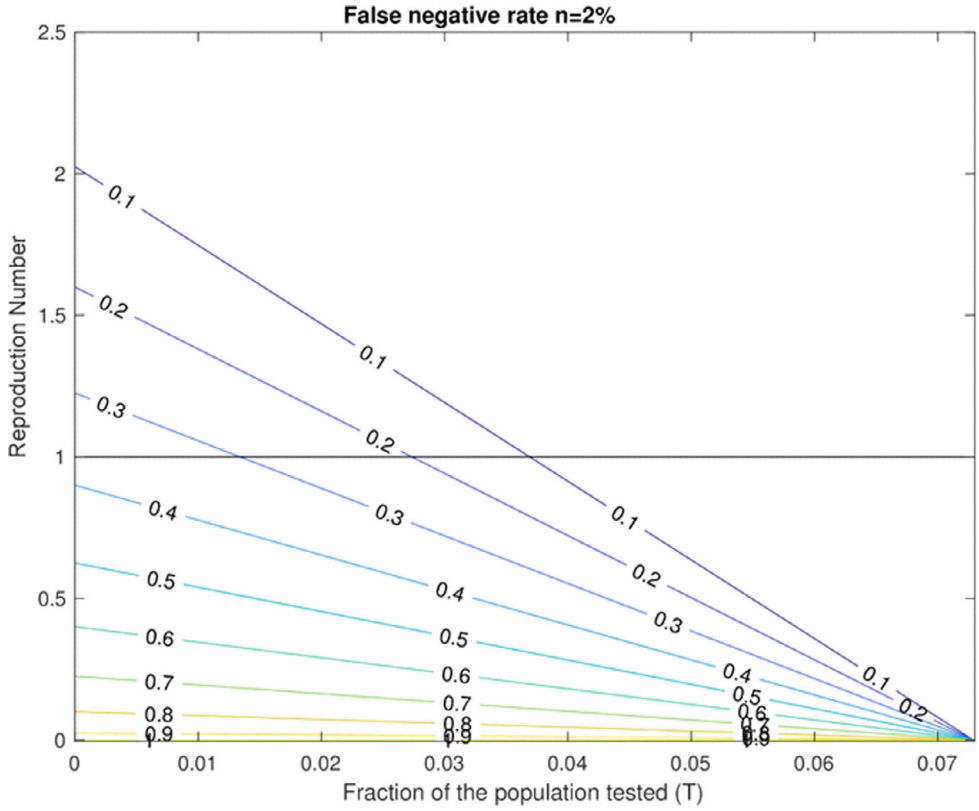

Figure 1. Trade-off between R and Testing, for different levels of Lockdown
Note: This figure illustrates the reproduction number (R_1) as a function of testing, for different levels of lockdown. The numerical illustration is based on real values: basic reproduction number $\frac{\beta}{\gamma} = 2.5$, number of days in isolation $l = 14$, and false negative rate of the test $n = 2\%$.

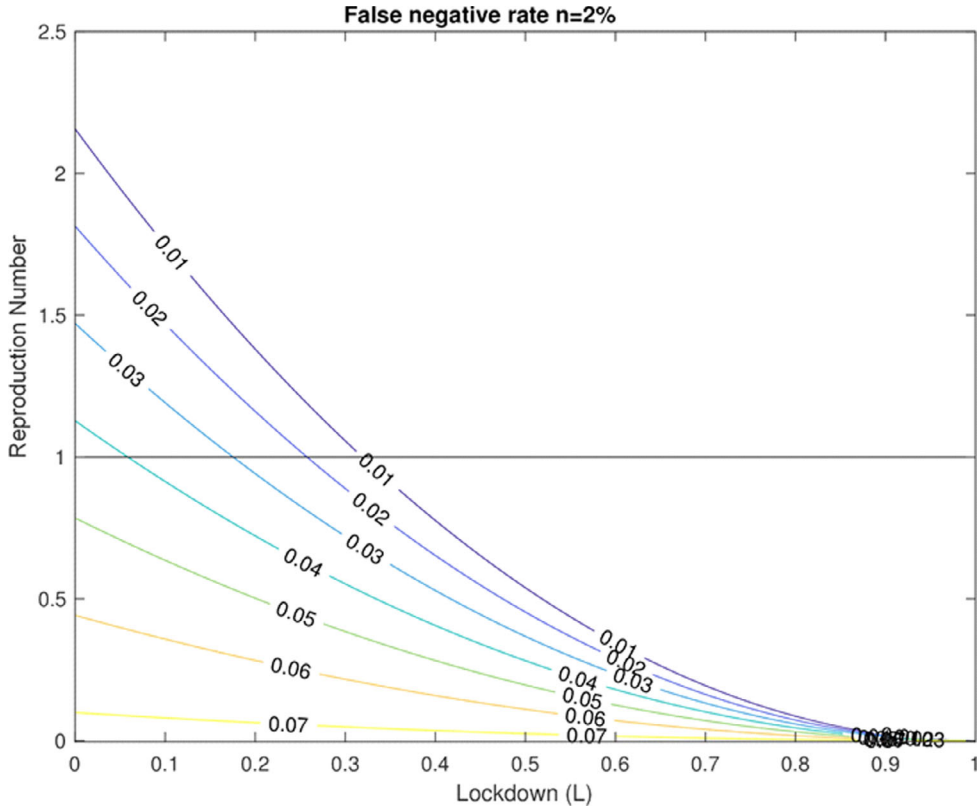

Figure 2. Trade-off between R and Lockdown, for different levels of Testing
Note: This figure illustrates the reproduction number (R_1) as a function of lockdown, for different levels of testing. Same real values as those reported in the footnote of Figure 1 are used.

between the horizontal line at one and the downward trending curves. When the testing capacity is above 4%, then the reproduction number will be below one, whatever the level of lockdown. However, the speed to wipe out the virus will depend on the level of the lockdown.

Another interesting result is reported in Figure 3, which illustrates the trade-off between L and T for different levels of R_1. This figure provides the sets of values of (T, L) required to reach certain levels of R_1. For example, the curve at the top of Figure 3 represents the set of combinations of T and L that a country needs to implement to reach an R_1 of 0.5. These sets are expanding when the targeted R_1 is low.

Dynamic testing policy

For a given time horizon (hereafter 3 months), we use the dynamic model in equation (11) to study how the minimum value of the reproduction number varies with the level of lockdown L and the testing policy represented by the parameters μ, ξ and η.

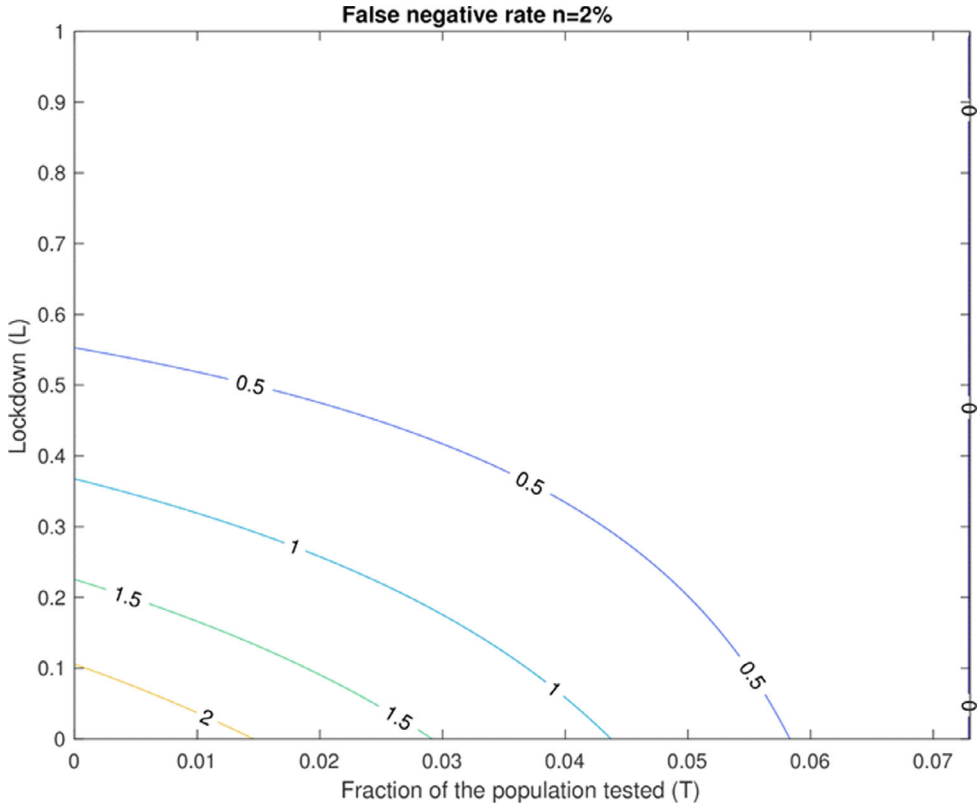

Figure 3. Trade-off between Testing and Lockdown, for different levels of R
Note: This figure illustrates the trade-off between testing and lockdown, for different levels of the reproduction number (R_1). Same real values as those reported in the footnote of Figure 1 are used.

We also investigate how this minimum changes when the false negative rate n and R-naught change. We focus on a 3-month horizon to mimic Covid-19 crisis in which governments around the world ran against the clock to control the infection and minimize the economic damage from the virus.

To obtain our numerical results, we use the same values of $\frac{\beta}{\gamma}$, n, and l as those chosen in section 'Static testing policy'. We rescale the time to the unit interval [0, 1] to reassure that the fraction of the population tested each day remains between zero and one. The value 1 in this interval represents 1-year time horizon. If, instead, we want to look at horizons of say 5 or 10 years, then the value 1 will represent a 5-year or 10-year time horizon respectively.

Using the rescaled time horizon, the drift parameter μ of the testing policy in equation (10) captures the fraction of the population tested at the beginning of the infection, which we suppose equal to zero (μ=0) to mimic Covid-19 situation when the tests did not exist at the start of the infection. To provide the results for different testing policies, we consider the following values of (ξ, η): (0.56, −1.12), (0.32, −0.64), (0.16, −0.32) and (0.008, −0.016). These values are selected to allow

TABLE 1

Minimum reproduction number and maximum fraction of the population tested each day

	L = *0.15*		L = *0.35*		L = *0.65*	
	T_{max}	R_{min}	T_{max}	R_{min}	T_{max}	R_{min}
$n = 2\%$						
$\xi = 0.56, \eta = -1.12$	7%	0.072	7%	0.041	7%	0.012
$\xi = 0.32, \eta = -0.64$	4%	0.815	4%	0.476	4%	0.138
$\xi = 0.16, \eta = -0.32$	2%	1.310	2%	0.766	2%	0.222
$\xi = 0.008, \eta = -0.016$	0.1%	1.781	0.1%	1.041	0.1%	0.302
$n = 10\%$						
$\xi = 0.56, \eta = -1.12$	7%	0.213	7%	0.124	7%	0.036
$\xi = 0.32, \eta = -0.64$	4%	0.895	4%	0.523	4%	0.151
$\xi = 0.16, \eta = -0.32$	2%	1.351	2%	0.790	2%	0.229
$\xi = 0.008, \eta = -0.016$	0.1%	1.783	0.1%	1.042	0.1%	0.302
$n = 50\%$						
$\xi = 0.56, \eta = -1.12$	7%	0.921	7%	0.538	7%	0.156
$\xi = 0.32, \eta = -0.64$	4%	1.300	4%	0.760	4%	0.220
$\xi = 0.16, \eta = -0.32$	2%	1.553	2%	0.908	2%	0.263
$\xi = 0.008, \eta = -0.016$	0.1%	1.793	0.1%	1.048	0.1%	0.304

Notes: For a targeted three month time horizon, different values of the level of lockdown (L), different dynamics of testing policy (represented by different values of ξ and η), and different false negative rates (n), this table reports the maximum fraction of the population tested each day ($T_{\max}$) and the corresponding minimum value of the reproduction number ($R_{\min}$). The natural (basic) reproduction number and the number of days in isolation are $\frac{\beta}{\gamma} = 2.5$ and $l = 14$, respectively.

for different maximum daily testing capacities that vary from 0.1% to 7%, representing scenarios ranging from small- to large-scale testing programs.

The numerical results are reported in Table 1. The latter shows that both the maximum daily testing capacity ($T_{\max}$) and the corresponding minimum value of the reproduction number ($R_{\min}$) depend on the dynamic of the testing policy. Large-scale testing policies with maximum daily capacities of 4% and 7% will lead to minimum reproduction numbers below one for the target time horizon of 3 months, and this is even when the lockdown level is low. For example, when the lockdown is of 15% level and the false negative rate is 2%, $R_{\min}$ will be equal to 0.072 and 0.815 when the implemented Covid-19 testing programs have $T_{\max}$ of 7% and 4% respectively. For the same levels of $T_{\max}$ and lockdown, the minimum reproduction number will remain below one even when the used test is highly unreliable. For a test that has 50% false negative rate, a testing program with a $T_{\max}$ of 7% will lead to an $R_{\min}$ of 0.921, thus to infection control and saving lives. Table 1 also shows that a small-scale testing program will lead to an undesirable outcome with a minimum reproduction number a way above one. For example, a testing policy with a maximum daily capacity of 0.1% will not help control the infection for the target time horizon of 3 months, except if a hard lockdown of at least 65% level is imposed, which will be very damaging for the economy.

V. Discussion

After comparing the effect of lockdown and testing policies on infection control, in the light of our findings, we will now compare the economic costs of these policies based on figures that have been released by governmental and non-governmental organizations, newspapers and some published work.

On the one hand, many whole sectors of the economy have been put on pause during the lockdown. This had a significant negative effect on the world economy and the wellbeing of populations. According to a recent world economic outlook report published by IMF in April 2020, due to the lockdowns of Covid-19 pandemic, the global economy is projected to contract sharply by 3% in 2020, much worse than during the 2008–09 financial crisis. The drop in economic growth is even worst for many advanced economies like Italy (−9.1%), France (−7.2%), Germany (−7%), United Kingdom (−6.5), Canada (−6.2), United States (−5.9%), etc. Even the emerging market and developing economies that used to enjoy high economic growth rates are also severely affected by Covid-19 pandemic. For example, China and India were projected to reach a modest 1% and 1.9% growth rates in 2020 after a 6.1% and 4.2% growth rates in 2019 respectively. Similar scenarios were projected for emerging and developing Europe, Latin America and the Caribbean, and Middle East and Central Asia economies. According to the ILO, out of an active population of 3.3 billion people, more than four of five are affected by the total or partial closure of workplaces.

In the Unites States, after the lockdown was imposed in many US states to prevent the spread of Covid-19, the Department of Labor announced that 3.3 million people had lost their job. In the first week of April 2020, it was reported that an additional 6.6 million of Americans filed for unemployment. Then on the first week of May 2020, the Bureau of Labor Statistics said that 20.5 million had lost their jobs in April 2020, which is by far the most largest decline in employment since the beginning of tracking the data on unemployment in 1939. Paul Romer said that the lockdown is costing the Unites States '*$350 billion to $450 billion a month on reduced output*'. In France, 5.8 million workers filled out unemployment benefit claims in April 2020 due to the lockdown. Furthermore, a study by OFCE, French Economic Observatory Centre at Sciences-Po, estimated that the lockdown is costing € 60 billion per month for France. In Germany, Ifo Institute for Economic Research estimates that the lockdown will cost € 729 billion for the year 2020. For the United Kingdom, according to the office for Budget Responsibility (OBR), if the lockdown lasts for 3 month this will cause a 35% fall in the UK GDP. In addition, the office projected that more than 2 million British workers could lose their jobs by June 2020. The British national newspaper The Times estimated the cost of the lockdown for the United Kingdom at £ 2.4 billion a day (£ 72 billion a month). A recent study by African Union (AU) suggests that in addition to increases in debt, 20 million jobs were lost in the continent due to the lockdown. The same study predicts that Covid-19 could cost Africa $500 billion, because of the effect of lockdown on tourism, foreign direct investment (FDI) and other sectors of the African economy. Many more official reports paint a dark picture for the other economies of the world.

On the other hand, the cost of a massive Covid-19 testing policy (4%–7% of the population each day following our findings) could depend on the country. Regarding the cost of an individual test, in a press conference convened by WHO's European Regional Office on the 17th of March 2020, Dr. Dorit Nitzan, WHO/European Coordinator of Health Emergencies, said that this cost depends on the country, but ranges from € 30 to € 60. Based on our findings (say testing 7% of the population each day) and using the **highest** price of € 60 per test, simple calculations show that a massive testing program will cost around $550.55 billions a year for the United States, € 125.5 billions a year for Germany, € 101.2 billions a year for France and around £ 89.36 billions a year for the United Kingdom. These costs are much lower than the economic costs of lockdowns that we discussed in the second and third paragraphs of this section. A yearly cost of lockdown for the United States is estimated to be between $4.2 and $5.4 trillions. Yearly costs of lockdowns for the United Kingdom, Germany and France are estimated at £ 864, € 729 and € 720 billions respectively.

To sum up, according to our findings, in terms of controlling the infection and saving lives, a large-scale testing policy with a soft lockdown can lead to better results than a small-scale testing policy with a hard lockdown. Given the previous discussion, it seems quite obvious that roadmaps for lifting lockdowns safely should seriously consider testing populations massively. Massive nationwide testing programs could help quickly slow the spread of Covid-19, hence save lives and economies.

VI. Conclusion

We studied the trade-off between infection control, lockdown and testing. An extended SIR epidemic model was developed to identify the set of testing and lockdown levels that lead to a reproduction number below one, thus to infection control and saving lives. Depending on whether the testing policy is static or dynamic, the model suggested that testing 4%–7% of the population daily is the way to safely reopen the economy and the society.

Moreover, we provided a general discussion of the economic costs of lockdown and testing policies and how comparing these costs – in light of our findings – can guide countries in terms of setting out roadmaps for easing their lockdowns. Using the results we obtained previously, we found that a massive testing policy would cost much less than a hard lockdown. According to our findings, in terms of controlling the infection and saving lives, a massive testing policy with a soft lockdown would lead to better results than a small-scale testing policy with a hard lockdown. Hence, roadmaps for lifting lockdowns safely should seriously consider testing populations massively.

Appendix: Proofs

Proof of Proposition 1. (i) The infection is under control and more likely to die out if:

$$R_1 = \frac{\beta}{\gamma}(1 - T(1-n)l)\psi_L < 1.$$

In a matrix form, this inequality can be re-written as follows:

$$\rho' B\rho + b'\rho + c < 0, \tag{A1}$$

where $\rho = (T, \psi_L)'$, $B = \begin{pmatrix} 0 & -l \\ nl & 0 \end{pmatrix}$, $b = (0, 1)'$, and $c = -\frac{\gamma}{\beta}$. Notice that the matrix B is asymmetric, but we can re-write the quadratic form in equation (A1) in terms of a symmetric matrix. To achieve this, observe that

$$\rho' B\rho = \rho' \frac{(B + B')}{2}\rho + \rho' \frac{(B - B')}{2}\rho,$$

with

$$\rho' \frac{(B + B')}{2}\rho = \frac{1}{2}\rho' \underbrace{\begin{pmatrix} 0 & nl - l \\ nl - l & 0 \end{pmatrix}}_{A}\rho \neq 0 \text{ and } \rho' \frac{(B - B')}{2}\rho = 0,$$

where the matrix A is symmetric. Consequently, the inequality in equation (A1) is equivalent to

$$\rho' A\rho + b'\rho + c < 0,$$

where now the quadratic form depends on symmetric matrix A. Hence, the set of all values of (T, ψ_L), say $S_{(T,\psi_L)}$, such that $R_1 < 1$ – that is, the infection is under control and will more likely to die out – can be represented by a quadratic form that depends on a symmetric matrix A, a vector b and a scalar c:

$$S_{(T,\psi_L)} = \left\{\rho = (T, \psi_L)' : \rho' A\rho + b'\rho + c < 0\right\}.$$

(ii) If the isolation period is not zero, $l \neq 0$, and if the false negative rate of the used test is strictly below one, $n < 1$, then the matrix A is not singular [$\det(A) = -l^2(1-n)^2 \neq 0$]. Hence, following Dufour and Taamouti (2005) [see the result (c) in Section 4.1 of Dufour and Taamouti (2005)], the set $S_{(T,\psi_L)}$ is not empty. Now, to show that the set $S_{(T,\psi_L)}$ is unbounded, we again follow the approach in Dufour and Taamouti (2005). In other words, to show that $S_{(T,\psi_L)}$ is unbounded it suffices to show that the matrix A has both positive and negative eigenvalues. We now calculate the eigenvalues of A:

$$\det|A - \lambda I| = \lambda^2 - (nl - l)^2.$$

The eigenvalues are $\lambda_1 = (1 - n)\, l > 0$, and $\lambda_2 = -(1 - n)\, l < 0$, as $n \in (0, 1)$, hence $S_{(T,\psi_L)}$ is unbounded.

Proof of Proposition 2. The proof of this proposition is straightforward. For a given level of lockdown L and known values of $\frac{\beta}{\gamma}$, n, and l, the reproduction number at horizon h, say R_h, is a decreasing function of $T_{h-1} = \mu + \xi\,(h - 1) + \eta\,(h-1)^2$. Thus, R_h reaches its minimum value when T_{h-1} reaches its maximum value. Now, notice that using equation (10), the maximum daily testing capacity occurs at $h_{\min} - 1 = -\xi/2\eta$,

which results in maximal value of the fraction of the tested population per day of $T_{\max} = \mu - \xi^2/4\eta$. Consequently, the minimum value of R_h is:

$$R_{\min} = \frac{\beta}{\gamma}[1 - (T_{\max})(1-n)l](1-L)^2 = \frac{\beta}{\gamma}\left[1 - \left(\mu - \xi^2/4\eta\right)(1-n)l\right](1-L)^2.$$

Finally, since the maximum daily testing capacity occurs at $h_{\min} - 1 = -\xi/2\eta$, the minimum value of R_h occurs one period later at $h_{\min} = 1 - \xi/2\eta$.

Proof of Proposition 3. The proof of part (i) of Proposition 3 is similar to the proof of part (i) of Proposition 1, hence the proof will be omitted. Regarding the part (ii) of Proposition 3 and since $\det(A^{dyn}) = 0$ (matrix A^{dyn} is singular), part (ii) can be proved using one of the results in Section 4.3 of Dufour and Taamouti (2005) for $b^{dyn} \neq 0$.

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