

Hawking-Moss transition with a black hole seed

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ABSTRACT: We extend the concept of Hawking-Moss, or up-tunnelling, transitions in the early universe to include black hole seeds. The black hole greatly enhances the decay amplitude, however, order to have physically consistent results, we need to impose a new condition (automatically satisfied for the original Hawking-Moss instanton) that the cosmological horizon area should not increase during tunnelling. We motivate this conjecture physically in two ways. First, we look at the energetics of the process, using the formalism of extended black hole thermodynamics; secondly, we extend the stochastic inflationary formalism to include primordial black holes. Both of these methods give a physical substantiation of our conjecture.

KEYWORDS: Black Holes, Solitons Monopoles and Instantons, Models of Quantum Gravity

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1 Introduction

It is inevitable that quantum processes played an important role in the very earliest stages of our universe. Possibly the most remarkable process of all is the decay of the quantum vacuum state. This is because the change in vacuum state can change the curvature of spacetime, and then vacuum decay becomes a fully non-perturbative quantum gravitational phenomenon. If we can provide a plausible understanding of vacuum decay in this context, then we may learn a little about quantum gravity.

Some time ago [1], Hawking and Moss noticed that the simple picture of vacuum decay in a system with a scalar field coupled to gravity produces strange results when the field has a very flat potential. The usual picture of a bubble of true vacuum nucleating inside false vacuum with a distinct bubble wall [2] no longer holds: as the potential becomes flatter, the bubble wall becomes thicker, and the field on either side of the wall becomes closer to either side of the maximum of the potential barrier, until the solution interpolating between each side of the potential maximum can no longer exist. Instead, it appears that the field ‘jumps’ to the top of the potential barrier and hence the universe undergoes a uniform jump in spacetime geometry in which everything, up to and including the cosmological horizon, is affected. In a previous paper, we looked at the way vacuum decay occurs in the presence of a primordial black hole as the uniform field limit was approached [3]. In this paper, we consider vacuum decay in the presence of a primordial black hole and a uniform scalar field. We are led to make a new proposal, that vacuum decay is only permitted when the cosmological horizon does not grow in size.

The set-up is as follows: consider a scalar field theory on a curved background geometry described by Einstein gravity, with a standard Lagrangian for the scalar field ϕ ,

$$\mathcal{L}_\phi = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi). \quad (1.1)$$

Since the specifics of the potential are not relevant to our discussion, let us take a toy potential for V of the form shown in figure 1. In particular, V has a false vacuum located

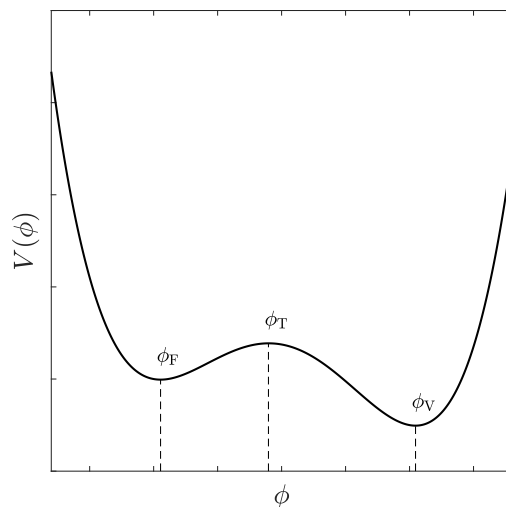


Figure 1. An example potential containing a true and a false vacuum, located at ϕ_V and ϕ_F respectively, separated by a barrier peaked at ϕ_T .

at $\phi = \phi_F$ and the true vacuum is at $\phi = \phi_V$. The top of the potential barrier separating these two regions is at $\phi = \phi_T$. If the potential is everywhere positive, then the stable, stationary solutions result in a de Sitter space.

If the field is initially in the false vacuum, there is a non-zero probability to tunnel through the barrier to the true vacuum. These are the bounce solutions of Coleman-de Luccia (CDL) [2, 4, 5], which describe the nucleation of a bubble of true vacuum within a sea of false vacuum, i.e. a first order phase transition. The bubble subsequently expands under the influence of gravitation, converting the false vacuum to true [2], at least within a Hubble volume.

The type of transition we are interested in here occurs when the field undergoes a fluctuation from the false vacuum up to the top of the potential barrier. This is known as the Hawking-Moss (HM) instanton [1], and involves an entire horizon volume of spacetime simultaneously undergoing a transition to a new state. In situations where the CDL bounce does not exist, the only non-perturbative way the system can evolve into the true vacuum is via a HM bounce.

In the formal theory of vacuum decay [4, 5], the bounce solution asymptotes to the false vacuum state as the imaginary time becomes infinite. However, once gravity is included, *all* of the bounce solutions with positive false vacuum energy violate this condition due to the *finite* volume of Euclidean de Sitter space, but none violate this condition more so than the HM instanton. Consequently, various attempts have been made to understand the role of this instanton better. An early proposal was that the instanton solution represents the ‘creation of the universe from nothing’ [6, 7]. If this were true, then the instanton should play some role in the quantum wave function of the universe, and indeed the HM instanton gives the leading saddle-point contribution to the Hartle-Hawking wave function [8].

The HM instanton also plays a role in a stochastic picture of vacuum decay. A particular feature of de Sitter space is that the large scale average of light fields, like the inflaton, satisfy a stochastic equation [9, 10]. It is therefore possible to evaluate the vacuum decay rate using stochastic techniques, and these reveal that the vacuum decay rate depends on the HM instanton in the WKB limit [11, 12].

In yet another picture, the HM bounce can be interpreted as contributing to the thermal ensemble of states at the Hawking temperature of de Sitter space [13]. Motivated by this thermodynamical picture, it is important to examine the HM transition in the presence of a primordial black hole, which has its own additional thermodynamic profile. Indeed, it has been shown [14–17] that the tunnelling rate for CDL bubbles is increased if a black hole is present. Thus, a natural question to ask is how the HM instanton picture and the stochastic formalism are altered in the presence of a black hole.

In this paper we answer this question for the case of a primordial black hole in a single Hubble volume of the inflationary universe. In section 2 we generalise the HM instanton to include a black hole, and comment on how this impacts on the instanton action. We discover that in order for the non-perturbative description to remain well-defined we need an additional constraint on the instanton. We therefore make the following conjecture:

Cosmological Area Conjecture. *In an up-tunnelling transition, the cosmological horizon area can never increase.*

Once we impose this constraint, the parameter space and instanton actions are remarkably reminiscent of the black hole bubbles of [14]. In the following two sections we turn to the physical explanation of our conjecture: in section 3 we consider the thermodynamical implications of the tunnelling transition, computing the internal energy of the false and HM states. It turns out that the internal energy inside the cosmological horizon is directly related to the horizon area, thus can only increase if energy is being pumped in from beyond the horizon. This would correspond to an unnatural and artificially tuned set-up, so we conclude that an un-triggered decay cannot increase horizon area. In section 4 we explore an alternate physical motivation, generalising the stochastic inflationary picture to include black holes. Using results from the analysis of slow roll inflation with black holes [18–20], we are again led to the conclusion that the area of the cosmological horizon cannot increase. We conclude in section 5, discussing possible extensions of our analysis.

Planck units are used throughout: $c = \hbar = k_B = G = 1$.

2 Hawking-Moss instanton with a black hole seed

The HM instanton represents a simultaneous up-tunnelling event from a false vacuum ϕ_F , to the top of a potential barrier ϕ_T . A natural way to generalise this picture is to include a seed primordial black hole in the false vacuum, and to allow a remnant black hole at the top of the potential. Typically, the masses of the black holes will be different, and this in turn will lead to a richer set of possibilities for the tunnelling process.

Consider a HM tunnelling event from the false vacuum at ϕ_F up to the top of the potential barrier ϕ_T , where the initial and final configurations contain a black hole. Assuming

positive vacuum energy density $V(\phi) \neq 0$ for both states, the initial and final configurations are described by the Schwarzschild-de Sitter (SdS) solution,

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega^2, \quad f = 1 - \frac{2m}{r} - \frac{r^2}{\ell^2}, \quad (2.1)$$

where the radius of curvature ℓ is given by,

$$\ell = \sqrt{\frac{3}{8\pi V(\phi)}}. \quad (2.2)$$

Note that since we are interested in up-tunnelling, we always have $\ell_F > \ell_T$. In these coordinates, the range for r goes from the black hole horizon r_h to the cosmological horizon r_c , i.e. $r \in [r_h, r_c]$, where $f(r_{c,h}) = 0$, and the roots can be expressed as:

$$r_c = \frac{2}{\sqrt{3}}\ell \cos\left(\frac{\pi}{3} - b\right), \quad r_h = \frac{2}{\sqrt{3}}\ell \cos\left(\frac{\pi}{3} + b\right), \quad b = \frac{1}{3} \cos^{-1}\left(\frac{3\sqrt{3}m}{\ell}\right). \quad (2.3)$$

The two horizons coincide at the Nariai mass m_N ,

$$m_N = \frac{\ell}{3\sqrt{3}}, \quad (2.4)$$

which places an upper bound on the mass parameter, $m \in [0, m_N]$.

The tunnelling rate from the false vacuum to the top of the potential has the form $\Gamma \approx Ae^{-B}$, where we focus on the tunnelling exponent B rather than the pre-factor A . We follow Coleman and de Luccia in assuming that the tunnelling exponent is related to the change in Euclidean action I ,

$$B = I_T - I_F. \quad (2.5)$$

As we stated in the introduction, there is some evidence in support of this result from quantum cosmology and from stochastic inflation. In SdS, the action is totally determined by the areas of the horizons \mathcal{A}_h and \mathcal{A}_c [14],

$$I = -\frac{1}{4}(\mathcal{A}_c + \mathcal{A}_h). \quad (2.6)$$

Since each horizon is associated with an entropy $\mathcal{S} = \mathcal{A}/4$, the tunnelling rate is related to the change in total entropy $\Delta\mathcal{S}$ by the Boltzmann formula $\Gamma = Ae^{\Delta\mathcal{S}}$. This links the tunnelling process to gravitational thermodynamics, and provides further support for the validity of the tunnelling formula.

The area of an horizon is $\mathcal{A} = 4\pi r^2$, so that using (2.3) the tunnelling exponent is,

$$B = \pi \left[\frac{4}{3}(\ell_F^2 - \ell_T^2) - \frac{2}{3}\ell_F^2 \cos(2b_F) + \frac{2}{3}\ell_T^2 \cos(2b_T) \right]. \quad (2.7)$$

Since ℓ_F and ℓ_T are fixed by the form of the potential V , we can consider the tunnelling exponent as a function of the seed and remnant masses, $B = B(m_F, m_T)$. The tunnelling

exponent at the extremes of the mass ranges are,

$$\begin{aligned}
B_{\text{HM}} &\equiv B(0, 0) = \pi (\ell_{\text{F}}^2 - \ell_{\text{T}}^2), \\
B(0, m_{\text{NT}}) &= \pi \left(\ell_{\text{F}}^2 - \frac{2}{3} \ell_{\text{T}}^2 \right), \\
B(m_{\text{NF}}, 0) &= \pi \left(\frac{2}{3} \ell_{\text{F}}^2 - \ell_{\text{T}}^2 \right), \\
B(m_{\text{NF}}, m_{\text{NT}}) &= \pi \left(\frac{2}{3} \ell_{\text{F}}^2 - \frac{2}{3} \ell_{\text{T}}^2 \right),
\end{aligned} \tag{2.8}$$

where the HM bounce is recovered for vanishing seed and remnant masses, and we note that the Nariai limits for the false vacuum and potential top are distinct, since $\ell_{\text{F}} \neq \ell_{\text{T}}$.

For a black hole seed of a given mass, the remnant mass can lie anywhere in the range $[0, m_{\text{NT}}]$. The most probable tunnelling event will therefore be the one with the smallest value of B . However, from (2.8) we see that if $\sqrt{\frac{2}{3}}\ell_{\text{F}} < \ell_{\text{T}} (< \ell_{\text{F}})$, it is possible for $B(m_{\text{F}}, 0)$ to become negative for masses close to the Nariai limit. Negativity of an instanton action (or indeed the action dropping below one in Planck units) indicates a breakdown of the semi-classical description underlying the calculation. We therefore need an additional constraint on the tunnelling process that prevents this catastrophe.

Our conjecture (that we motivate in the subsequent sections) is to impose that the area of the cosmological horizon should never increase during a transition,

$$\Delta \mathcal{A}_{\text{c}} = \mathcal{A}_{\text{cT}} - \mathcal{A}_{\text{cF}} \leq 0. \tag{2.9}$$

This is consistent with the idea that the instanton represents a thermal fluctuation, because the condition implies that the fluctuation can be contained entirely inside the original cosmological horizon. A fluctuation that was larger than the event horizon could not arise in a causal process. Once we impose this constraint, we find that there is a natural cut-off in parameter space that keeps the instanton solutions in the range consistent with the semi-classical approximation.

The parameter space of instantons is illustrated in figure 2, where the ratio of the *Black-Hole-Hawking-Moss* (BHHM) instanton action to the pure HM action is plotted as a function of the seed primordial black hole mass, m_{F} . As expected, for each seed mass there is a range of remnant masses with the action increasing as the remnant mass increases. The blue curves in the plot show how the action varies with seed mass, m_{F} , for a given remnant mass, m_{T} . As the seed mass increases, the action decreases until we reach the red curve boundary. This is the equal area curve, where the area of the cosmological horizon is the same for initial and final states. Note that in [3], we had imposed this as a constraint on the BHHM instantons for convenience. Above the red curve, all instantons have $\Delta \mathcal{A}_{\text{c}} < 0$, hence are allowed, but have higher action than the equal area curve, so are suppressed. Below the red curve, the cosmological horizon area would increase, which we argue is unphysical. Thus, the condition $\Delta \mathcal{A}_{\text{c}} \leq 0$ provides a lower bound on the allowed region of the parameter space, and is pleasingly familiar from the black hole bubbles of [14]. The remaining bounds in the plot are fixed by recalling that the allowed masses in each

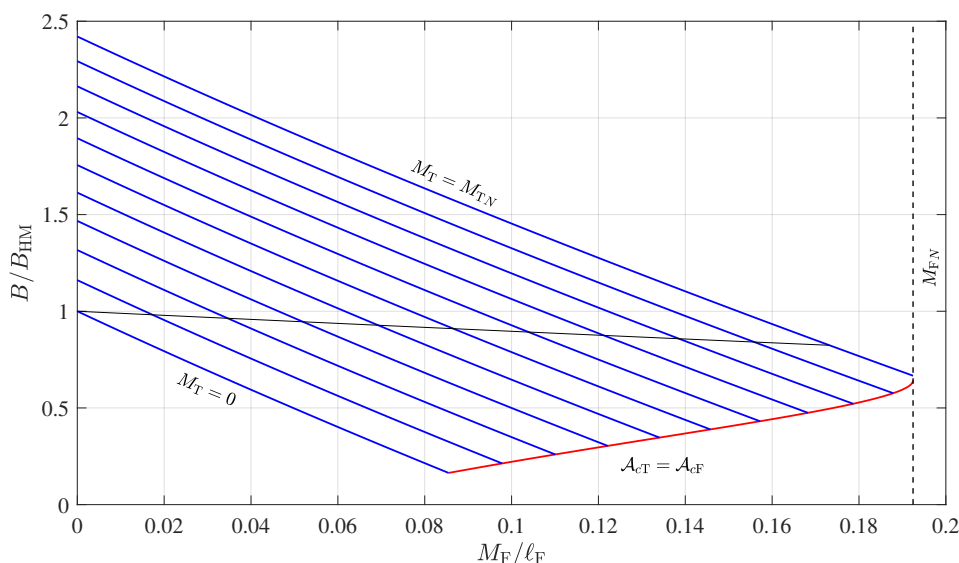


Figure 2. Dependence of the tunnelling exponent B on the seed mass m_F for $l_T/l_F = 0.9$. The blue lines correspond to different values of the remnant mass m_T , starting at zero and increasing in steps of $0.1m_{NT}$ up to the Nariai limit. The area of the cosmological horizon is conserved along the red curve and decreases above it. Above the broken black line, the remnant black hole has a larger mass parameter than the seed.

of the SdS spacetimes are bounded by the appropriate Nariai mass, $m \in [0, m_N]$. The maximal tunnelling rate occurs at the point where B is minimal, $m_F = m_C$, where the cosmological horizon areas are identical and the remnant mass is zero:

$$m_C = \frac{\ell_F(\ell_F^2 - \ell_T^2)}{2\ell_F^2}. \quad (2.10)$$

Simple analytic formulae are available in the small barrier approximation $\ell_F \approx l_T$. This approximation is equivalent to asserting that the height of the barrier relative to the false vacuum be small compared to its absolute value, i.e. $V(\phi_T) - V(\phi_F) \ll V(\phi_T)$. In this case, the maximal rate is obtained for the critical seed mass (2.10), $m_C \approx \ell_F - \ell_T$. The black hole horizon reduces approximately to the Schwarzschild value $r_{hF} \approx 2m_C$, and the value of B is given by,

$$B_C \approx 4\pi m_C^2 \quad \Rightarrow \quad \left(\frac{B}{B_{HM}} \right)_C \approx \frac{2(\ell_F - \ell_T)}{\ell_T}. \quad (2.11)$$

Before moving on to examine the physics of our conjecture, note that the line $m_T = m_{NT}$ in figure 2 does not close up with the equal area curve at $m_F = m_{NF}$. This is because the cosmological horizons in the Nariai limit are $r_{cF,T} = \ell_{F,T}/\sqrt{3}$, which clearly do not coincide for $\ell_F \neq \ell_T$.

3 Thermodynamics of the Hawking-Moss process

In the tunnelling scenario, we have provided additional motivation for our result on the probability of decay as Boltzmann suppression of an entropy-lowering transition. We now seek to explore further physical explanations for our results. Note that the BHHM transition between black hole spacetimes with differing cosmological constants before and after the transition suggests that we explore the *extended* black hole thermodynamical description [21–24], in which the cosmological ‘constant’ determines a thermodynamic pressure $P = -\Lambda/(8\pi)$. Of course, if P is to be truly dynamical, then Λ cannot just be a constant term in the gravitational Lagrangian, rather, as we have here, the vacuum energy is determined by the expectation value of a scalar field, thus can obviously be allowed to vary.

The thermodynamic pressure becomes a thermodynamic charge in the First Law,

$$\delta M = T\delta S + \mathcal{V}\delta P + \dots \quad (3.1)$$

with an associated potential — the thermodynamic volume \mathcal{V} — which can be computed for each of the horizons. This comes with the caveat that, although thermodynamical relationships exist for the individual horizons, the temperatures of the black hole and cosmological horizon are unequal and the total system cannot be in thermal equilibrium. Interestingly, as pointed out in [21], the black hole mass parameter, m , that is conventionally associated with the internal energy of the black hole in the original formulation of black hole thermodynamics, in this extended formulation leads to a variable M that has the interpretation of enthalpy, H , due to the first law above containing a $+\mathcal{V}\delta P$ term, rather than the $-P\delta\mathcal{V}$ term associated to “ dU ”. Although the extended thermodynamics of black holes is more conventionally explored in anti-de Sitter space, where the negative Λ gives rise to a positive pressure, the extended thermodynamics of black holes in *de Sitter* space can equally well be considered, and was explored in [23], (see also [19, 20]) with a Smarr relation and First Law (3.1) being derived.

Now let us summarise the picture for the SdS spacetime. Computing the thermodynamic parameters locally at each horizon yields

$$M = m, \quad S = \pi r_{h,c}^2, \quad T = \frac{1}{r_{h,c}} \left(1 - 3 \frac{r_{h,c}^2}{\ell^2} \right), \quad \mathcal{V} = \frac{4\pi}{3} r_{h,c}^3, \quad (3.2)$$

however, notice that this definition of the temperature yields a negative sign, and sometimes the modulus is taken. We will retain the signs here however for consistency of the expressions that follow. It proves useful to repackage these expressions in a ‘chemical’ form, following [22, 25] that uses only thermodynamic charges:

$$M = \sqrt{\frac{S}{4\pi}} \left(1 + \frac{8PS}{3} \right), \quad T = \frac{1}{4\sqrt{\pi S}} (1 + 8PS), \quad \mathcal{V} = \frac{4}{3} \sqrt{\frac{S^3}{\pi}}. \quad (3.3)$$

Let us now consider the internal energy bounded by the cosmological horizon; we can think of this as the total energy in the observable de Sitter universe. According to the thermodynamic expressions, this is

$$U = M - P\mathcal{V} = \sqrt{\frac{S}{4\pi}} \left(1 + \frac{8PS}{3} \right) - \frac{4P}{3} \sqrt{\frac{S^3}{\pi}} = \sqrt{\frac{S}{4\pi}}, \quad (3.4)$$

thus, the total internal energy of the SdS spacetime is determined by the entropy of the cosmological horizon.

During a decay, the only way we can imagine the internal energy of the spacetime to increase is if there is an influx of energy from beyond the cosmological horizon. This would therefore not represent a spontaneous transition between vacua, but would be more analogous to a stimulated decay (and one would also have to take account of this input in any computation of a decay amplitude). However, it is natural to imagine that energy can be dissipated beyond the horizon, or that the decay gives an energy neutral budget. We therefore posit that the internal energy of the spacetime must not increase in any decay, hence $\delta S_c \leq 0$. This provides a natural constraint on the space of HM instantons. As we see from figure 2, for a given seed mass, the preferred HM instanton either has no remnant black hole, or has a remnant, but conserves the internal energy of the observable de Sitter universe.

4 Stochastic tunnelling in the presence of a black hole

We now explore a very different approach, based on stochastic inflation, to support our premise that the cosmological horizon area decreases for the HM type of tunnelling process. We start from de Sitter space to review some of the basic premises, and then modify the stochastic formalism to include a population of primordial black holes.

In the stochastic inflationary formalism, the inflaton field is averaged over large spatial scales in a spatially flat universe to produce a ‘coarse grained’ effective cosmological model [9, 10]. The effective field $\phi(t)$ evolves by a stochastic equation

$$3H\partial_t\phi = -\partial_\phi V + \xi, \quad (4.1)$$

where ξ is a gaussian random function that arises from the effects of small-scale quantum fluctuations. The noise correlation function obtained from quantum field theory can be approximated by a local expression with diffusion coefficient D ,

$$\langle \xi(t)\xi(t') \rangle = 2D\delta(t-t') = \frac{9H^5}{4\pi^2}\delta(t-t'). \quad (4.2)$$

If the field is released in the false vacuum ϕ_F , and there is a potential barrier with the top at ϕ_T , then the stochastic source in eq. (4.1) pushes the field across the top of the barrier. The probability to remain inside the barrier falls, and therefore the false vacuum decays. The decay constant Γ is given by a general formula [12, 26]

$$\Gamma = \frac{1}{2\pi\gamma} (V''(\phi_F)V''(\phi_T))^{1/2} e^{-\gamma(V(\phi_T)-V(\phi_F))/D}, \quad (4.3)$$

where γ is the effective friction: in our case $\gamma = H$. This decay constant is of the form Ae^{-B} , with

$$B = \frac{8\pi^2\delta V}{3H^4}, \quad (4.4)$$

exactly as we have for the HM instanton when $\delta V = V(\phi_T) - V(\phi_F) \ll V$. Note that the tunnelling result only holds as long as $B \gg 1$. If the potential is very flat, the field

does a random walk and reaches ϕ_T on a timescale of ϕ_T^2/H^3 that does not depend on the barrier height.

We have a stochastic picture of the HM transition, but our concern is how the stochastic decay affects the cosmological horizon area, $\mathcal{A} = 4\pi/H^2$. According to the stochastic inflationary formalism [9, 10], the back reaction of the field on the metric implies that the Hubble expansion rate, $H(t)$, varies over large scales according to the Friedmann equation,

$$3H^2 = 8\pi V. \quad (4.5)$$

Starting with the field in the false vacuum, a change in potential δV induces a change in the horizon area $\delta\mathcal{A}$,

$$\delta\mathcal{A} = -\frac{8\pi\mathcal{A}}{3H^2}\delta V. \quad (4.6)$$

Therefore, stochastic evolution to the top of the potential barrier, which we have argued corresponds to the HM instanton transition, causes a decrease in horizon area.

To extend these ideas to black holes in de Sitter space, we need a notion of the slow roll equation with a black hole, together with a time and radially dependent counterpart to the Friedmann equation. Fortunately, this problem was addressed for a single black hole with a slowly evolving scalar field in a sequence of papers [18–20]. Physically, a black hole with a slowly evolving scalar field will be very close to a SdS spacetime, therefore the solution is expressed as a perturbation of SdS in time. This will not necessarily yield a solution for arbitrarily long timescales, but will give a good approximate solution in the same sense that slow roll inflation gives a good approximation to the inflationary universe.

The first step is to identify a “time” coordinate in the SdS spacetime that will asymptote cosmological time beyond the cosmological horizon. This is done by identifying the direction in which the scalar field rolls. The challenge is that this coordinate must be regular at each of the black hole and cosmological horizon radii, r_h and r_c respectively. This was identified in [18–20], and interpolates between the local advanced Eddington time v at the black hole horizon, and retarded time U at the cosmological horizon. The time coordinate takes the form $T = t + h(r)$, (although any rescaling of this combination by a constant factor will also work).

In [19, 20], it was shown that, provided the standard slow roll relations [27] for the potential V are satisfied, then ϕ approximately solves a modified slow roll equation:

$$3\gamma\frac{d\phi}{dT} = -\partial_\phi V, \quad (4.7)$$

where

$$\gamma = \frac{r_c^2 + r_h^2}{r_c^3 - r_h^3}. \quad (4.8)$$

As pointed out in [19], γ has the nice thermodynamical interpretation as being, up to a factor, the ratio of the total entropy divided by the thermodynamic volume of the intra-horizon SdS system.

In order to obtain a stochastic system, we divide space into cells, and average as in stochastic inflation, but now include one black hole in each cell. Small scale quantum

fluctuations will cause the field to evolve stochastically, and we replace eq. (4.7) with

$$3\gamma \frac{d\phi}{dT} = -\partial_\phi V + \xi. \quad (4.9)$$

By analogy with eq. (4.1), we expect the noise correlation function to be of the form

$$\langle \xi(T)\xi(T') \rangle = 2D\delta(T - T'). \quad (4.10)$$

However, the particular form of the noise correlation function does not affect the argument which follows.

We are particularly interested in how the scalar field back-reacts on the geometry, specifically, the area of the horizons. In [19], the evolution of the horizons was analysed, and to leading order it was found that

$$\delta\mathcal{A}_i = -\frac{8\pi\mathcal{A}_i}{3\gamma|\kappa_i|}\delta V, \quad (4.11)$$

where κ_i is the surface gravity of the horizon in question, explicitly:

$$\kappa_h = \frac{(r_c - r_h)(2r_h + r_c)}{2r_h(r_h^2 + r_c^2 + r_h r_c)}, \quad \kappa_c = \frac{(r_h - r_c)(2r_c + r_h)}{2r_c(r_h^2 + r_c^2 + r_h r_c)}. \quad (4.12)$$

We see therefore that (4.11) implies that under stochastic evolution from an initial false vacuum ϕ_F to $\phi_T > \phi_F$, with $\delta V > 0$, the horizon areas *decrease*. This confirms our general proposal that the cosmological horizon shrinks during the up-tunnelling type of vacuum decay.

An interesting corollary is that since the same qualitative behaviour of horizon area occurs at *each* horizon, the black hole violates the area theorem during HM vacuum decay. This could not happen for a purely classical process, and confirms the quantum nature of vacuum decay. The decoherence process of quantum fluctuations leads to entropy production by which the generalized second law may be satisfied [29].

5 Conclusion

We have seen that the Hawking-Moss, or up-tunnelling, types of transition extend naturally to vacuum decays seeded by black holes, as long as we impose a condition that the nucleation event can be contained within the original cosmological event horizon, i.e. the cosmological horizon does not increase in area. The vacuum decay rate is always enhanced by the black hole seed. The mass of the remnant black hole after vacuum decay can be zero, and the cosmological horizon shrinks, or the remnant mass is non-zero and the cosmological horizon stays the same size. Which of these outcomes occurs depends on the value of the seed black hole mass.

The theory of stochastic inflation was extended to include primordial black holes in section 4. In stochastic inflation, the Hawking-Moss instanton gives the leading order approximation for calculating the probability flux across the potential barrier. The new theory implies that both the black hole and cosmological horizon areas decrease during

up-tunnelling events. On the other hand, the stochastic picture allows less freedom in the choice of remnant mass than does the vacuum tunnelling picture.

It would be interesting to generalise these results to rotating black holes, as has been explored for black hole bubbles in [30]. It might seem that the outcomes would be similar, however there are several important technical differences. Positivity of the tunnelling action was ensured here by imposing the thermodynamic constraint of decreasing internal energy, alternately, the argument from stochastic inflation that the horizon area not increase. For rotating black holes, the action of the instanton, related to the free energy, now contains a $\beta\Omega J$ term [31–33], dependent on the angular momentum and a potentially arbitrary periodicity of Euclidean time. Further, the scalar field in the Kerr-de Sitter background will now have superradiant modes [34–36], that will likely have a stronger effect on the system than any putative tunnelling decay process. We plan to study this system further.

The aim of this paper has been to push the theory of vacuum decay to its limits, and yet we find nothing unreasonable in the results. It would be of interest to use the phenomena discussed here to test the scope of theories of quantum gravity. As for actual applications to our universe, vacuum decay during inflation can take place when there is a secondary, ‘spectator’ field present with a suitable false vacuum state. Since ‘flat’ potentials are a common feature of most inflationary models of the early universe, the Hawking-Moss, or up-tunnelling seems the most likely type of transition in this situation.

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