Bilevel Programming for Manufacturers Operating in an Omnichannel Retailing Environment

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Abstract: This paper presents a joint production, pricing and inventory control problem that can help manufacturers manage revenue in an omnichannel environment. The paper puts forward a multi-period Stackelberg game between a manufacturer and a retailer, assuming the former to be the leader. The leader–follower game is formulated as a mixed-integer non-linear bilevel optimization problem wherein both players seek to maximize their respective profits. The manuscript envisions an omnichannel retailing environment where online, offline, direct and drop-shipping channels coexist. It then investigates how enabling showrooming and webrooming on the drop-shipping channel account can affect supply chain profitabilities. Analyses suggest that when customers are aided with showrooming and webrooming services and are allowed to place drop-shipping orders from retailers' stores, additional profit can be generated in the supply chain. To solve the proposed bilevel optimization problem, we investigated a) single-level reduction using Karush–Kuhn–Tucker (KKT) conditions and b) hierarchical optimization technique based on Simulated Annealing and Randomized Decomposition Solver. To assess the efficacy of the solution techniques, a comparative analysis was carried out. Thereafter, with the aid of numerical experiments and sensitivity analyses, the paper draws key managerial insights for manufacturers.

Keywords: Retailing, price optimization, omnichannel, bilevel programming, Stackelberg game

Managerial Relevance Statement: The manuscript is relevant to the manufacturers who operate direct channels in omnichannel settings. The paper highlights that manufacturers often lack pricing and revenue management solutions in their operations; such solutions would prove beneficial in mitigating the impact of demand and production mismatch. Therefore, the paper proposed joint production, pricing and inventory control solution for the manufacturers of seasonal products.

The paper asserts drop-shipping service as a means of cooperation between manufacturers and retailers who otherwise are non-cooperative decision-makers in decentralized supply chains. It numerically demonstrates that drop-shipping can not only serve as an omnichannel service but can also increase supply chain's profitabilities. However, a drop-shipping channel can also cannibalize the direct channel.

Numerical experiments suggest that having a drop-shipping channel can bring positive changes to supply chain profitabilities in the case when production capacity is not sufficient. Whereas, in the case when production capacity is more, drop-shipping channel may not bring any significant change in the supply chain profitabilities. Finally, the results suggest that an increase in popularity of a drop-shipping channel can benefit the retailer more than the manufacturer.

I. INTRODUCTION

It is now common for retailers (particularly in fashion apparel and consumer electronics) to use pricing and revenue management. Revenue management helps influence demand and mitigate the effect of limited supply, limited inventory and/or finite selling seasons [1]. To do so, the point-of-sales data is intelligently processed to understand consumers' price sensitivity, preferences, and market trends. These insights are then used to actively influence consumer purchase decisions by changing the posted price of the product. Such active demand management (in contrast to passive management of inventory) reduces demand and supply mismatch, thereby yielding a better return on investment [2].

In 2020, when the entire world was dealing with the Covid-19 pandemic, Target Corporation (an American retailer) was able to grow its sales by 145% thanks to its newly adopted omnichannel fulfillment strategy [3]. In India, brands like Pepperfry, Zivame, Van Heusen, and Adidas have already introduced their omnichannel strategies. For example, Adidas' endless aisle and virtual footwear wall allow customers to search and order items that are not in stock at the physical stores [4]. A 2020 report by McKinsey states that companies that can provide omnichannel personalization can achieve 5–15 % revenue growth [5]. The report also suggests that because more than 80% of total sales occur at physical locations, providing better offline personalization is therefore vital for growth. In the following paragraphs, we discuss how omnichannel retailing became so ubiquitous.

Thanks to the internet, information technology, and competitive third-party logistics services, in 2012 online retailing became the fastest-growing retail sector in the United States [6]. Facing competition from e-commerce giants like Amazon, traditional retailers like Walmart, Costco Wholesale, Tesco and Metro added online channels to their channel mix. Likewise, e-retailers like Fab.com, Gilt.com and JD.com also transformed themselves into multi-channel retailers by incorporating brick-and-mortar stores [7]. At the same time, manufacturers like Dell, Nike and Apple started operating their own direct online channels. Although direct channels were initially for informational and sales support purposes only [8], they have become a means to balance the increasing power of multi-channel retailers and to improve supply chain performance [7].

Multi-channel retailing emerged as channels were progressively added to retailing portfolios. However, the abundance of retail channels and the omnipresence of the internet via smartphone also allowed consumers to search multiple channels concurrently. As per a report by McKinsey, in 2012 alone, about 20% of customers used mobile phones for product research—of whom about 40% did so while they were in the store [8]. Customers intentionally moved from one channel to the other to maximize their overall utility [9]. Specifically, when a customer moves from a physical store to an e-commerce website, that behavior is known as “showrooming” [6]; in contrast, when a customer moves from an online channel to a physical store, it is known as “webrooming” [7]. These customer activities collapsed the boundaries between multiple retail channels. A price change in one of the retail channels now affects the demand generated at all other consumer touchpoints, a phenomenon commonly known as “cross-channel demand substitution” [10]. Moreover, realizing the importance of providing a superior customer experience, many multi-channel retailers started deliberately integrating their retail channels—what we know as “omnichannel retailing.” Although different firms can have different omnichannel strategies, the underlying premise of omnichannel retailing remains the same, which is to integrate multiple available channels and thereby provide a superior customer experience.

A. Motivation

Observing the need for revenue management in omnichannel retailing, researchers recommended that retailers upgrade their pricing solutions to incorporate multi-channel interdependencies and cross-channel demand substitution [11], [12]. The need for price optimization and revenue management arises whenever a fixed and perishable set of resources are sold to a population that is sensitive to prices, and since outlets of both retailer and manufacturer face this problem, they are both free to incorporate pricing and revenue management solutions into their businesses.

Manufacturers who are actively adopting direct channels lack pricing and revenue management solutions in their operations; such solutions would prove beneficial in mitigating the impact of demand and production mismatch. It is not uncommon for a product to have short selling seasons [13] in which manufacturers are constrained by their limited production capacities [14]. Such industries include clothing, pharmaceuticals, toys, gifts, and firecrackers. During a selling season, there is a certain starting inventory as well as a limited production capacity, as production cannot be easily ramped up or down. This challenge calls for active demand management by these manufacturers to achieve a better return on investment. Furthermore, a revenue management solution must consider cross-channel demand substitution if the manufacturer is operating in an omnichannel retailing environment. The objective of this paper is to develop a joint production planning, pricing and inventory control solution for a manufacturer operating a direct channel in such an environment. Furthermore, the paper draws researchers' attention toward a relatively unexplored area where pricing and revenue management solution must be developed for manufacturers considering demand substitution and pricing competition from retailers' online/offline channels.

B. Stackelberg Game

While developing a pricing and revenue management solution for manufacturers, we must assume that the retailer will also be using price optimization to maximize their profit. During a selling season, a retailer seeks to maximize its profit by effectively pricing the product while considering the demand, wholesale price and fulfillment costs. Essentially, manufacturers must also price the product depending on anticipated demand, available inventory and limited production capacities [15]. Due to cross-channel demand substitution, a price change by the retailer can also affect the demand in the manufacturer's channels, and vice-versa [10]. Usually, revenue management models ignore channel substitution and assume that price affects demand only in their respective channels. However, cross-channel demand substitution cannot be ignored in omnichannel retailing. Since customers can easily evaluate prices and availability of the product in multiple available channels, neither retailer not manufacturer can set their price independently. This leads to a multi-period leader–follower Stackelberg game between manufacturer and retailer, wherein the manufacturer is traditionally seen as a leader.

Here, we present some of the research where similar scenarios are documented. In [14], the authors presented a bilevel problem where the retailer determines selling prices and advertising expenditures while the manufacturer optimizes wholesale price and cycle time for the products. Since the demand was nonlinearly influenced by pricing and advertising expenditures, the formulated problem was solved meta-heuristically using the imperialist competitive algorithm. The authors proposed a modified assimilation strategy along with a diversification approach to improve the performance of the given algorithm. The focus of the paper was on the methodology rather than on managerial insights. Authors in [16] investigated a decentralized production–distribution supply chain problem and presented a nonlinear bi-level programming problem that reflected Stackelberg games between manufacturers and distributors. In that paper, the authors used a hierarchical solution algorithm that combined genetic algorithm and particle swarm optimization to solve the formulation. The authors concluded that a leader always earns more than the follower and highlighted the finding that cooperation between the players can increase the supply chain profit. However, the paper did not explain how cooperation can be formulated and solved. In [17], the authors presented a competitive supply-chain network design problem. Authors assumed that rivals enter a new market and design their network to establish distribution centers. Both players sought to maximize their market share and minimize their costs using bilevel programming. The formulated problem was a nonconvex mixed-integer nonlinear problem that was convexified and solved using a two-step heuristic to attain global optimal. Authors found that when a rival "supply chain" is expected to enter a new market, the existing "supply chain" can structure itself non-cooperatively based on plant locations. The paper did not advise complete cooperation in the supply chain to generate better returns. Authors in [18] formulated and solved a problem of joint production planning, retailer selection and pricing for a manufacturer constrained by emissions' regulatory limits. Authors developed a mixed nonlinear bilevel programming problem where retailers' and manufacturer's profits were maximized. They solved the problem using a nested genetic algorithm, assuming the manufacturer as leader and retailers as followers in the multi-period Stackelberg game. They found that due to carbon emission regulations, a manufacturer will be obligated to select fewer (but the most profitable) retailers. Retailers, on the other hand, would increase the advertising expenditure and reduce their retail price if they want to be selected in the manufacturers' supply chain.

Many of these papers dealt with price competition between manufacturers and retailers when both players seek to maximize their respective profits. However, these papers were not tuned to deal with omnichannel retailing or situations in which multiple retail outlets are owned by both players. Our review suggests that there is a need to investigate aspects of price competition between manufacturer and retailer when the former operates a direct channel. Nevertheless, other aspects of the supply chain, such as production planning and inventory management, do exist as discussed in the previous paragraph.

Next, we discuss bilevel programming, a common theme that appeared in all these papers.

C. Bilevel Programming

Bilevel programming problems can be realized in several real-life scenarios, including supply chain coordination [14], toll optimization [19], the defense sector [20], production planning [21], facility location optimization [22], dynamic facility layout problems [23], road network design problems [24], bus rapid transit system design [25], and real-time pricing for smart grids [26].

Bilevel programming is employed to model decentralized management problem where two non-cooperative decision-makers are in a hierarchical structure [16]. Bilevel optimization problems are essentially a special case of multilevel optimization and are very closely related to Stackelberg's leader–follower games [27]. The optimization involves solving two problems simultaneously, where one of the problems acts as a constraint to the other [28]. It contains two types of decision variables: a) upper-level (leader's) decision variables x where $x \in X \subseteq R^n$ and b) lower-level (follower's) decision variables y where $y \in Y \subseteq R^m$. In general, such problems are formulated as follows:

$$\begin{aligned} &\max_{x\in X, y\in Y} \quad F(x,y) \\ &\text{subject to:} \\ &\qquad y \in \arg\max_{y\in Y}\left\{ f(x,y) : g_j(x,y) \geq 0,\ j=1,..,J \right\}, \\ &\qquad G_i(x,y) \geq 0,\ i=1,..,I \end{aligned}$$

$F(x,y)$ and $f(x,y)$ are the leader's and the follower's objective functions, respectively. $G_i(x,y) \geq 0$ for $i = 1,..,I$ and $g_j(x,y) \geq 0$ for $j = 1,..,J$ are the constraints for the upper-level and the lower-level optimization problems, respectively. The literature suggests that these problems are mostly solved using one of two approaches: a) single-level reduction technique and b) hierarchical optimization technique. For a detailed review of the solution techniques, readers are requested to refer to [29].

D. Contributions

The paper makes the following contributions to the literature. First, it presents a joint production planning, pricing and inventory control problem for manufacturers operating in an omnichannel retailing environment. Second, this paper formulates a bilevel price optimization problem where a make-to-stock manufacturer operates a direct online channel and competes with retailer's online and offline channels. Both the manufacturer and the retailer seek to maximize their respective profits before the season/inventory ends (assuming price and lead time-sensitive demand, cross-channel demand substitution and limited production capacity). Third, our model investigates the possibility of collaboration between the manufacturer and the retailer through a drop-shipping channel. Drop-shipping is considered as an omnichannel retailing service [30] where the retailer and the manufacturer share the profit earned through drop-shipping fulfillments. To solve the proposed problem, this paper evaluates several solution techniques including exact and heuristic approaches. Finally, based on the results obtained through numerical experiments, we generated some managerial insights for the manufacturer, specifically vis-à-vis drop-shipping channel.

The rest of the paper is structured as follows. In Section II, the problem scenario is described in greater detail; Section III presents the notations and formulations for the bilevel optimization problem. Section IV discusses and evaluates various solution techniques. Section V reports the numerical experiments and presents the results obtained using sensitivity analyses. Section VI discusses some of the managerial insights and outlines the scope for future research. Finally, Section VII concludes this paper.

II. PROBLEM DESCRIPTION

For this research, we consider that there is a make-to-stock manufacturer that is interested in maximizing the profit from the sale of its product. The manufacturer produces seasonal goods well in advance and sells them during the appropriate season. It can carry out production during the selling season, but production capacity during the selling season is constrained [15]. The manufacturer sells the products via a traditional retailer, i.e., a multi-location online-offline retailer like Walmart. Additionally, it also owns an online direct channel. In collaboration with the retailer, there is a provision to incorporate omnichannel retailing by providing the customers with additional drop-shipping alternative [30]. A customer can place an order directly to the manufacturer through any of the retailer-owned stores (online or offline). The manufacturer fulfills both direct-channel and drop-shipping orders with the help of a third-party logistics (3PL) provider. A vendor-managed inventory (VMI) model has been adopted as it enables the manufacturer to achieve better production planning by retrieving the downstream market information. VMI also helps retailers improve their fill rate and decrease inventory stock-outs. Therefore, it is assumed that inventories at the retailer-owned stores are managed by the manufacturer.

This problem is further elaborated with the help of Fig. 1. In this figure, it is shown that the market contains two zones (although there can be any number of zones). A customer can place an order through any of the four available channels present in a zone. They can buy the product directly from the store, place an online order through the retailer's e-commerce website, or place a drop-shipping order to the manufacturer through either of the retailer's store or online channels. When an order is routed through either of the retailer's channels, the retailer receives a share of the profit made by the manufacturer. A customer can also order the product through the manufacturer's direct channel, thus bypassing the retailer altogether. If the retailer receives an online order from a customer located in zone z, the product is delivered within a day using inventory available in the store located in zone z. However, if an order is placed to the manufacturer through either drop-shipping or direct channel, it is assumed that the product will be delivered after a certain delivery lead time. "Delivery lead time" is the time gap between the placement of an online order and the actual delivery of the product. In both drop-shipping and direct channel, delivery lead time can influence overall demand; therefore, demand is considered to be price- and time-sensitive. Nevertheless, it is assumed that the manufacturer can expedite the deliveries; however, expediting a delivery increases the transportation cost and decreases the profit margin for the manufacturer.

In summary, the problem considers that the manufacturer controls the wholesale price, the direct-channel and the drop-shipping prices, the production quantity, inventory at the retailer's stores across all the zones, and fulfillments through direct and drop-shipping channels. Additionally, the manufacturer can decide when to expedite the fulfillments of direct and drop-shipping orders. However, in order to improve the delivery lead times, the manufacturer must pay a relatively higher price to the 3PL partner.

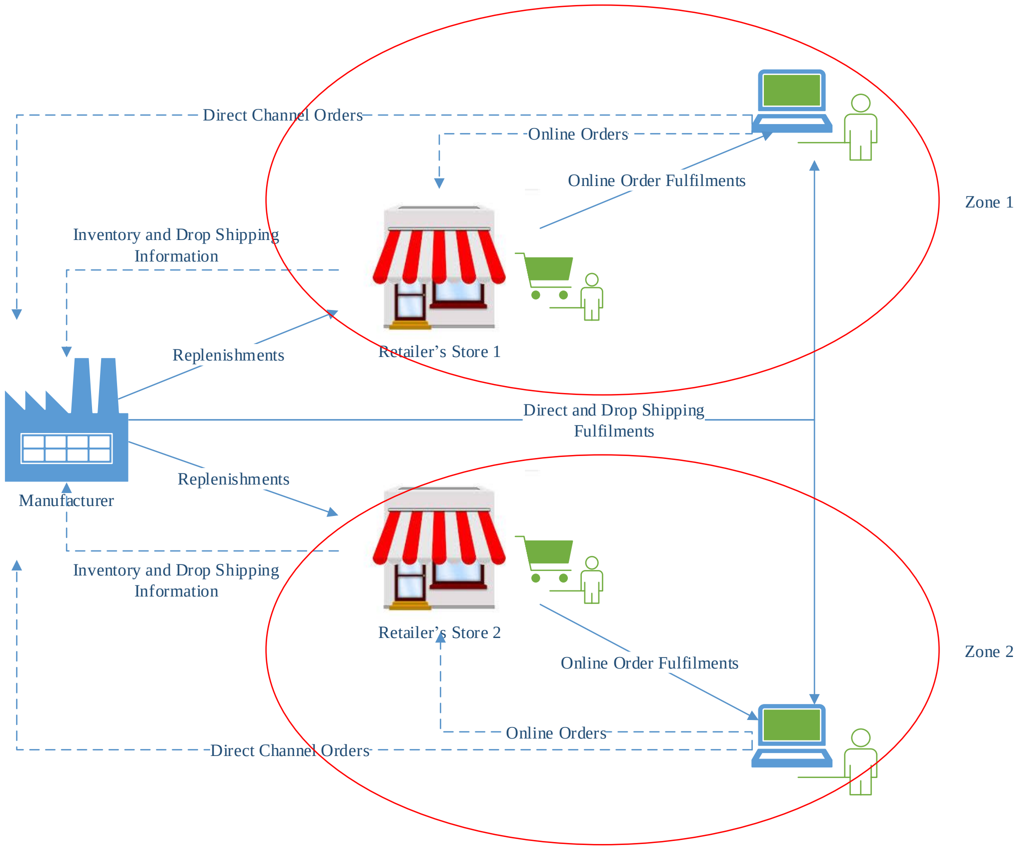

Fig. 1: Flow of information and the product in omnichannel retailing considered in this paper.

The retailer determines the online and offline prices, as well as the fulfillments through respective channels. As a business rule, at a given point in time, the retailer's online channel should offer a uniform price irrespective of the zone. But there can be a price difference between individual stores and online channel. Similarly, the manufacturer follows price-matching in the direct and drop-shipping channels at a given point in time. The wholesale price and retailer's share in the profit generated from the drop-shipping channel is fixed at the start of the selling season and remains so over the entire season.

III. MODEL FORMULATION

A. Notations

Sets:

Z: all market zones

T: all selling periods

Ω: the set of all prices

Indices:

z: index for a market zone, $z \in Z$

t: index for periods, $t \in T$

Input Parameters:

c_m: per-unit manufacturing cost of the product

M_z^t: size of the market in market zone z and time t

c_z: transportation cost for a unit of product from manufacturer to retailer's store in market zone z

c_z^f: fixed cost of replenishments of the retailer's store in market zone z, borne by the manufacturer

h_z: holding cost per unit time per unit product in the store located in market zone z

h_m: holding cost per unit time per unit product in the manufacturer's warehouse

c_{lz}: price charged by the 3PL partner for delivery to a customer in zone z

c_{oz}: least incurred fulfillment costs when the delivery lead time is very long for zone z

γ_z: constant representing the increase in cost for expediting delivery of an order

c_{erz}: cost of online demand fulfillment from the store to a customer in market zone z

α_{bz}: intrinsic attraction towards the product in retailer's store channel in zone z

β_{bz}^p: price sensitivity towards the product in retailer's store channel in zone z

α_{erz}: intrinsic attraction towards the product in retailer's online channel in zone z

β_{erz}^p: price sensitivity towards the product in retailer's online channel in zone z

α_{emz}: intrinsic attraction towards the product in manufacturer's direct channel in zone z

β_{emz}^p: price sensitivity towards the product in manufacturer's direct channel in zone z

β_{emz}^l: delivery lead-time sensitivity towards the product in manufacturer's direct channel in zone z

α_{bmz}: intrinsic attraction towards the product in the drop-shipping channel in zone z

β_{bmz}^p: price sensitivity towards the product in the drop-shipping channel in zone z

β_{bmz}^l: delivery lead-time sensitivity towards the product in the drop-shipping channel in zone z

Discretionary decision variables:

q_m: represents allocated production capacity per unit time t

W: wholesale price set by the manufacturer at the start of the selling season

θ: retailer's share in the profit earned through the drop-shipping demand fulfillments

Upper-level decision variables:

q_m^t: production during the period t where $q_m^t \leq q_m$

B_z^t: binary decision variable for the replenishment of the retailer's store in zone z at time t

l_z^t: delivery lead time for demand fulfillments in market zone z and time t

S_{emz}^t: total direct channel demand fulfillments by the manufacturer in zone z and time t

S_{bmz}^t: total drop-shipping demand fulfillments by the manufacturer in zone z and time t

Q_z^t: replenishment quantities to the retailer's store in zone z and time t

I_z^t: level of inventory in the retailer's store in zone z and at the end of time t

I_m^t: level of inventory in the manufacturer's warehouse at the end of time t

I_z^o: initial inventory made available in the retailer's store in zone z before sales start

I_m^o: initial inventory available in the manufacturer's warehouse at the start of the season

P_{em}^t: the price of the product in direct and drop-shipping channels at time t

Lower-level decision variables:

P_{bz}^t: price listed by the retailer in its store located in market zone z at time t

P_{er}^t: price listed by the retailer on its online channel at time t

S_{bz}^t: total store channel demand fulfilled by the retailer, in zone z and time t

S_{erz}^t: total online channel demand fulfilled by the retailer, in zone z and time t

B. Demand Estimation

In the presence of multiple channels and an almost negligible hassle of alternating between available channels, an increase in the price in one of the channels can decrease its share from the total sales. This decrease in sales often gets distributed among other channels; this phenomenon is known as "cross-channel demand substitution" [12]. In the past, researchers have adopted a variety of mathematical models to understand demand substitution; for a comprehensive review, please refer to [31]. We note that discrete choice models are some of the commonly used demand-modeling techniques [32]. Although cross-channel demand substitution can also be modeled using linear demand functions, using discrete choice models over the former yields the benefit of parsimony of the number of coefficients that must be estimated [33].

Whereas discrete choice models need O(|M|) coefficients, linear demand functions may require O(|M|^2) coefficients [12].

Discrete choice models like multinomial logit (MNL) and multiplicative competitive interaction (MCI) are extensively used in fields like transportation planning [34], assortment optimization [35], and marketing research [36]. They help model customer behavior where many substitutable alternatives are available. These models are based on the assumption that customers associate a utility with each available alternative, including a zero utility for not choosing any alternative [37]. Using a discrete choice model, demand in channel $n, n \in N$ and at location $z, z \in Z$ can be estimated using the following equation:

$$D_{nz} = M_z * \frac{A_{nz}}{1 + \sum_{n' \in N} A_{n'z}} \quad (1)$$

where M_z represents the size of the market; A_{nz} (also known as an attraction function) is a function of explanatory variables such as price, lead time and/or advertisement, and helps determine demand generated in channel n at location z. In this paper, we have employed an MNL discrete choice model to estimate the demand. In MNL models, attraction functions are formulated as $A_{nz} = \exp\left(\alpha_{nz} + \sum_{k=1}^{K} \beta_{nz}^{k} k_{nz}\right)$, where K represents total number of explanatory variables such as price, delivery lead time and quality; α_{nz} represents the intrinsic attraction constant of the product in channel n; β_{nz}^{k} represents the sensitivity of the customers toward the explanatory variable k. For a detailed discussion on techniques used to estimate these demand parameters, please refer to [32], [38].

C. Assumptions

Assumption 1: Both the manufacturer and the retailer are interested in profit maximization.

Assumption 2: The manufacturer is responsible for managing the retailer's inventory under the VMI strategy. This enables the manufacturer to have downstream market information.

Assumption 3: Demand is price- and lead-time-sensitive [39], and depending on the degree of product differentiation [7], attraction towards retailer's offline, retailer's online, manufacturer's direct and drop-shipping channels can be modeled as follows:

$A_{bz}^{t} = \exp\left(\alpha_{bz} - \beta_{bz}^{p} P_{bz}^{t}\right)$, where A_{bz}^{t} represents the attraction towards the product available in retailer's physical store present in zone z and time t; $A_{erz}^{t} = \exp\left(\alpha_{erz} - \beta_{erz}^{p} P_{er}^{t}\right)$, where A_{erz}^{t} represents the attraction towards the product in retailer's online channel in zone z and time t; $A_{emz}^{t} = \exp\left(\alpha_{emz} - \beta_{emz}^{p} P_{em}^{t} - \beta_{emz}^{l} l_{emz}^{t}\right)$, where A_{emz}^{t} represents the attraction towards the product in manufacturer's direct channel in zone z and time t; and, finally, $A_{bmz}^{t} = \exp\left(\alpha_{bmz} - \beta_{bmz}^{p} P_{em}^{t} - \beta_{bmz}^{l} l_{bmz}^{t}\right)$, where A_{bmz}^{t} signifies the attraction towards the product available via the drop-shipping channel in zone z and time t.

Assumption 4: The manufacturer can expedite direct and drop-shipping deliveries with the help of a 3PL partner. Cost of expediting deliveries can be calculated using $c_{lz} = c_{oz} + \gamma_z / l_z$ [39], where c_{lz} represents the delivery cost, l_z denotes the delivery lead time, γ_z signifies the marginal increase in the cost for expediting the deliveries, and c_{oz} represents the bare minimum fulfillment cost for zone z.

Assumption 5: Retailer's online orders are fulfilled from the nearest physical store, and because all the deliveries are local, a fixed cost is incurred for every such delivery.

Assumption 6: Retailer's online channel offers uniform price irrespective of the customer's location. Similarly, manufacturer's direct channel and drop-shipping channel offers uniform price irrespective of the customer's location.

D. Objective Functions and Constraints

Manufacturer's Profit Function (Upper-Level Problem):

$$\text{Maximize:} \quad \sum_{z=1}^{Z}\sum_{t=1}^{T}\left(W - c_z\right) B_z^t Q_z^t + \sum_{t=1}^{T}\sum_{z=1}^{Z}\left(P_{em}^{t} - c_{lz}^{t} - \frac{h_m}{2}\right) S_{emz}^{t} + \sum_{t=1}^{T}\sum_{z=1}^{Z}\left(P_{em}^{t} - c_m - c_{lz}^{t} - \frac{h_m}{2}\right)(1-\theta) S_{bmz}^{t}$$
$$- \sum_{t=1}^{T}\sum_{z=1}^{Z} c_z^f B_z^t - \sum_{t=1}^{T} I_m^t h_m - I_m^t \frac{h_m I_m^t}{2 q_{\mathrm{m}}} - c_m\left(\sum_{t=1}^{T} q_m^t - \sum_{t=1}^{T}\sum_{z=1}^{Z} S_{bmz}^{t}\right) - \sum_{z=1}^{Z} I_z^T W \qquad (2)$$

Subject to:

$$S_{emz}^{t} \le M_z^t \left(\frac{A_{emz}^{t}}{1 + A_{bz}^{t} + A_{erz}^{t} + A_{emz}^{t} + A_{bmz}^{t}}\right) \qquad \forall z \in Z,\ \forall t \in T \qquad (3)$$

$$S_{bmz}^{t} \le M_{z}^{t}\left(\frac{A_{bmz}^{t}}{1+A_{bz}^{t}+A_{erz}^{t}+A_{emz}^{t}+A_{bmz}^{t}}\right) \qquad \forall z\in Z,\ \forall t\in T \tag{4}$$

$$Q_{z}^{t}, S_{bmz}^{t}, S_{emz}^{t} \ge 0 \qquad \forall z\in Z,\ \forall t\in T \tag{5}$$

$$I_{m}^{t}, I_{z}^{t} > 0 \qquad \forall z\in Z,\ \forall t\in T \tag{6}$$

$$q_{m}^{t} \le q_{\mathrm{m}} \qquad \forall t\in T \tag{7}$$

$$B_{z}^{t} \in \{0,1\} \qquad \forall z\in Z,\ \forall t\in T \tag{8}$$

$$p_{em}^{t} \in \Omega \qquad \forall z\in Z,\ \forall t\in T \tag{9}$$

Retailer's Profit Function (Lower-Level Problem):

Maximize:

$$\sum_{t=1}^{T}\sum_{z=1}^{Z}\left(P_{bz}^{t}-W-\frac{h_{z}}{2}\right)S_{bz}^{t}+\sum_{t=1}^{T}\sum_{z=1}^{Z}\left(P_{er}^{t}-W-\frac{h_{z}}{2}\right)S_{erz}^{t}+\sum_{t=1}^{T}\sum_{z=1}^{Z}\left(P_{em}^{t}-c_{m}-c_{lz}^{t}-\frac{h_{m}}{2}\right)\theta S_{bmz}^{t}$$
$$-\sum_{z=1}^{Z}\sum_{t=1}^{T}c_{erz}S_{erz}^{t}-\sum_{t=1}^{T}\sum_{z=1}^{Z}I_{z}^{t}h_{z} \tag{10}$$

Subject to:

$$S_{bz}^{t} \le M_{z}^{t}\left(\frac{A_{bz}^{t}}{1+A_{bz}^{t}+A_{erz}^{t}+A_{emz}^{t}+A_{bmz}^{t}}\right) \qquad \forall z\in Z,\ \forall t\in T \tag{11}$$

$$S_{erz}^{t} \le M_{z}^{t}\left(\frac{A_{erz}^{t}}{1+A_{bz}^{t}+A_{erz}^{t}+A_{emz}^{t}+A_{bmz}^{t}}\right) \qquad \forall z\in Z,\ \forall t\in T \tag{12}$$

$$S_{bz}^{t}, S_{erz}^{t} \ge 0 \qquad \forall z\in Z,\ \forall t\in T \tag{13}$$

$$p_{er}^{t}, p_{bz}^{t} \in \Omega \qquad \forall z\in Z,\ \forall t\in T \tag{14}$$

Equation (2) represents the manufacturer's profit in the proposed bilevel omnichannel price optimization problem. The first term in the manufacturer's objective function is the profit earned by the manufacturer by supplying the product to the retailer at a wholesale price W. The second term is the profit earned by the manufacturer through direct-channel demand fulfillments. The third term is the manufacturer's share in the profit earned through drop-shipping demand fulfillments. It has been assumed that W and θ are determined by the manufacturer at the start of the selling season and remain constant for the entire season. The fourth term represents the total replenishment costs incurred by the manufacturer to replenish the retailer-owned stores. Inventory holding costs at the manufacturer's warehouse are represented by the next two terms in the objective function. The seventh term is the manufacturing cost incurred to the manufacturer. Since the manufacturing cost is already removed from the drop-shipping profits (see the third term), S_{bmz}^t for all $t, t \in T$ and all $z, z \in Z$ is reduced from the total quantity produced over the time $t, t \in T$. Finally, the last term in the manufacturer's objective function represents recoveries of unsold items from the retailer's stores at the end of the selling period t, where $t = T$.

Equations (3) and (4) are the constraints so that the fulfillments cannot be more than the direct and the drop-shipping demands, respectively. Constraints in (5) are non-negativity constraints and represent that production or fulfillments during an interval cannot be less than zero. Constraints in (6) signify non-negativity of the manufacturer and the retailer's inventories, where I_m^t and I_z^t are calculated using (15) and (16), respectively.

$$I_m^t = I_m^o + \sum_{n=1}^{t} q_m^n - \sum_{n=1}^{t}\sum_{z=1}^{Z}\left(B_z^n Q_z^n + S_{emz}^n + S_{bmz}^n\right) \qquad \forall z \in Z,\ \forall t \in T \tag{15}$$

$$I_z^t = I_z^o + \sum_{n=1}^{t}\left(B_z^n Q_z^n - S_{bz}^n - S_{erz}^n\right) \qquad \forall z \in Z,\ \forall t \in T \tag{16}$$

Constraint (7) imposes a limit on variation in the units produced per unit time considering allocated production capacity. We know that once the resources are assigned to a certain task, it is difficult to reallocate them to some other activities. Companies often use overtime when actual demand is high, but there is a limit to which the productivity of the resources can be extended. Reducing the volume of production is relatively easy, but that means keeping the resources under-utilized. In a way, q_m^t is a variable that is difficult to adjust once the resources are allocated for q_m units. Constraint (8) represents the binary decision variable B_z^t; if $B_z^t = 1$, the manufacturer replenishes the retailer's store present in zone z at time t with a quantity Q_z^t where $Q_z^t \geq 0$. Constraint (9) ensures that direct and drop-shipping prices belong to a set of predefined prices Ω.

Equation (10) defines the retailer's profit; the first two terms in the formulation represent the profit from the stores and the online channel, respectively. The third term represents the retailer's share in the profit from the drop-shipping fulfillments. One can see that the profit earned by the retailer through the drop-shipping channel depends on the manufacturer's decision variable θ. The fourth term is the total cost incurred by the retailer to fulfill the online orders. Finally, there is an inventory holding cost involved. Constraints (11) and (12) ensure that the fulfillments are not more than the demand in the store or the online channel, respectively. Equation (13) represents the non-negativity of fulfillments. Constraint in (14) ensures that the store and the online prices belong to a set of predefined prices Ω.

IV. SOLUTION APPROACH

Bilevel programming problems contain hierarchical optimization structures and are challenging to solve. Even if the objective functions of both players are linear, the overall problem remains NP-Hard [40]. Often, bilevel programming problems are solved using a single-level reduction technique, which uses the classical KKT approach while assuming the smoothness, linearity, or convexity of the objective functions. But the real-world bilevel optimization problem can contain non-differentiability and non-convexity [41]. This reduces our ability to convert the bilevel programming problems to a classic single-level optimization problem. These non-linearities force researchers to opt for heuristic and evolutionary approaches to solve the problem; for example, see [14], [16]–[18], [23].

It is known that MNL profit functions are not convex [42]. Therefore, employing KKT conditions directly on the retailer's problem for single-level reduction will not guarantee an optimal solution. Nonetheless, we can use single-level reduction using KKT conditions if we can convexify the retailer's problem.

Proposition 1: In the single-location multi-period bilevel price optimization problem, the retailer's objective function when reformulated to purchase probability space $(\lambda_{er}, \lambda_b)$ instead of price (P_{er}, P_b) is a concave function, given that all the demand generated at the retailer's stores is fulfilled.

Proof: See Appendix

This proposition states that if we consider a retailer that operates only one physical store and ensures that all the demand that gets generated is fulfilled, then the retailer's problem can be convexified. Convexification allows us to use the single-level reduction technique and thereby solve single-location bilevel optimization problems.

As a result, the problem is now divided into two cases: Case 1, the single-location, multi-period bilevel omnichannel price optimization problem, and Case 2, the multi-location, multi-period bilevel omnichannel price optimization problem.

Case 1: Single-Location, Multi-Period Bilevel Omnichannel Price Optimization Problem

Based on Proposition 1, the retailer's single location problem (Eqs. 10–14) can be replaced by the objective function show in (17), with no constraints to satisfy and assuming holding costs are negligible (for proof see (A.6) in the Appendix).

$$\text{Maximize:} \quad \Pi_r = \left(\frac{1}{\beta_b^p}\left(\alpha_b^p - \ln\left(\frac{\lambda_b}{1-\lambda_b-\lambda_{er}-\lambda_{em}-\lambda_{bm}}\right)\right) - W\right) M\lambda_b + \left(\frac{1}{\beta_{er}^p}\left(\alpha_{er}^p - \ln\left(\frac{\lambda_{er}}{1-\lambda_b-\lambda_{er}-\lambda_{em}-\lambda_{bm}}\right)\right) - W\right) M\lambda_{er} + \left(\frac{1}{\beta_{bm}^p}\left(\alpha_{bm}^p - \ln\left(\frac{\lambda_{bm}}{1-\lambda_b-\lambda_{er}-\lambda_{em}-\lambda_{bm}}\right) - \beta_{bm}^l l_{bm}\right) - c_m - c_l\right)\theta M\lambda_{bm} - c_{er} M\lambda_{er} \tag{17}$$

Furthermore, since (17) is concave in purchase probability space λ_{er} and λ_{b}, it can be replaced by its KKT conditions (18) and (19), where λ_{er} and λ_{b} are retailer's alternate decision variables.

$$\frac{\partial \Pi_r}{\partial \lambda_{\mathrm{er}}} = \frac{\theta M \lambda_{\mathrm{bm}}}{\beta_{\mathrm{bm}}(-1+\lambda_{\mathrm{bm}}+\lambda_{\mathrm{b}}+\lambda_{\mathrm{em}}+\lambda_{\mathrm{er}})} + \frac{M\lambda_{\mathrm{b}}}{\beta_{\mathrm{bz}}(-1+\lambda_{\mathrm{bm}}+\lambda_{\mathrm{b}}+\lambda_{\mathrm{em}}+\lambda_{\mathrm{er}})} - \frac{M(-1+\lambda_{\mathrm{bm}}+\lambda_{\mathrm{b}}+\lambda_{\mathrm{em}})}{\beta_{\mathrm{er}}(-1+\lambda_{\mathrm{bm}}+\lambda_{\mathrm{b}}+\lambda_{\mathrm{em}}+\lambda_{\mathrm{er}})} - \frac{M}{\beta_{\mathrm{er}}}\left(\mathrm{Log}\left[-\frac{\lambda_{\mathrm{er}}}{-1+\lambda_{\mathrm{bm}}+\lambda_{\mathrm{b}}+\lambda_{\mathrm{em}}+\lambda_{\mathrm{er}}}\right] - \alpha_{\mathrm{er}} + W\beta_{\mathrm{er}}\right) = 0 \tag{18}$$

$$\frac{\partial \Pi_r}{\partial \lambda_{\mathrm{b}}} = \frac{\theta M \lambda_{\mathrm{bm}}}{\beta_{\mathrm{bm}}(-1+\lambda_{\mathrm{bm}}+\lambda_{\mathrm{b}}+\lambda_{\mathrm{em}}+\lambda_{\mathrm{er}})} + \frac{M\lambda_{\mathrm{er}}}{\beta_{\mathrm{er}}(-1+\lambda_{\mathrm{bm}}+\lambda_{\mathrm{b}}+\lambda_{\mathrm{em}}+\lambda_{\mathrm{er}})} - \frac{M(-1+\lambda_{\mathrm{bm}}+\lambda_{\mathrm{em}}+\lambda_{\mathrm{er}})}{\beta_{\mathrm{b}}(-1+\lambda_{\mathrm{bm}}+\lambda_{\mathrm{b}}+\lambda_{\mathrm{em}}+\lambda_{\mathrm{er}})} - \frac{M}{\beta_{\mathrm{bz}}}\left(\mathrm{Log}\left[-\frac{\lambda_{\mathrm{b}}}{-1+\lambda_{\mathrm{bm}}+\lambda_{\mathrm{b}}+\lambda_{\mathrm{em}}+\lambda_{\mathrm{er}}}\right] - \alpha_{\mathrm{b}} + W\beta_{\mathrm{b}}\right) = 0 \tag{19}$$

It must be noted that, although we were able to simplify the bilevel problem to a single-level problem, the manufacturer's objective function remains a non-convex NP-Hard problem. Therefore, the manufacturer's problem, along with the newly added constraints from the retailer's problem (Eqs. 18–19) must be solved using stochastic search algorithms. We opt for Simulated Annealing (SA) [43] and Randomized Decomposition Solver (RDSolver)[44] because of the discrete nature of the decision variables we are dealing with.

Case 2: Multi-Location, Multi-Period Bilevel Omnichannel Price Optimization Problem

Since the retailer's online price across multiple locations is supposed to be equal, the retailer's problem cannot be convexified for the multi-location problem (refer to Proposition 2 in [12]). Therefore, in this case, the upper- and lower-level problems must be solved iteratively and cannot be simplified using the single-level reduction technique as in Case 1. This makes it computationally more challenging as compared to solving a single-level NP-Hard problem. Subsequently, to solve the proposed multi-location, multi-period bilevel price optimization problem, we employed a frequently used technique known as hierarchical bilevel optimization [16], [45].

A. *Hierarchical bilevel optimization*

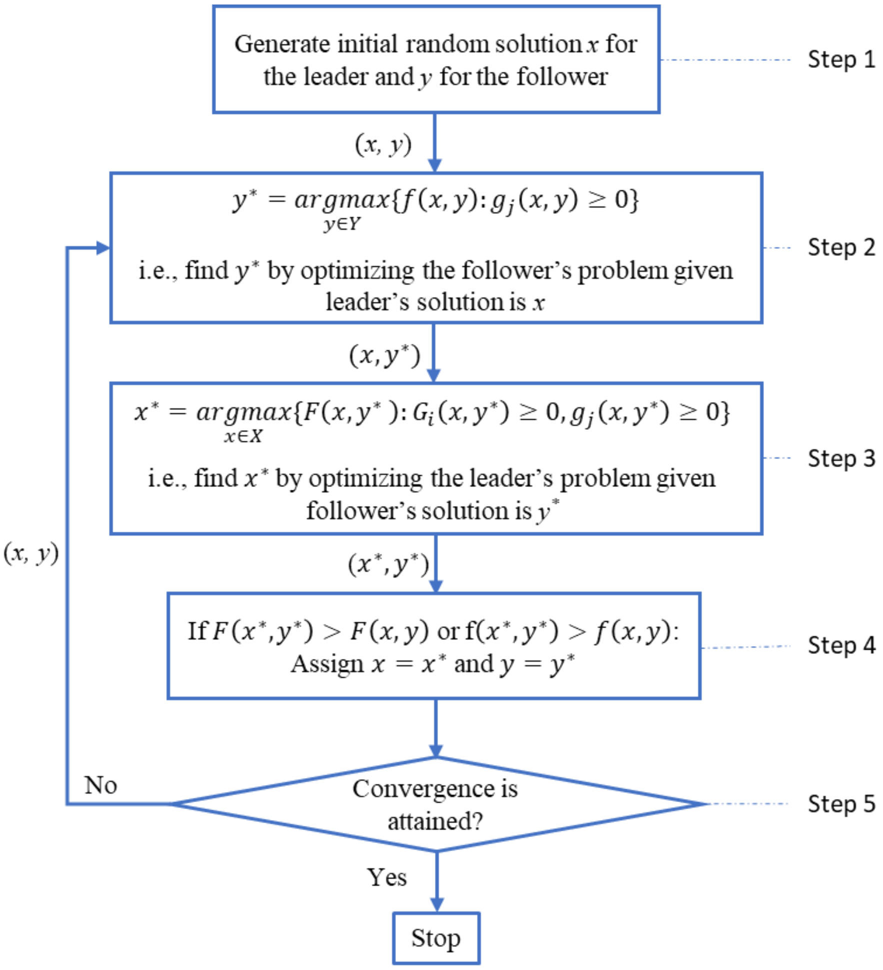

Fig. 2: The flowchart of hierarchical bilevel optimization.

Fig. 2 presents the flowchart of hierarchical bilevel optimization. In step 1, a random solution for each player is generated. Let x represent a random solution for the manufacturer and y represent a random solution for the retailer. In step 2, keeping the manufacturer's decision variables x as constants, the retailer's problem is solved completely. The optimal response y^* obtained for the retailer in step 2, along with the previously found solution x, is then forwarded to step 3. In step 3, the manufacturer's problem is solved completely, keeping the retailer's decision variables y^* as constants. The obtained solutions (x^*, y^*) are then sent to step 4. In step 4, if any of the players' objective function improves because of (x^*, y^*), we accept the solution and assign $x = x^*$ and $y = y^*$. Finally, in step 5 we check whether the algorithm has converged. It is presumed that the algorithm has converged if there is no change in either x or y for consecutive n number of iterations of steps 2 to 4. If the termination criterion is not satisfied, the flow returns to step 2 where the process starts with the updated values of x and y. To carry out optimization in steps 2 and 3, we once again used Simulated Annealing (SA) [43] and Randomized Decomposition Solver (RDSolver) [44] because of their ability to handle discrete decision variables. For more details on these optimization techniques, please see the next section.

B. SA and RDSolver

SA is a well-known algorithm originally proposed by [43]. It was inspired by the process of annealing in metals, in which a metal is heated to a temperature high enough for its molecular structure to break down. Subsequently, the metal is slowly cooled to obtain a desired molecular structure. SA is often considered to be a discrete optimization algorithm and therefore fits well into the requirements of our problem. Moreover, SA has been used by many researchers in the past to solve bilevel optimization problems; for an example, see [46].

RDSolver is a solution framework recently proposed by [44]. It was developed to solve nonlinear, nonconvex discrete optimization problems by combining two algorithms, namely Randomized Decomposition (RD) and RDPerturb. RD is a local search approach that starts from a feasible solution and is based on splitting the decision variables into random subsets. To escape local optima, RDPerturb is used along with RD. In the original paper, RDSolver was tested on 400 problem instances and the results were found to be comparable with that of the state-of-the-art solution techniques. As RDSolver needs no prior knowledge of the structure of the problem and can solve discrete optimization problems, the algorithm fits well with our requirements. For pseudo-codes for RDSolver, RD and RDPerturb, readers are invited to refer to algorithms 1, 2, and 3 in [44].

C. Comparative Analysis

Since the algorithms under consideration are known to provide near-optimal results, we must assess the quality of the solutions obtained. In Case 1, we discussed how to convert our single-zone bilevel problem into a single-level problem where only one objective function must be optimized. Because of its straightforward application, solutions obtained after single-level reduction (as in Case 1) can also serve as a benchmark for hierarchical bilevel optimization techniques (intended for multi-location problems). Hence, we carried out a comparative analysis to see how the objective functions converged, and the best/worst solutions obtained.

For benchmarking and comparative analysis, a problem instance consisting of one zone and three period was created. Parameter tuning was carried out for all the given algorithms. The problem instance was then solved multiple times using each of the algorithms. The results were recorded for statistical analysis. Figs. 3a and 3b present how the results converged when we used SA, and the problem was reduced to single-level versus when the problem was solved hierarchically for upper and lower levels separately. Figs. 3c and 3d present the convergence when RDSolver was used, and the problem was reduced to single-level versus when the problem was solved hierarchically for upper and lower levels separately. The convergence signifies that neither the manufacturer nor the retailer wants to deviate from the near-equilibrium attained, also known as "Stackelberg equilibrium."

In the one-zone three-period problem scenario, these algorithms took approximately five to ten minutes to converge. Since the algorithms are based on stochastic search techniques, the time taken by them to converge may vary. The speed of convergence also depends on how the algorithmic parameters are tuned. However, we observe that, on an average, when the problem was reduced to single level, the time to convergence was less than when the problem was solved using the hierarchical bilevel optimization route. This is because it is always straightforward and simple to solve a traditional single-level optimization problem than to solve a bilevel problem.

Table 1 shows the compiled results of mean, standard deviation, best, and worst values obtained using all the mentioned solution techniques. On average, the retailer's profit converged at approximately 200, whereas the manufacturer's profit converged at approximately 2000. Based on these results, it can be said that the single-level reduction technique provided marginally inferior optimal values as compared to the hierarchical optimization technique. The reason may be that for single-level reduction, we have simplified the original problem by relaxing the constraints (see Appendix).

Since the results obtained using all four solution techniques are almost identical, we can safely proceed to solve multi-location problems using the hierarchical bilevel optimization technique as shown in Fig. 2. Furthermore, as shown in Table 1, since hierarchical-RDSolver gave relatively better results, it may be preferred over hierarchical-SA.

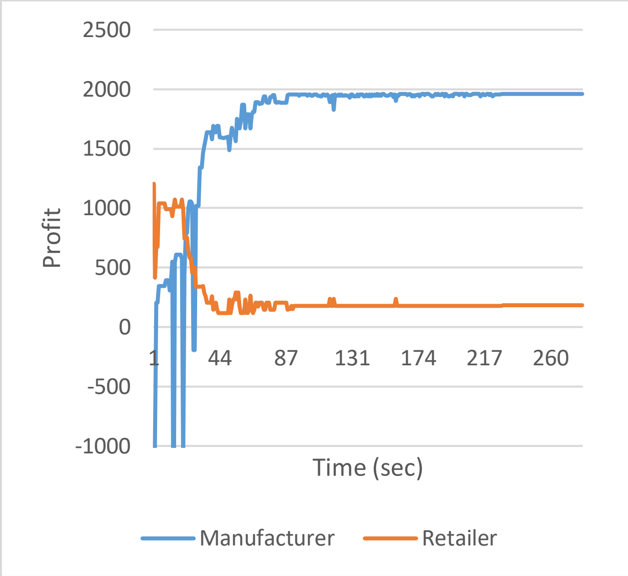

Fig. 3a: Convergence of manufacturer's and retailer's profit when the problem was reduced to single-level and solved using SA.

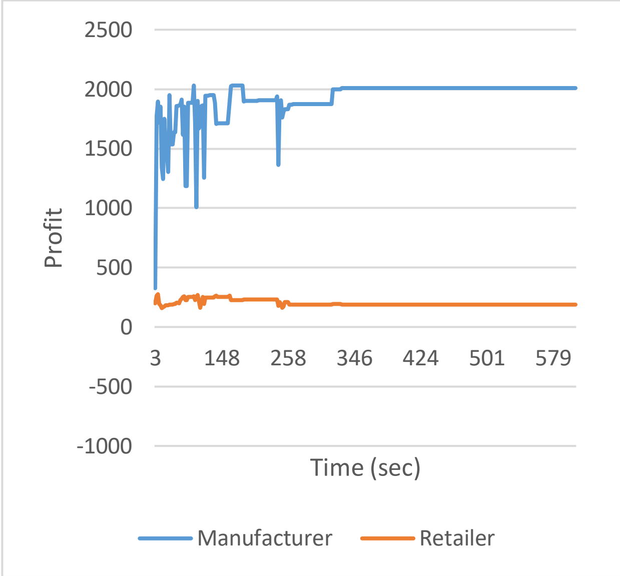

Fig. 3b: Convergence of manufacturer's and retailer's profit when the problem was solved using hierarchical bilevel optimization using SA.

Fig. 3c: Convergence of manufacturer's and retailer's profit when the problem was reduced to single-level and solved using RDSolver.

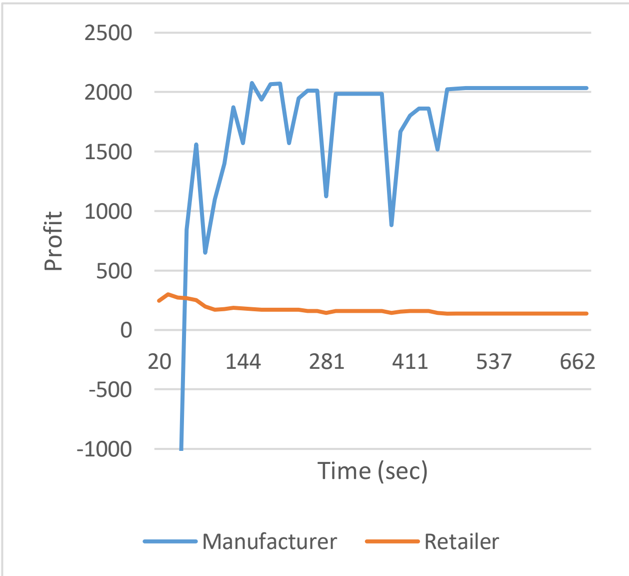

Fig. 3d: Convergence of manufacturer's and retailer's profit when the problem was solved using hierarchical bilevel optimization using RDSolver.

Table 1: Mean, standard deviation, best and worst values obtained using various solution techniques.

	SA on Single-Level Reduced Problem		SA for Bilevel Optimization		RDSolver on Single-Level Problem		RDSolver for Bilevel Optimization	
Player	Mfr.	Ret.	Mfr.	Ret.	Mfr.	Ret.	Mfr.	Ret.
Mean	1989.14	131.56	2022.56	169.64	1932.83	207.84	2242.12	201.68
Std. Dev.	124.98	48.52	136.79	41.06	65.72	51.95	61.67	48.65
Best	2120	182	2281.4	237.6	2011	213	2323	268
Worst	1796.3	118.2	1877.4	142.6	1817	287	2155	168

In the following section, we have compared hierarchical-SA and hierarchical-RDSolver based on the time they take to converge while solving different problem instances. Fig. 4a presents the number of decision variables for respective problem instances. Fig. 4b presents box plots for the time taken by these algorithms to converge. We want to highlight that all the experiments were done on an Intel i5-6500 with a 3.20 GHz processor and 8 GB RAM.

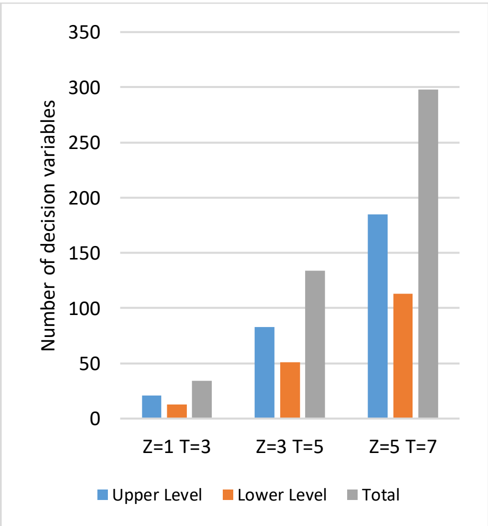

Fig. 4a: Problem instances vs the number of decision variables.

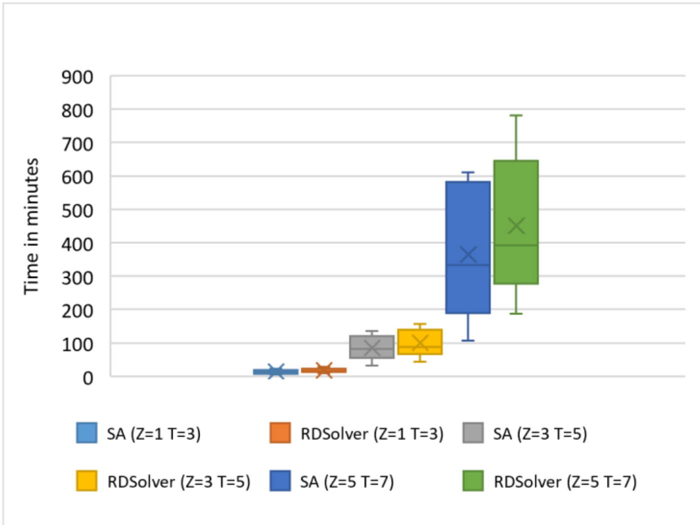

Fig. 4b: Time complexity of hierarchical bilevel optimization.

The results presented in Fig. 4b indicates an exponential growth in computational time as problem size increases. Hierarchical bilevel optimization based on RDSolver took relatively longer to converge (often more than 10 hours). Our analysis suggests that the reason for the slow convergence of larger problem instances relates to the nature of the problem itself. As we are solving a game-theory problem, for every value-combination of the manufacturer's decision-variables, the retailer has a corresponding optimal response.

Because both the players' objective functions are non-convex and non-differentiable, finding the retailer's optimal response that corresponds to that of the manufacturer's existing solution is itself computationally challenging. The algorithm will converge only when neither of the players has a better solution to counter the existing solution of the other player. Furthermore, every time either of the player's decision variables changes, the other player's solution must also be optimized to counter the change. In bilevel optimization, search space increases exponentially for every new decision variable added to either of the players' objective function [29]. This is because the algorithm is not optimizing both the functions concurrently (rather, it optimizes sequentially). That is the why we sought to reduce this bilevel problem to a single-level problem. However, we found that multi-location problems are difficult to simplify.

V. NUMERICAL EXPERIMENTS

This section reports the results obtained using the numerical experiments. Data and model parameters were adopted from the literature. Various scenarios concerning market variables, such as intrinsic utilities, price sensitivities and delivery lead-time sensitivity, were investigated. It is assumed that there are three zones ($z \in [1,3]$) and that the selling season is divided into five periods ($t \in [1,5]$). The approximate market size (M_z^t) is assumed to be known and is presented in Fig. 5.

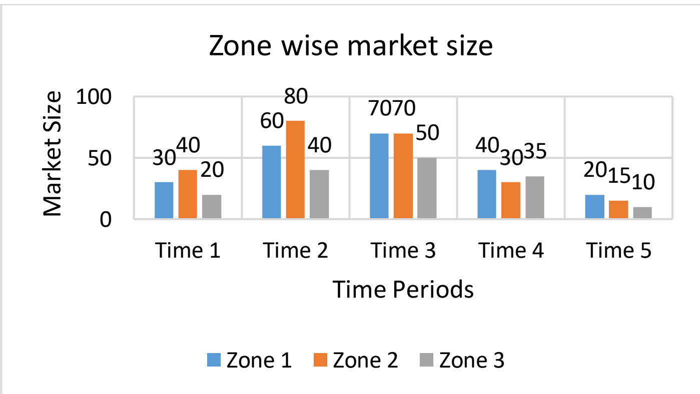

Fig. 5: Market size for the product for various zones and times.

The cost of manufacturing a unit of product was fixed and is assumed to be 110, i.e., c_m=110. To replenish the retailer's store located in zone $z \in [1,3]$, c_z^f was assumed to be 70, 120 and 150, respectively. An additional cost per unit transported from the manufacturer's warehouse to the retailer's store (c_z) was assumed to be 1. Holding costs per period (h_z) for $z \in [1,3]$ were assumed to be 3, 2.5 and 2, respectively.

Holding cost at the manufacturer's facility (h_m) was assumed to be 1 per unit time. For direct and drop-shipping fulfillments, extra cost for expediting deliveries was calculated using γ_z/l_z. Here γ_z for $z \in [1,3]$ were assumed to be 2, 4 and 6, respectively. The minimum amounts charged by 3PL for a delivery to a customer located in zone $z \in [1,3]$, i.e., c_{oz} were assumed to be 1, 2 and 3 per unit. An online demand fulfillment cost for the retailer (c_{erz}) was assumed to be 1 per unit. It was assumed to be minimal because the deliveries are performed locally from the store located in that zone.

Next, it was assumed that α_{bz}=124, α_{erz}=126, α_{bmz}=126 and α_{emz}=124 were the intrinsic utilities for retailer's stores and online channel, drop-shipping channel, and manufacturer's direct channel, respectively. Furthermore, β^p_{bz}=0.88, β^p_{erz}=0.9, β^p_{emz}=0.9 and β^p_{bmz}=0.9 were the assumed price sensitivities in the retailer's stores and online channel, drop-shipping channel, and manufacturer's direct channel, respectively. Finally, delivery lead-time sensitivities β^l_{emz} and β^l_{bmz} were assumed to be 0.25, whereas delivery lead times l^t_{emz} and l^t_{bmz} varied from 1 to 5 days. Later we increased these sensitivities towards 0.50 and 0.75 to evaluate their effect on profitabilities.

In the manufacturer's search space, there are discretionary variables and operational variables. Variables those are classified as discretionary (or strategic level) are W, θ, and q_m. These variables have a significant impact on other decision variables and ultimately on the convergence of the algorithms. Every time any of these three variables were modified, the prior solution (near-equilibrium) obtained became obsolete, and the algorithm took longer to converge. Therefore, we decomposed the problem by eliminating W and θ from the search space, although allowed q_m to be optimized by the algorithm. However, doing so we had to optimize for various values of W and θ to find a combination that suits the manufacturer. Initial inventory (I^o_m) depends on the number of periods the production of q_m units start in advance. It was assumed that the manufacturer begins the production five interval before the start of the selling season such that $I^o_m = 5q_m$. Keeping the problem simple we also assumed $q^t_m = q_m$.

A. *Finding Optimal Combination of W and θ*

First, we solved the given problem by removing drop-shipping as a viable retail alternative and $\theta = 0$. Fig. 6a presents the profit earned by the players and total supply chain profit as W, which varied from 131 to 139. We can see that the retailer's profit decreases almost linearly as W increases. At the same time, the manufacturer's profit increased to W = 135 and then started decreasing. At W = 135, the profit earned by the manufacturer is approximately 7450, while the retailer's profit is approximately 1350. The total supply chain profit stands at ≈ 8800. These values are treated as benchmarks to evaluate the performance of the supply chain when drop-shipping is incorporated.

Now considering that the players are willing to collaborate over drop-shipping channel, we varied θ from 10% to 50%. In Fig. 6b we present how the manufacturer's profit decreased as θ increased. Similarly, in Fig. 6c we present how the retailer's profit increased as θ increased. And finally, in Fig. 6d the change in total supply chain profit is shown.

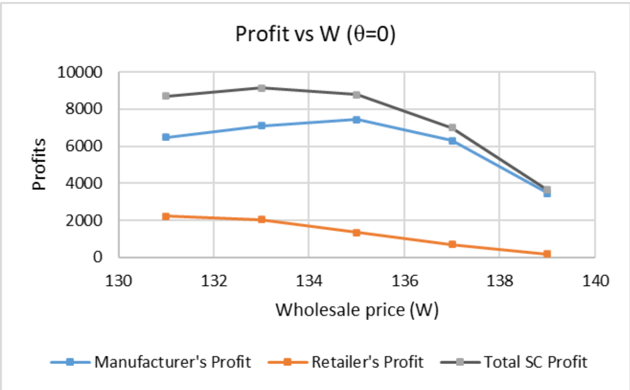

Fig. 6a: Change in supply-chain profit with respect to W without drop-shipping.

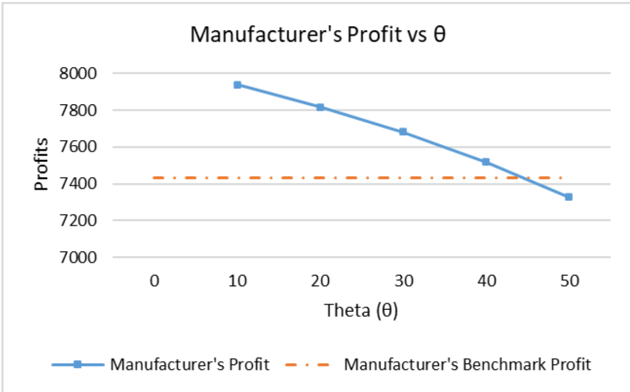

Fig. 6b: Change in manufacturer's profit with respect to θ for given W.

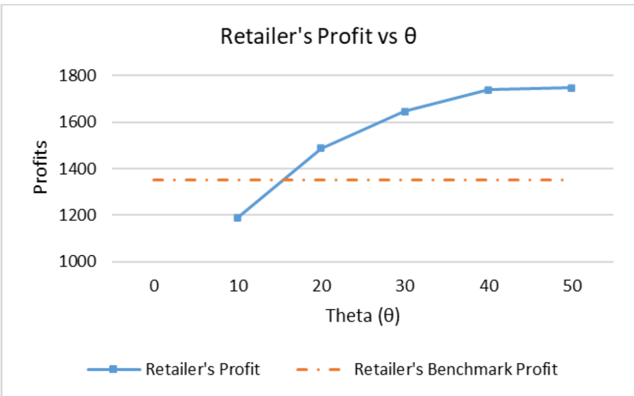

Fig. 6c: Change in retailer's profit with respect to θ for given W.

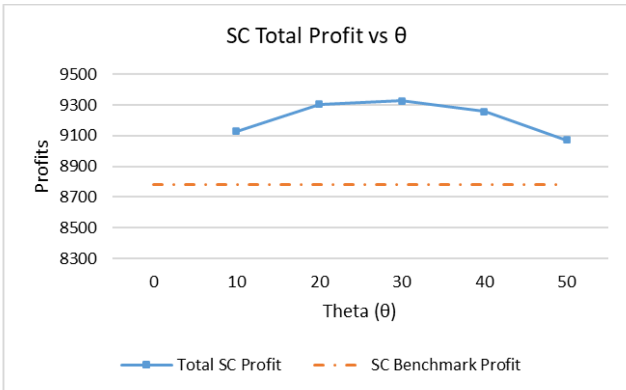

Fig. 6d: Change in supply-chain profit with respect to θ for given W.

In these figures (Figs. 6b–6d), we can that see when $\theta < 15\%$, the retailer's profit falls below the benchmark profit of 1350; therefore, in such a case, the retailer will not collaborate with the manufacturer. The two players will then operate in a competitive mode, and the supply-chain profit will drop to 8800.

When $\theta < 20\%$, the retailer reduces online and offline prices to compete with the now relatively less profitable drop-shipping channel. Although increasing θ from 20% to 30% seems to be counterproductive for the manufacturer, the change helps in terms of maximizing the overall profit. Total supply-chain profit increases to $\theta = 30\%$ but then decreases gradually. When $\theta \geq 40\%$, we see a lesser number of drop-shipping fulfillments. The retailer's profit does not increase significantly after $\theta = 40\%$. We also observed a slight increase in the direct and drop-shipping prices. This is because having a drop-shipping channel becomes less profitable for the manufacturer when a large portion of the channel's profit goes to the retailer. When θ is more than 45%, the manufacturer's profit falls below the benchmark profit of 7450, rendering drop-shipping a non-viable solution once again.

For the manufacturer, it is optimal to select $\theta \approx 20\%$ where the retailer also has some incentive to collaborate. At this point, the overall fulfillment through drop-shipping was recorded to be the highest. However, the total supply-chain profit was maximized at $\theta = 30\%$. Our analysis suggests that: a) there is a scope for coordination between the two players when it comes to determining the value for θ, and b) the inclusion of drop-shipping channel in the channel mix can not only serve as an omnichannel feature but can also assist in generating higher returns, given the value of θ is selected judiciously.

B. Constrained Production Capacity

In the previous section, it was assumed that the manufacturer first determines the values of W and θ, and accordingly allocates the resources for production. It was found that for $W = 135$ and $\theta = 20\%$, $q_m = 50$ represents optimal production quantity per unit time. This value of q_m corresponds to approximately 82% of the market potential being satisfied. But what if the manufacturer is dealing with underproduction and overproduction because of its inability to allocate and reallocate resources for the same? In consideration of this question, to simulate over-production we assumed $q_m = 55$, and to simulate under-production we assumed $q_m = 40$ and solved for W and θ.

Case A: Under-production

When q_m was fixed at 40 units, optimal values of W and θ changed to 137 and 10%, respectively. For $q_m = 40$, it became difficult for the manufacturer to meet the retailer's demand while keeping the wholesale price at $W = 135$. $W < 135$ became infeasible because in VMI settings the supplier can be penalized for not fulfilling the retailer's demand [47].

Fig. 7a presents how drop-shipping affects the profitabilities in the supply chain in the case of under-production. At $\theta = 10\%$, drop-shipping helped increase the manufacturer's profit by up to 14% and that of the retailer by 4%. While at θ = 20%, the manufacturer's profit was 12% higher, the retailer's profit was 27% higher, and the total supply-chain profit was maximized.

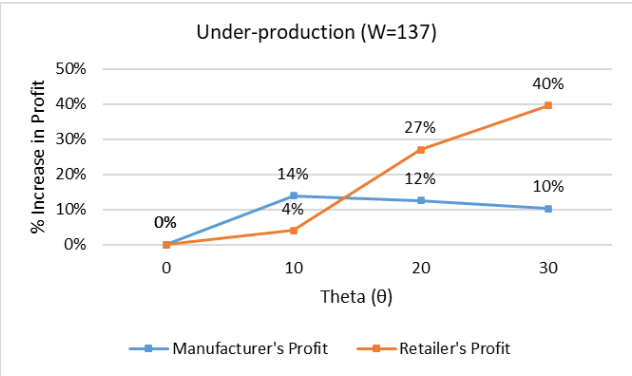

Fig. 7a: Percentage change in profits with respect of θ for given W in the case of under-production.

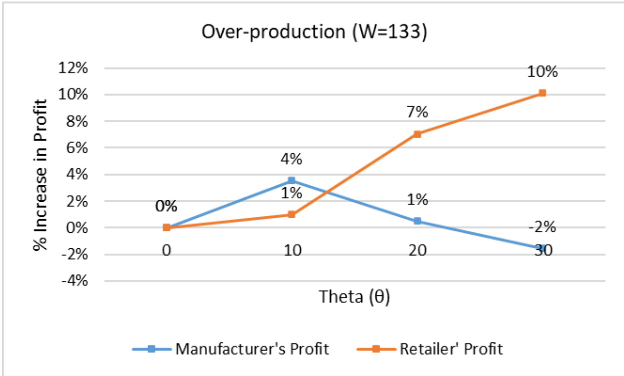

Fig. 7b: Percentage change in profits with respect of θ for given W in the case of over-production.

Case B: Over-production

When q_m was fixed at 55 units, the optimal value for W changed to 133. In this case, since the manufacturer is under pressure to dump a larger volume of the product at lower W, it is interesting to see how the retailer perceived the drop-shipping channel.

Fig. 7b presents how drop-shipping affects the profitabilities in the supply chain in the case of over-production. At $\theta = 10\%$, the retailer's profit increased by only 1%. At $\theta = 20\%$, where the retailer's profit is relatively high (increased 7%), the increase in the manufacturer's profit plummets back to just 1%. Since supporting a drop-shipping channel does not substantially increase the retailer's profit, the retailer may not find drop-shipping a game-changing proposition in the case of overproduction.

From the above analysis, we can infer that in the case that production capacity is not sufficient, a manufacturer can obtain better returns with the help of a drop-shipping channel. Our analysis suggests that by increasing W, the manufacturer induces the retailer to promote webrooming and showrooming.

However, a drop-shipping channel may not be so useful in the case of surplus production. In that case, because of relatively low W, the retailer already has an advantage and may not benefit significantly from the collaboration.

C. Sensitivity Analysis on Demand Parameters

Assuming the values of W and θ were fixed before the season starts, we carried out several analyses. Market variables like market size, attraction towards a channel, price sensitivities, and lead time sensitivities are susceptible to change. The purpose of this section is to reflect upon the inherent uncertainties in such market variables. Keeping W and θ constant at 135 and 20% respectively, we present the findings from the analyses in the following section.

Effect of increased intrinsic attractions:

Fig. 8 presents the effect of an increase in the values of intrinsic attractions α_{bmz} , α_{emz}, α_{erz}, and α_{bz} on manufacturer's profit, retailer's profit, production, and delivery lead times.

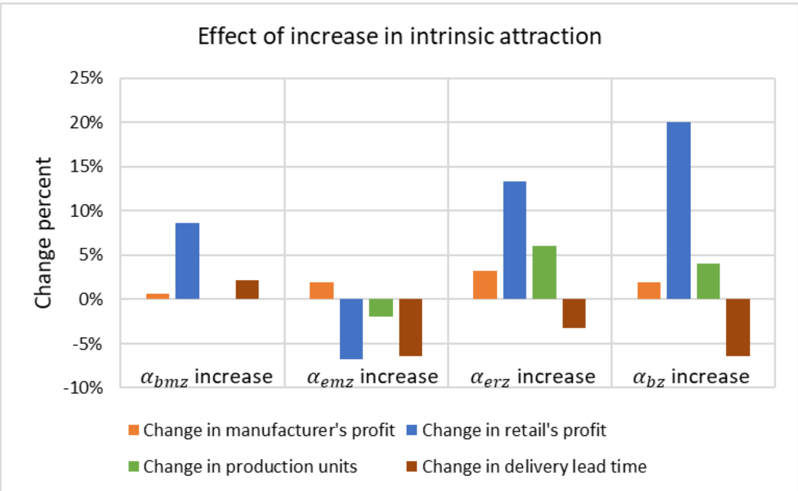

Fig. 8: Effect of increase in intrinsic attractions on profitabilities, production and lead time.

When the drop-shipping channel became more attractive, i.e., when α_{bmz} was increased by two points, the manufacturer's profit increased marginally while the retailer's profit increased by 8%. There is no change in production per unit time, but delivery lead time increased on average. It is counter-intuitive to see the manufacturer's profit increase only marginally. Also, this is the only scenario where delivery lead time increased as though the manufacturer were trying to make this channel less attractive.

When α_{emz} was increased by two points, i.e., if the manufacturer's direct channel becomes slightly more attractive, the retailer's profit decreased by 7%, production per unit time decreased by 2%, and manufacturer's profit increased by approximately 2%. This is the only scenario where production quantity per unit time decreased marginally. This suggests that the manufacturer can extract more profit from fewer sales when the direct channel is more attractive.

When either α_{erz} or α_{bz} were increased by two points, i.e., when retailer's channels became relatively more attractive, the retailer's profit increased by 14% and 20%, respectively; production per unit time increased by 6% and 4%, respectively; manufacturer's profit increased marginally by 4% and 2%, respectively. In both scenarios, delivery lead time decreased on average. This suggests that when a retailer's outlets become more attractive, the manufacturer may try to improve its service in terms of how fast it can deliver the product to the customer.

From Fig. 8, it can be inferred that a) the retailer's profit decreases when the direct channel becomes more attractive, and the manufacturer's dependence on the retailer decreases and it can generate more profit for itself; and b) if there is an increase in attraction towards drop-shipping channel, the retailer benefits more than the manufacturer; the manufacturer can make this channel less attractive by increasing delivery lead time.

Effect of decrease in price sensitivities:

Fig. 9 presents the effect of the decrease in price sensitivities β^{p}_{bmz} , β^{p}_{emz}, β^{p}_{erz}, and β^{p}_{bz} on manufacturer's profit, retailer's profit, production, and delivery lead times.

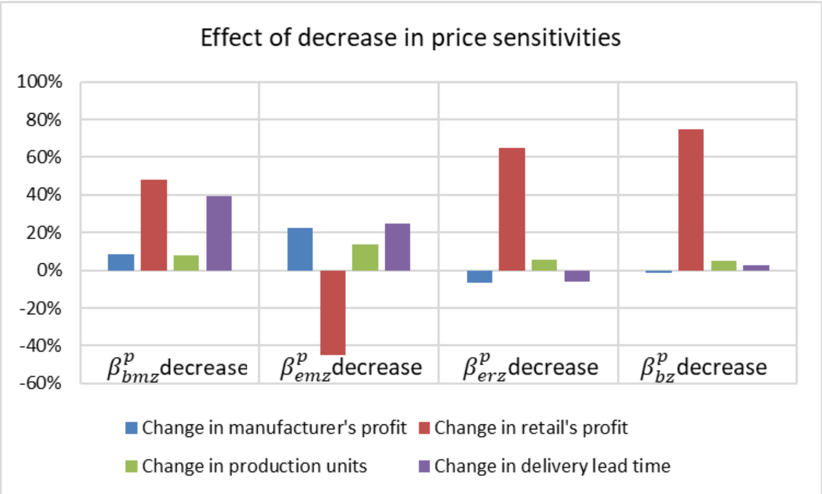

Fig. 9: Effect of decrease in price sensitivities on profitabilities, production and lead time.

When customers are assumed to be less sensitive to the drop-shipping channel prices, i.e., when β^{p}_{bmz} was reduced by 4%, the manufacturer's profit increased by 10% and the retailer's profit increased by about 50%. The delivery lead time increased on average. The results are identical to the case when there was an increase in intrinsic attraction towards the drop-shipping channel.

When customers are less sensitive to the direct channel prices, i.e., when β^{p}_{emz} was reduced by 4%, the manufacturer's profit increased by 20% and the retailer's profit decreased by 45%. However, this case is slightly different from the case in which the intrinsic attraction was increased for this channel. Here, production per unit time and delivery lead time has increased on an average. We can concur that a manufacturer can relax delivery times on account of decreased price sensitivities. Nevertheless, decreased price sensitivities lead to higher demand.

When either β^{p}_{erz} or β^{p}_{bz} are decreased by 4%, we see a sharp jump in the retailer's profits. However, the manufacturer's profit decreases marginally in both cases. There is no significant change in delivery lead times, and there is only a marginal increase in production per unit time. This result is slightly different from what we observed when intrinsic attraction towards the retailer's channels was increased. The retailer's profit has increased, but there is no significant change in production. Also, the manufacturer's profit decreases instead of increasing. We can say that when β^{p}_{erz} or β^{p}_{bz} increases, a retailer has an advantage, as it can generate more profit for itself without increasing the sales volume.

From Fig. 9, it can be inferred that a) when attraction towards retailer's channels increases, the manufacturer's profit increases (see Fig. 8), whereas when price sensitivity towards retailer's channels decreases, the manufacturer's profit decreases; b) decreased price sensitivity towards drop-shipping channel benefits the retailer more than the manufacturer; and c) on account of decreased price sensitivities, higher overall demand can be expected.

Effect of increase in lead time sensitivities:

In Fig. 10, we present how the manufacturer's profit, retailer's profit, production, and delivery lead times changed when delivery lead-time sensitivities were increased. The results are obvious as when the sensitivities increased, the delivery lead time decreased significantly. Profitabilities in the supply chain decreased due to lesser demand and so did production. When lead-time sensitivity towards drop-shipping was relatively greater (i.e., β^{l}_{bmz}=0.75 and β^{l}_{emz}=0.5), the retailer's profit decreased by approximately 5%. Otherwise, the retailer's profit did not decrease significantly when the sensitivities were increased.

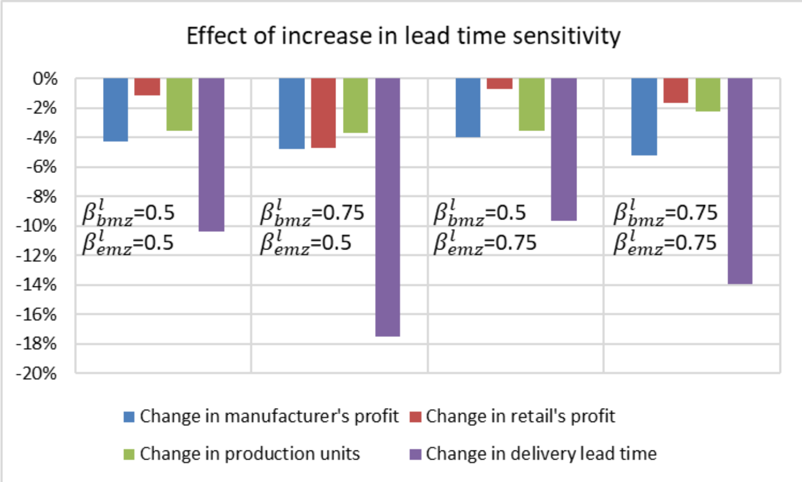

Fig. 10: Effect of increase in lead-time sensitivities on profitabilities, production and lead time.

VI. DISCUSSION

It is important to underline that many of the results obtained in the numerical experiments section are controlled by demand parameters that we chose at the start of the analysis. Interdependencies between profitabilities, production quantity, intrinsic attractions, price sensitivities and lead-time sensitivities can be too complex to unequivocally declare the existence of any specific pattern. That said, our analysis in Section A of the numerical experiments strongly suggests that the inclusion of a drop-shipping channel in the channel mix can assist in generating higher returns apart from serving as an omnichannel service. Additionally, there is a scope for coordination between the two players with respect to determining the value for θ, assuming the players are interested in maximizing the total supply-chain profit.

From the analysis in Section B of the numerical experiments, we can infer that adding a drop-shipping channel may become an appealing option in cases when production capacity is not sufficient—that is, in the case of under-production, the manufacturer can obtain better returns using a drop-shipping channel. However, a drop-shipping channel may not be so useful in the case of surplus production. Because of over-production, the manufacturer is forced to keep W relatively low. The retailer already has an advantage and may not benefit significantly from the collaboration.

In the analyses we carried out in Section C, we consistently found that the increased popularity of the drop-shipping channel benefits the retailer more than the manufacturer. The results also indicate that the drop-shipping channel cannibalizes the direct channel because of the non-differentiability of prices in both these channels. We recommend that a manufacturer operating a direct channel consider these results before opting for full-scale integration of a drop-shipping channel.

Although an array of sensitivity analyses could be carried out, we rest our case highlighting that our primary aim was to draw researchers' attention towards revenue management for manufacturers operating in omnichannel retailing environments. We explored how Stackelberg equilibria can be obtained when objective functions are non-convex. The likeness of the insights generated in this paper with the recent publication [48], in which a manufacturer–retailer problem was solved using a traditional approach, strengthens the utility of the proposed bilevel programming for revenue management problems.

Limitations and Future Scope:

In the section on comparative analysis, Figs. 4a–4b shows the time complexity of the hierarchical bilevel optimization technique. As the number of variables increased, the algorithms took exponentially longer to converge. Therefore, for larger problem instances, hierarchical optimization techniques are not suitable. In the past, researchers have also used population-based solution techniques such as the genetic algorithm (GA), particle swarm intelligence (PSO), and several co-evolutionary approaches. We anticipate that those algorithms may take less time. Another approach to reducing the time complexity of multi-location larger problem instances is to convexify or linearize the retailer's objective function. This paper presented how a single-location problem can be simplified and solved using a single-level reduction technique. In the future, researchers may seek to do the same for multi-zonal problems.

Drop-shipping is a sustainable alternative when time sensitivities are not very high. We can see that when time sensitivities are high, the manufacturer seeks to expedite the deliveries, thereby leading to higher delivery costs. It is understood that when time sensitivities are too high, buying online and picking from store (BOPS) [4] and shipping from store (SFS) [49] are suitable alternatives. Extending the research further, it would be interesting to investigate how the players would collaborate when the manufacturer is ready to fulfill direct channel orders via BOPS and SFS routes, and how these facilities would affect profitabilities in the supply chain. Another avenue of research may examine how a retailer would price the product, given that a consumer would visit the store to collect the package, or whether the manufacturer should match the price posted by the retailer.

In this paper, we assumed that the demand is deterministic and that its parameters do not change over time. This assumption is restrictive as it can be difficult to have accurate information on actual demand well before the selling season starts. This calls for dynamic bilevel optimization that proactively optimizes production, prices and fulfillments based on recent demand and market trends. To cater to uncertainties, the same problem can also be reformulated using stochastic programming instead of assuming demand to be deterministic. Finally, we assumed the manufacturer to be the leader; however, the scenario where a retailer is the leader has not been considered. The paper focused on profit maximization for both retailer and manufacturer; however, there are scenarios where retailers follow "everyday low pricing" with little regard for profit maximization in the short term.

VII. CONCLUSION

This paper proposed a joint production, pricing and inventory control problem for manufacturers operating in a competitive omnichannel environment; the premise is revenue management for manufacturers. An increasing number of manufacturers are operating direct channels to have immediate access to the market and to reduce their dependency on retailers. Direct channels play a crucial role in generating profits in the supply chain. However, while formulating a revenue management problem for a manufacturer, the direct channel must not be seen to be independent of the retailer's online and offline channels. A mere suggestion for revenue management for the manufacturers leads to bilevel optimization that essentially relates to Stackelberg leader–follower games. Keeping in view that drop-shipping may become a necessary omnichannel service, the paper simultaneously investigated the possibility of collaboration between the two players via a drop-shipping channel.

To solve the formulated problem, the paper first examined the single-level reduction technique commonly used in bilevel optimization literature. For single-level reduction, it was necessary to convexify the lower-level problem so that the necessary KKT conditions could be applied. In this paper, the lower-level problem was convexified assuming the retailer operated only one physical store. Even after reducing the bilevel problem to a single-level problem, the overall problem remained non-linear and non-convex. As shown in the paper, the resulting problems can be solved directly using stochastic search algorithms. Nevertheless, research is required to convexify the lower-level problem if the retailer operates from multiple locations. For now, the paper proposed a hierarchical optimization technique coupled with stochastic search algorithms to solve such problems.

The paper carried out numerical experiments to generate some insights as presented in the Discussion section. Our results suggest that in the absence of a drop-shipping channel, the players would operate in an entirely competitive mode, leading to suboptimal supply-chain profit. Therefore, the inclusion of a drop-shipping channel in the channel mix not only serves as an omnichannel feature but can also assist in generating a higher profit. The paper shows how, in the presence of drop-shipping services, retailers' existing online and offline channels can effortlessly become webrooms and showrooms, respectively. The paper further suggested that adding a drop-shipping channel can a particularly appealing option in cases when the production capacity is not sufficient. However, the results also suggest that the introduction of a drop-shipping channel benefits the retailer more than the manufacturer.

APPENDIX

Proof for Proposition 1:

In the single-zone case ($Z = 1$), it is assumed that the retailer has an offline store and an online channel (that caters to the demand generated locally). Let $\frac{A_b^t}{1+A_b^t+A_{er}^t+A_{em}^t+A_{bm}^t}=\lambda_b^t$, $\frac{A_{er}^t}{1+A_b^t+A_{er}^t+A_{em}^t+A_{bm}^t}=\lambda_{er}^t$ and $\frac{A_{bm}^t}{1+A_b^t+A_{er}^t+A_{em}^t+A_{bm}^t}=\lambda_{bm}^t$ where λ_b^t, λ_{er}^t and λ_{bm}^t represent purchase probabilities at time *t* in the store, retailer's online channels and drop-shipping channels, respectively. For simplicity of expression, let us assume that holding costs are negligible and $T = 1$ such that the retailer's objective function and constraints (Eqs. 10–14) appear as follows:

Maximize: $(P_b - W)S_b + (P_{er} - W)S_{er} + (P_{em} - c_m - c_l)\theta S_{bm} - c_{er}S_{er}$ (A.1)

Subject to: $S_b \leq M\lambda_b$ (A.2)

$S_{er} \leq M\lambda_{er}$ (A.3)

$$S_b, S_{er} \geq 0 \tag{A.4}$$

$$P_{er}, P_b \in \Omega \tag{A.5}$$

Next, we employ reformulation and constraint relaxation. It can be proved that $P_{em} = \frac{1}{\beta^p_{bm}}\left(\alpha^p_{bm} - \ln\left(\frac{\lambda_{bm}}{1-\lambda_b-\lambda_{er}-\lambda_{em}-\lambda_{bm}}\right) - \beta^l_{bm} l_{bm}\right)$, $P_b = \frac{1}{\beta^p_b}\left(\alpha^p_b - \ln\left(\frac{\lambda_b}{1-\lambda_b-\lambda_{er}-\lambda_{em}-\lambda_{bm}}\right)\right)$ and $P_{er} = \frac{1}{\beta^p_{er}}\left(\alpha^p_{er} - \ln\left(\frac{\lambda_{er}}{1-\lambda_b-\lambda_{er}-\lambda_{em}-\lambda_{bm}}\right)\right)$. Furthermore, assuming that all the demand generated at the retailer's end is satisfied, constraints (A.2) and (A.3) can be eliminated such that $S_b = M\lambda_b$ and $S_{er} = M\lambda_{er}$. By doing so, constraint (A.4) is also eliminated because of the non-negativity of λ_b and λ_{er}. Finally, non-differentiability due to the discrete nature of the decision variables (such as price) is also neglected. Thus, the retailer's objective function, i.e., (A.1), becomes (A.6), with no constraints to satisfy.

Maximize: $\Pi_r =$

$$\left(\frac{1}{\beta^p_b}\left(\alpha^p_b - \ln\left(\frac{\lambda_b}{1-\lambda_b-\lambda_{er}-\lambda_{em}-\lambda_{bm}}\right)\right) - W\right)M\lambda_b + \left(\frac{1}{\beta^p_{er}}\left(\alpha^p_{er} - \ln\left(\frac{\lambda_{er}}{1-\lambda_b-\lambda_{er}-\lambda_{em}-\lambda_{bm}}\right)\right) - W\right)M\lambda_{er}$$
$$+\left(\frac{1}{\beta^p_{bm}}\left(\alpha^p_{bm} - \ln\left(\frac{\lambda_{bm}}{1-\lambda_b-\lambda_{er}-\lambda_{em}-\lambda_{bm}}\right) - \beta^l_{bm} l_{bm}\right) - c_m - c_l\right)\theta M\lambda_{bm} - c_{er} M\lambda_{er} \tag{A.6}$$

Eigenvalues for first, second and third terms can be calculated as $\left\{0, -\frac{M(\lambda_b^2+(-1+\lambda_{em}+\lambda_{\mathrm{bm}}+\lambda_{er})^2)}{\beta^p_b\lambda_b(-1+\lambda_b+\lambda_{em}+\lambda_{\mathrm{bm}}+\lambda_{er})^2}\right\}$, $\left\{0, -\frac{M(\lambda_{er}^2+(-1+\lambda_b+\lambda_{em}+\lambda_{\mathrm{bm}})^2)}{\beta^p_{er}\lambda_{er}(-1+\lambda_b+\lambda_{em}+\lambda_{\mathrm{bm}}+\lambda_{er})^2}\right\}$, $\left\{-\frac{2M\theta\lambda_{\mathrm{bm}}}{\beta^p_{bm}(-1+\lambda_b+\lambda_{em}+\lambda_{\mathrm{bm}}+\lambda_{er})^2}, 0\right\}$, which are all negative semidefinite. This means that first, second and third terms are concave in purchase probability space $(\lambda_{er}, \lambda_b)$. The fourth term is linear. Since a non-negative weighted sum of concave functions is itself concave, the retailer's objective function, when the retailer is operating in just one market zone, is proved to be concave.

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