Higgs decay to fermion pairs at NLO in SMEFT

Jonathan M. Cullen and Benjamin D. Pecjak

Institute for Particle Physics Phenomenology, Durham University,
Durham DH1 3LE, U.K.

E-mail: jonathan.m.cullen@durham.ac.uk, ben.pecjak@durham.ac.uk

Abstract: The calculation of next-to-leading order (NLO) perturbative corrections at fixed operator dimension in Standard Model Effective Field Theory (SMEFT) has been a topic of much recent interest. In this paper we obtain the NLO corrections from dimension-6 operators to the Higgs boson decays $h \to f\bar{f}$, where the fermions $f \in \{\mu, \tau, c\}$. This extends previous results for $h \to b\bar{b}$ to all phenomenologically relevant Higgs boson decays into fermions, and provides the basis for future precision analyses of these decays within effective field theory. We point out the benefits of studying ratios of decay rates into different fermions in SMEFT, the most surprising of which is enhanced sensitivity to anomalous $h\gamma\gamma$ and hgg couplings induced by flavor-universal SMEFT operators, especially in scenarios where flavor-dependent Wilson coefficients are constrained by Minimal Flavor Violation.

Contents

1 Introduction

Higgs decay into a fermion anti-fermion pair ($h \to f\bar{f}$) is a benchmark process in the Standard Model (SM). At present, the LHC has measured Higgs decays into b-quarks [1, 2] and τ-leptons [3, 4], both with an $\mathcal{O}(10\%)$ precision, and put limits on decays into muons [5, 6] and charm quarks [7, 8]. At lepton colliders, decays into b-quarks, c-quarks, and τ-leptons should reach percent-level precision, with somewhat worse performance for decays into muons [9–13].

As measurements become more precise, $h \to f\bar{f}$ decays will allow the measurement of the SM Yukawa interactions and place strong constraints on non-standard Higgs couplings. Assuming that any potential new physics (NP) contributions come only from particles with masses much heavier than the electroweak (EW) scale, Standard Model Effective Theory (SMEFT) offers a model-independent, systematically improvable quantum-field theoretical framework for analyses of Higgs decays beyond the SM. One possible improvement, which has received much interest of late, is to include next-to-leading order (NLO) perturbative corrections at a fixed dimension in the operator expansion [14–41].

In [42], we built upon the results of [43, 44] to calculate the full set of NLO corrections from dimension-6 SMEFT operators to the decay rate $h \to b\bar{b}$. On the phenomenological side, those results provide a baseline for future precision studies of that decay mode using

effective field theory. From the perspective of SMEFT development, that paper laid out a hybrid renormalization scheme appropriate for combining EW and QCD corrections. The main idea, outlined in section 2, was to use the "FJ tadpole scheme" [45] within SMEFT to define gauge-independent $\overline{\text{MS}}$ renormalized light fermion masses and QED/QCD gauge couplings, while all other masses are renormalized on-shell. Expressing the decay rate in terms of the corresponding $\overline{\text{MS}}$ parameters defined in a five-flavor version of QCD×QED requires one to add on to the decay rate virtual corrections involving the top quark and heavy electroweak bosons characteristic of on-shell renormalization through terms involving perturbatively calculable decoupling constants. The benefit of this procedure is that a set of large logarithms of the ratio m_f/m_H appearing in QCD×QED corrections is resummed into the definitions of the $\overline{\text{MS}}$ parameters, while spuriously large tadpole corrections from top-quark loops cancel from the decay rate, as in a purely on-shell scheme.

The goal of the present work is to apply that framework to obtain the NLO dimension-6 SMEFT corrections to Higgs decays into charm quarks, τ leptons, and muons, thus extending the results of [42] to cover the full spectrum of phenomenologically relevant $h \to f\bar{f}$ decays. While these cases are conceptually similar to $h \to b\bar{b}$, the explicit Feynman diagrams and dimension-6 Wilson coefficient entering the calculation differ, necessitating us to calculate from scratch the large majority of EW corrections. After setting some notation for mass-basis Wilson coefficients, also in Minimal Flavor Violation (MFV), in section 3, we give a small subset of the analytical results in section 4, accompanied by illustrative numerical studies in section 5. The complete analytical results are rather lengthy and included in electronic form with the electronic submission of this article. In addition to our analysis of individual decay channels, we emphasize in section 6 the advantages of studying ratios of decay modes in SMEFT. In particular, since a large number of Wilson coefficients drop out of ratios, particularly if MFV is imposed, ratios of decay rates may allow for stronger constraints on Wilson coefficients than decay rates alone. We summarize our findings in section 7, and in appendix A we list results for the decoupling constants used in the renormalization procedure.

2 Calculational set-up

In this section we outline the procedure used to obtain the NLO corrections to the decay rates $h \to f\bar{f}$, with h the Higgs boson and f a fermion, in dimension-6 SMEFT. The results depend on a set of input parameters and the renormalization scheme in which they are defined. A main outcome of the NLO SMEFT calculation of $h \to b\bar{b}$ decays in [42] was the development of a hybrid renormalization scheme that minimizes the impact of potentially large higher-order corrections that appear when naively combining QCD and EW corrections. It uses as input the following parameters:

$$m_{EW}, \overline{m}_f, \overline{\alpha}, \alpha_s, C_i, V_{ij} \,. \tag{2.1}$$

Some comments on the input parameters and notation are in order:

- The massive electroweak bosons and top quark are considered as heavy particles with masses denoted collectively by $m_{EW} \in \{m_W, m_Z, m_H, m_t\}$. These are renormalized in the on-shell scheme.

- The light fermion masses $\overline{m}_f \equiv \overline{m}_f(\mu)$, with f any fermion other than the top quark, are renormalized in the $\overline{\text{MS}}$ scheme in a 5-flavor version of QED×QCD, where the top quark is integrated out. The same is true of the electromagnetic fine-structure and strong coupling constants, $\overline{\alpha} = \overline{e}^2/(4\pi) \equiv \overline{\alpha}(\mu)$ and $\alpha_s = g_s^2/(4\pi) \equiv \alpha_s(\mu)$. In all cases $\overline{m}_f \ll m_{EW}$, and to simplify results we work to first non-vanishing order in this "small-mass limit".[1] The decay rates are finite in this limit, with the exception of double logarithms in the ratio $\overline{m}_f/m_H$ appearing in NLO SMEFT corrections proportional to effective $h\gamma\gamma$ and hgg vertices, see section 4.1. We have checked that corrections to the small-mass limit are typically at or below the percent level, depending on the particular Wilson coefficient.

  In [42] the alternative notation $\overline{m}_f^\ell$ and $\overline{\alpha}^\ell$ was used for $\overline{\text{MS}}$ parameters renormalized in 5-flavor QED×QCD, in order to make explicit the difference between the corresponding parameters in the full SM, where heavy particles also contribute to the running. We omit the superscript ℓ in the present work, with the understanding that the running of $\overline{\text{MS}}$ parameters is driven exclusively by light particles. Effects of heavy particles are integrated out and added on to the decay rate perturbatively through terms involving decoupling constants as discussed in appendix A.

- The $C_i \equiv C_i(\mu)$ are Wilson coefficients of the dimension-6 SMEFT operators $Q_i \equiv Q_i(\mu)$, for which we use the Warsaw basis [46] in table 3. The Wilson coefficients are renormalized in the $\overline{\text{MS}}$ scheme, with all SM particles contributing to the running. We use the notation where the dimension-6 piece of the SMEFT Lagrangian is $\mathcal{L}^{(6)} = \sum C_i Q_i$, so the C_i have mass dimension minus 2.

- As standard in EW calculations, we use the numerical approximation where the CKM matrix $V_{ij} = \delta_{ij}$.

The decay rate up to NLO in perturbation theory is written as

$$\Gamma(h \to f\bar{f}) \equiv \Gamma_f = \Gamma_f^{(0)} + \Gamma_f^{(1)}, \tag{2.2}$$

where the superscripts (0) and (1) refer to the LO and NLO contribution in perturbation theory. We split these contributions into an SM part and a piece containing exactly one Wilson coefficient of a dimension-6 operator as

$$\Gamma_f^{(0)} = \Gamma_f^{(4,0)} + \Gamma_f^{(6,0)},$$
$$\Gamma_f^{(1)} = \Gamma_f^{(4,1)} + \Gamma_f^{(6,1)}, \tag{2.3}$$

---

[1]Decay into first-generation fermions is not considered and we set their masses to zero throughout the paper.

so that the double superscripts (i, j) refer to the dimension-i contribution at j-th order in perturbation theory. It is useful to subdivide the NLO corrections into two pieces as

$$\Gamma_f^{(i,1)} = \Gamma_{f,(g,\gamma)}^{(i,1)} + \Gamma_{f,\text{weak}}^{(i,1)}. \qquad (2.4)$$

The $\Gamma_{f,(g,\gamma)}^{(i,1)}$ contain 1-loop diagrams involving at least one photon or gluon propagator, in addition to contributions from the three-body processes $h \to f\bar{f}(g,\gamma)$, whereas $\Gamma_{f,\text{weak}}^{(i,1)}$ contains the remaining NLO corrections.

The procedure for obtaining $\Gamma_f^{(1)}$ in the hybrid renormalization scheme described above was set forth in [42], and applied to the case of $h \to b\bar{b}$.[2] It included a careful treatment of tadpoles in the FJ-tadpole scheme, and the calculation of decoupling constants relating $\overline{\text{MS}}$-renormalized quantities in 5-flavor QCD×QED to the corresponding quantities in the full SM. The benefit of this renormalization scheme is threefold. First, it treats contributions from light-particle loops, which are sensitive both to m_{EW} and $\overline{m}_f$, in the $\overline{\text{MS}}$ scheme, thus resumming a set of corrections of the form $\ln \overline{m}_f/m_H$ that appear in $\Gamma_{f,(g,\gamma)}^{(i,1)}$ through the use of the running mass $\overline{m}_f$. Second, contributions from heavy-particle loops depending on m_{EW} appear in $\Gamma_{f,\text{weak}}^{(i,1)}$ through decoupling constants for $\overline{m}_f$ and $\overline{\alpha}$; these pieces are effectively calculated in the on-shell scheme, where tadpoles cancel between different terms in the decay rate, so enhanced EW corrections scaling as $m_t^4/m_H^2\overline{v}^2$ (where $\overline{v}$ is defined in eq. (4.1) below) due to these tadpoles are absent. Finally, the running of the parameters $\overline{m}_f$ and $\overline{\alpha}$ does not depend on the SMEFT Wilson coefficients C_i to leading order in $\overline{m}_f/m_{EW}$, so these are easily related to $\overline{\text{MS}}$-renormalized parameters extracted from low-energy experiments and LEP.

It is worth mentioning that the main benefits of our renormalization scheme (cancellation of heavy-particle tadpole contributions and resummation of small-mass logarithms through the use of running light-fermion masses) would be retained in the "α_{G_μ}" scheme, where one uses as inputs $\{G_\mu, M_W, M_Z\}$ in the EW sector by trading $\overline{\alpha}$ for a function of G_μ, M_W, M_Z, and C_i, with G_μ the Fermi constant. Within SMEFT, the α_{G_μ} scheme has been implemented in widely used tools for tree-level calculations such as [48]. Moreover, partial results needed for NLO applications have been given in [19], which however makes certain flavor assumptions and neglects tadpoles as appropriate in a purely on-shell scheme. General comments about the relative merits of the different schemes within the SM can be found, for instance, in [49], but no systematic study exists in SMEFT. One potential benefit is that NLO corrections scaling as m_t^2/M_W^2 in the large-m_t limit are often smaller in the α_{G_μ} scheme. We have explored this issue by adapting the results of [42, 43] to compare the large-m_t limit corrections to $h \to f\bar{f}$ in the two schemes, and found that in most cases the corrections from SMEFT Wilson coefficients common to the two cases are indeed somewhat milder in the α_{G_μ} scheme. On the other hand, the number of Wilson coefficients appearing in the decay rate is larger in the α_{G_μ} scheme, due to the appearance of four-fermion operators unrelated to Higgs decays. Given that a full implementation of the α_{G_μ} scheme for $h \to f\bar{f}$ at NLO would require to generalize the calculation of [19], and

---

[2]The SM result $\Gamma_f^{(4,1)}$ was first calculated in on-shell scheme for all parameters in [47].

that the large-m_t corrections are not anomalously large in either scheme, we do not pursue this issue further. An important observation is that the scheme dependence largely drops out when studying ratios of decay rates into different fermion species as in section 6.

There are no conceptual complications in extending the methods for $h \to b\bar{b}$ to other cases such as $h \to c\bar{c}$ or $h \to \tau\bar{\tau}$. Indeed, the NLO results for $\Gamma^{(i,1)}_{f,(g,\gamma)}$, as well as the contributions of four-fermion operators to $\Gamma^{(i,1)}_{f,\text{weak}}$ can be obtained straightforwardly from [42–44]. We give these partial NLO results in analytic form in section 4. However, the same is not true of $\Gamma^{(i,1)}_{f,\text{weak}}$. These weak corrections receive flavor-dependent contributions from a large set of one-loop diagrams entering mass renormalization, the decoupling constants, and the bare one-loop matrix elements, which we must calculate from scratch. To this end, we have altered the in-house code developed to automate the one-loop $h \to b\bar{b}$ calculation [42]. It implements the SMEFT Lagrangian in the mass basis, including ghosts, into `FeynRules` [50], and then uses `FeynArts` [51] and `FormCalc` [52] to generate and calculate the one-loop diagrams. `Package-X` [53] has also been used for analytic cross-checks of certain loop integrals.

The full NLO results are rather lengthy, and are included in computer files in the arXiv submission of this work. They involve contributions from 40 different dimension-6 operators for $h \to \tau\bar{\tau}$, 40 for $h \to \mu\bar{\mu}$, and 47 for $h \to c\bar{c}$. Electronic results for the decoupling constants are given as well, although these are compact enough to list in appendix A. We have performed the standard consistency checks on our results. In particular, we calculate the weak corrections in both Feynman and unitary gauge, check the cancellation of UV and IR poles in the dimensional regulator, and make sure that the decay rate is independent of unphysical renormalization scales up to NLO.

3 Wilson coefficients and MFV

The dimension-6 SMEFT operators in table 3 are defined in the weak basis, but the physical decay rates and on-shell renormalization conditions are defined in the mass basis. Moreover, with the exception of the top quark, Higgs couplings to fermions in the SM are suppressed by small and hierarchical Yukawa couplings, a feature not inherited by generic SMEFT interactions. In order to avoid pushing the UV scale of the effective theory to values far above the TeV scale to avoid flavor constraints, one often considers the SMEFT Wilson coefficients to be constrained by MFV, and we ourselves will study this scenario at the level of decay-rate ratios in section 6. The purpose of this section is to briefly outline both of these issues, and to set up the notation required to implement them in our results.

We first explain our treatment of Wilson coefficients in the mass basis. As an example, consider the term in the Lagrangian involving the operator Q_{uH} defined in the weak basis in table 3. The explicit form of this term, which we denote by $\mathcal{L}_{uH}$, is

$$\mathcal{L}_{uH} = C^w_{uH_{rs}}(H^\dagger H)(\bar{q}^w_r u^w_s \widetilde{H}). \tag{3.1}$$

The superscript w indicates that the Wilson coefficient and fermion fields are defined in the weak basis, and the subscripts r, s are flavor indices. The mass-basis fermion fields are

related to the weak-basis fields via the unitary transformations

$$u_R^w = U_{u_R} u_R^m \, ,$$
$$q_L^w = U_{u_L} \begin{pmatrix} u_L^m \\ V d_L^m \end{pmatrix} \, . \tag{3.2}$$

Using that the CKM matrix $V_{ij} \equiv (U_{u_L}^\dagger U_{d_L})_{ij} \approx \delta_{ij}$, the Lagrangian contribution can be written as

$$\mathcal{L}_{uH} = \underset{rs}{C_{uH}^m} (H^\dagger H)(\bar{q}_r^m u_s^m \widetilde{H}) \, , \tag{3.3}$$

where $q^m = (u_L^m, d_L^m)^T$ is a doublet of mass-basis fields and the mass-basis Wilson coefficient is defined as

$$\underset{rs}{C_{uH}^m} \equiv [U_{u_L}^{-1}]_{ri} \underset{ij}{C_{uH}^w} [U_{u_R}]_{js} \, . \tag{3.4}$$

This pattern holds in general: the SMEFT Lagrangian in terms of mass-basis fermion fields and Wilson coefficients is obtained by interpreting the fermion fields in the list of operators table 3 to be in the mass basis, and multiplying it with a corresponding mass-basis Wilson coefficient, which can be related to the weak-basis one through rotations such as eq. (3.4).[3]

In quoting analytic and numerical results in the coming sections, we always work with mass-basis quantities, and for simplicity drop the superscripts m on the fields and Wilson coefficients. Moreover, within our approximations, $h \to f\bar{f}$ decay is sensitive to a group of generation-diagonal operators involving right-handed fermion fields. For these, we use the shorthand notation where e.g. $C_{cH} \equiv \underset{22}{C_{uH}}$, thus allowing us to suppress flavor indices. In fact, the only Wilson coefficients appearing in our calculation which require explicit flavor indices are the class-7 quantities $C_{Hl}^{(1,3)}$ and $C_{Hq}^{(1,3)}$, in addition to the coefficients of the class-8 four-fermion operators.

We next turn to our implementation of MFV. A pedagogical discussion of MFV can be found in, for instance, [55]. Its implementation in SMEFT was discussed in detail in [56]. In short, imposing MFV in SMEFT constrains the flavor indices of the Wilson coefficients to be carried by certain combinations of Yukawa matrices. Upon rotation to the mass basis, Yukawa couplings are converted to powers of the fermion masses, which for light fermions f can be expanded in the small-mass limit $\overline{m}_f \ll m_{EW}$.

As an explicit example we consider the class-5 Wilson coefficients, starting with C_{uH}. MFV implies that the weak-basis coefficient takes the form [56]

$$\underset{rs}{C_{uH}^w} = \left[Y_u G_{uH} \left(Y_d Y_d^\dagger, Y_u Y_u^\dagger \right) \right]_{rs} \, , \tag{3.5}$$

where the function G_{uH} is regular in the limit that its arguments go to zero, but otherwise arbitrary. In the approximation where the CKM matrix is the unit matrix, the MFV

---

[3]The definition of mass-basis Wilson coefficients beyond the approximation $V_{ij} \approx \delta_{ij}$ is not unique. For one possible choice see [54].

scaling for the mass basis coefficient is obtained by making the replacement $Y_u \rightarrow M_u$, where the mass-basis Yukawa matrices for $f \in \{e, u, d\}$ are given by

$$[M_f]_{ij} = \sqrt{2} \frac{[m_f]_{ij}}{\overline{v}},$$
$$[m_e] = \mathrm{diag}(0, \overline{m}_\mu, \overline{m}_\tau),$$
$$[m_u] = \mathrm{diag}(0, \overline{m}_c, m_t),$$
$$[m_d] = \mathrm{diag}(0, \overline{m}_s, \overline{m}_b), \tag{3.6}$$

with $\overline{v}$ defined in eq. (4.1). The mass-basis Yukawas are diagonal matrices and their elements vanish in the small-mass limit, with the exception of $[M_u]_{33}$ which is proportional to the top-quark mass and thus order one in that limit. Therefore, to leading order in the small-mass limit, we can write

$$\left[G_{uH}\left(Y_d Y_d^\dagger, Y_u Y_u^\dagger\right) \right]_{ks} = \delta_{ks} G_{uH}(0,0) + \delta_{k3}\delta_{s3} \sum_{k=1}^\infty \frac{y_t^{2k}}{k!} \left(\left. \frac{d^k}{(dy_t^2)^k} G_{uH}(0, y_t^2) \right|_{y_t \to 0} \right), \quad (3.7)$$

where $y_t^2 = 2m_t^2/\overline{v}^2$. It follows that the expansion of the mass-basis coefficient C_{uH} in the small-mass limit within MFV is given by

$$C_{uH}_{rs} \approx [M_u]_{rk} \left[\mathcal{C}^1_{uH}_{ks} + \mathcal{O}\left(\frac{m^2}{\overline{v}^2}\right) \right], \tag{3.8}$$

where m is any light-fermion mass and the explicit expression for $\mathcal{C}^1_{uH}$ can be read off by matching with eq. (3.7). Here and below the superscript j on the calligraphic Wilson coefficients $\mathcal{C}^j_i$ indicates that they multiply j explicit powers of mass-basis Yukawa matrices. Note that the object $\mathcal{C}^1_{uH}_{ks}$ is flavor-diagonal, but non-universal in the sense that $\mathcal{C}^1_{uH}_{11} = \mathcal{C}^1_{uH}_{22} \neq \mathcal{C}^1_{uH}_{33}$. Similar statements hold for the MFV version of C_{dH} in the small-mass limit, which can be obtained from the C_{uH} result by the replacement $u \rightarrow d$.

For the corresponding leptonic operator the MFV expression is

$$C^w_{eH}_{rs} = \left[Y_e G_{eH}\left(Y_e Y_e^\dagger\right) \right]_{rs}. \tag{3.9}$$

All elements of the mass-basis Yukawa coupling M_e vanish in the small-mass limit, so the mass-basis coefficient is given by

$$C_{eH}_{rs} \approx [M_e]_{rs} \left[\mathcal{C}^1_{eH} + \mathcal{O}\left(\frac{m^2}{\overline{v}^2}\right) \right], \tag{3.10}$$

where $\mathcal{C}^1_{eH} = G_{eH}(0)$ carries no flavor indices and is thus universal, in contrast to the quark cases.

It is a straightforward exercise to obtain analogous results for the other Wilson coefficients in MFV in the small-mass limit. For our analysis in section 6, the important point is whether, after factoring out j overall Yukawa factors, the calligraphic Wilson coefficients $\mathcal{C}^j_i$ are flavor universal (as in the case of $i = eH$), or flavor non-universal due to contributions

from top-quark Yukawas (as in the case of $i = dH, uH$). The flavor-universal cases used in this paper are

$$
\begin{aligned}
C_{eH}_{pr} &\approx [M_e]_{pr}\mathcal{C}^1_{eH}\,, & C_{Hd}_{pr} &\approx \delta_{pr}\mathcal{C}^0_{Hd}\,, \\
C_{eF}_{pr} &\approx [M_e]_{pr}\mathcal{C}^1_{eF}\,, & C_{He}_{pr} &\approx \delta_{pr}\mathcal{C}^0_{He}\,, \\
C^{(1,3)}_{H\ell}_{pr} &\approx \delta_{pr}\mathcal{C}^{(1,3),0}_{H\ell}\,, & C_{le}_{prst} &\approx \delta_{pr}\delta_{st}\mathcal{C}^0_{le}\,,
\end{aligned}
\tag{3.11}
$$

where F is any gauge field appearing in the class-6 operators. The flavor non-universal cases are

$$
\begin{aligned}
C_{uH}_{pr} &\approx [M_u]_{pr}\mathcal{C}^1_{uH}_{pr}\,, & C_{Hud}_{pr} &\approx [M_u]_{pr}[M_d]_{pr}\mathcal{C}^2_{Hud}_{pr}\,, \\
C_{dH}_{pr} &\approx [M_d]_{pr}\mathcal{C}^1_{dH}_{pr}\,, & C^{(1,8)}_{qu}_{prst} &\approx \delta_{pr}\delta_{st}\mathcal{C}^{(1,8),0}_{qu}_{prst}\,, \\
C_{uF}_{pr} &\approx [M_u]_{pr}\mathcal{C}^1_{uF}_{pr}\,, & C^{(1,8)}_{qd}_{prst} &\approx \delta_{st}\delta_{pr}\mathcal{C}^{(1,8),0}_{qd}_{pr}\,, \\
C_{dF}_{pr} &\approx [M_d]_{pr}\mathcal{C}^1_{dF}_{pr}\,, & C_{ledq}_{prst} &\approx [M_e]_{pr}[M_d]_{st}\mathcal{C}^2_{ledq}_{st}\,, \\
C^{(1,3)}_{Hq}_{pr} &\approx \delta_{pr}\mathcal{C}^{(1,3),0}_{Hq}_{pr}\,, & C^{(1,8)}_{quqd}_{prst} &\approx [M_u]_{pr}[M_d]_{st}\mathcal{C}^{(1,8),2}_{quqd}_{prst}\,, \\
C_{Hu}_{pr} &\approx \delta_{pr}\mathcal{C}^0_{Hu}_{pr}\,, & C^{(1,3)}_{lequ}_{prst} &\approx [M_e]_{pr}[M_u]_{st}\mathcal{C}^{(1,3),2}_{lequ}_{st}\,,
\end{aligned}
\tag{3.12}
$$

where there is no implied summation on repeated indices on the right-hand side of the approximations; this notation makes clear that the calligraphic coefficients are flavor diagonal in the pairs of indices pr and st. We note that while the Wilson coefficients $C^{(1,8)}_{qd}$, C_{ledq} and $C^{(1,3)}_{lequ}$ carry four flavor indices, their corresponding small-mass MFV expansion functions, $\mathcal{C}_i$, are a function of (and therefore only carry) two flavor indices.

It is worth mentioning that all the SMEFT coefficients can also depend on Yukawas through functions of flavor invariants such as

$$
\mathrm{Tr}\left(Y_e Y_e^\dagger\right),\ \mathrm{Tr}\left(Y_d Y_d^\dagger\right),\ \mathrm{Tr}\left(Y_u Y_u^\dagger\right)\,.
\tag{3.13}
$$

In the small-mass limit these are either constants, or functions of y_t^2. They can thus be absorbed into the definitions of the $\mathcal{C}_i^j$ above. For consistency of notation, we make explicit that Wilson coefficients in classes 1-4 depend on the above invariants and should also be expanded in the small-mass limit. Since those Wilson coefficients carry no flavor indices, this amounts to a change of notation $C_i \to \mathcal{C}_i^0$, where the superscript indicates that the small-mass limit has been taken in the flavor invariants on which the coefficients can depend.

4 Analytical results

In this section we provide a subset of analytic results for the decay rates $h \to f\bar{f}$. This also allows us to fix some notation used in the rest of the paper. For instance, in order

to express results in terms of the input parameters eq. (2.1), we introduce the derived quantities

$$\overline{v} \equiv \frac{2M_W \hat{s}_w}{\overline{e}}, \qquad \hat{c}_w^2 \equiv \frac{M_W^2}{M_Z^2}, \qquad \hat{s}_w^2 \equiv 1 - \hat{c}_w^2 \,. \tag{4.1}$$

We can then write the LO dimension-4 and dimension-6 contributions to the $h \to f\bar{f}$ decay rates defined in eq. (2.3) as

$$\Gamma_f^{(4,0)} = \frac{N_c^f m_H}{8\pi} \frac{\overline{m}_f^2}{\overline{v}^2} \,,$$
$$\Gamma_f^{(6,0)} = 2\Gamma_f^{(4,0)} \left[C_{H\Box} - \frac{C_{HD}}{4}\left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2}\right) + \frac{\hat{c}_w}{\hat{s}_w}C_{HWB} - \frac{\overline{v}}{\overline{m}_f}\frac{C_{fH}}{\sqrt{2}} \right]\overline{v}^2 \,, \tag{4.2}$$

where $N_c^f = 1$ if f is a lepton and $N_c^f = 3$ if f is a quark. To NLO in dimension-6 SMEFT, the decay rates depend only on the real parts of the dimension-6 Wilson coefficients. We leave this implicit, such that $C_i \equiv \mathrm{Re}(C_i)$ for any Wilson coefficient C_i that appears in the decay rates, not only above but in the rest of the paper. Furthermore, we work with the mass-basis Wilson coefficients discussed in section 3 (without imposing MFV) such that C_{fH} multiplies an operator that alters the hff vertex in that basis.

4.1 QED×QCD corrections

The QED×QCD corrections may be deduced from the results in [42]. The results are

$$\Gamma_{f,(g,\gamma)}^{(4,1)} = \Gamma_f^{(4,0)} \left(\frac{\delta_{f,q}C_F\alpha_s + Q_f^2\alpha}{\pi}\right)\left[\frac{17}{4} + \frac{3}{2}\ln\left(\frac{\mu^2}{m_H^2}\right)\right] \,,$$

$$\Gamma_{f,(g,\gamma)}^{(6,1)} = \Gamma_f^{(6,0)}\frac{\Gamma_{f,(g,\gamma)}^{(4,1)}}{\Gamma_f^{(4,0)}} + \frac{\overline{v}^2}{\pi}\Gamma_f^{(4,0)}$$

$$\times \left\{ \frac{m_H^2}{\sqrt{2}\overline{v}\overline{m}_f}\left(\delta_{f,q}\frac{C_F}{g_s}\alpha_s C_{fG} + \frac{Q_f}{\overline{e}}\alpha\left(C_{fB}\hat{c}_w + 2T_f^3 C_{fW}\hat{s}_w\right)\right) \right.$$

$$+ \left(\delta_{f,q}C_F\alpha_s C_{HG} + Q_f^2\alpha\, c_{h\gamma\gamma}\right)\left[19 - \pi^2 + \ln^2\left(\frac{\overline{m}_f^2}{m_H^2}\right) + 6\ln\left(\frac{\mu^2}{m_H^2}\right)\right]$$

$$\left. + c_{h\gamma Z}\, v_f Q_f \alpha\, F_{h\gamma Z}\left(\frac{M_Z^2}{m_H^2}, \frac{\mu^2}{m_H^2}, \frac{\overline{m}_f^2}{m_H^2}\right) \right\} \,, \tag{4.3}$$

where $v_f = (T_f^3 - 2Q_f\hat{s}_w^2)/(2\hat{s}_w\hat{c}_w)$ is the vector coupling of f to the Z-boson, T_f^3 is the weak isospin of fermion f (i.e. $T_\tau^3 = -\frac{1}{2}$ and $T_c^3 = \frac{1}{2}$), $\delta_{f,q} = 1$ if f is a quark and $\delta_{f,q} = 0$ if f is a lepton, $C_F = (N_c^2 - 1)/(2N_c)$ with $N_c = 3$, and the combinations of Wilson coefficients multiplying the $h\gamma\gamma$ and $h\gamma Z$ vertices are

$$c_{h\gamma\gamma} = C_{HB}\hat{c}_w^2 + C_{HW}\hat{s}_w^2 - C_{HWB}\hat{c}_w\hat{s}_w \,,$$
$$c_{h\gamma Z} = 2(C_{HB} - C_{HW})\hat{c}_w\hat{s}_w + C_{HWB}(\hat{c}_w^2 - \hat{s}_w^2) \,. \tag{4.4}$$

In the small-mass limit the function $F_{h\gamma Z}$ takes the form

$$F_{h\gamma Z}\left(z, \hat{\mu}^2, 0\right) = -12 + 4z - \frac{4}{3}\pi^2 \bar{z}^2 + \left(3 + 2z + 2\bar{z}^2 \ln(\bar{z})\right)\ln(z) + 4\bar{z}^2 \text{Li}_2(z) - 6\ln(\hat{\mu}^2),$$
(4.5)

where $\bar{z} = 1 - z$.

An interesting feature of eq. (4.3) is the double logarithm in the ratio $\overline{m}_f^2/m_H^2$ multiplying C_{HG} and $c_{h\gamma\gamma}$. In the SM, logarithms of this type first appear at NNLO, and are related to diagrams where the Higgs couples to a top-quark loop which in the large-m_t limit can be shrunk to an effective hAA vertex, where $A = \gamma, g$, multiplied by an m_t-dependent matching coefficient. These SM corrections, not only the logarithms but also the finite parts, are thus proportional to the SMEFT corrections given above (see for instance eq. (8) of [57]). As noted already in [58], these double logarithms cancel against corresponding terms in the $h \to AA$ decay rate, such that total Higgs decay width remains finite in the limit of vanishing fermion masses. In a less inclusive quantity such as $h \to f f$, they introduce sizeable flavor-dependent contributions to the decay rate, even though they multiply flavor-universal Wilson coefficients. We return to this issue when studying ratios of decay rates (in MFV) in section 6.

4.2 Four-fermion operators

Contributions from four-fermion operators ("class-8" in table 3) are obtained as in [42, 43]. In this section we generalize those results to the generic decay $h \to f\bar{f}$, including contributions from second-generation fermions which were neglected in previous analyses of $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$.

We first introduce some notation. In cases where top-quark loops contribute, the results involve the functions

$$F_{8S}\left(\frac{m_t^2}{m_H^2}, \frac{\mu^2}{m_H^2}\right) = \beta_t^2 \left(2\beta_t \text{arccot}\left(\beta_t\right) - \ln\frac{\mu^2}{m_t^2} - 2\right),$$

$$F_{8V}\left(\frac{m_t^2}{m_H^2}, \frac{\mu^2}{m_H^2}\right) = \beta_t^2 \left(-8\beta_t \text{arccot}\left(\beta_t\right) + 4\ln\frac{\mu^2}{m_t^2} + 6\right),$$
(4.6)

where

$$\beta_t \equiv \sqrt{\frac{4m_t^2}{m_H^2} - 1}.$$
(4.7)

Contributions from other fermions involve the real part of the above functions in the limit $m_t \to 0$, given by

$$F_{8S}\left(0, \frac{\mu^2}{m_H^2}\right) = 2 + \ln\frac{\mu^2}{m_H^2},$$

$$F_{8V}\left(0, \frac{\mu^2}{m_H^2}\right) = -6 - 4\ln\frac{\mu^2}{m_H^2}.$$
(4.8)

The functions with subscripts $8V$ arise from four-fermion operators of the form $(\bar{L}L)(\bar{R}R)$ in table 3, whereas those with subscripts $8S$ arise from four-fermion operators of the form $(\bar{L}R)(\bar{R}L)$ or $(\bar{L}R)(\bar{L}R)$ given in the last row of that table.

In terms of the above functions, the results for $h \to \tau\bar{\tau}$ are

$$
\Gamma_{\tau 8} = \frac{m_H^2 \Gamma_\tau^{(4,0)}}{16\pi^2} \Bigg\{ \left[C_{\substack{le \\ 3333}} + \frac{\overline{m}_\mu}{\overline{m}_\tau} C_{\substack{le \\ 2332}} \right] F_{8V}\left(0, \frac{\mu^2}{m_H^2}\right) - 2N_c \frac{m_t}{\overline{m}_\tau} C_{\substack{lequ \\ 3333}}^{(1)} F_{8S}\left(\frac{m_t^2}{m_H^2}, \frac{\mu^2}{m_H^2}\right)
$$

$$
+ 2N_c \left[\frac{\overline{m}_b}{\overline{m}_\tau} C_{\substack{ledq \\ 3333}} - \frac{\overline{m}_c}{\overline{m}_\tau} C_{\substack{lequ \\ 3322}}^{(1)} + \frac{\overline{m}_s}{\overline{m}_\tau} C_{\substack{ledq \\ 3322}} \right] F_{8S}\left(0, \frac{\mu^2}{m_H^2}\right) \Bigg\}. \tag{4.9}
$$

The expression for $h \to \mu\bar{\mu}$ is obtained after obvious replacements and reads

$$
\Gamma_{\mu 8} = \frac{m_H^2 \Gamma_\mu^{(4,0)}}{16\pi^2} \Bigg\{ \left[C_{\substack{le \\ 2222}} + \frac{\overline{m}_\tau}{\overline{m}_\mu} C_{\substack{le \\ 2332}} \right] F_{8V}\left(0, \frac{\mu^2}{m_H^2}\right) - 2N_c \frac{m_t}{\overline{m}_\mu} C_{\substack{lequ \\ 2233}}^{(1)} F_{8S}\left(\frac{m_t^2}{m_H^2}, \frac{\mu^2}{m_H^2}\right)
$$

$$
+ 2N_c \left[\frac{\overline{m}_b}{\overline{m}_\mu} C_{\substack{ledq \\ 2233}} - \frac{\overline{m}_c}{\overline{m}_\mu} C_{\substack{lequ \\ 2222}}^{(1)} + \frac{\overline{m}_s}{\overline{m}_\mu} C_{\substack{ledq \\ 2222}} \right] F_{8S}\left(0, \frac{\mu^2}{m_H^2}\right) \Bigg\}. \tag{4.10}
$$

For $h \to c\bar{c}$, we find instead

$$
\Gamma_{c8} = \frac{m_H^2 \Gamma_c^{(4,0)}}{16\pi^2} \Bigg\{ \left[C_{\substack{qu \\ 2222}}^{(1)} + C_F C_{\substack{qu \\ 2222}}^{(8)} \right] F_{8V}\left(0, \frac{\mu^2}{m_H^2}\right)
$$

$$
+ \frac{m_t}{\overline{m}_c} \left[C_{\substack{qu \\ 2332}}^{(1)} + C_F C_{\substack{qu \\ 2332}}^{(8)} \right] F_{8V}\left(\frac{m_t^2}{m_H^2}, \frac{\mu^2}{m_H^2}\right) + \left(\frac{\overline{m}_b}{\overline{m}_c} \left[C_{\substack{quqd \\ 3223}}^{(1)} + C_F C_{\substack{quqd \\ 3223}}^{(8)} + 2N_c C_{\substack{quqd \\ 2233}}^{(1)} \right] \right.
$$

$$
\left. + \frac{\overline{m}_s}{\overline{m}_c} \left[(1+2N_c) C_{\substack{quqd \\ 2222}}^{(1)} + C_F C_{\substack{quqd \\ 2222}}^{(8)} \right] - 2\frac{\overline{m}_\tau}{\overline{m}_c} C_{\substack{lequ \\ 3322}}^{(1)} - 2\frac{\overline{m}_\mu}{\overline{m}_c} C_{\substack{lequ \\ 2222}}^{(1)} \right) F_{8S}\left(0, \frac{\mu^2}{m_H^2}\right) \Bigg\}. \tag{4.11}
$$

To simplify the results, we have used relations such as $C_{\substack{qu \\ 2332}}^{(k)} = C_{\substack{qu \\ 3223}}^{(k)\dagger}$ (with $k = 1, 8$) which follow from the Hermiticity of the SMEFT Lagrangian. Finally, the result for $h \to b\bar{b}$ is

$$
\Gamma_{b8} = \frac{m_H^2 \Gamma_b^{(4,0)}}{16\pi^2} \Bigg\{ \left[C_{\substack{qd \\ 3333}}^{(1)} + C_F C_{\substack{qd \\ 3333}}^{(8)} + \frac{\overline{m}_s}{\overline{m}_b} \left(C_{\substack{qd \\ 2332}}^{(1)} + C_F C_{\substack{qd \\ 2332}}^{(8)} \right) \right] F_{8V}\left(0, \frac{\mu^2}{m_H^2}\right)
$$

$$
+ \frac{m_t}{\overline{m}_b} \left[(1 + 2N_c) C_{\substack{quqd \\ 3333}}^{(1)} + C_F C_{\substack{quqd \\ 3333}}^{(8)} \right] F_{8S}\left(\frac{m_t^2}{m_H^2}, \frac{\mu^2}{m_H^2}\right)
$$

$$
+ \left(\frac{\overline{m}_c}{\overline{m}_b} \left[C_{\substack{quqd \\ 3223}}^{(1)} + C_F C_{\substack{quqd \\ 3223}}^{(8)} + 2N_c C_{\substack{quqd \\ 2233}}^{(1)} \right] + 2\frac{\overline{m}_\tau}{\overline{m}_b} C_{\substack{ledq \\ 3333}} + 2\frac{\overline{m}_\mu}{\overline{m}_b} C_{\substack{ledq \\ 2233}} \right) F_{8S}\left(0, \frac{\mu^2}{m_H^2}\right) \Bigg\}. \tag{4.12}
$$

In all cases, contributions where the Higgs couples to a top-quark loop are enhanced by a large factor of $m_t/\overline{m}_f$. If MFV is imposed, the Wilson coefficients multiplying these contributions pick up factors proportional to $\overline{m}_f$, thus removing this enhancement. Moreover, contributions from top loops to decay into charm quarks vanish in MFV, under the approximation where the CKM matrix is the unit matrix, as can be seen by using MFV scalings for the Wilson coefficients in section 3.

m_H	$125\,\text{GeV}$	$\overline{e}(m_H)$	$\sqrt{4\pi/128}$
m_t	$173\,\text{GeV}$	$\alpha_s\,(m_H)$	0.1
M_W	$80.4\,\text{GeV}$	$\overline{m}_b(m_H)$	$3.0\,\text{GeV}$
M_Z	$91.2\,\text{GeV}$	$\overline{m}_\tau(m_H)$	$1.7\,\text{GeV}$
$\overline{v}(m_H)$	$240\,\text{GeV}$	$\overline{m}_c(m_H)$	$0.7\,\text{GeV}$

Table 1. Input parameters used in numerics. The derived quantity $\overline{v}(m_H) = 2M_W \hat{s}_w/\overline{e}(m_H)$ is listed for convenience.

5 Numerical results

In this section we give numerical results for the $h \to \tau\bar{\tau}$ and $h \to c\bar{c}$ decay rates. We list contributions from the full set of Wilson coefficients at the scale $\mu = m_H$ in section 5.1, and then examine scale uncertainties for the numerically dominant ones in section 5.2. The numerical inputs used throughout the paper are given in table 1.

5.1 Decay rates at $\mu = m_H$

It is convenient to normalize all results to the LO SM decay rate at the scale $\mu = m_H$. To this end, in analogy with eq. (2.3) we define the ratios

$$\Delta_f^{(i,j)}(\mu) \equiv \frac{\Gamma_f^{(i,j)}(\mu)}{\Gamma_f^{(4,0)}(m_H)}\,, \tag{5.1}$$

as well as the LO and NLO ratios

$$\Delta_f^{\text{LO}}(\mu) \equiv \Delta_f^{(4,0)}(\mu) + \Delta_f^{(6,0)}(\mu)\,,$$
$$\Delta_f^{\text{NLO}}(\mu) \equiv \Delta_f^{\text{LO}}(\mu) + \Delta_f^{(4,1)}(\mu) + \Delta_f^{(6,1)}(\mu)\,. \tag{5.2}$$

We also define dimensionless Wilson coefficients as

$$\tilde{C}_i(\mu) \equiv \Lambda_{\text{NP}}^2 C_i(\mu)\,, \tag{5.3}$$

where Λ_{NP} is a UV scale characteristic of heavy new physics beyond the SM. Contributions to the decay rate from dimension-6 operators are suppressed by a factor of $\overline{v}^2/\Lambda_{\text{NP}}^2 \approx 5\%$, where the numerical value refers to $\Lambda_{\text{NP}} = 1\,\text{TeV}$ and $\tilde{C}_i \sim 1$.

The LO result for the decay $h \to f\bar{f}$ at the scale $\mu = m_H$ is

$$\Delta_f^{\text{LO}}(m_H) = 1 + \frac{\overline{v}^2}{\Lambda_{\text{NP}}^2} \left[3.74 \tilde{C}_{HWB} + 2.00 \tilde{C}_{H\square} - 1.41 \frac{\overline{v}}{\overline{m}_f} \tilde{C}_{fH} + 1.24 \tilde{C}_{HD} \right]\,, \tag{5.4}$$

where here and below all $\overline{\text{MS}}$-renormalized quantities (in this case $\overline{v}, \tilde{C}_i, \overline{m}_f$) are evaluated at the scale m_H. The contribution from $\tilde{C}_{fH}$ is enhanced by a factor of $\overline{v}/\overline{m}_f$ compared to other contributions. We have left this factor symbolic, such that the numerical coefficient multiplying it is finite in the limit $m_f \to 0$. In a theory which respects MFV, the enhancement factor would indeed be compensated by an implicit scaling of the Wilson coefficient. While we are not necessarily advocating MFV, we choose to write all our results in such a way that all numerical factors multiplying dimension-6 Wilson coefficients are free of large enhancement factors. This affects several more coefficients at NLO, and for the same

reason we leave factors of $1/\overline{e}$ and $1/g_s$ appearing in operators such as $Q_{f(B,W)}$ and Q_{fG}, where gauge fields couple to fermions through field-strength tensors rather than covariant derivatives, symbolic.

The NLO corrections depend on the flavor of the fermion f. For $h \to \tau\bar{\tau}$, we find

$$
\Delta_\tau^{\mathrm{NLO}}(m_H) = 0.98 + \frac{\overline{v}^2}{\Lambda_{\mathrm{NP}}^2}\left\{ 3.63\tilde{C}_{HWB} + 2.11\tilde{C}_{H\Box} - 1.50\frac{\overline{v}}{\overline{m}_\tau}\tilde{C}_{\tau H} + 1.20\tilde{C}_{HD} + 0.16\tilde{C}_{HB} \right.
$$

$$
+ \left(-9.0\tilde{C}_{Hq}^{(3)} - 7.9\tilde{C}_{Ht} + 6.8\tilde{C}_{HW} + 5.2\tilde{C}_{Hq}^{(1)} - 4.6\tilde{C}_{tH} - 3.8\frac{m_t}{\overline{m}_\tau}\tilde{C}_{lequ}^{(1)} + 2.4\tilde{C}_H \right.
$$
$$
 {}_{33} \phantom{7.9\tilde{C}_{Ht} + 6.8\tilde{C}_{HW} + 5.2} {}_{33} \phantom{4.6\tilde{C}_{tH} - 3.8} {}_{3333}
$$

$$
+ 2.0\frac{\overline{m}_b}{\overline{m}_\tau}\tilde{C}_{ledq} + 2.0\frac{\overline{m}_s}{\overline{m}_\tau}\tilde{C}_{ledq} - 2.0\frac{\overline{m}_c}{\overline{m}_\tau}\tilde{C}_{lequ}^{(1)} + 1.5\tilde{C}_{Hl}^{(3)} - 1.0\tilde{C}_{le} - 1.0\frac{\overline{m}_\mu}{\overline{m}_\tau}\tilde{C}_{le} \bigg)
$$
$$
\phantom{+ 2.0\frac{\overline{m}_b}{\overline{m}_\tau}} {}_{3333} \phantom{+ 2.0\frac{\overline{m}_s}{\overline{m}_\tau}} {}_{3322} \phantom{- 2.0\frac{\overline{m}_c}{\overline{m}_\tau}} {}_{3322} {}_{33} {}_{3333} \phantom{- 1.0\frac{\overline{m}_\mu}{\overline{m}_\tau}} {}_{2332}
$$

$$
\times 10^{-2} + \left(-9\left[\frac{\tilde{C}_{tB}}{\overline{e}} + \tilde{C}_{Hq}^{(3)} + \tilde{C}_{Hq}^{(3)} - \tilde{C}_{Hl}^{(1)} + \tilde{C}_{Hu} + \tilde{C}_{Hc} \right] - 7\tilde{C}_W + 4\left[\tilde{C}_{Hl}^{(1)} + \tilde{C}_{Hl}^{(1)} \right. \right.
$$
$$
\phantom{\times 10^{-2} + \left(-9\left[\frac{\tilde{C}_{tB}}{\overline{e}} + \tilde{C}_{Hq}^{(3)} \right.\right.} {}_{11} \phantom{+ \tilde{C}_{Hq}^{(3)}} {}_{22} \phantom{- \tilde{C}_{Hl}^{(1)}} {}_{33} \phantom{7\tilde{C}_W + 4\left[\tilde{C}_{Hl}^{(1)} \right.} {}_{11} \phantom{+ \tilde{C}_{Hl}^{(1)}} {}_{22}
$$

$$
+ \tilde{C}_{He} + \tilde{C}_{H\mu} - \tilde{C}_{Hq}^{(1)} - \tilde{C}_{Hq}^{(1)} + \tilde{C}_{Hd} + \tilde{C}_{Hs} + \tilde{C}_{Hb} \bigg] - 3\left[\tilde{C}_{Hl}^{(3)} + \tilde{C}_{Hq}^{(3)} \right] + 2\frac{\tilde{C}_{tW}}{\overline{e}}
$$
$$
\phantom{+ \tilde{C}_{He} + \tilde{C}_{H\mu} - \tilde{C}_{Hq}^{(1)}} {}_{11} \phantom{- \tilde{C}_{Hq}^{(1)}} {}_{22} \phantom{\bigg] - 3\left[\tilde{C}_{Hl}^{(3)} \right.} {}_{11} \phantom{+ \tilde{C}_{Hq}^{(3)}} {}_{22}
$$

$$
\left. + \frac{\overline{v}}{\overline{m}_\tau}\frac{\tilde{C}_{\tau W}}{\overline{e}} \right) \times 10^{-3} + (4 \times 10^{-4})\left(\tilde{C}_{H\tau} - \frac{\overline{v}}{\overline{m}_\tau}\frac{\tilde{C}_{\tau B}}{\overline{e}} \right) \bigg\}. \tag{5.5}
$$

While our main focus in this section is $h \to \tau\bar{\tau}$, we mention in passing that results for $h \to \mu\bar{\mu}$ can be obtained from the above result by exchanging $\overline{m}_\tau \leftrightarrow \overline{m}_\mu$ along with appropriate changes on flavor indices of the Wilson coefficients. The only implicit m_τ dependence in the small-mass limit is in the $\alpha \ln(\overline{m}_\tau^2/m_H^2)$ terms in eq. (4.3); for $h \to \mu\bar{\mu}$, these change the coefficients of $\tilde{C}_{HWB}$, $\tilde{C}_{HW}$, $\tilde{C}_{HB}$ from the values listed above to 3.49, 0.14, 0.41, respectively.

The results for $h \to c\bar{c}$ are given by

$$
\Delta_c^{\mathrm{NLO}}(m_H) = 1.16 + \frac{\overline{v}^2}{\Lambda_{\mathrm{NP}}^2}\left\{ 4.95\tilde{C}_{HG} + 4.31\tilde{C}_{HWB} + 2.46\tilde{C}_{H\Box} - 1.75\frac{\overline{v}}{\overline{m}_c}\tilde{C}_{cH} \right.
$$

$$
+ 1.41\tilde{C}_{HD} + \left(9.4\tilde{C}_{HB} - 8.9\tilde{C}_{Hq}^{(3)} - 7.9\tilde{C}_{Ht} - 6.3\frac{m_t}{\overline{m}_c}\tilde{C}_{qu}^{(8)} + 5.4\tilde{C}_{HW} \right.
$$
$$
\phantom{+ 1.41\tilde{C}_{HD} + \left(9.4\tilde{C}_{HB} - 8.9\tilde{C}_{Hq}^{(3)} \right.} {}_{33} \phantom{- 7.9\tilde{C}_{Ht} - 6.3\frac{m_t}{\overline{m}_c}\tilde{C}_{qu}^{(8)}} {}_{2332}
$$

$$
+ 5.2\tilde{C}_{Hq}^{(1)} - 4.8\frac{m_t}{\overline{m}_c}\tilde{C}_{qu}^{(1)} - 4.6\tilde{C}_{tH} + 2.4\tilde{C}_H + 2.4\frac{\overline{m}_s}{\overline{m}_c}\tilde{C}_{quqd}^{(1)} + 2.0\frac{\overline{m}_b}{\overline{m}_c}\tilde{C}_{quqd}^{(1)} - 1.3\tilde{C}_{qu}^{(8)}
$$
$$
\phantom{+ 5.2\tilde{C}_{Hq}^{(1)}} {}_{33} \phantom{- 4.8\frac{m_t}{\overline{m}_c}\tilde{C}_{qu}^{(1)}} {}_{2332} \phantom{- 4.6\tilde{C}_{tH} + 2.4\tilde{C}_H + 2.4\frac{\overline{m}_s}{\overline{m}_c}\tilde{C}_{quqd}^{(1)}} {}_{2222} \phantom{+ 2.0\frac{\overline{m}_b}{\overline{m}_c}\tilde{C}_{quqd}^{(1)}} {}_{2233} \phantom{- 1.3\tilde{C}_{qu}^{(8)}} {}_{2222}
$$

$$
- 1.0\tilde{C}_{qu}^{(1)} + 1.0\tilde{C}_{Hq}^{(3)} - 1.0\tilde{C}_{Hq}^{(1)} \bigg) \times 10^{-2} + \left(-9\left[\frac{\tilde{C}_{tB}}{\overline{e}} + \tilde{C}_{Hq}^{(3)} + \tilde{C}_{Hu} \right] + 8\frac{\overline{v}}{\overline{m}_c}\frac{\tilde{C}_{cG}}{g_s} \right.
$$
$$
\phantom{- 1.0\tilde{C}_{qu}^{(1)}} {}_{2222} \phantom{+ 1.0\tilde{C}_{Hq}^{(3)}} {}_{22} \phantom{- 1.0\tilde{C}_{Hq}^{(1)}} {}_{22} \phantom{\bigg) \times 10^{-2} + \left(-9\left[\frac{\tilde{C}_{tB}}{\overline{e}} + \tilde{C}_{Hq}^{(3)} \right.\right.} {}_{11}
$$

$$
- 7\frac{\overline{m}_\tau}{\overline{m}_c}\tilde{C}_{lequ}^{(1)} - 7\frac{\overline{m}_\mu}{\overline{m}_c}\tilde{C}_{lequ}^{(1)} - 7\tilde{C}_W - 5\tilde{C}_{Hc} + 4\left[\frac{\overline{m}_b}{\overline{m}_c}\tilde{C}_{quqd}^{(8)} + \frac{\overline{m}_s}{\overline{m}_c}\tilde{C}_{quqd}^{(8)} + \tilde{C}_{Hl}^{(1)} + \tilde{C}_{Hl}^{(1)} \right.
$$
$$
\phantom{- 7\frac{\overline{m}_\tau}{\overline{m}_c}\tilde{C}_{lequ}^{(1)}} {}_{3322} \phantom{- 7\frac{\overline{m}_\mu}{\overline{m}_c}\tilde{C}_{lequ}^{(1)}} {}_{2222} \phantom{- 7\tilde{C}_W - 5\tilde{C}_{Hc} + 4\left[\frac{\overline{m}_b}{\overline{m}_c}\tilde{C}_{quqd}^{(8)} \right.} {}_{3223} \phantom{+ \frac{\overline{m}_s}{\overline{m}_c}\tilde{C}_{quqd}^{(8)}} {}_{2222} \phantom{+ \tilde{C}_{Hl}^{(1)}} {}_{11} \phantom{+ \tilde{C}_{Hl}^{(1)}} {}_{22}
$$

$$
+ \tilde{C}_{Hl}^{(1)} - \tilde{C}_{Hq}^{(1)} + \tilde{C}_{He} + \tilde{C}_{H\mu} + \tilde{C}_{H\tau} + \tilde{C}_{Hd} + \tilde{C}_{Hs} + \tilde{C}_{Hb} \bigg] - 3\left[-\frac{\overline{m}_b}{\overline{m}_c}\tilde{C}_{quqd}^{(1)} + \tilde{C}_{Hl}^{(3)} \right.
$$
$$
\phantom{+ \tilde{C}_{Hl}^{(1)}} {}_{33} \phantom{- \tilde{C}_{Hq}^{(1)}} {}_{11} \phantom{+ \tilde{C}_{He} + \tilde{C}_{H\mu} + \tilde{C}_{H\tau} + \tilde{C}_{Hd} + \tilde{C}_{Hs} + \tilde{C}_{Hb} \bigg] - 3\left[-\frac{\overline{m}_b}{\overline{m}_c}\tilde{C}_{quqd}^{(1)} \right.} {}_{3223} \phantom{+ \tilde{C}_{Hl}^{(3)}} {}_{11}
$$

$$
\left. + \tilde{C}_{Hl}^{(3)} + \tilde{C}_{Hl}^{(3)} \right] + 2\frac{\tilde{C}_{tW}}{\overline{e}} + \frac{\overline{v}}{\overline{m}_c}\frac{\tilde{C}_{cW}}{\overline{e}} \bigg) \times 10^{-3} + (2 \times 10^{-4})\frac{\overline{v}}{\overline{m}_c}\frac{\tilde{C}_{cB}}{\overline{e}} \bigg\}. \tag{5.6}
$$
$$
\phantom{\left. + \tilde{C}_{Hl}^{(3)} \right.} {}_{22} \phantom{+ \tilde{C}_{Hl}^{(3)}} {}_{33}
$$

$h \to \tau\bar{\tau}$	SM	$\tilde{C}_{HWB}$	$\tilde{C}_{H\square}$	$\tilde{C}_{fH}$	$\tilde{C}_{HD}$
NLO QED	1.1%	−1.3%	1.1%	1.1%	1.1%
NLO weak	−2.8%	−1.5%	4.5%	5.3%	−5.0%
NLO correction	−1.7%	−2.9%	5.5%	6.3%	−3.9%
$h \to c\bar{c}$					
NLO QCD-QED	18.5%	17.0%	18.5%	18.5%	18.5%
NLO weak	−2.8%	−1.6%	4.4%	5.3%	−5.2%
NLO correction	15.7%	15.3%	22.9%	23.8%	13.3%

Table 2. Size of NLO corrections to different terms in LO decay rate, split into QED(-QCD) and weak corrections for $h \to \tau\bar{\tau}$ (top) and $h \to c\bar{c}$ (bottom). See text for further explanation.

Compared to LO, a total of 36 new Wilson coefficients appear at NLO in $h \to \tau\bar{\tau}$ and 43 in $h \to c\bar{c}$. While the size of these contributions depends on the values of the in general unknown Wilson coefficients, it is possible to make some general statements.

First, in all cases the potentially largest NLO contributions are from those coefficients which carry a non-trivial scaling with $\overline{m}_f$ in MFV. Higgs decays thus offer an interesting testing ground for MFV, which becomes more involved beyond LO. Second, since leptonic decays $h \to \ell\ell$ have no QCD corrections to this order, the NLO corrections are generally mild, the most sizeable (apart from MFV-breaking enhancements) being that from $\tilde{C}_{HB}$, which is enhanced by the double logarithm in $\overline{m}_\ell/m_H$ in eq. (4.3). For $h \to c\bar{c}$ this double logarithm also multiplies the QCD correction from $\tilde{C}_{HG}$, and the correction is so large that actually dominates over coefficients appearing at LO. Finally, the corrections to the operators appearing at LO are much larger in $h \to c\bar{c}$ than in $h \to \ell\ell$ due to QCD corrections, although in neither case are these well approximated by applying an SM-like K-factor to the LO results. To illustrate this latter point we show in table 2 the NLO corrections to coefficients appearing at LO in $h \to \tau\bar{\tau}$ and $h \to c\bar{c}$ decays, split into the QED-QCD and weak corrections, i.e. $\Gamma^{(i,1)}_{f,(g,\gamma)}$ and $\Gamma^{(i,1)}_{f,\text{weak}}$ in eq. (2.4), respectively.

5.2 Scale uncertainties

The truncation of the perturbative series to finite order introduces dependence of the decay rates on unphysical renormalization scales. The change in the decay rate under ranges of scale choices is often used to estimate uncertainties associated with uncalculated, higher-order corrections. Ideally, the uncertainty bands shrink upon adding higher-order perturbative corrections, and also show good overlap between orders.

In this section, we study scale uncertainties in the LO and NLO SMEFT calculations of $h \to \tau\bar{\tau}$ and $h \to c\bar{c}$ decay rates. This requires knowledge of how the $\overline{\text{MS}}$-renormalized input parameters depend on renormalization scale. In the SM this is simple, as the light-fermion masses and QED/QCD couplings have been extracted numerically at certain reference scales, and can be evolved to any other scale using RG equations.

In SMEFT, on the other hand, the dimension-6 Wilson coefficients have no accepted numerical values. In order to study scale uncertainties in a meaningful way, we must therefore express decay rates at different scales in terms of Wilson coefficients evaluated at a fixed reference scale. We choose this reference scale to be m_H. The Wilson coefficients at an arbitrary scale μ_C can then be determined from their values at m_H through use of RG equations. Since in our analysis the μ_C will be at most a factor of two away from the reference value $\mu_C = m_H$, we can use the fixed-order solutions to the RG equations. In fact, the same is true of the $\overline{\text{MS}}$-renormalized masses and couplings, so to study scale variations we will need only the following equations:

$$C_i(\mu_C) = C_i(m_H) + \ln\left(\frac{\mu_C}{m_H}\right)\dot{C}_i(m_H),$$
$$\overline{m}_f(\mu_R) = \overline{m}_f(m_H)\left[1 + \gamma_f(m_H)\ln\left(\frac{\mu_R}{m_H}\right)\right],$$
$$\overline{\alpha}(\mu_R) = \overline{\alpha}(m_H)\left[1 + 2\gamma_e(m_H)\ln\left(\frac{\mu_R}{m_H}\right)\right],$$
$$\alpha_s(\mu_R) = \alpha_s(m_H)\left[1 - 2\gamma_g(m_H)\ln\left(\frac{\mu_R}{m_H}\right)\right]. \tag{5.7}$$

Expressions for $\dot{C}_i \equiv dC_i/d\ln\mu$ at one loop can be found in [56, 59, 60], and we have made use of their electronic implementation from [61]. The corresponding SM expressions are

$$\gamma_f(\mu) = -\frac{3}{2\pi}\left[\delta_{f,q}\alpha_s(\mu)C_F + \overline{\alpha}(\mu)Q_f^2\right],$$
$$\gamma_e(\mu) = \frac{\overline{\alpha}(\mu)}{3\pi}\left[3Q_\ell^2 + N_c\left(2Q_u^2 + 3Q_b^2\right)\right],$$
$$\gamma_g(\mu) = \frac{\alpha_s(\mu)}{4\pi}\left(\frac{11}{3}C_A - \frac{2}{3}n_l\right), \tag{5.8}$$

where $n_l = 5$ is the number of light fermions and $C_A = 3$. The notation makes clear that we are free to choose the two renormalization scales μ_C and μ_R independently. We will indeed do so, such that the expansion coefficients in eq. (5.1) become functions of two scales, $\Delta_f^{(i,j)}(\mu) \to \Delta_f^{(i,j)}(\mu_C, \mu_R)$. The explicit logarithmic dependence on the two scales can be deduced through an RG analysis and is given in eq. (5.7) of [42].

With these expressions at hand, we can fix a procedure for quantifying scale uncertainties. We follow the method advocated in [42]. First, we set $\mu_C = \mu_R = m_H$ by default. We then vary μ_C to the values $m_H/2$ and $2m_H$, keeping $\mu_R = m_H$, and use the results to calculate upper and lower uncertainties from μ_C variations. We obtain uncertainties from μ_R variations analogously, and add the μ_C and μ_R uncertainties in quadrature to get a total uncertainty.

For $h \to \tau\bar{\tau}$, the LO result obtained in this way is

$$\Delta_\tau^{\text{LO}}(m_H, m_H) = (1^{+0.002}_{-0.003}) + \frac{\overline{v}^2}{\Lambda_{\text{NP}}^2}\Bigg\{(3.74 \pm 0.14)\tilde{C}_{HWB} + (2.00 \pm 0.12)\tilde{C}_{H\Box}$$

$$- (1.41 \pm 0.06)\frac{\overline{v}}{\overline{m}_\tau}\tilde{C}_{\tau H} + (1.24 \pm 0.09)\tilde{C}_{HD} \pm 0.19\tilde{C}^{(1)}_{\substack{Hq \\ 33}} \pm 0.18\tilde{C}_{Ht} \pm 0.09\frac{m_t}{\overline{m}_\tau}\tilde{C}^{(1)}_{\substack{lequ \\ 3333}}$$

$$\pm 0.05\tilde{C}_{tH} \pm 0.05\tilde{C}^{(3)}_{\substack{Hq \\ 33}} + \dots\Bigg\}, \tag{5.9}$$

while at NLO we find

$$\Delta_\tau^{\text{NLO}}(m_H, m_H) = (0.98^{+0.0001}_{-0.0002}) + \frac{\overline{v}^2}{\Lambda_{\text{NP}}^2}\left\{(3.62^{+0.00}_{-0.01})\tilde{C}_{HWB} + (2.11^{+0.00}_{-0.02})\tilde{C}_{H\square}\right.$$

$$+ (-1.50^{+0.01}_{-0.00})\frac{\overline{v}}{\overline{m}_\tau}\tilde{C}_{\tau H} + (1.20^{+0.00}_{-0.01})\tilde{C}_{HD} + (0.16^{+0.00}_{-0.00})\tilde{C}_{HB} + (-0.09^{+0.02}_{-0.00})\tilde{C}^{(3)}_{\substack{Hq\\33}}$$

$$\left.+ (-0.08^{+0.03}_{-0.00})\tilde{C}_{Ht} + (0.07^{+0.00}_{-0.00})\tilde{C}_{HW} + (0.05^{+0.00}_{-0.03})\tilde{C}^{(1)}_{\substack{Hq\\33}} + (-0.05^{+0.01}_{-0.00})\tilde{C}_{tH} + \ldots\right\}.$$

$$(5.10)$$

In the above equations and in the results for $h \to c\bar{c}$ that follow, the ellipses denote terms whose magnitude is smaller than 5%.[4]

The uncertainties are significantly reduced when including the NLO corrections, and in general the LO uncertainty ranges encompass the true value of the NLO correction, showing a good convergence of the perturbative series. There are two notable exceptions. The first is the SM itself, where the LO and NLO uncertainty bands show little overlap. This is mainly due to the fact that there is a correlation between the running of $\overline{m}_\tau$ and $\overline{v}$, such that the ratio $\overline{m}_f(\mu_R)/\overline{v}(\mu_R)$ is more stable under scale variations that the numerator and denominator alone. A more conservative option would be to introduce and vary independently separate renormalization scales for the fermion masses and the electromagnetic coupling constant, but we do not pursue that option here. The second exception is C_{HB} and C_{HW}, which get NLO corrections proportional to $\alpha \ln^2(\overline{m}_\tau^2/m_H^2)$ that are inaccessible to an RG analysis.

For $h \to c\bar{c}$, the LO result is

$$\Delta_c^{\text{LO}}(m_H, m_H) = (1 \pm 0.08) + \frac{\overline{v}^2}{\Lambda_{\text{NP}}^2}\left\{(3.74^{+0.37}_{-0.36})\tilde{C}_{HWB} + (2.00^{+0.22}_{-0.21})\tilde{C}_{H\square}\right.$$

$$+ (-1.41 \pm 0.07)\frac{\overline{v}}{\overline{m}_c}\tilde{C}_{cH} + (1.24^{+0.15}_{-0.14})\tilde{C}_{HD} \pm 0.35\tilde{C}_{HG} \pm 0.19\tilde{C}^{(1)}_{\substack{Hq\\33}}$$

$$\left.\pm 0.18\tilde{C}_{Ht} \pm 0.08\frac{m_t}{\overline{m}_c}\tilde{C}^{(8)}_{\substack{qu\\2332}} \pm 0.06\frac{m_t}{\overline{m}_c}\tilde{C}^{(1)}_{\substack{qu\\2332}} \pm 0.05\tilde{C}_{tH} \pm 0.05\tilde{C}^{(3)}_{\substack{Hq\\33}} + \ldots\right\}, \quad (5.11)$$

and at NLO we have

$$\Delta_c^{\text{NLO}}(m_H, m_H) = (1.16^{+0.02}_{-0.04}) + \frac{\overline{v}^2}{\Lambda_{\text{NP}}^2}\left\{(4.95^{+0.89}_{-0.84})\tilde{C}_{HG} + (4.31^{+0.06}_{-0.15})\tilde{C}_{HWB}\right.$$

$$+ (2.46^{+0.05}_{-0.10})\tilde{C}_{H\square} + (-1.75^{+0.04}_{-0.03})\frac{\overline{v}}{\overline{m}_c}\tilde{C}_{cH} + (1.41^{+0.02}_{-0.05})\tilde{C}_{HD} + (0.00^{+0.12}_{-0.08})\frac{\tilde{C}_{tG}}{g_s}$$

$$+ (0.09^{+0.01}_{-0.01})\tilde{C}_{HB} + (-0.09^{+0.03}_{-0.01})\tilde{C}^{(3)}_{\substack{Hq\\33}} + (-0.08^{+0.06}_{-0.01})\tilde{C}_{Ht} + (-0.06^{+0.01}_{-0.02})\frac{m_t}{\overline{m}_c}\tilde{C}^{(8)}_{\substack{qu\\2332}}$$

$$\left.+ (0.05^{+0.01}_{-0.01})\tilde{C}_{HW} + (0.05^{+0.01}_{-0.06})\tilde{C}^{(1)}_{\substack{Hq\\33}} + (-0.05^{+0.01}_{-0.02})\frac{m_t}{\overline{m}_c}\tilde{C}^{(1)}_{\substack{qu\\2332}} + (-0.05^{+0.02}_{-0.02})\tilde{C}_{tH} + \ldots\right\}.$$

$$(5.12)$$

---

[4]More precisely, for a term of the form x^{+y}_{-z}, we neglect terms where both $|x + y|$ and $|x - z|$ are smaller than 5%.

The qualitative behavior is similar to the case of $h \to \tau\bar{\tau}$. There are however some differences for decay into quarks rather than leptons. There is a worse overlap between the LO and NLO results for the SM and dimension-6 coefficients appearing at tree level. This is mainly due to the fact that the NLO QCD corrections in the SM are slightly larger than what one gets by varying the scale in $\overline{m}_c$. More significantly, the correction proportional to $\alpha_s \ln^2(\overline{m}_c^2/m_H^2)$ multiplying the C_{HG} contribution is much larger than the corresponding terms in $h \to \tau\bar{\tau}$, due to the appearance of the QCD coupling, and is well outside the LO uncertainty estimate. In fact, the running of C_{HG} introduces a new coefficient, C_{tG}, into the NLO uncertainty estimate, which is an $\alpha_s^2 \ln^2(\overline{m}_c^2/m_H^2)$ correction to the LO result and is numerically similar to the NLO corrections for other Wilson coefficients.

Since C_{HG} itself gets NNLO corrections of order $\alpha_s^2 \ln^4(\overline{m}_c^2/m_H^2)$, which are substantial, it is clear that more reliable predictions for $h \to c\bar{c}$ would involve a resummation of the logarithmic terms. While techniques for such a resummation exist for the virtual $h \to AA$ amplitudes (where $A = \gamma, g$) [62–66], it is not clear how to translate them to the inclusive $h \to f\bar{f}$ decay rate, which receives double-logarithmic corrections from both real and virtual contributions.

6 Ratios of decay rates

LHC measurements are to a large extent limited to $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$. At lepton colliders, on the other hand, measurements of Higgs decays into muons and charm quarks should also be possible, which motivates studying ratios of decay rates into different fermions. This is a benefit in the SM, as ratios of decay rates are more stable under perturbative corrections than the decay rates themselves. It is even more advantageous in SMEFT, as many flavor-universal dimension-6 contributions drop out of ratios. This also eliminates, to a large extent, dependence on the choice of input parameters used in the renormalization procedure, for instance on the freedom to trade $\overline{\alpha}$ for the Fermi constant G_F as is often done in SM calculations. Finally, as we shall see in a moment, ratios of decay rates offer interesting tests of MFV, as well as probes of anomalous hgg and $h\gamma\gamma$ couplings induced by SMEFT operators.

From the theoretical side, the cleanest possible ratios are those involving fermions with the same charges under the SM gauge group. Lepton colliders should be able to measure the tau to muon ratio, so we focus on this as a concrete possibility. Let us first define

$$R_{\tau/\mu} = \frac{\overline{m}_\mu^2}{\overline{m}_\tau^2} \frac{\Gamma_\tau}{\Gamma_\mu} \,. \tag{6.1}$$

In the SM, up to NLO in perturbation theory and ignoring terms suppressed by m_ℓ^2/m_H^2, $R_{\tau/\mu} = 1$. In SMEFT, keeping only dimension-6 terms in the ratio up to NLO in perturba-

tion theory and using $\overline{m}_\mu(m_H) = 100\,\mathrm{MeV}$, we find the following result at the scale $\mu = m_H$:

$$R_{\tau/\mu} - 1 = \Delta_\tau^{\mathrm{NLO}}(m_H) - \Delta_\mu^{\mathrm{NLO}}(m_H) - \Delta_\tau^{(4,1)}\left(\Delta_\tau^{(6,0)} - \Delta_\mu^{(6,0)}\right)$$

$$= \frac{\overline{v}^2}{\Lambda_{\mathrm{NP}}^2}\Bigg\{ -1.53\left(\frac{\overline{v}}{\overline{m}_\tau}\tilde{C}_{\tau H} - \frac{\overline{v}}{\overline{m}_\mu}\tilde{C}_{\mu H}\right) - 0.25\tilde{C}_{HB} + 0.13\tilde{C}_{HWB} + \Bigg[-7.2\tilde{C}_{HW}$$

$$-3.8\left(\frac{m_t}{\overline{m}_\tau}\tilde{C}^{(1)}_{\substack{lequ\\3333}} - \frac{m_t}{\overline{m}_\mu}\tilde{C}^{(1)}_{\substack{lequ\\2233}}\right) + 2.0\left(\frac{\overline{m}_b}{\overline{m}_\tau}\tilde{C}_{\substack{ledq\\3333}} - \frac{\overline{m}_b}{\overline{m}_\mu}\tilde{C}_{\substack{ledq\\2233}} + \frac{\overline{m}_s}{\overline{m}_\tau}\tilde{C}_{\substack{ledq\\3322}} - \frac{\overline{m}_s}{\overline{m}_\mu}\tilde{C}_{\substack{ledq\\2222}}\right.$$

$$\left. -\frac{\overline{m}_c}{\overline{m}_\tau}\tilde{C}^{(1)}_{\substack{lequ\\3322}} + \frac{\overline{m}_c}{\overline{m}_\mu}\tilde{C}^{(1)}_{\substack{lequ\\2222}}\right) + 1.8\left(\tilde{C}^{(3)}_{\substack{Hl\\33}} - \tilde{C}^{(3)}_{\substack{Hl\\22}}\right) - 1.0\left(\left(\frac{\overline{m}_\mu}{\overline{m}_\tau} - \frac{\overline{m}_\tau}{\overline{m}_\mu}\right)\tilde{C}_{\substack{le\\2332}} + \tilde{C}_{\substack{le\\3333}}\right.$$

$$\left.\left. -\tilde{C}_{\substack{le\\2222}}\right)\right] \times 10^{-2} + \left[4\left(\tilde{C}^{(1)}_{\substack{Hl\\33}} - \tilde{C}^{(1)}_{\substack{Hl\\22}} - \tilde{C}_{H\tau} + \tilde{C}_{H\mu}\right) + \left(\frac{\overline{v}}{\overline{m}_\tau}\frac{\tilde{C}_{\tau W}}{\overline{e}}\right.\right.$$

$$\left.\left. -\frac{\overline{v}}{\overline{m}_\mu}\frac{\tilde{C}_{\mu W}}{\overline{e}}\right)\right] \times 10^{-3} - (4\times10^{-4})\left(\frac{\overline{v}}{\overline{m}_\tau}\frac{\tilde{C}_{\tau B}}{\overline{e}} - \frac{\overline{v}}{\overline{m}_\mu}\frac{\tilde{C}_{\mu B}}{\overline{e}}\right)\Bigg\}, \tag{6.2}$$

where have used that $\Delta_\mu^{(4,1)} = \Delta_\tau^{(4,1)}$ in the first equality.

The results simplify if some form of universality is assumed for the generation-dependent Wilson coefficients. As a concrete realization, we study the scenario where the SMEFT Wilson coefficients are constrained by MFV. The expressions and notation needed in the analysis are given in section 3. Dropping contributions to the ratio that are suppressed by powers of $\overline{m}_f/m_H$ after taking the MFV limit, we find the simple result

$$\left[R_{\tau/\mu} - 1\right]_{\mathrm{MFV}} = \frac{\alpha\overline{v}^2}{\pi}\mathcal{C}_{h\gamma\gamma}\left(\ln^2\frac{\overline{m}_\tau^2}{m_H^2} - \ln^2\frac{\overline{m}_\mu^2}{m_H^2}\right)$$

$$= \frac{\overline{v}^2}{\Lambda_{\mathrm{NP}}^2}\left(-0.25\tilde{\mathcal{C}}^0_{HB} + 0.13\tilde{\mathcal{C}}^0_{HWB} - 0.072\tilde{\mathcal{C}}^0_{HW}\right), \tag{6.3}$$

where $\mathcal{C}_{h\gamma\gamma}$ has been obtained from $c_{h\gamma\gamma}$ in eq. (4.4) by replacing the Wilson coefficients by their LO expansion in the small-mass limit as explained at the end of section 3. We have listed the analytic expression in order to emphasize that the deviation of the ratio from unity is due to the double logarithmic corrections generated by the effective $h\gamma\gamma$ vertex and given explicitly in eq. (4.3). Uncertainties from higher orders estimated from scale variations using the method described in the previous section are found to be less than 1% for all dimension-6 coefficients involved.

The above results illustrate important features of decay-rate ratios in SMEFT. First, for a generic flavor structure, whereas $\Gamma_\tau + \Gamma_\mu$ depends on 48 Wilson coefficients, $R_{\tau/\mu}$ depends on 26 Wilson coefficients, as almost all operators that do not contain fermion fields drop out of the ratio; in an MFV scenario the ratio depends only on three coefficients. Second, for a generic flavor structure it is possible to have significant deviations from the SM in Γ_μ, Γ_τ, and $R_{\tau/\mu}$ simultaneously. However, in an MFV scenario, even if there were significant deviations in both Γ_τ and Γ_μ due to modified Higgs couplings to fermions, they would not propagate into $R_{\tau/\mu}$. In this sense, the ratio not only offers a very clean test of MFV, but also an independent constraint on the flavor-independent Wilson coefficients appearing in eq. (6.3) in such a scenario, beyond what is currently used in global fits. To

illustrate the sensitivity of $R_{\tau/\mu}$ to these coefficients, we have varied them to the extreme edges of the 95% C.L. limits from a recent global fit marginalized over a set of 19 SMEFT Wilson coefficients in [67], and found that deviations in $R_{\tau/\mu}$ from the SM value on the order of a few percent are not ruled out.

As a second example, involving quarks instead of leptons, we consider the ratio $R_{c/b}$, defined in analogy to eq. (6.1). The result is not as simple as for $R_{\tau/\mu}$, since the b and c quarks have different couplings to the electroweak gauge bosons. QCD corrections cancel from the SM result up to NLO, so deviations from unity are due to both to SM electroweak corrections and dimension-6 effects. In particular, we find

$$
\begin{aligned}
R_{c/b}-1 = {} & \Delta_c^{\mathrm{NLO}} - \Delta_b^{\mathrm{NLO}} - \Delta_c^{(4,1)}\Delta_b^{(6,0)} - \Delta_b^{(4,1)}\Delta_c^{(6,0)} + 2\Delta_b^{(4,1)}\Delta_b^{(6,0)} \\
= {} & 0.03 + \frac{\overline{v}^2}{\Lambda_{\mathrm{NP}}^2}\Bigg\{ 2.20\tilde{C}_{HG} - 1.57\frac{\overline{v}}{\overline{m}_c}\tilde{C}_{cH} + 1.59\frac{\overline{v}}{\overline{m}_b}\tilde{C}_{bH} + \Bigg[8.5\tilde{C}_{HB} - 6.3\frac{m_t}{\overline{m}_c}\tilde{C}^{(8)}_{\substack{qu\\2332}} \\
& -4.8\frac{m_t}{\overline{m}_c}\tilde{C}^{(1)}_{\substack{qu\\2332}} - 4.4\frac{m_t}{\overline{m}_b}\tilde{C}^{(1)}_{\substack{quqd\\3333}} + 4.4\tilde{C}_{HWB} + 4.2\tilde{C}_{HD} + 2.7\tilde{C}_{HW} + 2.6\tilde{C}^{(3)}_{\substack{Hq\\33}} \\
& +2.4\frac{\overline{m}_s}{\overline{m}_c}\tilde{C}^{(1)}_{\substack{quqd\\2222}} + 2.0\left(\frac{\overline{m}_b}{\overline{m}_c} - \frac{\overline{m}_c}{\overline{m}_b}\right)\tilde{C}^{(1)}_{\substack{quqd\\2233}} + 1.9\tilde{C}^{(3)}_{\substack{Hq\\22}} + 1.9\frac{\overline{v}}{\overline{m}_b}\frac{\tilde{C}_{bW}}{\overline{e}} - 1.5\tilde{C}_{tH} + 1.5\frac{\tilde{C}_{tW}}{\overline{e}} \\
& -1.3\left(\tilde{C}^{(8)}_{\substack{qu\\2222}} - \tilde{C}^{(8)}_{\substack{qd\\3333}} - \frac{\overline{m}_s}{\overline{m}_b}\tilde{C}^{(8)}_{\substack{qd\\2332}}\right) - 1.0\left(\tilde{C}^{(1)}_{\substack{qu\\2222}} - \tilde{C}^{(1)}_{\substack{qd\\3333}} - \frac{\overline{m}_s}{\overline{m}_b}\tilde{C}^{(1)}_{\substack{qd\\2332}}\right)\Bigg] \times 10^{-2} \\
& + \Bigg[8\left(\frac{\overline{v}}{\overline{m}_c}\frac{\tilde{C}_{cG}}{g_s} - \frac{\overline{v}}{\overline{m}_b}\frac{\tilde{C}_{bG}}{g_s}\right) - 8\frac{m_t}{\overline{m}_b}\tilde{C}^{(8)}_{\substack{quqd\\3333}} - 7\left(\frac{\overline{m}_\tau}{\overline{m}_c}\tilde{C}^{(1)}_{\substack{lequ\\3322}} + \frac{\overline{m}_\tau}{\overline{m}_b}C_{\substack{ledq\\3333}} + \frac{\overline{m}_\mu}{\overline{m}_c}\tilde{C}^{(1)}_{\substack{lequ\\2222}}\right. \\
& \left.+ \frac{\overline{m}_\mu}{\overline{m}_b}C_{\substack{ledq\\2233}}\right) - 6\tilde{C}^{(1)}_{\substack{Hq\\33}} - 5\tilde{C}^{(1)}_{\substack{Hq\\22}} + 4\left(\frac{\overline{m}_b}{\overline{m}_c} - \frac{\overline{m}_c}{\overline{m}_b}\right)\tilde{C}^{(8)}_{\substack{quqd\\3223}} + 4\frac{\overline{m}_s}{\overline{m}_c}\tilde{C}^{(8)}_{\substack{quqd\\2222}} + 4\frac{\overline{v}}{\overline{m}_b}\tilde{C}_{Htb} \\
& + 3\left(\frac{\overline{m}_b}{\overline{m}_c} - \frac{\overline{m}_c}{\overline{m}_b}\right)\tilde{C}^{(1)}_{\substack{quqd\\3223}} + 3\tilde{C}_{Hc} + 2\tilde{C}_{Hb} + \frac{\overline{v}}{\overline{m}_c}\frac{\tilde{C}_{cW}}{\overline{e}}\Bigg] \times 10^{-3} + (2\times10^{-4})\frac{\overline{v}}{\overline{m}_c}\frac{\tilde{C}_{cB}}{\overline{e}} \\
& + (4\times10^{-5})\frac{\overline{v}}{\overline{m}_b}\frac{\tilde{C}_{bB}}{\overline{e}}\Bigg\}. \tag{6.4}
\end{aligned}
$$

The deviation of the ratio from unity is only 3% in the SM, and while $\Gamma_b + \Gamma_c$ depends on 60 Wilson coefficients, $R_{c/b}$ depends on 41 Wilson coefficients. Again imposing MFV we find

$$
\begin{aligned}
\left[R_{c/b}-1\right]_{\mathrm{MFV}} = {} & 0.03 + \frac{\overline{v}^2}{\Lambda_{\mathrm{NP}}^2}\Bigg\{ 2.24\tilde{\mathcal{C}}^1_{bH} - 2.22\tilde{\mathcal{C}}^1_{cH} + 2.20\tilde{\mathcal{C}}^0_{HG} + \Bigg[8.5\tilde{\mathcal{C}}^0_{HB} - 4.5\tilde{\mathcal{C}}^{(1),2}_{\substack{quqd\\3333}} \\
& + 4.4\tilde{\mathcal{C}}^0_{HWB} + 4.2\tilde{\mathcal{C}}^0_{HD} + 2.7\frac{\tilde{\mathcal{C}}^1_{bW}}{\overline{e}} + 2.7\tilde{\mathcal{C}}^0_{HW} + 2.6\tilde{\mathcal{C}}^{(3),0}_{\substack{Hq\\33}} + 1.9\tilde{\mathcal{C}}^{(3),0}_{\substack{Hq\\22}} - 1.6\tilde{\mathcal{C}}^1_{tH} \\
& + 1.5\frac{\tilde{\mathcal{C}}^1_{tW}}{\overline{e}} - 1.3\left(\tilde{\mathcal{C}}^{(8),0}_{\substack{qu\\2222}} - \tilde{\mathcal{C}}^{(8),0}_{\substack{qd\\33}}\right) + 1.1\left(\frac{\tilde{\mathcal{C}}^1_{cG}}{g_s} - \frac{\tilde{\mathcal{C}}^1_{bG}}{g_s}\right) \\
& - 1.0\left(\tilde{\mathcal{C}}^{(1),0}_{\substack{qu\\2222}} - \tilde{\mathcal{C}}^{(1),0}_{\substack{qd\\33}}\right)\Bigg] \times 10^{-2} + \Bigg[-9\tilde{\mathcal{C}}^{(8),2}_{\substack{quqd\\3333}} - 6\tilde{\mathcal{C}}^{(1),0}_{\substack{Hq\\33}} - 5\tilde{\mathcal{C}}^{(1),0}_{\substack{Hq\\22}} + 5\tilde{\mathcal{C}}^2_{Htb} \\
& + 3\tilde{\mathcal{C}}^0_{Hc} + 2\tilde{\mathcal{C}}^0_{Hb} + 2\frac{\tilde{\mathcal{C}}^1_{cW}}{\overline{e}}\Bigg] \times 10^{-3} + (3\times10^{-4})\frac{\tilde{\mathcal{C}}^1_{cB}}{\overline{e}} + (6\times10^{-5})\frac{\tilde{\mathcal{C}}^1_{bB}}{\overline{e}}\Bigg\}. \tag{6.5}
\end{aligned}
$$

where we have used that $\mathcal{C}_{Hq}^{(1,3),0} = \mathcal{C}_{Hq}^{(1,3),0}$. The result still depends on 28 dimension-6 Wilson coefficients, a number which can only be reduced by assuming universality for Wilson coefficients of operators containing up and down type quarks of different generations not present in MFV. Still, regardless of any flavor assumptions by far the largest numerical contributions multiply $\tilde{\mathcal{C}}_{HG}^0$, $\tilde{\mathcal{C}}_{bH}^1$, and $\tilde{\mathcal{C}}_{cH}^1$. Contributions from the additional coefficients $\tilde{\mathcal{C}}_{HD}^0$, $\tilde{\mathcal{C}}_{HWB}^0$, $\tilde{\mathcal{C}}_{H\square}^0$, which appear at LO in the decay rates, either begin at NLO in the ratio, or else drop out entirely. The contribution from $\tilde{\mathcal{C}}_{HG}^0$ is due to terms of the form $\alpha_s \ln^2(\overline{m}_f^2/m_H^2)$, so as remarked above it would be desirable to resum such terms to obtain a more reliable prediction. However, this coefficient is quite constrained by LHC data, again using the 95% C.L. limits from the recent global fit [67] we find contributions to $R_{c/b}$ that are below the 1% level.

7   Conclusions

We have computed the NLO corrections from dimension-6 SMEFT operators to the decays $h \to f\bar{f}$, where $f \in \{\mu, \tau, c\}$, thus extending the results for $h \to b\bar{b}$ from [42] to cover the full spectrum of phenomenologically viable Higgs decays into fermions.

Many lessons learned from $h \to b\bar{b}$ carry over to the generic decay modes $h \to f\bar{f}$. For instance, it is advantageous to use a hybrid renormalisation scheme which avoids spuriously large tadpole corrections while resumming a series of single-logarithmic corrections in the ratio $\overline{m}_f/m_H$ through the use of running masses $\overline{m}_f$, defined in five-flavor version of QED×QCD. At the same time, it is a poor approximation to estimate NLO corrections to the dimension-6 operators appearing in the LO result by assuming they are proportional to the NLO correction in the SM.

On the other hand, considering several modes at once also opens up the possibility to study ratios of decay modes, which turns out to be even more advantageous in SMEFT than in the SM, since many flavor-universal dimension-6 Wilson coefficients drop out of the ratios. Amusingly, even in scenarios where the Wilson coefficients are constrained by MFV, the flavor-universal SMEFT coefficients which alter the hgg and $h\gamma\gamma$ couplings introduce at NLO flavor-dependence in ratios of decay rates normalised to their respective LO SM results. This dependence is from double logarithmic corrections of the form $(\alpha, \alpha_s) \ln^2(\overline{m}_f^2/m_H^2)$, which cancel in total decay width but are present in the exclusive decay into a particular fermion pair.

The analytic expressions for the $h \to f\bar{f}$ decay rates, given as computer files with the arXiv submission of this work, will be useful in future precision analysis of these decay modes in effective field theory, as well as for benchmarking all-purpose tools for automated NLO SMEFT calculations as they become available.

Acknowledgments

A Decoupling constants

The $\overline{\text{MS}}$-renormalized input parameters in eq. (2.1) are defined in a five-flavor version of QED×QCD, where particles with masses at the EW scale are integrated out. These are related to the corresponding quantities in SMEFT through decoupling constants. Adapting the notation of [42] to the current paper, we define these decoupling constants as

$$\overline{m}_f^{\text{SM}}(\mu) = \zeta_f(\mu, m_t, m_H, M_W, M_Z)\overline{m}_f(\mu),$$
$$\overline{e}^{\text{SM}}(\mu) = \zeta_e(\mu, m_t, m_H, M_W, M_Z)\overline{e}(\mu), \tag{A.1}$$

where the parameters with the superscript "SM" include all particles in the renormalization procedure. The decoupling constants are calculated as a double series in the operator expansion and in perturbation theory. In order to obtain the decay rate in terms of our chosen input parameters, the prescription is to first do the calculation in the full SM, then include terms involving decoupling constants to convert to the $\overline{\text{MS}}$-renormalized parameters in five-flavor QED×QCD. The exact relations are

$$\Gamma_f^{(4,0)} = \Gamma_{f,\text{SM}}^{(4,0)},$$
$$\Gamma_f^{(6,0)} = \Gamma_{f,\text{SM}}^{(6,0)},$$
$$\Gamma_f^{(4,1)} = \Gamma_{f,\text{SM}}^{(4,1)} + 2\Gamma_f^{(4,0)}\left(\zeta_f^{(4,1)} + \zeta_e^{(4,1)}\right),$$
$$\Gamma_f^{(6,1)} = \Gamma_{f,\text{SM}}^{(6,1)} + 2\Gamma_f^{(4,0)}\left(\zeta_f^{(6,1)} + \zeta_e^{(6,1)}\right) + 2\Gamma_f^{(6,0)}\zeta_f^{(4,1)}$$
$$+ \sqrt{2}C_{fH}\frac{\overline{v}^3}{\overline{m}_f}\Gamma_f^{(4,0)}\left(\zeta_f^{(4,1)} + \zeta_e^{(4,1)}\right), \tag{A.2}$$

where in all cases the expressions on the right-hand side are evaluated using the parameters in eq. (2.1) directly.

The decoupling constants for the electric charge and the b-quark mass were given in [42]. Here we give results for the decoupling constants for the τ-lepton and c-quark masses. In writing the results, it is convenient to split up tadpole and non-tadpole contributions. This separation is gauge dependent, although the sum of the terms and the decoupling constants themselves are gauge independent. We define the tadpole contributions in Feynman gauge, making use of the quantities

$$T_{\text{Feyn.}}^{(4)} = \frac{1}{32\pi^2\overline{v}}\left\{\hat{A}_0(M_W^2)(12M_W^2 + 2m_H^2) - 8M_W^4 + \hat{A}_0(M_Z^2)(6M_Z^2 + m_H^2)\right.$$
$$\left. - 4M_Z^4 + 3m_H^2\hat{A}_0(m_H^2) - 8N_c m_t^2\hat{A}_0(m_t^2)\right\},$$

$$T_{\text{Feyn.}}^{(6)} = \frac{\overline{v}}{32\pi^2}\left\{4\left(C_{H\Box} - \frac{C_{HD}}{4}\right)m_H^2\left(\hat{A}_0(m_H^2) - \hat{A}_0(M_W^2)\right) - 2C_{H\Box}m_H^2\hat{A}_0(M_Z^2)\right.$$
$$- 6C_H\overline{v}^2\hat{A}_0(m_H^2) + \left(24\hat{A}_0(M_W^2) - 16M_W^2\right)C_{HW}M_W^2 + \left(3\hat{A}_0(M_Z^2) - 2M_Z^2\right)M_Z^2$$
$$\times \left[C_{HD} + 4\left(C_{HW}\hat{c}_w^2 + C_{HB}\hat{s}_w^2 + C_{HWB}\hat{c}_w\hat{s}_w\right)\right] + 4N_c\sqrt{2}\overline{v}m_t C_{tH}\hat{A}_0(m_t^2)\right\}$$
$$+ \overline{v}^2\left[C_{H\Box} - \frac{C_{HD}}{4} + \frac{\hat{c}_w}{\hat{s}_w}\left(C_{HWB} + \frac{\hat{c}_w}{4\hat{s}_w}C_{HD}\right)\right]T_{\text{Feyn.}}^{(4)}, \tag{A.3}$$

where the superscript represents a contribution at mass dimension i and

$$\hat{A}_0(m^2) = m^2 + m^2 \log\left(\frac{\mu^2}{m^2}\right). \tag{A.4}$$

For the τ-lepton and charm-quark masses in the SM, the decoupling constants are

$$\zeta_{m_\tau}^{(4,1)} = \frac{1}{64\pi^2\overline{v}^2}\left\{58M_W^2 - 40M_W^2\hat{c}_w^2 - 21M_Z^2 + 4M_W^2\ln\left(\frac{\mu^2}{M_W^2}\right) + \left(72M_W^2\right.\right.$$
$$\left.\left. - 48M_W^2\hat{c}_w^2 - 22M_Z^2\right)\ln\left(\frac{\mu^2}{M_Z^2}\right)\right\} - \frac{1}{\overline{v}m_H^2}T_{\text{Feyn.}}^{(4)}, \tag{A.5}$$

$$\zeta_{m_c}^{(4,1)} = \frac{1}{576\pi^2\overline{v}^2}\left\{182M_W^2 - 160M_W^2\hat{c}_w^2 - 49M_Z^2 + 36M_W^2\ln\left(\frac{\mu^2}{M_W^2}\right) + 6\left(40M_W^2\right.\right.$$
$$\left.\left. - 32M_W^2\hat{c}_w^2 - 5M_Z^2\right)\ln\left(\frac{\mu^2}{M_Z^2}\right)\right\} - \frac{1}{\overline{v}m_H^2}T_{\text{Feyn.}}^{(4)}. \tag{A.6}$$

The dimension-6 contributions to the m_f decoupling constants can be written in the generic form

$$\zeta_{m_f}^{(6,1)} = -\frac{1}{m_H^2\overline{v}}T_{\text{Feyn.}}^{(6)} - \frac{1}{2m_H^2\overline{v}}\frac{\Gamma_f^{(6,0)}}{\Gamma_f^{(4,0)}}T_{\text{Feyn.}}^{(4)} + \zeta_{m_f,\text{ no-tad.}}^{(6,1)}, \tag{A.7}$$

$$\zeta_{m_f,\text{no-tad}}^{(6,1)} = \zeta_{m_f,\text{NL}}^{(6,1)} + \zeta_{m_f,\text{LH}}^{(6,1)}\log\left(\frac{\mu^2}{m_H^2}\right) + \zeta_{m_f,\text{LW}}^{(6,1)}\log\left(\frac{\mu^2}{M_W^2}\right) + \zeta_{m_f,\text{LZ}}^{(6,1)}\log\left(\frac{\mu^2}{M_Z^2}\right)$$
$$+ \zeta_{m_f,\text{Lt}}^{(6,1)}\log\left(\frac{\mu^2}{m_t^2}\right). \tag{A.8}$$

For $f = \tau$ we find

$$\zeta_{m_\tau,\text{NL}}^{(6,1)} = -\frac{1}{128\pi^2 M_W M_Z^3\hat{s}_w^3}\left\{-3C_{HD}M_W M_Z\hat{s}_w\left(6M_W^4 + 7M_Z^4 - 14M_W^2 M_Z^2\right)\right.$$
$$+ 12C_{HWB}M_W^2 M_Z^2\hat{s}_w^2\left(4M_W^2 - 3M_Z^2\right) + 2M_Z^2\hat{s}_w^3\left[\frac{6\sqrt{2}}{\hat{s}_w}\frac{\overline{ve}}{\overline{m}_\tau}C_{\tau W}M_W M_Z\right.$$
$$\times\left(2M_W^2 - M_Z^2\right) + 2\sqrt{2}\frac{\overline{ve}}{\overline{m}_\tau}C_{\tau B}M_Z^2\left(4M_W^2 - 3M_Z^2\right) + 8N_c\frac{m_t^3}{\overline{m}_\tau}C_{\underset{3333}{lequ}}^{(1)}M_W M_Z$$
$$+ \sqrt{2}\frac{\overline{v}}{\overline{m}_\tau}C_{\tau H}M_W M_Z\left(3m_H^2 + 2M_W^2 + M_Z^2\right) + 2M_W M_Z\left(\left(10M_W^2 - 7M_Z^2\right)C_{\underset{33}{Hl}}^{(1)}\right.$$
$$\left.\left.\left. + \left(16M_W^2 - 7M_Z^2\right)C_{\underset{33}{Hl}}^{(3)} + \left(10M_W^2 - 8M_Z^2\right)C_{H\tau}\right)\right]\right\}, \tag{A.9}$$

$$\zeta_{m_\tau,\text{LH}}^{(6,1)} = -\frac{3\sqrt{2}m_H^2}{64\pi^2}\frac{\overline{v}}{\overline{m}_\tau}C_{\tau H}, \tag{A.10}$$

$$\zeta_{m_\tau,\text{LW}}^{(6,1)} = \frac{\hat{c}_w^2}{32\pi^2 M_Z\hat{s}_w^3}\left\{4C_{HWB}M_W M_Z^2\hat{s}_w^2 + C_{HD}M_W^2 M_Z\hat{s}_w\right.$$
$$\left. - \sqrt{2}\frac{\overline{v}}{\overline{m}_\tau}M_Z^3\hat{s}_w^2\left(6C_{\tau W}\overline{e} + C_{\tau H}\hat{s}_w\right)\right\}, \tag{A.11}$$

– 22 –

$$
\begin{aligned}
\zeta_{m_\tau,\mathrm{LZ}}^{(6,1)} = & -\frac{1}{64\pi^2 M_W M_Z^3 \hat{s}_w^3} \Big\{ C_{HD} M_W M_Z \hat{s}_w \left(22 M_W^2 M_Z^2 - 12 M_W^4 - 11 M_Z^4 \right) \\
& + 4 C_{HWB} M_W^2 M_Z^2 \hat{s}_w^2 \left(6 M_W^2 - 7 M_Z^2 \right) + \sqrt{2}\,\frac{\overline{v}}{\overline{m}_\tau} C_{\tau H} M_W M_Z^5 \hat{s}_w^3 \\
& + 6\sqrt{2}\,\frac{\overline{ve}}{\overline{m}_\tau} M_Z^4 \hat{s}_w^2 \left(C_{\tau B} \hat{s}_w + C_{\tau W} \hat{c}_w \right) \left(4 M_W^2 - 3 M_Z^2 \right) \\
& - 24 \left(C_{Hl}^{(1)} + C_{Hl}^{(3)} \right) M_W M_Z^5 \hat{s}_w^5 + 12 C_{H\tau} M_W M_Z^3 \hat{s}_w^3 \left(2 M_W^2 - M_Z^2 \right) \Big\}, \quad (\mathrm{A}.12)
\end{aligned}
$$

$$
\zeta_{m_\tau,\mathrm{Lt}}^{(6,1)} = -\frac{N_c}{8\pi^2}\frac{m_t^3}{\overline{m}_\tau} C_{lequ}^{(1)}{}_{3333}. \quad (\mathrm{A}.13)
$$

For $f = c$ one finds

$$
\begin{aligned}
\zeta_{m_c,\mathrm{NL}}^{(6,1)} = & -\frac{1}{1152\pi^2 M_Z^3 \hat{s}_w^3} \Big\{ C_{HD} M_Z \hat{s}_w \left(98 M_W^2 M_Z^2 - 49 M_Z^4 - 22 M_W^4 \right) \\
& - 12 C_{HWB} M_W \left(17 M_Z^4 + 26 M_W^4 - 43 M_W^2 M_Z^2 \right) + 18\sqrt{2}\,\frac{\overline{v}}{\overline{m}_c} C_{cH} M_Z^3 \hat{s}_w^3 \\
& \times \left(3 m_H^2 + 2 M_W^2 + M_Z^2 \right) + 12 M_Z^3 \hat{s}_w^3 \Big[-\frac{\sqrt{2}}{\hat{c}_w}\frac{\overline{ev}}{\overline{m}_c} C_{cB} \left(8 M_W^2 - 5 M_Z^2 \right) \\
& + \frac{\sqrt{2}}{\hat{s}_w}\frac{\overline{ev}}{\overline{m}_c} C_{cW} \left(14 M_W^2 - 5 M_Z^2 \right) - C_{Hq}^{(1)}{}_{22} \left(20 M_W^2 - 11 M_Z^2 \right) \\
& + C_{Hq}^{(3)}{}_{22} \left(38 M_W^2 - 11 M_Z^2 \right) - 2 C_{Hc} \left(10 M_W^2 - 7 M_Z^2 \right) \\
& + 12 \frac{m_t^3}{\overline{m}_c} \left(C_{qu}^{(1)}{}_{2332} + C_F C_{qu}^{(8)}{}_{2332} \right) \Big] \Big\}, \quad (\mathrm{A}.14)
\end{aligned}
$$

$$
\zeta_{m_c,\mathrm{LH}}^{(6,1)} = -\frac{3\sqrt{2}\,m_H^2}{64\pi^2}\frac{\overline{v}}{\overline{m}_c} C_{cH}, \quad (\mathrm{A}.15)
$$

$$
\begin{aligned}
\zeta_{m_c,\mathrm{LW}}^{(6,1)} = & \frac{1}{32\pi^2 M_Z^5 \hat{s}_w^3} \Big\{ C_{HD} M_W^4 M_Z^3 \hat{s}_w + 4 C_{HWB} M_W^3 M_Z^4 \hat{s}_w^2 \\
& - \sqrt{2}\,\frac{\overline{v}}{\overline{m}_c} C_{cH} M_W^2 M_Z^5 \hat{s}_w^3 - 6\sqrt{2}\,\frac{\overline{ev}}{\overline{m}_c} C_{cW} M_W^2 M_Z^5 \hat{s}_w^2 \Big\}, \quad (\mathrm{A}.16)
\end{aligned}
$$

$$
\begin{aligned}
\zeta_{m_c,\mathrm{LZ}}^{(6,1)} = & \frac{1}{192\pi^2 M_W M_Z^5 \hat{s}_w^3} \Big\{ C_{HD} M_W M_Z^3 \hat{s}_w \left(8 M_W^4 + 5 M_Z^4 - 10 M_W^2 M_Z^2 \right) \\
& + 12 C_{HWB} M_W^2 M_Z^4 \hat{s}_w^2 \left(5 M_Z^2 - 4 M_W^2 \right) - 3\sqrt{2}\,\frac{\overline{v}}{\overline{m}_c} C_{cH} M_W M_Z^7 \hat{s}_w^3 \\
& + 6\sqrt{2}\,\frac{\overline{ev}}{\overline{m}_c} M_Z^6 \hat{s}_W^2 \left(8 M_W^2 - 5 M_Z^2 \right) \left(C_{cB} \hat{s}_w - C_{cW} \hat{c}_w \right) \\
& + 48 M_W M_Z^7 \hat{s}_w^5 \left(C_{Hq}^{(3)}{}_{22} - C_{Hq}^{(1)}{}_{22} \right) + 12 C_{Hc} M_W M_Z^5 \hat{s}_w^3 \left(4 M_W^2 - M_Z^2 \right) \Big\}, \quad (\mathrm{A}.17)
\end{aligned}
$$

$$
\zeta_{m_c,\mathrm{Lt}}^{(6,1)} = -\frac{1}{4\pi^2}\frac{m_t^3}{\overline{m}_c} \Big\{ C_{qu}^{(1)}{}_{2332} + C_F C_{qu}^{(8)}{}_{2332} \Big\}. \quad (\mathrm{A}.18)
$$

1 : X^3	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$

2 : H^6	
Q_H	$(H^\dagger H)^3$

3 : H^4D^2	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^*\left(H^\dagger D_\mu H\right)$

5 : $\psi^2 H^3$ + h.c.	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

4 : X^2H^2	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu}B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$
Q_{HWB}	$H^\dagger \sigma^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \sigma^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

6 : $\psi^2 XH$ + h.c.	
Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\sigma^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\sigma^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\sigma^I H W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)H\, B_{\mu\nu}$

7 : $\psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p\gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{l}_p\sigma^I\gamma^\mu l_r)$
Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p\gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{q}_p\sigma^I\gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p\gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p\gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p\gamma^\mu d_r)$

8 : $(\bar{L}L)(\bar{L}L)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\sigma^I q_r)(\bar{q}_s\gamma^\mu\sigma^I q_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\sigma^I l_r)(\bar{q}_s\gamma^\mu\sigma^I q_t)$

8 : $(\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$
Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$
Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$

8 : $(\bar{L}L)(\bar{R}R)$	
Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

8 : $(\bar{L}R)(\bar{R}L)$ + h.c.	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

8 : $(\bar{L}R)(\bar{L}R)$ + h.c.	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\epsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)$

Table 3. The 59 independent baryon number conserving dimension-6 operators built from Standard Model fields, in the notation of [59]. The subscripts p, r, s, t are flavor indices, and σ^I are Pauli matrices.

References

[1] ATLAS collaboration, *Observation of $H \to b\bar{b}$ decays and VH production with the ATLAS detector*, *Phys. Lett. B* **786** (2018) 59 [`arXiv:1808.08238`] [INSPIRE].

[2] CMS collaboration, *Observation of Higgs boson decay to bottom quarks*, *Phys. Rev. Lett.* **121** (2018) 121801 [`arXiv:1808.08242`] [INSPIRE].

[3] CMS collaboration, *Observation of the Higgs boson decay to a pair of τ leptons with the CMS detector*, *Phys. Lett. B* **779** (2018) 283 [`arXiv:1708.00373`] [INSPIRE].

[4] ATLAS collaboration, *Cross-section measurements of the Higgs boson decaying into a pair of τ-leptons in proton-proton collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector*, *Phys. Rev. D* **99** (2019) 072001 [`arXiv:1811.08856`] [INSPIRE].

[5] ATLAS collaboration, *Search for the dimuon decay of the Higgs boson in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector*, *Phys. Rev. Lett.* **119** (2017) 051802 [`arXiv:1705.04582`] [INSPIRE].

[6] CMS collaboration, *Search for the Higgs boson decaying to two muons in proton-proton collisions at $\sqrt{s} = 13$ TeV*, *Phys. Rev. Lett.* **122** (2019) 021801 [`arXiv:1807.06325`] [INSPIRE].

[7] ATLAS collaboration, *Search for the Decay of the Higgs Boson to Charm Quarks with the ATLAS Experiment*, *Phys. Rev. Lett.* **120** (2018) 211802 [`arXiv:1802.04329`] [INSPIRE].

[8] CMS collaboration, *A search for the standard model Higgs boson decaying to charm quarks*, *JHEP* **03** (2020) 131 [`arXiv:1912.01662`] [INSPIRE].

[9] M. Ruan, *Higgs Measurement at e^+e^- Circular Colliders*, *Nucl. Part. Phys. Proc.* **273-275** (2016) 857 [`arXiv:1411.5606`] [INSPIRE].

[10] K. Fujii et al., *Physics Case for the International Linear Collider*, [`arXiv:1506.05992`] [INSPIRE].

[11] H. Abramowicz et al., *Higgs physics at the CLIC electron-positron linear collider*, *Eur. Phys. J. C* **77** (2017) 475 [`arXiv:1608.07538`] [INSPIRE].

[12] J. Ellis, P. Roloff, V. Sanz and T. You, *Dimension-6 Operator Analysis of the CLIC Sensitivity to New Physics*, *JHEP* **05** (2017) 096 [`arXiv:1701.04804`] [INSPIRE].

[13] G. Durieux, C. Grojean, J. Gu and K. Wang, *The leptonic future of the Higgs*, *JHEP* **09** (2017) 014 [`arXiv:1704.02333`] [INSPIRE].

[14] G.M. Pruna and A. Signer, *The $\mu \to e\gamma$ decay in a systematic effective field theory approach with dimension 6 operators*, *JHEP* **10** (2014) 014 [`arXiv:1408.3565`] [INSPIRE].

[15] C. Hartmann and M. Trott, *On one-loop corrections in the standard model effective field theory; the $\Gamma(h \to \gamma\gamma)$ case*, *JHEP* **07** (2015) 151 [`arXiv:1505.02646`] [INSPIRE].

[16] C. Hartmann and M. Trott, *Higgs Decay to Two Photons at One Loop in the Standard Model Effective Field Theory*, *Phys. Rev. Lett.* **115** (2015) 191801 [`arXiv:1507.03568`] [INSPIRE].

[17] C. Hartmann, W. Shepherd and M. Trott, *The Z decay width in the SMEFT: y_t and λ corrections at one loop*, *JHEP* **03** (2017) 060 [arXiv:1611.09879] [INSPIRE].

[18] M. Ghezzi, R. Gomez-Ambrosio, G. Passarino and S. Uccirati, *NLO Higgs effective field theory and κ-framework*, *JHEP* **07** (2015) 175 [arXiv:1505.03706] [INSPIRE].

[19] S. Dawson and P.P. Giardino, *Higgs decays to ZZ and $Z\gamma$ in the standard model effective field theory: An NLO analysis*, *Phys. Rev. D* **97** (2018) 093003 [arXiv:1801.01136] [INSPIRE].

[20] S. Dawson and P.P. Giardino, *Electroweak corrections to Higgs boson decays to $\gamma\gamma$ and W^+W^- in standard model EFT*, *Phys. Rev. D* **98** (2018) 095005 [arXiv:1807.11504] [INSPIRE].

[21] S. Dawson and A. Ismail, *Standard model EFT corrections to Z boson decays*, *Phys. Rev. D* **98** (2018) 093003 [arXiv:1808.05948] [INSPIRE].

[22] S. Dawson and P.P. Giardino, *Electroweak and QCD corrections to Z and W pole observables in the standard model EFT*, *Phys. Rev. D* **101** (2020) 013001 [arXiv:1909.02000] [INSPIRE].

[23] N. Deutschmann, C. Duhr, F. Maltoni and E. Vryonidou, *Gluon-fusion Higgs production in the Standard Model Effective Field Theory*, *JHEP* **12** (2017) 063 [*Erratum ibid.* **02** (2018) 159] [arXiv:1708.00460] [INSPIRE].

[24] M. Grazzini, A. Ilnicka and M. Spira, *Higgs boson production at large transverse momentum within the SMEFT: analytical results*, *Eur. Phys. J. C* **78** (2018) 808 [arXiv:1806.08832] [INSPIRE].

[25] R. Boughezal, C.-Y. Chen, F. Petriello and D. Wiegand, *Top quark decay at next-to-leading order in the Standard Model Effective Field Theory*, *Phys. Rev. D* **100** (2019) 056023 [arXiv:1907.00997] [INSPIRE].

[26] A. Dedes, K. Suxho and L. Trifyllis, *The decay $h \to Z\gamma$ in the Standard-Model Effective Field Theory*, *JHEP* **06** (2019) 115 [arXiv:1903.12046] [INSPIRE].

[27] E. Vryonidou and C. Zhang, *Dimension-six electroweak top-loop effects in Higgs production and decay*, *JHEP* **08** (2018) 036 [arXiv:1804.09766] [INSPIRE].

[28] J. Baglio, S. Dawson and I.M. Lewis, *An NLO QCD effective field theory analysis of W^+W^- production at the LHC including fermionic operators*, *Phys. Rev. D* **96** (2017) 073003 [arXiv:1708.03332] [INSPIRE].

[29] S. Dawson, P.P. Giardino and A. Ismail, *Standard model EFT and the Drell-Yan process at high energy*, *Phys. Rev. D* **99** (2019) 035044 [arXiv:1811.12260] [INSPIRE].

[30] C. Degrande, B. Fuks, K. Mawatari, K. Mimasu and V. Sanz, *Electroweak Higgs boson production in the standard model effective field theory beyond leading order in QCD*, *Eur. Phys. J. C* **77** (2017) 262 [arXiv:1609.04833] [INSPIRE].

[31] C. Zhang, *Effective field theory approach to top-quark decay at next-to-leading order in QCD*, *Phys. Rev. D* **90** (2014) 014008 [arXiv:1404.1264] [INSPIRE].

[32] C. Zhang and F. Maltoni, *Top-quark decay into Higgs boson and a light quark at next-to-leading order in QCD*, *Phys. Rev. D* **88** (2013) 054005 [arXiv:1305.7386] [INSPIRE].

[33] C. Zhang, *Single Top Production at Next-to-Leading Order in the Standard Model Effective Field Theory*, *Phys. Rev. Lett.* **116** (2016) 162002 [arXiv:1601.06163] [INSPIRE].

[34] C. Degrande, F. Maltoni, K. Mimasu, E. Vryonidou and C. Zhang, *Single-top associated production with a Z or H boson at the LHC: the SMEFT interpretation*, *JHEP* **10** (2018) 005 [arXiv:1804.07773] [INSPIRE].

[35] O. Bessidskaia Bylund, F. Maltoni, I. Tsinikos, E. Vryonidou and C. Zhang, *Probing top quark neutral couplings in the Standard Model Effective Field Theory at NLO in QCD*, *JHEP* **05** (2016) 052 [arXiv:1601.08193] [INSPIRE].

[36] F. Maltoni, E. Vryonidou and C. Zhang, *Higgs production in association with a top-antitop pair in the Standard Model Effective Field Theory at NLO in QCD*, *JHEP* **10** (2016) 123 [arXiv:1607.05330] [INSPIRE].

[37] R. Grober, M. Muhlleitner, M. Spira and J. Streicher, *NLO QCD Corrections to Higgs Pair Production including Dimension-6 Operators*, *JHEP* **09** (2015) 092 [arXiv:1504.06577] [INSPIRE].

[38] T. Neumann and Z.E. Sullivan, *Off-Shell Single-Top-Quark Production in the Standard Model Effective Field Theory*, *JHEP* **06** (2019) 022 [arXiv:1903.11023] [INSPIRE].

[39] D. de Florian, I. Fabre and J. Mazzitelli, *Higgs boson pair production at NNLO in QCD including dimension 6 operators*, *JHEP* **10** (2017) 215 [arXiv:1704.05700] [INSPIRE].

[40] A. Crivellin, S. Najjari and J. Rosiek, *Lepton Flavor Violation in the Standard Model with general Dimension-Six Operators*, *JHEP* **04** (2014) 167 [arXiv:1312.0634] [INSPIRE].

[41] A. Dedes, M. Paraskevas, J. Rosiek, K. Suxho and L. Trifyllis, *The decay $h \to \gamma\gamma$ in the Standard-Model Effective Field Theory*, *JHEP* **08** (2018) 103 [arXiv:1805.00302] [INSPIRE].

[42] J.M. Cullen, B.D. Pecjak and D.J. Scott, *NLO corrections to $h \to b\bar{b}$ decay in SMEFT*, *JHEP* **08** (2019) 173 [arXiv:1904.06358] [INSPIRE].

[43] R. Gauld, B.D. Pecjak and D.J. Scott, *One-loop corrections to $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ decays in the Standard Model Dimension-6 EFT: four-fermion operators and the large-m_t limit*, *JHEP* **05** (2016) 080 [arXiv:1512.02508] [INSPIRE].

[44] R. Gauld, B.D. Pecjak and D.J. Scott, *QCD radiative corrections for $h \to b\bar{b}$ in the Standard Model Dimension-6 EFT*, *Phys. Rev. D* **94** (2016) 074045 [arXiv:1607.06354] [INSPIRE].

[45] J. Fleischer and F. Jegerlehner, *Radiative Corrections to Higgs Decays in the Extended Weinberg-Salam Model*, *Phys. Rev. D* **23** (1981) 2001 [INSPIRE].

[46] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, *Dimension-Six Terms in the Standard Model Lagrangian*, *JHEP* **10** (2010) 085 [arXiv:1008.4884] [INSPIRE].

[47] B.A. Kniehl, *Radiative corrections for $H \to f\bar{f}(\gamma)$ in the standard model*, *Nucl. Phys. B* **376** (1992) 3 [INSPIRE].

[48] I. Brivio, Y. Jiang and M. Trott, *The SMEFTsim package, theory and tools*, *JHEP* **12** (2017) 070 [arXiv:1709.06492] [INSPIRE].

[49] A. Denner and S. Dittmaier, *Electroweak Radiative Corrections for Collider Physics*, *Phys. Rept.* **864** (2020) 1 [arXiv:1912.06823] [INSPIRE].

[50] A. Alloul, N.D. Christensen, C. Degrande, C. Duhr and B. Fuks, *FeynRules 2.0 — A complete toolbox for tree-level phenomenology*, *Comput. Phys. Commun.* **185** (2014) 2250 [arXiv:1310.1921] [INSPIRE].

[51] T. Hahn, *Generating Feynman diagrams and amplitudes with FeynArts 3*, *Comput. Phys. Commun.* **140** (2001) 418 [hep-ph/0012260] [INSPIRE].

[52] T. Hahn and M. Pérez-Victoria, *Automatized one loop calculations in four-dimensions and D-dimensions*, *Comput. Phys. Commun.* **118** (1999) 153 [hep-ph/9807565] [INSPIRE].

[53] H.H. Patel, *Package-X: A Mathematica package for the analytic calculation of one-loop integrals*, *Comput. Phys. Commun.* **197** (2015) 276 [arXiv:1503.01469] [INSPIRE].

[54] A. Dedes, W. Materkowska, M. Paraskevas, J. Rosiek and K. Suxho, *Feynman rules for the Standard Model Effective Field Theory in R_ξ-gauges*, *JHEP* **06** (2017) 143 [arXiv:1704.03888] [INSPIRE].

[55] B. Grinstein, *TASI-2013 Lectures on Flavor Physics*, in *Theoretical Advanced Study Institute in Elementary Particle Physics: Particle Physics: The Higgs Boson and Beyond*, (2015) [arXiv:1501.05283] [INSPIRE].

[56] R. Alonso, E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators III: Gauge Coupling Dependence and Phenomenology*, *JHEP* **04** (2014) 159 [arXiv:1312.2014] [INSPIRE].

[57] K.G. Chetyrkin, B.A. Kniehl and M. Steinhauser, *Virtual top quark effects on the $H \to b\bar{b}$ decay at next-to-leading order in QCD*, *Phys. Rev. Lett.* **78** (1997) 594 [hep-ph/9610456] [INSPIRE].

[58] S.A. Larin, T. van Ritbergen and J.A.M. Vermaseren, *The Large top quark mass expansion for Higgs boson decays into bottom quarks and into gluons*, *Phys. Lett. B* **362** (1995) 134 [hep-ph/9506465] [INSPIRE].

[59] E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators I: Formalism and lambda Dependence*, *JHEP* **10** (2013) 087 [arXiv:1308.2627] [INSPIRE].

[60] E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators II: Yukawa Dependence*, *JHEP* **01** (2014) 035 [arXiv:1310.4838] [INSPIRE].

[61] A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, *DsixTools: The Standard Model Effective Field Theory Toolkit*, *Eur. Phys. J. C* **77** (2017) 405 [arXiv:1704.04504] [INSPIRE].

[62] R. Akhoury, H. Wang and O.I. Yakovlev, *Resummation of large QCD logarithms in $H \to \gamma\gamma$*, *Phys. Rev. D* **64** (2001) 113008 [hep-ph/0102105] [INSPIRE].

[63] T. Liu and A.A. Penin, *High-Energy Limit of QCD beyond the Sudakov Approximation*, *Phys. Rev. Lett.* **119** (2017) 262001 [arXiv:1709.01092] [INSPIRE].

[64] T. Liu and A. Penin, *High-Energy Limit of Mass-Suppressed Amplitudes in Gauge Theories*, *JHEP* **11** (2018) 158 [arXiv:1809.04950] [INSPIRE].

[65] Z.L. Liu and M. Neubert, *Factorization at subleading power and endpoint-divergent convolutions in $h \to \gamma\gamma$ decay*, *JHEP* **04** (2020) 033 [arXiv:1912.08818] [INSPIRE].

[66] J. Wang, *Resummation of double logarithms in loop-induced processes with effective field theory*, arXiv:1912.09920 [INSPIRE].

[67] S. Dawson, S. Homiller and S.D. Lane, *Putting standard model EFT fits to work*, *Phys. Rev. D* **102** (2020) 055012 [arXiv:2007.01296] [INSPIRE].