

## Quasi ML estimation of dynamic panel models with endogeneity

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### ONLINE APPENDIX

#### S1. PROOF OF THEOREM 2 IN KRUINIGER, 2013 (CONSISTENCY OF FE/RE QMLE FOR A PANEL AR(1) MODEL)

We first prove consistency of the FE QMLE for  $\rho$  by verifying the conditions of Theorem 2.1 in Newey and McFadden (1994, henceforth NMcf).<sup>1</sup> The FE QMLE for  $\rho$  is based on the quasi likelihood function corresponding to the following auxiliary model

$$y_i - y_{i,1}\iota = \rho(y_{i,-1} - y_{i,1}\iota) + u_i, \quad (\text{S.1})$$

where  $-1 < \rho \leq 1$  and  $u_i = \tilde{v}_{i,1}\iota + \varepsilon_i \sim N(0, \Phi)$  with  $\tilde{v}_{i,1} = (\rho - 1)v_{i,1}$ ,  $v_{i,1} = y_{i,1} - \mu_i$  and  $\Phi = \Phi(\varphi) = \tilde{\sigma}_v^2\iota\iota' + \Psi$ , where  $\varphi$  is the vector comprising all (co-)variance parameters.

The Basic Assumptions imply that  $\text{plim}_{N \rightarrow \infty} N^{-1} \sum_{i=1}^N \varepsilon_i \varepsilon_i' = \Psi = \text{diag}(\bar{\sigma}_2^2, \dots, \bar{\sigma}_T^2)$ ,  $\text{plim}_{N \rightarrow \infty} N^{-1} \sum_{i=1}^N \tilde{v}_{i,1}^2 = \tilde{\sigma}_v^2 < \infty$  and  $\text{plim}_{N \rightarrow \infty} N^{-1} \sum_{i=1}^N \tilde{v}_{i,1} \varepsilon_{i,t} = 0$  for  $t = 2, \dots, T$ , where  $\bar{\sigma}_s^2 = \lim_{N \rightarrow \infty} N^{-1} \sum_{i=1}^N \sigma_{i,s}^2$  with  $\sigma_{i,s}^2 = E(\varepsilon_{i,s}^2)$  for  $s = 2, \dots, T$ .

The quasi log-likelihood for the conditional panel AR(1) model with FE is given by

$$l_{FE}(r, f) = -\frac{1}{2}N(T-1)\ln 2\pi - \frac{N}{2}\ln|F| - \frac{1}{2}\sum_{i=1}^N (\tilde{\Delta}y_i - r\tilde{\Delta}y_{i,-1})'F^{-1}(\tilde{\Delta}y_i - r\tilde{\Delta}y_{i,-1}), \quad (\text{S.2})$$

where  $F = F(f)$ . We can express  $\tilde{\Delta}y_{i,-1}$  in terms of  $v_{i,1}$  and  $\varepsilon_i$

$$\tilde{\Delta}y_{i,-1} = Pu_i = P\iota(\rho - 1)v_{i,1} + P\varepsilon_i, \quad (\text{S.3})$$

where

$$P = P(\rho) = \begin{pmatrix} 0 & . & . & 0 & 0 & 0 \\ 1 & 0 & & & 0 & 0 \\ \rho & 1 & 0 & & & 0 \\ . & \rho & 1 & 0 & & . \\ . & . & \rho & 1 & 0 & . \\ \rho^{T-3} & . & . & \rho & 1 & 0 \end{pmatrix}. \quad (\text{S.4})$$

Next, we can rewrite (S.2) using that

$$\begin{aligned} \tilde{\Delta}y_i - r\tilde{\Delta}y_{i,-1} &= (\rho - r)\tilde{\Delta}y_{i,-1} + u_i = ((\rho - r)P + I)u_i = \\ &((\rho - r)P\iota + \iota)(\rho - 1)v_{i,1} + ((\rho - r)P + I)\varepsilon_i. \end{aligned} \quad (\text{S.5})$$

<sup>1</sup>The proof assumes that the parameter space is compact. If one does not like this assumption, then one can prove consistency of the QMLE for  $\rho$  by using a version of Theorem 2.7 in NMcf after a reparametrization so that the likelihood function is concave, see NMcf for details.

It follows from (S.5), our assumptions and the Markov Law of Large Numbers that  $N^{-1}l_{FE}(r, f)$  converges uniformly in probability to a nonrandom function,  $\bar{l}_{FE}(r, f)$  say.  $N^{-1}\bar{l}_{FE}(r, f)$  would converge uniformly in probability to exactly the same function  $\bar{l}_{FE}(r, f)$  if the  $\tilde{v}_{i,1} = (\rho - 1)v_{i,1}$  and  $\varepsilon_i$  were i.i.d. and normal with  $E(\tilde{v}_{i,1}^2) = \tilde{\sigma}_v^2$ ,  $E(\tilde{v}_{i,1}\varepsilon_i) = 0$ , and  $E(\varepsilon_i\varepsilon_i') = \Psi$ . Therefore to verify the other conditions of Theorem 2.1 of NMcf we can use Theorem 2.5 of NMcf.

To show that  $\rho$  and  $\varphi$  are uniquely identified when  $-1 < \rho \leq 1$  we proceed as follows: Let  $g(\tilde{\Delta}y_i|\rho, \varphi)$  be the normal pdf of  $\tilde{\Delta}y_i$ .

From (S.5) we obtain  $(\tilde{\Delta}y_i - r\tilde{\Delta}y_{i,-1})'F^{-1}(\tilde{\Delta}y_i - r\tilde{\Delta}y_{i,-1}) = u_i'((\rho - r)P + I)'F^{-1}((\rho - r)P + I)u_i$ .

Note that  $F = [f_{i,j}] = F(f)$  is PD as long as  $\bar{s}_t^2 > 0$  for some  $t \geq 2$ , and that  $((\rho - r)P + I)$  is nonsingular for any  $-1 < \rho, r \leq 1$ . Hence  $((\rho - r)P + I)'F^{-1}((\rho - r)P + I)$  is PD.

Furthermore, given the specific structure of  $F$  and  $P$ ,  $((\rho - r)P + I)'F^{-1}((\rho - r)P + I) = \Phi^{-1}$  if and only if  $f = \varphi$  and  $r = \rho$ , unless  $\rho = 1$  and either  $T < 4$  or  $\bar{\sigma}_s^2/\bar{\sigma}_{s-1}^2 = \bar{\sigma}_3^2/\bar{\sigma}_2^2 \neq 1$  for all  $s \in \{3, \dots, T-1\}$ . If  $\rho = 1$ ,  $T \geq 4$  and  $\bar{\sigma}_s^2/\bar{\sigma}_{s-1}^2 = \bar{\sigma}_3^2/\bar{\sigma}_2^2 \neq 1$  for all  $s \in \{3, \dots, T-1\}$ , both  $((\rho - r)P + I)'F^{-1}((\rho - r)P + I) = \Phi^{-1}$  and  $\det(F) = \det(\Phi)$  not only hold for  $r = 1$ ,  $f_{i,i} = \bar{\sigma}_{i+1}^2$ ,  $i = 1, \dots, T-1$ ,  $f_{i,j} = 0$ ,  $i \neq j$  but also for  $r = \bar{\sigma}_3^2/\bar{\sigma}_2^2$ ,  $f_{i,i} = \bar{\sigma}_2^2 - \bar{\sigma}_3^2 + \bar{\sigma}_2^2(\bar{\sigma}_3^2/\bar{\sigma}_2^2)^i$ ,  $i = 1, \dots, T-2$ ,  $f_{T-1,T-1} = \bar{\sigma}_2^2 - \bar{\sigma}_3^2 + \bar{\sigma}_2^2(\bar{\sigma}_3^2/\bar{\sigma}_2^2)^{T-2}((\bar{\sigma}_3^2/\bar{\sigma}_2^2) - 1) + \bar{\sigma}_T^2$ ,  $f_{i,j} = \bar{\sigma}_2^2 - \bar{\sigma}_3^2$ ,  $i \neq j$ . If  $\rho = 1$  and  $T = 3$ ,  $((\rho - r)P + I)'F^{-1}((\rho - r)P + I) = \Phi^{-1}$  and  $\det(F) = \det(\Phi)$  hold for any value of  $r$  such that  $-1 < r \leq 1$ ,  $f_{1,1} = \bar{\sigma}_2^2$ ,  $f_{2,2} = \bar{\sigma}_2^2(1 + \bar{\sigma}_3^2/\bar{\sigma}_2^2 + r^2 - 2r)$  and  $f_{1,2} = \bar{\sigma}_2^2(1 - r)$ .

It follows that  $\Pr(\tilde{\Delta}y_i : g(\tilde{\Delta}y_i|r, F) \neq g(\tilde{\Delta}y_i|\rho, \Phi)) = 1$  if  $r \neq \rho$  and  $f \neq \varphi$  unless  $\rho = 1$  and either  $T < 4$  or  $\bar{\sigma}_s^2/\bar{\sigma}_{s-1}^2 = \bar{\sigma}_3^2/\bar{\sigma}_2^2 \neq 1$  for all  $s \in \{3, \dots, T-1\}$ . To establish unique identification in the last case, we can use similar arguments as at the end of the proof of theorem 1 in Kruiniger (2013wp). We conclude that  $\rho$  and  $\varphi$  are uniquely identified if and only if  $T \geq 3$  when  $|\rho| < 1$ , or  $T \geq 4$  when  $\rho = 1$ .

$E\left(\sup_{\theta \in \Theta} |\ln g(\tilde{\Delta}y_i|r, f)|\right) < \infty$  by standard arguments where  $\theta = (r \ f')'$  and  $\Theta$  is the parameter space. Finally  $\bar{l}_{FE}(r, f)$  is continuous in  $r$  and  $f$ . We conclude that the FE QMLE for  $\rho$  is consistent if and only if  $T \geq 3$  when  $|\rho| < 1$ , or  $T \geq 4$  when  $\rho = 1$ .

Consistency of the RE QMLE for  $\rho$  in the conditional panel AR(1) model can be proved along similar lines. However, instead of (S.5), one should use  $y_i - ry_{i,-1} - p(1-r)y_{i,1}\iota = ((\rho - r)P + I)u_i + (\rho - r)P\iota(1 - \rho)\pi y_{i,1} + (\pi(1 - \rho) - p(1 - r))y_{i,1}\iota$ .  $\square$

PROOF OF LEMMA 2.1

## S2. SOME OF THE MONTE CARLO RESULTS.

**Table 1.** Estimators and t-tests for  $\rho$  and  $\beta$ ; Design I-A; 2500 replications.

estimator	param.	N=100, T=5		N=500, T=5		N=100, T=10		N=500, T=10	
		bias	MSE	bias	MSE	bias	MSE	bias	MSE
AB	$\rho$	-0.034	0.909	-0.008	0.153	-0.039	0.344	-0.009	0.052
	$\beta$	-0.015	0.570	-0.003	0.106	-0.026	0.241	-0.005	0.041
ABAS	$\rho$	-0.022	0.888	-0.005	0.140	-0.035	0.363	-0.003	0.042
	$\beta$	-0.012	0.629	-0.003	0.106	-0.023	0.288	-0.002	0.040
OPAS	$\rho$	-0.027	0.851	-0.006	0.141	-0.037	0.345	-0.005	0.043
	$\beta$	-0.014	0.595	-0.003	0.106	-0.024	0.254	-0.003	0.039
OPASH	$\rho$	-0.028	0.913	-0.006	0.144	-0.039	0.347	-0.006	0.044
	$\beta$	-0.016	0.632	-0.005	0.109	-0.024	0.249	-0.004	0.042
SYS	$\rho$	0.006	0.479	0.000	0.106	0.014	0.123	0.002	0.028
	$\beta$	0.008	0.425	0.003	0.084	0.012	0.120	0.002	0.028
INFLIML	$\rho$	-0.017	0.369	-0.004	0.080	-0.003	0.111	-0.001	0.025
	$\beta$	-0.009	0.295	-0.003	0.059	-0.003	0.101	0.000	0.020
ABLIML	$\rho$	-0.025	0.446	-0.006	0.095	-0.012	0.130	-0.003	0.029
	$\beta$	-0.019	0.403	-0.005	0.080	-0.015	0.133	-0.003	0.026
ABASLIML	$\rho$	-0.024	0.442	-0.006	0.094	-0.011	0.132	-0.002	0.028
	$\beta$	-0.017	0.402	-0.004	0.080	-0.013	0.134	-0.001	0.025
OPASLIML	$\rho$	-0.025	0.442	-0.006	0.094	-0.011	0.131	-0.002	0.029
	$\beta$	-0.018	0.401	-0.005	0.080	-0.014	0.133	-0.002	0.025
ssLIML	$\rho$	-0.034	0.539	-0.011	0.109	-0.013	0.149	-0.001	0.029
	$\beta$	-0.026	0.495	-0.009	0.095	-0.011	0.134	-0.001	0.029
ssLIMLh	$\rho$	-0.035	0.566	-0.012	0.109	NA	NA	NA	NA
	$\beta$	-0.026	0.489	-0.009	0.096	NA	NA	NA	NA
HPTFLIML	$\rho$	-0.027	0.540	-0.033	0.229	-0.012	0.090	-0.013	0.041
	$\beta$	-0.033	0.444	-0.037	0.222	-0.012	0.080	-0.016	0.049
HPT	$\rho$	-0.080	1.078	-0.079	0.702	-0.051	0.371	-0.052	0.282
	$\beta$	-0.104	1.379	-0.102	1.114	-0.077	0.672	-0.076	0.597
FIML	$\rho$	0.005	0.622	0.001	0.123	0.002	0.162	0.000	0.028
	$\beta$	0.002	0.431	0.000	0.084	0.002	0.133	-0.000	0.024
		s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.
AB	$\rho$	0.089	0.081	0.039	0.060	0.044	0.145	0.021	0.068
	$\beta$	0.074	0.063	0.032	0.050	0.041	0.115	0.019	0.066
INFLIML	$\rho$	0.059	0.067	0.029	0.054	0.033	0.062	0.016	0.051
	$\beta$	0.054	0.052	0.025	0.054	0.031	0.054	0.014	0.044
ABLIML	$\rho$	NA	0.026	0.039	0.019	0.038	0.062	0.018	0.043
	$\beta$	NA	0.028	0.032	0.026	0.035	0.079	0.016	0.046
FIML	$\rho$	0.080	0.078	0.037	0.054	0.039	0.065	0.017	0.045
	$\beta$	0.068	0.079	0.032	0.050	0.036	0.069	0.016	0.050

actual MSE = MSE/100; s.e.: standard error; rej.f.: rejection frequency.

**Table 2.** Estimators and t-tests for  $\rho$  and  $\beta$ ; Design I-B; 2500 replications.

estimator	param.	N=100, T=5		N=500, T=5		N=100, T=10		N=500, T=10	
		bias	MSE	bias	MSE	bias	MSE	bias	MSE
AB	$\rho$	-0.070	1.979	-0.013	0.301	-0.060	0.629	-0.013	0.080
	$\beta$	-0.007	0.604	-0.001	0.102	-0.015	0.214	-0.003	0.041
ABAS	$\rho$	-0.044	1.601	-0.007	0.255	-0.054	0.630	-0.004	0.057
	$\beta$	-0.006	0.679	-0.001	0.104	-0.015	0.277	-0.001	0.043
OPAS	$\rho$	-0.054	1.618	-0.009	0.258	-0.057	0.613	-0.007	0.059
	$\beta$	-0.007	0.642	-0.001	0.104	-0.015	0.237	-0.002	0.042
OPASH	$\rho$	-0.058	1.705	-0.011	0.269	-0.057	0.621	-0.007	0.060
	$\beta$	-0.008	0.662	-0.002	0.106	-0.015	0.222	-0.003	0.043
SYS	$\rho$	-0.003	0.720	0.000	0.138	0.011	0.142	0.001	0.039
	$\beta$	0.001	0.479	0.002	0.096	0.013	0.144	0.004	0.034
INFLIML	$\rho$	-0.033	0.656	-0.009	0.139	-0.008	0.144	-0.001	0.034
	$\beta$	-0.002	0.327	-0.001	0.063	0.002	0.108	-0.001	0.021
ABLIML	$\rho$	-0.039	0.700	-0.010	0.147	-0.013	0.157	-0.002	0.036
	$\beta$	-0.012	0.437	-0.003	0.087	-0.009	0.136	-0.003	0.028
ABASLIML	$\rho$	-0.038	0.695	-0.010	0.146	-0.012	0.157	-0.001	0.036
	$\beta$	-0.009	0.444	-0.002	0.087	-0.008	0.146	-0.001	0.028
OPASLIML	$\rho$	-0.038	0.697	-0.010	0.146	-0.013	0.157	-0.002	0.036
	$\beta$	-0.010	0.438	-0.002	0.087	-0.008	0.139	-0.002	0.028
ssLIML	$\rho$	-0.058	0.932	-0.020	0.174	-0.017	0.182	-0.001	0.036
	$\beta$	-0.015	0.536	-0.007	0.106	-0.008	0.157	-0.001	0.035
ssLIMLh	$\rho$	-0.067	1.009	-0.021	0.175	NA	NA	NA	NA
	$\beta$	-0.014	0.492	-0.007	0.106	NA	NA	NA	NA
HPTFLIML	$\rho$	-0.009	1.366	-0.012	0.302	-0.013	0.203	-0.011	0.045
	$\beta$	-0.028	0.422	-0.023	0.123	-0.019	0.139	-0.020	0.056
HPT	$\rho$	0.012	3.229	-0.021	0.355	-0.016	0.252	-0.017	0.066
	$\beta$	-0.053	0.625	-0.051	0.331	-0.046	0.308	-0.046	0.224
FIML	$\rho$	0.013	1.366	0.004	0.247	-0.002	0.211	0.001	0.038
	$\beta$	0.001	0.443	0.001	0.086	0.002	0.143	-0.000	0.024
		s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.
AB	$\rho$	0.123	0.107	0.053	0.061	0.053	0.202	0.025	0.086
	$\beta$	0.075	0.066	0.032	0.046	0.043	0.075	0.020	0.055
INFLIML	$\rho$	0.076	0.093	0.038	0.078	0.038	0.071	0.019	0.065
	$\beta$	0.057	0.055	0.026	0.042	0.032	0.051	0.014	0.050
ABLIML	$\rho$	NA	0.034	0.058	0.018	0.044	0.056	0.020	0.040
	$\beta$	NA	0.026	0.033	0.034	0.037	0.068	0.017	0.048
FIML	$\rho$	0.110	0.126	0.050	0.059	0.045	0.066	0.020	0.052
	$\beta$	0.068	0.066	0.031	0.048	0.037	0.060	0.016	0.048

actual MSE = MSE/100; s.e.: standard error; rej.f.: rejection frequency.

**Table 3.** Estimators and t-tests for  $\rho$  and  $\beta$ ; Design II-A; 2500 replications.

estimator	param.	N=100, T=5		N=500, T=5		N=100, T=10		N=500, T=10	
		bias	MSE	bias	MSE	bias	MSE	bias	MSE
AB	$\rho$	-0.038	1.137	-0.008	0.184	-0.046	0.425	-0.011	0.063
	$\beta$	-0.023	0.694	-0.003	0.124	-0.031	0.285	-0.007	0.045
ABAS	$\rho$	-0.027	1.101	-0.005	0.170	-0.041	0.438	-0.004	0.051
	$\beta$	-0.018	0.735	-0.002	0.125	-0.028	0.337	-0.003	0.044
OPAS	$\rho$	-0.032	1.080	-0.006	0.170	-0.043	0.421	-0.006	0.052
	$\beta$	-0.020	0.711	-0.003	0.125	-0.030	0.300	-0.004	0.044
OPASH	$\rho$	-0.036	1.139	-0.006	0.177	-0.046	0.428	-0.007	0.054
	$\beta$	-0.020	0.740	-0.004	0.130	-0.031	0.291	-0.005	0.046
SYS	$\rho$	0.002	0.370	0.000	0.067	0.010	0.104	0.001	0.026
	$\beta$	0.008	0.342	0.000	0.061	0.011	0.100	0.001	0.025
INFLIML	$\rho$	-0.021	0.383	-0.008	0.086	-0.008	0.126	-0.001	0.024
	$\beta$	-0.012	0.320	-0.003	0.062	-0.003	0.103	-0.001	0.023
ABLIML	$\rho$	-0.032	0.475	-0.011	0.105	-0.018	0.161	-0.004	0.029
	$\beta$	-0.024	0.439	-0.006	0.087	-0.017	0.148	-0.004	0.034
ABASLIML	$\rho$	-0.030	0.464	-0.010	0.103	-0.017	0.159	-0.002	0.028
	$\beta$	-0.022	0.435	-0.005	0.086	-0.016	0.148	-0.002	0.032
OPASLIML	$\rho$	-0.031	0.468	-0.011	0.104	-0.018	0.160	-0.003	0.028
	$\beta$	-0.023	0.437	-0.006	0.087	-0.017	0.147	-0.003	0.032
ssLIML	$\rho$	-0.044	0.602	-0.015	0.130	-0.016	0.161	-0.002	0.034
	$\beta$	-0.034	0.555	-0.012	0.110	-0.013	0.158	-0.002	0.031
ssLIMLh	$\rho$	-0.049	0.635	-0.016	0.137	NA	NA	NA	NA
	$\beta$	-0.035	0.559	-0.013	0.111	NA	NA	NA	NA
HPTFLIML	$\rho$	-0.043	0.793	-0.044	0.310	-0.017	0.116	-0.023	0.079
	$\beta$	-0.045	0.598	-0.050	0.317	-0.019	0.128	-0.027	0.103
HPT	$\rho$	-0.100	1.402	-0.098	1.021	-0.065	0.542	-0.066	0.454
	$\beta$	-0.122	1.763	-0.123	1.552	-0.093	0.970	-0.093	0.872
FIML	$\rho$	0.003	0.737	0.001	0.144	0.003	0.175	0.001	0.030
	$\beta$	-0.001	0.523	0.004	0.092	0.005	0.162	0.000	0.031
		s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.
AB	$\rho$	0.096	0.090	0.042	0.054	0.047	0.163	0.023	0.077
	$\beta$	0.079	0.072	0.035	0.061	0.043	0.115	0.020	0.062
INFLIML	$\rho$	0.061	0.064	0.029	0.060	0.033	0.064	0.016	0.043
	$\beta$	0.056	0.061	0.026	0.046	0.031	0.059	0.015	0.048
ABLIML	$\rho$	NA	0.024	0.042	0.018	0.039	0.070	0.019	0.039
	$\beta$	NA	0.024	0.035	0.025	0.037	0.073	0.018	0.047
FIML	$\rho$	0.094	0.080	0.046	0.051	0.043	0.077	0.018	0.044
	$\beta$	0.079	0.089	0.040	0.046	0.040	0.076	0.017	0.054

actual MSE = MSE/100; s.e.: standard error; rej.f.: rejection frequency.

**Table 4.** Estimators and t-tests for  $\rho$  and  $\beta$ ; Design II-B; 2500 replications.

estimator	param.	N=100, T=5		N=500, T=5		N=100, T=10		N=500, T=10	
		bias	MSE	bias	MSE	bias	MSE	bias	MSE
AB	$\rho$	-0.086	2.574	-0.016	0.396	-0.070	0.789	-0.015	0.092
	$\beta$	-0.016	0.642	-0.004	0.127	-0.020	0.245	-0.005	0.047
ABAS	$\rho$	-0.050	1.913	-0.008	0.319	-0.062	0.770	-0.006	0.063
	$\beta$	-0.012	0.719	-0.003	0.128	-0.018	0.314	-0.002	0.049
OPAS	$\rho$	-0.064	1.948	-0.011	0.323	-0.066	0.759	-0.008	0.067
	$\beta$	-0.014	0.667	-0.004	0.128	-0.019	0.268	-0.003	0.048
OPASH	$\rho$	-0.068	2.167	-0.011	0.349	-0.066	0.782	-0.008	0.069
	$\beta$	-0.015	0.696	-0.002	0.131	-0.020	0.254	-0.003	0.050
SYS	$\rho$	-0.001	0.525	0.002	0.093	0.005	0.120	-0.000	0.034
	$\beta$	0.012	0.431	0.002	0.087	0.012	0.125	0.002	0.030
INFLIML	$\rho$	-0.039	0.678	-0.012	0.146	-0.013	0.152	-0.003	0.035
	$\beta$	-0.006	0.359	-0.001	0.069	-0.001	0.122	-0.001	0.024
ABLIML	$\rho$	-0.045	0.746	-0.014	0.158	-0.020	0.178	-0.004	0.039
	$\beta$	-0.018	0.496	-0.004	0.099	-0.016	0.171	-0.004	0.035
ABASLIML	$\rho$	-0.045	0.735	-0.013	0.156	-0.019	0.178	-0.003	0.038
	$\beta$	-0.014	0.496	-0.003	0.099	-0.014	0.177	-0.002	0.034
OPASLIML	$\rho$	-0.045	0.738	-0.014	0.157	-0.020	0.178	-0.004	0.038
	$\beta$	-0.016	0.492	-0.003	0.099	-0.015	0.172	-0.003	0.034
ssLIML	$\rho$	-0.077	1.190	-0.027	0.251	-0.022	0.199	-0.003	0.041
	$\beta$	-0.023	0.602	-0.010	0.123	-0.009	0.178	-0.001	0.035
ssLIMLh	$\rho$	-0.080	1.239	-0.030	0.274	NA	NA	NA	NA
	$\beta$	-0.019	0.605	-0.010	0.120	NA	NA	NA	NA
HPTFLIML	$\rho$	-0.022	1.344	-0.013	0.351	-0.016	0.212	-0.014	0.061
	$\beta$	-0.034	0.510	-0.031	0.172	-0.025	0.168	-0.027	0.098
HPT	$\rho$	0.017	3.560	-0.028	0.407	-0.030	0.258	-0.026	0.101
	$\beta$	-0.070	0.932	-0.066	0.506	-0.064	0.508	-0.064	0.427
FIML	$\rho$	0.016	1.593	0.002	0.296	0.003	0.248	-0.001	0.047
	$\beta$	0.003	0.495	0.000	0.098	0.003	0.164	0.001	0.031
		s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.
AB	$\rho$	0.137	0.116	0.060	0.066	0.056	0.232	0.027	0.084
	$\beta$	0.079	0.061	0.034	0.065	0.045	0.075	0.021	0.062
INFLIML	$\rho$	0.078	0.084	0.039	0.072	0.037	0.062	0.019	0.066
	$\beta$	0.059	0.057	0.026	0.048	0.033	0.056	0.015	0.056
ABLIML	$\rho$	NA	0.031	0.067	0.019	0.045	0.061	0.022	0.033
	$\beta$	NA	0.028	0.036	0.032	0.038	0.067	0.018	0.049
FIML	$\rho$	0.120	0.124	0.055	0.057	0.050	0.074	0.021	0.056
	$\beta$	0.071	0.073	0.033	0.056	0.041	0.068	0.017	0.057

actual MSE = MSE/100; s.e.: standard error; rej.f.: rejection frequency.

**Table 5.** Estimators and t-tests for  $\rho$  and  $\beta$ ; Design III-A; 2500 replications.

estimator	param.	N=100, T=5		N=500, T=5		N=100, T=10		N=500, T=10	
		bias	MSE	bias	MSE	bias	MSE	bias	MSE
AB	$\rho$	-0.055	1.527	-0.010	0.251	-0.051	0.498	-0.012	0.073
	$\beta$	-0.028	0.752	-0.005	0.127	-0.031	0.282	-0.007	0.048
ABAS	$\rho$	-0.027	1.170	-0.005	0.192	-0.045	0.498	-0.005	0.052
	$\beta$	-0.018	0.773	-0.003	0.124	-0.029	0.334	-0.004	0.047
OPAS	$\rho$	-0.037	1.212	-0.007	0.195	-0.048	0.486	-0.007	0.054
	$\beta$	-0.023	0.749	-0.004	0.124	-0.030	0.297	-0.005	0.047
OPASH	$\rho$	-0.042	1.300	-0.008	0.187	-0.051	0.497	-0.007	0.054
	$\beta$	-0.024	0.792	-0.006	0.124	-0.032	0.290	-0.006	0.049
SYS	$\rho$	0.000	0.536	-0.000	0.085	0.026	0.234	0.000	0.047
	$\beta$	-0.000	0.420	0.001	0.067	-0.015	0.148	-0.001	0.046
INFLIML	$\rho$	-0.001	0.639	0.000	0.136	0.002	0.162	-0.001	0.030
	$\beta$	-0.001	0.397	0.000	0.082	0.002	0.131	0.000	0.024
ABLIML	$\rho$	-0.016	0.667	-0.002	0.154	-0.010	0.169	-0.003	0.034
	$\beta$	-0.017	0.481	-0.002	0.108	-0.013	0.154	-0.003	0.030
ABASLIML	$\rho$	-0.013	0.641	-0.001	0.153	-0.009	0.169	-0.002	0.034
	$\beta$	-0.012	0.468	-0.001	0.106	-0.011	0.157	-0.001	0.030
OPASLIML	$\rho$	-0.014	0.657	-0.002	0.153	-0.009	0.169	-0.002	0.034
	$\beta$	-0.014	0.472	-0.002	0.106	-0.012	0.154	-0.002	0.030
ssLIML	$\rho$	0.012	1.116	0.002	0.194	0.002	0.226	0.001	0.040
	$\beta$	0.008	0.731	0.001	0.130	0.001	0.210	0.000	0.040
ssLIMLh	$\rho$	0.014	1.121	0.002	0.192	NA	NA	NA	NA
	$\beta$	0.009	0.733	0.001	0.128	NA	NA	NA	NA
HPTFLIML	$\rho$	-0.043	0.793	-0.044	0.310	-0.017	0.116	-0.023	0.079
	$\beta$	-0.045	0.598	-0.050	0.317	-0.019	0.128	-0.027	0.103
HPT	$\rho$	-0.100	1.402	-0.098	1.021	-0.065	0.542	-0.066	0.454
	$\beta$	-0.122	1.763	-0.123	1.552	-0.093	0.970	-0.093	0.872
FIML	$\rho$	0.003	0.806	0.002	0.188	0.003	0.190	0.001	0.033
	$\beta$	-0.001	0.520	0.000	0.141	0.002	0.166	-0.000	0.028
		s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.
AB	$\rho$	0.112	0.079	0.049	0.059	0.050	0.178	0.024	0.084
	$\beta$	0.082	0.079	0.036	0.052	0.043	0.120	0.020	0.062
INFLIML	$\rho$	0.079	0.098	0.037	0.052	0.039	0.049	0.017	0.050
	$\beta$	0.063	0.068	0.029	0.053	0.034	0.062	0.015	0.046
ABLIML	$\rho$	NA	0.098	0.043	0.073	0.043	0.077	0.018	0.070
	$\beta$	NA	0.097	0.031	0.098	0.039	0.086	0.017	0.066
FIML	$\rho$	0.088	0.087	0.040	0.053	0.042	0.067	0.019	0.048
	$\beta$	0.072	0.084	0.032	0.053	0.039	0.072	0.017	0.047

actual MSE = MSE/100; s.e.: standard error; rej.f.: rejection frequency.

**Table 6.** Estimators and t-tests for  $\rho$  and  $\beta$ ; Design III-B; 2500 replications.

estimator	param.	N=100, T=5		N=500, T=5		N=100, T=10		N=500, T=10	
		bias	MSE	bias	MSE	bias	MSE	bias	MSE
AB	$\rho$	-0.111	3.579	-0.023	0.513	-0.077	0.944	-0.018	0.114
	$\beta$	-0.017	0.644	-0.004	0.122	-0.019	0.227	-0.005	0.045
ABAS	$\rho$	-0.053	2.109	-0.008	0.366	-0.070	0.904	-0.006	0.065
	$\beta$	-0.011	0.730	-0.002	0.124	-0.017	0.306	-0.003	0.047
OPAS	$\rho$	-0.075	2.360	-0.013	0.373	-0.073	0.902	-0.010	0.071
	$\beta$	-0.014	0.677	-0.003	0.123	-0.018	0.256	-0.004	0.046
OPASH	$\rho$	-0.083	2.646	-0.013	0.393	-0.074	0.928	-0.010	0.074
	$\beta$	-0.016	0.703	-0.003	0.122	-0.018	0.239	-0.004	0.047
SYS	$\rho$	0.007	0.569	-0.001	0.098	0.016	0.156	0.001	0.034
	$\beta$	0.003	0.486	0.001	0.086	-0.003	0.121	0.001	0.029
INFLIML	$\rho$	-0.024	0.821	-0.001	0.228	-0.003	0.210	0.000	0.045
	$\beta$	-0.004	0.369	-0.001	0.074	0.000	0.118	-0.001	0.022
ABLIML	$\rho$	-0.031	0.852	-0.003	0.234	-0.009	0.217	-0.002	0.050
	$\beta$	-0.017	0.492	-0.003	0.106	-0.013	0.156	-0.004	0.032
ABASLIML	$\rho$	-0.030	0.836	-0.002	0.231	-0.009	0.217	-0.001	0.050
	$\beta$	-0.013	0.494	-0.002	0.106	-0.012	0.161	-0.002	0.033
OPASLIML	$\rho$	-0.030	0.843	-0.003	0.232	-0.009	0.217	-0.001	0.050
	$\beta$	-0.015	0.489	-0.003	0.106	-0.013	0.157	-0.002	0.032
ssLIML	$\rho$	0.007	2.225	0.007	0.483	0.002	0.316	0.001	0.048
	$\beta$	0.008	0.655	0.000	0.130	-0.000	0.219	0.001	0.043
ssLIMLh	$\rho$	-0.012	1.456	0.005	0.411	NA	NA	NA	NA
	$\beta$	0.001	0.542	0.000	0.127	NA	NA	NA	NA
HPTFLIML	$\rho$	-0.022	1.344	-0.013	0.351	-0.016	0.212	-0.014	0.061
	$\beta$	-0.034	0.510	-0.031	0.172	-0.025	0.168	-0.027	0.098
HPT	$\rho$	0.017	3.560	-0.028	0.407	-0.030	0.258	-0.026	0.101
	$\beta$	-0.070	0.932	-0.066	0.506	-0.064	0.508	-0.064	0.427
FIML	$\rho$	0.017	1.971	0.005	0.347	0.001	0.282	0.001	0.049
	$\beta$	0.001	0.523	0.001	0.103	0.003	0.167	-0.001	0.028
		s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.
AB	$\rho$	0.153	0.138	0.068	0.074	0.060	0.250	0.029	0.089
	$\beta$	0.078	0.066	0.034	0.057	0.045	0.079	0.021	0.056
INFLIML	$\rho$	0.094	0.117	0.049	0.149	0.044	0.081	0.021	0.053
	$\beta$	0.060	0.062	0.027	0.044	0.034	0.056	0.015	0.053
ABLIML	$\rho$	NA	0.057	0.057	0.046	0.048	0.069	0.021	0.065
	$\beta$	NA	0.039	0.032	0.056	0.039	0.076	0.018	0.059
FIML	$\rho$	0.125	0.139	0.059	0.066	0.049	0.073	0.022	0.053
	$\beta$	0.071	0.061	0.032	0.060	0.040	0.074	0.017	0.045

actual MSE = MSE/100; s.e.: standard error; rej.f.: rejection frequency.

**Table 7.** Estimators and t-tests for  $\rho$  and  $\beta$ ; Design IV-A; 2500 replications.

estimator	param.	N=100, T=5		N=500, T=5		N=100, T=10		N=500, T=10	
		bias	MSE	bias	MSE	bias	MSE	bias	MSE
AB	$\rho$	-0.032	0.868	-0.003	0.144	-0.033	0.278	-0.007	0.040
	$\beta$	-0.013	0.508	-0.003	0.097	-0.019	0.187	-0.004	0.035
ABAS	$\rho$	-0.021	0.860	-0.002	0.138	-0.030	0.305	-0.003	0.036
	$\beta$	-0.011	0.573	-0.003	0.101	-0.016	0.233	-0.002	0.036
OPAS	$\rho$	-0.026	0.852	-0.002	0.137	-0.032	0.288	-0.004	0.038
	$\beta$	-0.013	0.554	-0.003	0.099	-0.018	0.205	-0.003	0.036
SYS	$\rho$	0.035	0.577	0.046	0.307	0.036	0.233	0.040	0.195
	$\beta$	0.039	0.500	0.044	0.279	0.036	0.222	0.042	0.204
INFLIML	$\rho$	-0.021	0.369	-0.006	0.072	-0.010	0.100	-0.003	0.021
	$\beta$	-0.008	0.307	-0.004	0.062	-0.003	0.094	-0.001	0.018
ABLIML	$\rho$	-0.026	0.411	-0.008	0.077	-0.015	0.118	-0.004	0.022
	$\beta$	-0.015	0.377	-0.006	0.075	-0.012	0.114	-0.003	0.022
ABASLIML	$\rho$	-0.025	0.407	-0.008	0.077	-0.015	0.117	-0.004	0.022
	$\beta$	-0.013	0.378	-0.005	0.075	-0.011	0.115	-0.002	0.021
OPASLIML	$\rho$	-0.026	0.411	-0.008	0.077	-0.015	0.118	-0.004	0.022
	$\beta$	-0.015	0.379	-0.006	0.075	-0.011	0.114	-0.002	0.022
ssLIML	$\rho$	-0.035	0.514	-0.013	0.099	-0.015	0.135	-0.001	0.025
	$\beta$	-0.022	0.478	-0.010	0.088	-0.010	0.117	-0.001	0.024
ssLIMLh	$\rho$	-0.040	0.553	-0.015	0.107	NA	NA	NA	NA
	$\beta$	-0.022	0.469	-0.010	0.087	NA	NA	NA	NA
HPTFLIML	$\rho$	-0.018	0.626	-0.021	0.159	-0.015	0.140	-0.014	0.041
	$\beta$	-0.026	0.451	-0.029	0.149	-0.017	0.130	-0.018	0.050
HPT	$\rho$	-0.052	0.738	-0.049	0.353	-0.036	0.242	-0.035	0.142
	$\beta$	-0.075	0.822	-0.073	0.578	-0.054	0.384	-0.053	0.299
FIML	$\rho$	0.005	0.623	0.003	0.114	0.001	0.143	-0.001	0.027
	$\beta$	0.002	0.423	-0.000	0.072	0.001	0.117	-0.001	0.022
		s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.
AB	$\rho$	0.087	0.082	0.037	0.055	0.041	0.134	0.019	0.068
	$\beta$	0.071	0.060	0.030	0.058	0.038	0.093	0.018	0.076
SYS	$\rho$	0.067	0.108	0.030	0.333	0.032	0.239	0.018	0.619
	$\beta$	0.060	0.125	0.026	0.396	0.030	0.238	0.018	0.656
INFLIML	$\rho$	0.057	0.073	0.027	0.051	0.031	0.066	0.015	0.053
	$\beta$	0.053	0.062	0.024	0.059	0.030	0.064	0.014	0.054
ABLIML	$\rho$	NA	0.026	0.037	0.018	0.038	0.046	0.017	0.033
	$\beta$	NA	0.026	0.030	0.031	0.034	0.052	0.015	0.037
FIML	$\rho$	0.076	0.074	0.035	0.058	0.039	0.064	0.019	0.053
	$\beta$	0.063	0.072	0.030	0.058	0.036	0.054	0.018	0.064

actual MSE = MSE/100; s.e.: standard error; rej.f.: rejection frequency.

**Table 8.** Estimators and t-tests for  $\rho$  and  $\beta$ ; Design IV-B; 2500 replications.

estimator	param.	N=100, T=5		N=500, T=5		N=100, T=10		N=500, T=10	
		bias	MSE	bias	MSE	bias	MSE	bias	MSE
AB	$\rho$	-0.071	1.876	-0.011	0.288	-0.047	0.495	-0.009	0.066
	$\beta$	-0.002	0.586	-0.000	0.106	-0.006	0.165	-0.001	0.034
ABAS	$\rho$	-0.048	1.579	-0.004	0.245	-0.041	0.514	-0.002	0.053
	$\beta$	-0.003	0.677	-0.000	0.107	-0.006	0.224	-0.000	0.037
OPAS	$\rho$	-0.059	1.610	-0.007	0.247	-0.044	0.498	-0.005	0.054
	$\beta$	-0.003	0.638	-0.000	0.107	-0.006	0.189	-0.001	0.036
SYS	$\rho$	0.027	0.685	0.043	0.304	0.036	0.216	0.037	0.173
	$\beta$	0.041	0.563	0.045	0.283	0.037	0.246	0.043	0.220
INFLIML	$\rho$	-0.042	0.649	-0.015	0.127	-0.010	0.136	-0.003	0.029
	$\beta$	0.001	0.350	-0.001	0.064	-0.003	0.101	0.000	0.020
ABLIML	$\rho$	-0.045	0.663	-0.016	0.128	-0.012	0.143	-0.003	0.030
	$\beta$	-0.004	0.429	-0.002	0.082	-0.010	0.125	-0.001	0.026
ABASLIML	$\rho$	-0.046	0.662	-0.016	0.128	-0.012	0.143	-0.003	0.029
	$\beta$	-0.003	0.434	-0.001	0.083	-0.009	0.129	-0.000	0.026
OPASLIML	$\rho$	-0.046	0.664	-0.016	0.128	-0.012	0.143	-0.003	0.029
	$\beta$	-0.004	0.432	-0.001	0.082	-0.010	0.127	-0.001	0.026
ssLIML	$\rho$	-0.063	0.940	-0.027	0.201	-0.021	0.178	-0.005	0.033
	$\beta$	-0.003	0.540	-0.003	0.102	-0.004	0.142	-0.000	0.029
ssLIMLh	$\rho$	-0.067	0.966	-0.028	0.214	NA	NA	NA	NA
	$\beta$	-0.003	0.524	-0.003	0.104	NA	NA	NA	NA
HPTFLIML	$\rho$	-0.005	1.291	0.002	0.325	-0.004	0.204	-0.000	0.041
	$\beta$	-0.003	0.381	-0.005	0.074	-0.006	0.104	-0.003	0.022
HPT	$\rho$	0.033	2.921	0.005	0.420	-0.002	0.193	-0.001	0.045
	$\beta$	-0.011	0.353	-0.010	0.072	-0.010	0.093	-0.009	0.024
FIML	$\rho$	0.008	1.425	0.004	0.231	0.002	0.193	0.002	0.040
	$\beta$	0.002	0.443	-0.001	0.080	-0.002	0.123	0.001	0.024
		s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.
AB	$\rho$	0.122	0.101	0.052	0.060	0.049	0.172	0.023	0.085
	$\beta$	0.075	0.065	0.032	0.060	0.041	0.056	0.018	0.051
SYS	$\rho$	0.077	0.106	0.034	0.285	0.034	0.194	0.018	0.484
	$\beta$	0.063	0.116	0.028	0.376	0.032	0.237	0.017	0.675
INFLIML	$\rho$	0.074	0.087	0.034	0.064	0.034	0.070	0.016	0.062
	$\beta$	0.057	0.071	0.026	0.053	0.031	0.059	0.014	0.039
ABLIML	$\rho$	NA	0.039	0.047	0.027	0.042	0.038	0.019	0.032
	$\beta$	NA	0.023	0.030	0.028	0.035	0.049	0.016	0.041
FIML	$\rho$	0.114	0.120	0.049	0.058	0.043	0.063	0.019	0.054
	$\beta$	0.066	0.062	0.032	0.047	0.036	0.058	0.016	0.049

actual MSE = MSE/100; s.e.: standard error; rej.f.: rejection frequency.

**Table 9.** Estimators and t-tests for  $\rho$  and  $\beta$ ; Design II-C; 2500 replications.

estimator	param.	N=100, T=5		N=500, T=5		N=100, T=10		N=500, T=10	
		bias	MSE	bias	MSE	bias	MSE	bias	MSE
AB	$\rho$	-0.130	4.133	-0.029	0.592	-0.103	1.417	-0.026	0.156
	$\beta$	-0.018	0.618	-0.001	0.119	-0.030	0.285	-0.008	0.053
ABAS	$\rho$	-0.078	2.765	-0.019	0.485	-0.094	1.345	-0.013	0.100
	$\beta$	-0.017	0.698	-0.001	0.120	-0.028	0.353	-0.005	0.053
OPAS	$\rho$	-0.100	2.908	-0.023	0.509	-0.098	1.340	-0.017	0.110
	$\beta$	-0.018	0.638	-0.002	0.119	-0.028	0.303	-0.006	0.051
SYS	$\rho$	0.000	0.414	0.001	0.071	-0.004	0.088	0.001	0.020
	$\beta$	0.004	0.371	0.001	0.067	0.004	0.086	0.001	0.022
INFLIML	$\rho$	-0.051	0.965	-0.021	0.199	-0.023	0.180	-0.008	0.032
	$\beta$	-0.003	0.318	-0.001	0.061	0.000	0.088	-0.001	0.018
ABLIML	$\rho$	-0.060	1.100	-0.025	0.227	-0.038	0.290	-0.014	0.056
	$\beta$	-0.021	0.478	-0.004	0.097	-0.029	0.219	-0.010	0.046
ABASLIML	$\rho$	-0.058	1.089	-0.025	0.223	-0.037	0.286	-0.012	0.048
	$\beta$	-0.017	0.484	-0.003	0.095	-0.027	0.224	-0.006	0.041
OPASLIML	$\rho$	-0.058	1.087	-0.025	0.226	-0.037	0.287	-0.012	0.051
	$\beta$	-0.019	0.476	-0.004	0.096	-0.028	0.217	-0.008	0.042
ssLIML	$\rho$	-0.092	1.581	-0.036	0.339	-0.029	0.237	-0.008	0.045
	$\beta$	-0.013	0.522	-0.006	0.111	-0.006	0.157	-0.002	0.031
HPTFLIML	$\rho$	-0.034	1.479	-0.017	0.507	-0.031	0.325	-0.026	0.119
	$\beta$	-0.036	0.452	-0.031	0.164	-0.034	0.214	-0.037	0.152
HPT	$\rho$	0.037	3.735	-0.010	1.029	-0.035	0.408	-0.034	0.164
	$\beta$	-0.059	0.708	-0.056	0.382	-0.059	0.414	-0.058	0.354
FIML	$\rho$	0.011	1.985	0.006	0.448	-0.004	0.317	0.001	0.063
	$\beta$	-0.005	0.439	0.000	0.089	0.001	0.139	-0.001	0.033
		s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.
AB	$\rho$	0.168	0.136	0.072	0.067	0.063	0.378	0.031	0.121
	$\beta$	0.078	0.058	0.034	0.047	0.045	0.118	0.021	0.075
INFLIML	$\rho$	0.082	0.114	0.038	0.076	0.035	0.084	0.017	0.058
	$\beta$	0.056	0.062	0.025	0.058	0.030	0.066	0.013	0.048
ABLIML	$\rho$	NA	0.050	0.064	0.038	0.051	0.082	0.027	0.042
	$\beta$	NA	0.019	0.033	0.027	0.039	0.088	0.020	0.073
FIML	$\rho$	0.134	0.170	0.078	0.075	0.050	0.120	0.025	0.067
	$\beta$	0.074	0.070	0.047	0.053	0.037	0.080	0.019	0.063

actual MSE = MSE/100; s.e.: standard error; rej.f.: rejection frequency.

**Table 10.** Estimators and t-tests for  $\rho$  and  $\beta$ ; Design III-C; 2500 replications.

estimator	param.	N=100, T=5		N=500, T=5		N=100, T=10		N=500, T=10	
		bias	MSE	bias	MSE	bias	MSE	bias	MSE
AB	$\rho$	-0.165	5.774	-0.036	0.780	-0.119	1.864	-0.029	0.213
	$\beta$	-0.020	0.629	-0.004	0.124	-0.031	0.291	-0.008	0.049
ABAS	$\rho$	-0.086	2.946	-0.016	0.527	-0.109	1.675	-0.010	0.105
	$\beta$	-0.016	0.709	-0.003	0.125	-0.031	0.355	-0.006	0.050
OPAS	$\rho$	-0.119	3.472	-0.023	0.569	-0.114	1.726	-0.017	0.125
	$\beta$	-0.019	0.651	-0.004	0.124	-0.031	0.309	-0.007	0.048
SYS	$\rho$	0.003	0.452	0.001	0.076	0.006	0.102	0.000	0.021
	$\beta$	-0.000	0.396	0.000	0.073	-0.007	0.093	-0.000	0.021
INFLIML	$\rho$	-0.050	0.986	-0.018	0.245	-0.009	0.172	-0.002	0.048
	$\beta$	-0.001	0.344	-0.001	0.065	-0.002	0.089	0.000	0.019
ABLIML	$\rho$	-0.057	1.090	-0.022	0.269	-0.025	0.267	-0.010	0.063
	$\beta$	-0.019	0.500	-0.006	0.103	-0.033	0.223	-0.009	0.047
ABASLIML	$\rho$	-0.059	1.072	-0.022	0.264	-0.025	0.261	-0.007	0.058
	$\beta$	-0.015	0.502	-0.004	0.101	-0.032	0.229	-0.005	0.041
OPASLIML	$\rho$	-0.058	1.075	-0.022	0.267	-0.025	0.263	-0.007	0.060
	$\beta$	-0.017	0.492	-0.005	0.101	-0.032	0.222	-0.007	0.042
ssLIML	$\rho$	0.025	2.542	0.013	0.945	0.018	0.699	0.003	0.085
	$\beta$	0.014	0.551	0.003	0.120	0.001	0.185	0.001	0.037
HPTFLIML	$\rho$	-0.045	1.400	-0.022	0.519	-0.026	0.304	-0.028	0.119
	$\beta$	-0.033	0.464	-0.033	0.169	-0.038	0.215	-0.036	0.151
HPT	$\rho$	0.038	6.471	-0.004	1.034	-0.034	0.385	-0.033	0.170
	$\beta$	-0.056	0.829	-0.058	0.382	-0.059	0.432	-0.059	0.349
FIML	$\rho$	0.009	2.239	0.003	0.531	0.003	0.319	0.000	0.075
	$\beta$	-0.005	0.455	-0.001	0.095	-0.002	0.145	0.001	0.035
		s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.
AB	$\rho$	0.187	0.156	0.083	0.078	0.067	0.417	0.034	0.149
	$\beta$	0.078	0.062	0.034	0.058	0.045	0.124	0.021	0.058
INFLIML	$\rho$	0.086	0.130	0.050	0.150	0.042	0.098	0.022	0.112
	$\beta$	0.057	0.054	0.026	0.044	0.030	0.058	0.014	0.050
ABLIML	$\rho$	NA	0.066	0.061	0.055	0.049	0.086	0.025	0.078
	$\beta$	NA	0.024	0.032	0.046	0.038	0.114	0.019	0.096
FIML	$\rho$	0.121	0.195	0.073	0.115	0.054	0.101	0.028	0.078
	$\beta$	0.066	0.069	0.032	0.051	0.038	0.076	0.019	0.073

actual MSE = MSE/100; s.e.: standard error; rej.f.: rejection frequency.

**Table 11.** Estimators and t-tests for  $\rho$  and  $\beta$ ; Design II-D; 2500 replications.

estimator	param.	N=100, T=5		N=500, T=5		N=100, T=10		N=500, T=10	
		bias	MSE	bias	MSE	bias	MSE	bias	MSE
AB	$\rho$	-0.363	20.52	-0.093	2.507	-0.177	3.787	-0.050	0.413
	$\beta$	-0.003	0.528	-0.001	0.108	-0.016	0.204	-0.004	0.042
ABAS	$\rho$	-0.150	4.846	-0.040	0.952	-0.164	3.412	-0.024	0.170
	$\beta$	-0.004	0.668	-0.001	0.116	-0.016	0.260	-0.003	0.044
OPAS	$\rho$	-0.245	9.388	-0.061	1.247	-0.169	3.524	-0.034	0.229
	$\beta$	-0.003	0.579	-0.001	0.111	-0.016	0.221	-0.004	0.042
SYS	$\rho$	0.000	0.286	0.000	0.050	-0.003	0.058	-0.000	0.014
	$\beta$	0.002	0.293	0.001	0.051	0.001	0.055	0.001	0.013
INFLIML	$\rho$	-0.076	1.569	-0.040	0.460	-0.029	0.231	-0.013	0.053
	$\beta$	0.001	0.310	0.000	0.059	0.002	0.072	0.000	0.014
ABLIML	$\rho$	-0.079	1.617	-0.043	0.493	-0.038	0.304	-0.018	0.077
	$\beta$	-0.012	0.433	-0.004	0.093	-0.025	0.177	-0.009	0.043
ABASLIML	$\rho$	-0.081	1.615	-0.043	0.484	-0.037	0.299	-0.017	0.068
	$\beta$	-0.007	0.472	-0.002	0.093	-0.024	0.188	-0.005	0.038
OPASLIML	$\rho$	-0.079	1.595	-0.043	0.486	-0.037	0.301	-0.017	0.071
	$\beta$	-0.009	0.446	-0.003	0.092	-0.024	0.179	-0.007	0.038
ssLIML	$\rho$	-0.114	2.169	-0.059	0.623	-0.036	0.283	-0.012	0.053
	$\beta$	0.006	0.472	0.004	0.103	0.007	0.139	0.003	0.027
HPTFLIML	$\rho$	-0.069	1.711	-0.034	0.548	-0.027	0.319	-0.014	0.099
	$\beta$	-0.009	0.319	-0.009	0.067	-0.014	0.096	-0.013	0.031
HPT	$\rho$	0.033	2.792	0.012	1.112	0.006	1.020	-0.009	0.181
	$\beta$	-0.014	1.163	-0.016	0.086	-0.022	0.205	-0.018	0.043
FIML	$\rho$	0.022	2.465	0.012	0.893	0.002	0.412	-0.000	0.114
	$\beta$	-0.004	0.425	-0.002	0.086	-0.005	0.134	-0.001	0.031
		s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.	s.e.	rej.f.
AB	$\rho$	0.270	0.306	0.129	0.138	0.080	0.614	0.041	0.211
	$\beta$	0.074	0.049	0.033	0.056	0.042	0.076	0.020	0.066
INFLIML	$\rho$	0.083	0.149	0.048	0.103	0.038	0.107	0.019	0.075
	$\beta$	0.054	0.054	0.024	0.036	0.027	0.048	0.012	0.053
ABLIML	$\rho$	NA	0.091	0.066	0.076	0.056	0.067	0.033	0.040
	$\beta$	NA	0.010	0.031	0.018	0.037	0.050	0.019	0.039
FIML	$\rho$	0.113	0.291	0.084	0.186	0.057	0.171	0.033	0.109
	$\beta$	0.064	0.069	0.035	0.040	0.037	0.083	0.019	0.071

actual MSE = MSE/100; s.e.: standard error; rej.f.: rejection frequency.