

# Cusp or core? Revisiting the globular cluster timing problem in Fornax

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## ABSTRACT

We use  $N$ -body simulations to revisit the globular cluster (GC) ‘timing problem’ in the Fornax dwarf spheroidal (dSph). In agreement with earlier work, we find that, due to dynamical friction, GCs sink to the centre of dark matter haloes with a cuspy inner density profile but ‘stall’ at roughly  $1/3$  of the core radius ( $r_{\text{core}}$ ) in haloes with constant-density cores. The time-scales to sink or stall depend strongly on the mass of the GC and on the initial orbital radius, but are essentially the same for either cuspy (Navarro–Frenk–White) or cored haloes normalized to have the same total mass within  $r_{\text{core}}$ . Arguing against a cusp on the basis that GCs have not sunk to the centre is thus no different from arguing against a core, unless all clusters are today at  $\sim (1/3)r_{\text{core}}$ . This would imply a core radius exceeding  $\sim 3$  kpc, much larger than seems plausible in any core-formation scenario. (The average projected distance of Fornax GCs is  $\langle R_{\text{GC, Fnx}} \rangle \sim 1$  kpc and its effective radius is  $\sim 700$  pc.) A simpler explanation is that Fornax GCs have only been modestly affected by dynamical friction, as expected if clusters started orbiting at initial radii of the order of  $\sim 1$ – $2$  kpc, just outside Fornax’s present-day half-light radius but well within the tidal radius imprinted by Galactic tides. This is not entirely unexpected. Fornax GCs are significantly older and more metal-poor than most Fornax stars, and such populations in dSphs tend to be more spatially extended than their younger and more metal-rich counterparts. Contrary to some earlier claims, our simulations further suggest that GCs do not truly ‘stall’ at  $\sim 0.3 r_{\text{core}}$ , but rather continue decaying towards the centre, albeit at reduced rates. We conclude that dismissing the presence of a cusp in Fornax based on the spatial distribution of its GC population is unwarranted.

**Key words:** galaxies: clusters: general – galaxies: dwarfs – galaxies: haloes.

## 1 INTRODUCTION

The globular cluster (GC) system of the Fornax dwarf spheroidal (dSph) is often cited as evidence for the presence of a constant-density core in the dark matter halo density profile. The issue has been addressed repeatedly in the literature, starting with the early work of Hernandez & Gilmore (1998), who were among the first to describe how the spatial distribution of GCs may be used to gain insight into the dark matter density distribution in dSphs. This elaborated on the earlier work of Tremaine (1976), who puzzled about the lack of a central stellar ‘nucleus’ in Fornax, expected from the orbital decay and subsequent fusion of its GCs. Indeed, the five GCs in Fornax are widely spread through the galaxy, with an average projected radius<sup>1</sup> of  $\langle R_{\text{GC, Fnx}} \rangle \sim 1$  kpc (Mackey & Gilmore

2003), despite the fact that their orbital decay time-scales, inferred at the time from simple analytical dynamical friction estimates (Chandrasekhar 1943), were substantially shorter than their ages.

This puzzle is widely referred to as the Fornax ‘GC timing problem’ and has elicited the proposal of a number of possible solutions, ranging from the ‘dynamical stirring’ of GC orbits by Galactic tides or massive black holes (Oh, Lin & Richer 2000) to more straightforward options, such as assuming that GCs in Fornax started decaying from initial radii somewhat larger than where they are currently at (Angus & Diaferio 2009; Boldrini, Mohayaee & Silk 2019).

An alternative solution was proposed by Goerdt et al. (2006), who reported some of the first fully self-consistent  $N$ -body simulations of the problem. These authors found that analytical predictions for dynamical friction-induced orbital decay fail in the case of haloes with constant-density cores. Instead of continually decaying, GCs ‘stall’ once they are well inside the core, at a radius that is roughly independent of GC mass. In cuspy haloes, such as the Navarro–Frenk–White profiles of cold dark matter (CDM) haloes (NFW;

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<sup>1</sup>For a comparison, Fornax’s effective radius is  $R_{\text{eff, Fnx}} \sim 700$  pc (Irwin & Hatzidimitriou 1995).

Navarro, Frenk & White 1996b, 1997), GCs do not stall but rather sink until they reach either the centre or a radius where the enclosed dark mass is comparable to that of the cluster (Goerdt et al. 2010).

The ‘stalling radius’ result has been reproduced in subsequent work (see e.g. Read et al. 2006; Inoue 2009; Petts, Gualandris & Read 2015; Kaur & Sridhar 2018), and has become an often cited argument for the presence of a core in Fornax: If GCs ‘stall at the core radius’, as is often claimed, then for  $\langle R_{GC, \text{FnX}} \rangle \approx r_{\text{core}} \sim 1$  kpc the timing problem would be solved.

A core radius of that size would be comparable to Fornax’s effective radius, as expected if cores are carved out of cuspy, NFW haloes by baryonic inflows/outflows during the formation of the galaxy (see e.g. Navarro, Eke & Frenk 1996a; Pontzen & Governato 2012; Di Cintio et al. 2014, and references therein). It would also be commensurate with the core size expected for Fornax in models where cores are produced by ‘self-interactions’ between dark matter particles (Spergel & Steinhardt 2000; Rocha et al. 2013; Kaplinghat, Tulin & Yu 2016), at least for self-interacting cross-sections in the preferred range of  $0.1\text{--}1\text{ cm}^2\text{ g}^{-1}$ . These coincidences have helped galvanize support for the ‘core’ solution to the Fornax GC timing problem.

One problem with this solution is that the stalling radius is actually well inside the core<sup>2</sup>, i.e.  $r_{\text{stall}} \sim 0.3 r_{\text{core}}$ . Taken at face value, this would imply that a core radius as large as  $\sim 3$  kpc would be needed to solve the timing problem, a value that seems, in principle, much larger than can be reasonably accommodated by current core-formation models.

One reason why cores remain a viable solution is that subsequent simulation work uncovered a rather puzzling phenomenon that affects clusters that reach the inner regions of the core. In the simulations reported by Cole et al. (2012), clusters well inside the core tend to *gain* orbital energy, and are pushed out by ‘dynamical buoyancy’, a mechanism whose detailed origin remains unclear but which apparently counteracts dynamical friction in the innermost regions. The combination of friction and buoyancy could, in principle, lead to a stationary ‘shell-like’ distribution of globulars near the core radius, where the two effects would presumably cancel out. Although appealing, this result relies on a mechanism that is still poorly understood, and that urgently needs theoretical underpinning and independent numerical confirmation.

We address some of these issues here using a series of  $N$ -body simulations of the decay of GCs in cuspy or cored haloes. We focus on the difference in the time-scales needed for clusters to ‘sink’ (i.e. to reach the centre, in the case of cusps), or to ‘stall’ (in the case of cores). We also follow the long-term evolution of several clusters after they stall, in order to learn about the possible effects of dynamical buoyancy on these systems.

This paper is organized as follows. Section 2 describes our numerical setup, while our main results are presented in Section 3. We conclude with a discussion of the applicability of these results to Fornax and to the ongoing cusp versus core debate in Section 4.

## 2 NUMERICAL SIMULATIONS

The simulations follow the evolution of a GC (represented by a softened point mass) in two spherical  $N$ -body halo models. The first

<sup>2</sup>We shall hereafter define the core radius,  $r_{\text{core}}$ , as the (3D) radius where the dark matter density drops by a factor of 2 from its central value. Since this convention is not always followed, care is needed when comparing quantitative results from different authors.

model is a cuspy, NFW halo (hereafter, ‘NFW’) with parameters consistent with those expected in a Planck-normalized  $\Lambda$ CDM cosmology (Ludlow et al. 2016). The second model is a non-singular isothermal sphere (hereafter, ‘ISO’) normalized to have the same mass as the NFW profile within its core radius.

### 2.1 Halo models

The cuspy halo model follows an NFW profile,

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}, \quad (1)$$

and is fully specified by two parameters, e.g. a scale density,  $\rho_s$ , and a scale radius,  $r_s$ , or, alternatively, a maximum circular velocity,  $V_{\text{max}}$ , and the radius at which it is achieved,  $r_{\text{max}}$ . The two radial scales are related by  $r_{\text{max}} = 2.16 r_s$ .

The cored halo is modelled as a non-singular ISO (see e.g. Binney & Tremaine 1987, p.228). Although there is no simple algebraic formula to describe this model, it is also fully specified by two parameters, usually expressed as the central density,  $\rho_0$ , and the core radius,  $r_{\text{core}}$ . To prevent divergences, the models are truncated with an exponential taper in the outer regions, but this should be of little consequence for our analysis.

The models are assumed to have isotropic velocity distributions and are normalized to have the same enclosed mass within the de-projected (3D) half-light radius of Fornax,  $M(< 1\text{ kpc}) = 10^8 M_{\odot}$ , inferred from observations of the line-of-sight velocity dispersion and projected light profile of Fornax (Walker, Mateo & Olszewski 2009; Wolf et al. 2010). This is widely agreed to be the most robust dynamical mass estimate available for this system [see the discussion of fig. 1 in Fattahi et al. (2016), and references therein].

Fig. 1 contrasts the density,  $\rho(r)$ , circular velocity,  $V_c(r)$ , enclosed mass,  $M(r)$ , and radial velocity dispersion,  $\sigma_r(r)$ , profiles of the two models. The NFW profile has  $r_s = 2.11$  kpc and  $\rho(r_s) = (\rho_s/4) = 3 \times 10^6 M_{\odot} \text{ kpc}^{-3}$ . This corresponds to a ‘virial’<sup>3</sup> mass  $M_{200} = 2.7 \times 10^9 M_{\odot}$  and  $c = r_{200}/r_s = 14$ . The isothermal profile has  $\rho_0 = 3 \times 10^7 M_{\odot} \text{ kpc}^{-3}$  and  $r_{\text{core}} = 1$  kpc.

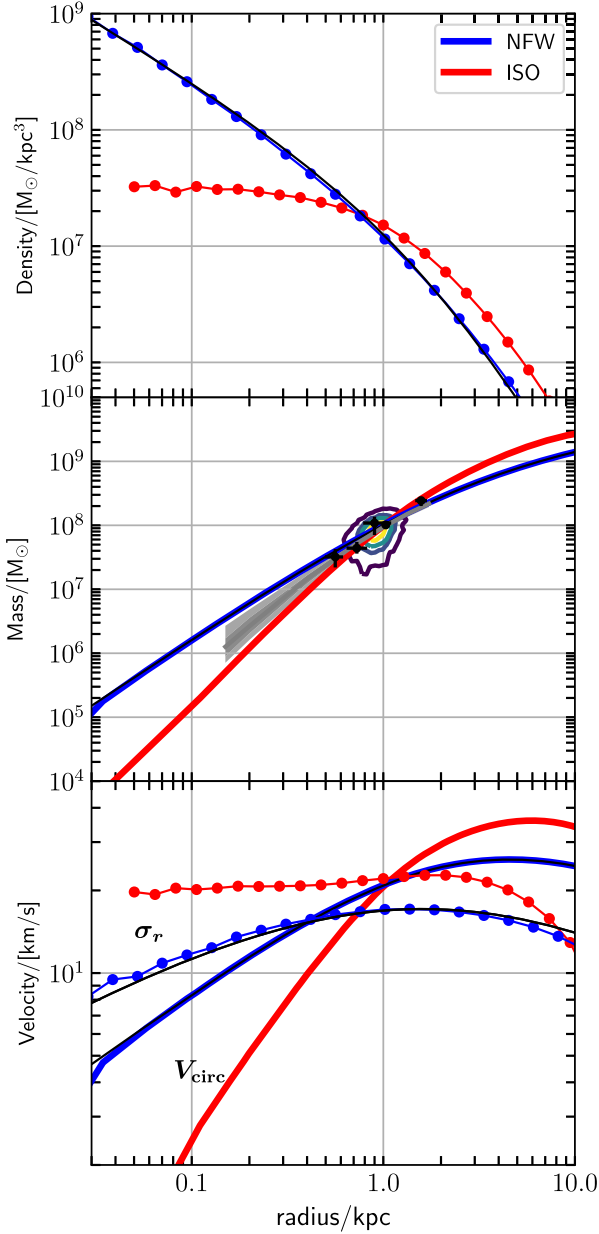
The contours in the middle panel of Fig. 1 indicate the constraints derived by Fattahi et al. (2016) on  $M(< 1\text{ kpc})$ . For a comparison, we also indicate with crosses the constraints at various radii from Walker & Peñarrubia (2011) and Amorisco et al. (2013). The grey shaded band corresponds to the results of the recent kinematic analysis of Fornax’s stellar component of Read et al. (2019). Note how all of these estimates concur at  $r \sim 1$  kpc to a mass close to what is assumed in our models.

For reference, the circular orbit time-scale is  $t_{\text{circ}} \approx 3 \times 10^8$  yr at  $r = 1$  kpc for both models; at  $r = 0.1$  kpc,  $t_{\text{circ}} = 8 \times 10^7$  yr for the NFW case, and  $t_{\text{circ}} = 2.2 \times 10^8$  yr for the cored halo.

### 2.2 GC models

GCs are modelled as single softened point masses. Three different masses were chosen in our runs: a fiducial value of  $M_{\text{GC}} = 3 \times 10^5 M_{\odot}$ , similar to Fornax GC3 (NGC 1049), the most massive cluster orbiting Fornax (Mackey & Gilmore 2003). We also explored models with  $M_{\text{GC}} = 10^5 M_{\odot}$ , comparable to GC2, GC4,

<sup>3</sup>Virial quantities are conventionally defined as those measured at a radius where the mean enclosed density equals 200× the critical density for closure, and are identified with a ‘200’ subscript.



**Figure 1.** Density (top), enclosed mass (middle) and circular velocity/radial velocity dispersion (bottom) profiles of the halo models used in this study. The profiles (shown with circles/thick curves) correspond to the 16M-particle  $N$ -body realization of each model, and are plotted after the halo has been run for  $\sim 4$  Gyr to allow it to relax to equilibrium. The blue corresponds to the cuspy NFW halo, and the red corresponds to the non-singular isothermal (cored) halo. The analytic NFW profile is shown with thin black lines. The contours in the middle panel are constraints on the enclosed mass within  $\sim 1$  kpc, derived from the stellar velocity dispersion and density profiles [see Fattahi et al. (2016), for details]. In the same panel, the crosses indicate the estimates of Walker & Peñarrubia (2011) and Amorisco, Agnello & Evans (2013). The grey shaded band corresponds to the recent kinematic analysis of Read, Walker & Steger (2019). All of these estimates coincide at  $r \sim 1$  kpc.

and GC5. The other GCs in Fornax have much lower mass (GC1,  $3.7 \times 10^4 M_\odot$ ). Recall that dynamical friction times scale inversely with mass. In the absence of other complicating factors, and in the regime where the GC mass is small compared to that enclosed

within its orbit, the orbital decay of different clusters should be similar, once their times are inversely scaled by cluster mass. We assume that GC masses remain constant during the evolution. This neglects possible mass losses due to internal collisional processes within the cluster. Including this effect would result in even longer orbital decay time-scales than the ones reported here, so our results may be regarded as conservative from that point of view.

### 2.3 $N$ -body models

Equilibrium  $N$ -body models with 1.6 and 16 million particles are generated for each halo using the software package *Zeno*<sup>4</sup> developed by Josh Barnes at the University of Hawaii. This package allows for the creation of a number of systems in virial equilibrium by Monte Carlo, sampling the appropriate distribution function.

The simulations were run with the publicly available *Gadget2* code (Springel 2005), with standard numerical integration parameters. Pairwise interactions between  $N$ -body particles are softened with a Plummer-equivalent softening length of  $\epsilon_P = 66.4$  and 210 pc, for the 16M- and 1.6M-particle haloes, respectively. The halo particle mass is  $1.78 \times 10^2 M_\odot$  (cusp) and  $1.99 \times 10^2 M_\odot$  (core) for the 16M-particle realizations. Particle masses are  $10\times$  larger for the 1.6M-particle haloes.

Each halo model is run for  $\sim 4$  Gyr in isolation to allow them to equilibrate and fully relax before introducing the GC. The profiles shown in Fig. 1 are measured at the end of these equilibration runs. Careful centring is required to obtain robust results; we use in our analysis the reference frame given by the gravitational potential-weighted centre of all halo particles, i.e.  $\vec{x}_C = \sum \Phi_i \vec{x}_i / \sum \Phi_i$ ;  $\vec{v}_C = \sum \Phi_i \vec{v}_i / \sum \Phi_i$ .

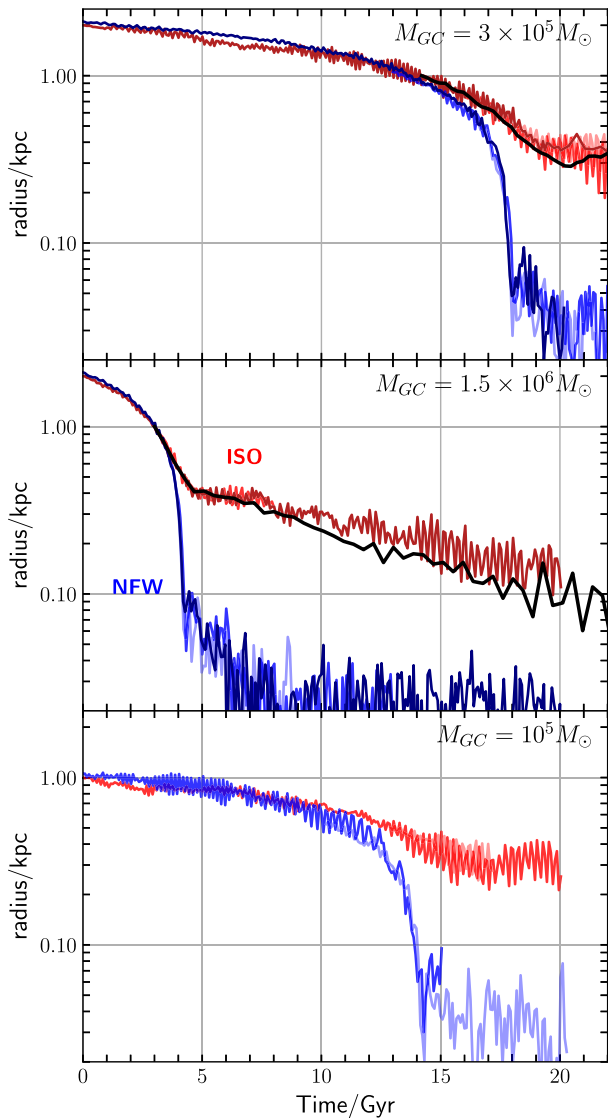
GC particles are softened with  $\epsilon_{P,GC} = 13$  pc and are introduced at the end of the equilibration period. They are placed at various radii (typically  $r_{\text{init}} = 0.5, 1,$  and  $2$  kpc) on circular orbits with random orientations. Their radial evolution is then monitored as a function of time. Most of the runs reported here correspond to the 1.6M model; a representative sample of those have been repeated with the 16M-particle model, with indistinguishable results. We have also repeated several runs varying  $\epsilon_{P,GC}$ . No significant variations were seen in the GC orbital evolution for values of  $\epsilon_{P,GC}$  smaller than adopted for our runs, although substantially longer dynamical friction decay times were seen for (unrealistically) large values of  $\epsilon_{P,GC}$ . For  $\epsilon_{P,GC} \sim 10$  pc, for example, GCs take roughly twice as long to decay than for our fiducial value of 13 pc.

## 3 RESULTS

### 3.1 Orbital decay time-scales

The time evolution of the fiducial mass GC ( $M_{GC} = 3 \times 10^5 M_\odot$ , similar to the most massive Fornax GC, NGC 1049) is shown in the top panel of Fig. 2. The figure shows the evolution of three different runs per halo, each with different starting radii,  $r_{\text{init}} = 2, 1,$  and  $0.5$  kpc. Curves for the latter two have been shifted horizontally so that they coincide in radius and time, at the beginning, with the  $r_{\text{init}} = 2$  kpc case. All three curves are essentially indistinguishable from each other. This highlights the fact that the GC evolution is independent of starting radius, as expected if orbits remain roughly circular throughout the evolution.

<sup>4</sup><http://www.ifa.hawaii.edu/faculty/barnes/zeno/>



**Figure 2.** Evolution of the radial distance of an  $M_{GC} = 3 \times 10^5 M_{\odot}$  (top),  $M_{GC} = 1.5 \times 10^6 M_{\odot}$  (middle), and  $M_{GC} = 10^5 M_{\odot}$  (bottom) globular cluster. The evolution is followed for roughly 20 Gyr. The cuspy, NFW halo case is shown in blue; the core case is shown in red. (The curves in black correspond to the 16M-particle halo model.) The different hues correspond to independent runs with different initial radii,  $r_{\text{init}} = 2, 1,$  and  $0.5$  kpc, respectively, and are shown after shifting their time origin so that their starting radii coincide. The near-perfect overlap between different curves shows that the numerical results are independent of starting radii, as expected if clusters remain on a nearly circular orbit as they decay. Clusters either sink to the centre (cusp) or stall (core), but do so on similar time-scales. The top panel corresponds to a cluster with mass comparable to the most massive GC in Fornax (GC3/NGC 1049). Its orbit decays from 2 to 1 kpc in  $\sim 13$  Gyr, before stalling (core) or sinking (cusp) after  $\sim 18$  Gyr. The middle panel represents a cluster  $5 \times$  more massive than NGC 1049. The bottom panel corresponds to a mass comparable to GC2, GC4, and GC5.

This figure illustrates a few interesting points. One is that, if NGC 1049 had formed at 2 kpc from the centre, then it would only have decayed to a distance of  $\sim 1$  kpc after a Hubble time. The orbital decay accelerates once the cluster reaches 1 kpc, and the cluster

quickly sinks to the centre in the case of the cusp, or ‘stalls’ at  $r_{\text{stall}} \sim 0.3 r_{\text{core}} = 300$  pc in the case of the core.<sup>5</sup>

This behaviour is consistent with earlier work (see e.g. Goerdt et al. 2006; Read et al. 2006; Cole et al. 2012): GCs always stall at  $\sim 0.3 r_{\text{core}}$ , when the core radius is defined as that where the density drops to half its central value.

Interestingly, the time the cluster takes to either sink or stall is approximately the same,  $\sim 18$  Gyr ( $\sim 4$  Gyr since the cluster reached 1 kpc) in both cases. In other words, *dynamical friction time-scales in cored or cuspy haloes are essentially indistinguishable* for haloes normalized as in Fig. 1. The difference is in the final radius reached by the cluster:  $\sim 300$  pc in the case of the core, or the centre in the case of the cusp.

The middle panel of Fig. 2 confirms this conclusion for the case of a cluster  $5 \times$  more massive,  $M_{GC} = 1.5 \times 10^6 M_{\odot}$ . The evolution of this cluster is exactly analogous to that of its less massive counterpart shown in the top panel. The time-scales to sink or stall are still roughly the same for cusp or core, albeit  $5 \times$  shorter than in the former case, just as expected from the mass ratio between those clusters.

Conversely, for clusters less massive than our fiducial mass, the decay time-scales are substantially longer. The results for  $M_{GC} = 10^5 M_{\odot}$  (comparable to Fornax GC2, GC4, and GC5) are shown in the bottom panel of Fig. 2, and show that clusters with  $r_{\text{init}} = 1$  kpc take more than 13 Gyr to either sink or stall. This is as expected from the fiducial ( $3 \times 10^5 M_{\odot}$ ) mass case, which takes  $\sim 4$  Gyr to sink or stall from a radius of 1 kpc. Placed at  $r_{\text{init}} > 1$  kpc, GC2, GC4, and GC5 would have barely evolved over 13 Gyr. Again, the evolution shown in the three panels of Fig. 2 is all analogous and consistent with each other, once times are scaled by the mass of a cluster, and comparisons are made for the same starting radius.

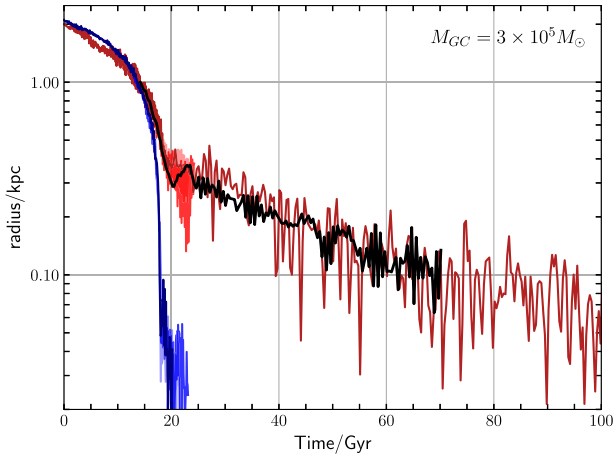
### 3.2 Long-term evolution

The middle panel of Fig. 2 follows the evolution of the most massive cluster in our series for  $\sim 20$  Gyr, quite a long period of time after its initial stall/sink. This allows us to probe the long-term evolution of the clusters once they reach the inner regions of the halo. In the case of the cusp, once the cluster sinks to the centre it stays there. In the case of the core, after its initial stall the cluster keeps losing energy and slowly drops deeper inside the core. At the end of the simulation the cluster has reached a radius of  $\sim 200$  pc, roughly where the halo enclosed mass is comparable to its own (see Fig. 1). Note that we find the same result for the 1.6M- and 16M-particle haloes, so the long-term sinking behaviour seems robust.

This long-term evolution is not unique to this massive cluster. The fiducial mass GC also keeps losing energy after its initial stall, as shown in Fig. 3. The main difference is that this long-term trend takes, as expected,  $5 \times$  longer, and is therefore only noticeable in simulations that follow the evolution for roughly  $\sim 100$  Gyr. Indeed, after that time the cluster has shrunk its orbit to roughly 100 pc, which is about the radius where the enclosed mass of the halo matches that of the cluster.

These results seem to disagree with those of Cole et al. (2012), who report that clusters that drop deep into the core of a halo are pushed out by a mechanism they call ‘dynamical buoyancy’. This

<sup>5</sup>Note that GCs do not truly stall at  $r_{\text{stall}}$ ; rather, the rate of their inspiraling slows down when clusters reach that radius. See Section 3.2 below for details.



**Figure 3.** As Fig. 2, but following the evolution for  $t = 100$  Gyr. Note that the GC keeps decaying inside the core, but on an  $\sim 5\times$  longer time-scale than in Fig. 2, as expected given the mass ratio between the clusters.

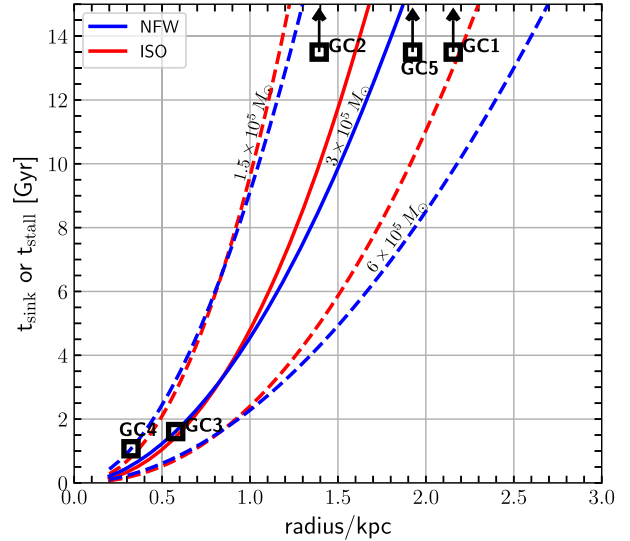
effect was only seen in the case of their ‘large-core’ (LC) halo, which is actually quite similar to the ISO halo we adopt here. Indeed, the LC halo has  $r_{\text{core}} \sim 1.2$  kpc (only 20 per cent larger than ISO’s) and  $\rho_0 \sim 4 \times 10^7 M_{\odot} \text{ kpc}^{-3}$  (about 33 per cent larger than ISO’s). The main difference is that the LC density profile steepens faster than ISO’s: at  $r_{\text{core}}$ , the logarithmic slope is  $d \log \rho / d \log r = -1.8$  for LC and  $-1.1$  for ISO. This difference seems, at face value, too small to explain why we do not see ‘dynamical buoyancy’ in our runs. At this point, it is unclear what the origin of the discrepancy might be, but it is something that we plan to investigate in future work.

#### 4 DISCUSSION AND CONCLUSIONS

In agreement with earlier work, our simulations indicate that the GC population of Fornax is expected to evolve continuously due to dynamical friction. For the halo models considered here (Section 2.1), our results are summarized in Fig. 4, where we plot the time it would take for clusters of various masses to sink (NFW, blue) or stall (ISO, red). These times are computed by fitting the results of our simulations with simple power laws and therefore, in a strict sense, are rough estimates that apply only to clusters in circular orbits. However, these times are not expected to differ much from those for clusters in non-circular orbits with comparable average radii (Angus & Diaferio 2009).

We compare these results with Fornax GCs (shown with open squares in Fig. 4), taking into account the masses of individual clusters and assuming that they are at radii  $4/3\times$  their current projected distance. These results indicate that GC1, GC2, and GC5 are either too far, or have too little mass, to decay significantly, even over a time span as long as the next 10–15 Gyr. It is thus highly unlikely that all clusters are today at a common radius dictated by dynamical friction effects.

On the other hand, both GC3 and GC4 (the two closest to the centre, with projected distances of 0.43 and 0.24 kpc, respectively; Mackey & Gilmore 2003) should either sink or stall over the next few Gyr, according to Fig. 4. Could it be that Fornax has a core and these two clusters have ‘stalled’ at a common radius? This possibility may, in principle, be checked using the radial velocities of these clusters relative to Fornax. GC4, in particular, has a well-defined radial velocity offset of nearly  $\sim 10 \text{ km s}^{-1}$  relative to Fornax



**Figure 4.** Time to stall (core, in red) or sink (cusp, in blue) as a function of GC mass and initial radius. Note that, at given radius and mass, the time-scales are similar for cuspy and cored haloes normalized to the same mass within  $r_{\text{core}} = 1$  kpc (Fig. 1). As expected, time-scales scale inversely with mass, and are strongly dependent on initial radius. Fornax GCs are placed on this figure at radii equal to  $4/3\times$  the present-day projected distance, and at a location consistent with its mass. Note that only GC3 and GC4 are expected to evolve significantly due to dynamical friction over the next few Gyr. See the text for a full discussion.

(Hendricks et al. 2016). This is significantly higher than the expected circular velocity at its present deprojected radius (in the case of a core), so it is highly unlikely that this cluster is actually close to its stalling radius.

This leaves GC3, which, if ‘stalled’, would imply  $r_{\text{stall}} \sim 600$  pc, its inferred 3D distance from the centre.<sup>6</sup> This implies  $r_{\text{core}} \sim 2$  kpc (i.e. at least twice as large as its stellar half-mass radius; recall that  $r_{\text{stall}} \approx 0.3 r_{\text{core}}$ ).

A core radius this large seems difficult to accommodate in either of the two leading scenarios for core creation, i.e. baryonic outflow-induced cores, or dark matter self-interactions. Indeed, if cores are carved out of CDM haloes through stellar feedback, then it would be difficult to explain a core size at least twice as large as the half-light radius of the galaxy (Pontzen & Governato 2014; Oman et al. 2016).

On the other hand, if cores are due to self-interacting dark matter, these would be expected to be of sub-kpc scale in galaxies as small as Fornax, even for extreme values of the cross-section. Elbert et al. (2015), for example, report sub-kpc core radii<sup>7</sup> even for haloes substantially more massive than Fornax, and for all values of the cross-section in the plausible range of  $0.1\text{--}1 \text{ cm}^2 \text{ g}^{-1}$  (the same is true even for larger cross-sections; see e.g. Sameie et al. 2019).

<sup>6</sup>There is, of course, also the possibility that this cluster is much further away in distance and lie, by chance, only in projection near the centre of Fornax. This would make the case for a core even weaker, and could be checked by inspecting the relative proper motion of GC3 relative to Fornax, an issue we are currently working on.

<sup>7</sup>Recall that our definition of core radius follows the traditional convention of designating the distance where the density drops by a factor of 2 from the central value.

If, on the other hand, Fornax has a cusp, then GC3 and GC4 must be on their way to sinking to the centre, having started their decay from  $r_{\text{init}} \sim 1.5$  kpc (GC3) and  $r_{\text{init}} \sim 1$  kpc (GC4) about  $\sim 10$  Gyr ago (their typical ages; see e.g. Buonanno et al. 1998). These initial radii are quite plausible, as they lie well within the inferred tidal radius of Fornax imposed by the Galactic tides, which is estimated to be of the order of 1.8–2.8 kpc (Angus & Diaferio 2009; Cole et al. 2012).

It could be argued that, because the sinking accelerates once GCs reach the inner regions of the halo, this represents a ‘fine-tuning’ problem. In other words, why are we observing GC3 and GC4 at such radii and not at the centre if they are at a rapidly evolving stage of their decay? The same fine-tuning argument may be used against a core, however, since in that case GCs also accelerate their decay before stalling, and the time-scales to sink or stall are very similar. This argument only favours a core if both clusters have stalled, which, as discussed above, is disfavoured by the radial velocity of GC4 and requires an implausibly large core radius of at least 2 kpc.

In the case of the cusp, the disadvantage of a scenario where GC3 and GC4 formed at slightly larger initial radii and are at present on their way to sinking to the centre is that all clusters would then have formed outside the present-day half-light radius of the dwarf. In the absence of a well-defined theory of GC formation it is difficult to assess the severity of this objection, but it should be noted that Fornax GCs are older and more metal-poor than most stars in the dwarf. Such populations tend to be more spatially extended than younger and more metal-rich ones, in Fornax (Battaglia et al. 2006; Walker & Peñarrubia 2011) as well as in other dwarfs such as Sextans (Battaglia et al. 2011) and Sculptor (Tolstoy et al. 2004).

Some of these differences could indicate an ancient merger, which would have dispersed the old stellar component, and allowed the enriched gas to sink further in before forming stars (Benítez-Llambay et al. 2016; Genina et al. 2019). This would provide a plausible explanation for the radial offset between the original distribution of Fornax GCs and the present-day distributions of its stars.

We end by noting that our simulations show no clear evidence of the ‘dynamical buoyancy’ effects reported by Cole et al. (2012). It is thus unclear at this point what the origin of the difference might be, but it does underscore the need for further study of the effect, including a theoretical explanation and an exploration of its dependence on cluster mass and on the detailed dynamical properties of the core.

The Fornax GC spatial distribution is thus unlikely to help discern between cusp and core. In this sense, the GC timing problem is no different from dynamical analyses that use the spherical Jeans equations to derive mass profiles from velocity dispersion and density profile data. These models suffer from well-known degeneracies that prevent a conclusive determination of the shape of the inner density profile (see e.g. the reviews by Strigari 2013; Walker 2013, and references therein). Indeed, data for several dSphs are consistent with NFW cusps and cores (e.g. Gilmore et al. 2007; Strigari, Frenk & White 2010).

Using higher order moments of the line-of-sight velocity distribution offers in some cases the possibility of breaking the degeneracy. Recently, Read et al. (2019) applied this method to Fornax and concluded that the dark matter density drops by about an order of magnitude (from  $\sim 10^8$  to  $10^7 M_{\odot} \text{ kpc}^{-3}$ ) over the range 0.1–1 kpc (see the middle panel of their fig. 3). This is close to what is expected for a  $\rho \propto r^{-1}$  NFW cusp and is only slightly less concentrated than the model we analyse here (see the middle panel of Fig. 1).

Our overall conclusion is that it is unclear how or whether the spatial distribution of GCs in Fornax may be used to discern between the core and cusp scenarios. What is clear, though, is that it cannot be used to argue convincingly against the presence of a cusp in the inner density profile of the Fornax dSph.

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## REFERENCES

- Amorisco N. C., Agnello A., Evans N. W., 2013, *MNRAS*, 429, L89  
 Angus G. W., Diaferio A., 2009, *MNRAS*, 396, 887  
 Battaglia G. et al., 2006, *A&A*, 459, 423  
 Battaglia G., Tolstoy E., Helmi A., Irwin M., Parisi P., Hill V., Jablonka P., 2011, *MNRAS*, 411, 1013  
 Benítez-Llambay A., Navarro J. F., Abadi M. G., Gottlöber S., Yepes G., Hoffman Y., Steinmetz M., 2016, *MNRAS*, 456, 1185  
 Binney J., Tremaine S., 1987, *Galactic Dynamics*. Princeton Univ. Press, Princeton, NJ  
 Boldrini P., Mohayaee R., Silk J., 2019, *MNRAS*, 485, 2546  
 Buonanno R., Corsi C. E., Zinn R., Pecci F. F., Hardy E., Suntzeff N. B., 1998, *ApJ*, 501, L33  
 Chandrasekhar S., 1943, *ApJ*, 97, 255  
 Cole D. R., Dehnen W., Read J. I., Wilkinson M. I., 2012, *MNRAS*, 426, 601  
 Di Cintio A., Brook C. B., Macciò A. V., Stinson G. S., Knebe A., Dutton A. A., Wadsley J., 2014, *MNRAS*, 437, 415  
 Elbert O. D., Bullock J. S., Garrison-Kimmel S., Rocha M., Oñorbe J., Peter A. H. G., 2015, *MNRAS*, 453, 29  
 Fattahi A., Navarro J. F., Sawala T., Frenk C. S., Sales L. V., Oman K., Schaller M., Wang J., 2016, preprint ([arXiv:1607.06479](https://arxiv.org/abs/1607.06479))  
 Genina A., Frenk C. S., Benítez-Llambay A. r., Cole S., Navarro J. F., Oman K. A., Fattahi A., 2019, *MNRAS*, 488, 2312  
 Gilmore G., Wilkinson M. I., Wyse R. F. G., Kleya J. T., Koch A., Evans N. W., Grebel E. K., 2007, *ApJ*, 663, 948  
 Goerdt T., Moore B., Read J. I., Stadel J., Zemp M., 2006, *MNRAS*, 368, 1073  
 Goerdt T., Moore B., Read J. I., Stadel J., 2010, *ApJ*, 725, 1707  
 Hendricks B., Boeche C., Johnson C. I., Frank M. J., Koch A., Mateo M., Bailey J. I., 2016, *A&A*, 585, A86  
 Hernandez X., Gilmore G., 1998, *MNRAS*, 297, 517  
 Inoue S., 2009, *MNRAS*, 397, 709  
 Irwin M., Hatzidimitriou D., 1995, *MNRAS*, 277, 1354  
 Kaplinghat M., Tulin S., Yu H.-B., 2016, *Phys. Rev. Lett.*, 116, 041302  
 Kaur K., Sridhar S., 2018, *ApJ*, 868, 134  
 Ludlow A. D., Bose S., Angulo R. E., Wang L., Hellwing W. A., Navarro J. F., Cole S., Frenk C. S., 2016, *MNRAS*, 460, 1214  
 Mackey A. D., Gilmore G. F., 2003, *MNRAS*, 340, 175  
 Navarro J. F., Eke V. R., Frenk C. S., 1996a, *MNRAS*, 283, L72  
 Navarro J. F., Frenk C. S., White S. D. M., 1996b, *ApJ*, 462, 563  
 Navarro J. F., Frenk C. S., White S. D. M., 1997, *ApJ*, 490, 493  
 Oh K. S., Lin D. N. C., Richer H. B., 2000, *ApJ*, 531, 727  
 Oman K. A., Navarro J. F., Sales L. V., Fattahi A., Frenk C. S., Sawala T., Schaller M., White S. D. M., 2016, *MNRAS*, 460, 3610  
 Petts J. A., Gualandris A., Read J. I., 2015, *MNRAS*, 454, 3778  
 Pontzen A., Governato F., 2012, *MNRAS*, 421, 3464  
 Pontzen A., Governato F., 2014, *Nature*, 506, 171  
 Read J. I., Goerdt T., Moore B., Pontzen A. P., Stadel J., Lake G., 2006, *MNRAS*, 373, 1451  
 Read J. I., Walker M. G., Steger P., 2019, *MNRAS*, 484, 1401  
 Rocha M., Peter A. H. G., Bullock J. S., Kaplinghat M., Garrison-Kimmel S., Oñorbe J., Moustakas L. A., 2013, *MNRAS*, 430, 81

- Sameie O., Yu H.-B., Sales L. V., Vogelsberger M., Zavala J., 2019, preprint ([arXiv:1904.07872](https://arxiv.org/abs/1904.07872))
- Spergel D. N., Steinhardt P. J., 2000, *Phys. Rev. Lett.*, 84, 3760
- Springel V., 2005, *MNRAS*, 364, 1105
- Strigari L. E., 2013, *Phys. Rep.*, 531, 1
- Strigari L. E., Frenk C. S., White S. D. M., 2010, *MNRAS*, 408, 2364
- Tolstoy E. et al., 2004, *ApJ*, 617, L119
- Tremaine S. D., 1976, *ApJ*, 203, 345
- Walker M., 2013, in Oswald T. D., Gilmore G., eds, *Planets, Stars and Stellar Systems*. Springer Science+Business Media, Dordrecht, p. 1039
- Walker M. G., Peñarrubia J., 2011, *ApJ*, 742, 20
- Walker M. G., Mateo M., Olszewski E. W., 2009, *AJ*, 137, 3100
- Wolf J., Martinez G. D., Bullock J. S., Kaplinghat M., Geha M., Muñoz R. R., Simon J. D., Avedo F. F., 2010, *MNRAS*, 406, 1220

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