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Large-scale messengers from massive higher spin fields

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Abstract: We show that nonperturbative production of massive higher spin fields from vacuum will accompany the generation of non-vanishing macroscopic energy-momentum tensor correlators. This argument is based on the general causal field formalism, which gives a manifestly covariant description of higher spin particles without any reference to gauge redundancy. Our findings are direct consequence of the Poincaré covariance and analyticity of the Green's functions and independent of any detailed particle physics model. Further, we discuss the idea that any mechanism causing imbalance between the on-shell production of left- and right-handed fields leads to a helical structure in the energy momentum correlators and violation of the macroscopic parity symmetry. We check our method for fields with spin $\frac{1}{2}$ and show that it correctly reproduces previous results. The formalism, however, suffers from pathologies related to non-localities that appear for massless particles with spin ≥ 1 in flat space. We discuss the origin of these pathologies and the relevance of our findings to cosmology.

Keywords: Higher Spin Symmetry, Nonperturbative Effects, Cosmology of Theories beyond the SM

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Contents

1 Introduction

Higher spin fields is among the most challenging topics in theoretical physics. Irrespective of the theoretical difficulties, there are a few examples where massive higher spin fields naturally arise in physical contexts. For example, hadronic resonances have masses of order the strong scale and spins $s \geq 1/2$. String theory, in addition, has a spectrum of massive higher spin fields with masses above the Planck scale. QCD excitations can be easily produced in terrestrial high energy experiments. The production of particles with masses larger than a few TeV, however, is infeasible given today's limited technology. Fortunately enough, massive higher spin particles could be produced in the early Universe, e.g., during

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inflation, electroweak, or QCD phase transitions. This will happen provided there is a mechanism that leads to the production of these particle, e.g., parametric resonance [1]. Generally, the non-perturbative production of particles from vacuum will be accompanied by the generation of gravitational waves (GW) [2, 3]. Despite the fact that heavy particles will immediately decay once they are produces, GW accompanying their production might be detected today as a signal from the early epoch of the Universe. One hopes that these GW will have distinct features that distinguish them among other early Universe signals.[1]

A systematic study of higher spins started as early as quantum field theory itself. However, this subject remains a largely unexplored territory until today. In 1939, Fierz and Pauli were the first to write a system of Lorentz covariant equations that respect unitarity and describe the motion of higher spin fields in flat background [7]. In their studies, they noticed that turning on interactions among higher spin particles results in inconsistencies in the field theories describing them. This led them to suggest that a Lagrangian formulation of higher spin fields might evade these difficulties and renders the theory consistent. It was not until the mid 1970s when Singh, Hagen, Fronsdal, and Fang, completed the program of Feirz and Pauli by writing a Lagrangian formulation of fields with arbitrary spins [8–12]. This formulation (known as Fronsdal formulation) extends the concept of gauge field theory of electromagnetism and gravity to an arbitrary spin and demands the introduction of auxiliary fields that are used to eliminate the spurious degrees of freedom. This Lagrangian formulation, however, was limited to free field theories. In the meanwhile, many no-go theorems appeared in the period from 1960s to 1980s that forbid the minimal coupling between higher spin fields and electromagnetism or gravity, see [13–15] for reviews. In addition, Weinberg and Witten proved that there cannot be a consistent massless field theory with spin $s \geq 2$ that has a gauge invariant and conserved energy-momentum tensor [16]. However, starting from 1980 many yes-go examples of interacting higher spin fields appeared. In all these examples non-minimal coupling, e.g., [17], and/or formulating the theory on a curved background, e.g., [18], was a necessary ingredient for a consistent description of higher spin particles.

Independently, Bragmann and Wigner [19] and Weinberg [20, 21] wrote down a free field description of higher spin particles based on the higher dimensional irreducible representations of Poincaré algebra. These fields are known as the *general causal fields* [22]. This construction generalizes the Weyl equation, which describes the motion of a spin half particle belonging to $(1/2, 0)$ or $(0, 1/2)$ representation of the Lorentz algebra, to higher spin fields that transform as $(s, 0)$ or $(s, 0)$ under the Lorentz group. Alternatively, we can also use the Dirac representation $(s, 0) \oplus (0, s)$, which is particularly important in the case of massive higher spin particles. Here, one does not introduce any gauge redundancy since the fields used in this description are the physical ones. Therefore, no question of inconsistency or unphysical states can arise. The price one pays by working with the physical fields, however, is the lack of a Lagrangian formulation which makes their coupling to a background field, like electromagnetism or gravity, a tricky business.

[1] Non-Gaussianities from higher spin fields was discussed in [4]. See also [5, 6] for recent discussions of the imprints of higher spin fields on cosmological perturbations.

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In this work we examine the generation of large scale signals from the nonperturbative production of general causal fields. In particular, we seek a model-independent setup that can provide simplified answers about the generation of GW that accompany the parametric-resonance production of higher spin fields. A precise account of the details of such scenario is a daunting task, specially that coupling higher spin fields to gravity (not to mention their coupling to other fields, which is necessary to provide the parametric resonance mechanism) is still an open question. In our setup, however, we do not need to know about the specific particle-physics model that explains the generation of higher spin fields from vacuum. We just assume that these fields are somehow produced, and therefore, we only need to compute their on-shell energy-momentum tensor correlators. This should be translated into large scale GW signals via multiplying by appropriate transfer, i.e., Green's functions. The normal procedure to compute the energy-momentum tensor correlator, which accompanies the production of particles in the presence of an external field or time varying background, e.g., in a cosmological context, is to start from a Lagrangian, vary it with respect to the metric tensor to obtain the energy-momentum tensor, and then use the canonical quantization to write down the fields in terms of creations and annihilation operators. Next, we evolve these operators via the Bogoliubov transformation to finally obtain an expression of the energy-momentum tensor and its correlators. The absence of a Lagrangian formulation for the general causal fields, however, makes it necessary to find an alternative route that enables us tackle the problem indirectly. Motivated by the effective action of $s = \frac{1}{2}$ field along with the principle of Poincaré covariance, we postulate a definition of the energy-momentum tensor of general causal fields in a flat or curved background. Then, we use the analytic properties of Green's function to show that the nonperturbative production of massive higher spin fields from vacuum will accompany the emergence of non-vanishing energy-momentum tensor correlators.

As a check on our new formalism, we compute the energy-momentum tensor correlator of $s = 1/2$ fields (both massive and massless) in flat and FRW backgrounds reproducing previous results that were obtained using the canonical formalism. Further, we apply our method to compute the energy-momentum tensor correlators of massive spin particles with spin $s = 1$ and show that these correlators respect unitarity. This can be shown by projecting the correlators along helicity-2 eigenbasis, which is a succinct way to directly check that $\langle T_-(k)T_-(-k)\rangle > 0$ and $\langle T_+(k)T_+(-k)\rangle > 0$, where $T_\pm$ are the energy-momentum tensor along the positive (negative) helicity-basis. Interestingly enough, these two correlators may not be equal in the presence of a mechanism that favors one helicity over the other, which can be envisaged directly from our construction without the need to provide a detailed particle-physics model to explain it. We also give general expressions of the energy-momentum tensor correlators for massive fields with $s \geq 2$. The explicit computations of these correlators as well as checking their unitarity is a cumbersome task that is left for future investigations.

We also show that the production of massless fields with spin $s \geq 1$ in a flat background is pathological within the general causal framework. This is attributed to the fact that the energy-momentum tensors of these fields are highly non-local functions of momentum.

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Our work in organized as follows. In section 2 we provide the necessary mathematical background of the general causal fields in both flat and curved backgrounds. In particular, we review the Lorentz group and Weinberg construction of Green's functions of the general causal fields in flat space. This includes both massless and massive fields. Next, we generalize this construction to curved background using the vierbein formalism. We also provide a review section on spinor calculus. In section 3 we use the effective action of the spin $\frac{1}{2}$ field along with the principle of Poincaré covariance as a motivation to postulate a definition of the energy-momentum tensor of higher spin general causal fields. The energy-momentum tensor and its correlators contain vertex functions that we determine in section 4. In section 5 we use the formalism of sections 2 to 4 along with the analyticity of Green's functions to show that the production of higher spin fields from vacuum accompanies the emergence of macroscopic energy-momentum tensor correlator. We calculate this correlator for $s = \frac{1}{2}$ and $s = 1$ in flat background and also for $s = \frac{1}{2}$ in FRW background. Using the energy-momentum tensor correlator, we review the gravitational waves power spectrum in section 6. We conclude in section 7 by discussing the implications and limitations of our work.

2 Theory and formulation

2.1 General causal fields

This section aims to introduce the formalism and notation used throughout this work. We use the general causal fields (also called the Weinberg-type fields) to describe higher spin particles. This formalism is heavily based on the higher dimensional representations of the Lorentz algebra $so(3,1)$ or its double cover $sl(2,\mathbb{C})$ that we will review momentarily. In this work we use the signature $\eta_{ab} = (-,+,+,+)$. The Greek letters are used to denote the curved spacetime coordinates, the latin letters a, b, c, d are used for the flat spacetime coordinates, the latin letters i, j, k label the spatial flat coordinates, while the latin letters M, N, P, Q are used to label the matrix elements of $sl(2,\mathbb{C})$. The Lorentz algebra generators $\mathcal{T}_{ab}$ satisfy the commutation relations [23]

$$[\mathcal{T}_{ab}, \mathcal{T}_{cd}] = i\left(\eta_{cb}\mathcal{T}_{ad} - \eta_{ca}\mathcal{T}_{bd} + \eta_{db}\mathcal{T}_{ca} - \eta_{da}\mathcal{T}_{cb}\right). \tag{2.1}$$

In particular we have $\mathcal{T}_{ab} = -\mathcal{T}_{ba}$ such that

$$\mathcal{T}_{i0} = K_i\,, \quad \frac{1}{2}\epsilon_{ijk}\mathcal{T}_{jk} = J_i\,, \tag{2.2}$$

where K_i and J_i are respectively the generators of boosts and $so(3)$ rotations. Then, it is a simple exercise to use (2.1) to show that the combinations

$$\mathcal{A}_i = \frac{1}{2}\left(J_i + iK_i\right), \quad \mathcal{B}_i = \frac{1}{2}\left(J_i - iK_i\right) \tag{2.3}$$

satisfy the commutation relations of two independent copies of $su(2)$ algebra:

$$[\mathcal{A}_i, \mathcal{A}_j] = i\epsilon_{ijk}\mathcal{A}_i\,, \quad [\mathcal{B}_i, \mathcal{B}_j] = i\epsilon_{ijk}\mathcal{B}_i\,, \quad [\mathcal{A}_i, \mathcal{B}_j] = 0 \text{ for all } i, j\,. \tag{2.4}$$

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The two algebras are called $su_L(2)$ and $su_R(2)$ for $\{\mathcal{A}_i\}$ and $\{\mathcal{B}_i\}$ generators, respectively. Let the set of generators $\{\mathcal{C}_i\}$ denotes either $\{\mathcal{A}_i\}$ or $\{\mathcal{B}_i\}$. Then for a representation of spin s we have

$$\left[\mathcal{C}_1^{(s)} \pm i\mathcal{C}_2^{(s)}\right]_M^N = \delta_{M,N\pm M}\left[(s\mp N)(s\pm N+1)\right]^{1/2}\,, \quad \left[\mathcal{C}_3^{(s)}\right]_M^N = M\delta_{MN}\,, \tag{2.5}$$

where $M, N = s, s-1, \ldots, -s+1, -s$ and $s = 0, 1/2, 1, 3/2, \ldots$

The complexified generators of $so(3,1)$ are related to $su(2)_L \oplus su(2)_R$, and hence, any representation of the Lorentz algebra can be designated by a pair of numbers (s_1, s_2), where s_1 and s_2 are the spins of the representations of $su(2)_L$ and $su(2)_R$, respectively. Thus, the generators $\mathcal{T}_{ab}$ can be written as the direct sum of two sets of generators transforming under $su(2)_L$ and $su(2)_R$ such that $\mathcal{T}_{ab} \equiv I^{su(2)_L} \otimes \mathcal{T}_{ab}^{su(2)_R} + \mathcal{T}_{ab}^{su(2)_L} \otimes I^{su(2)_R}$, or explicitly using the matrix elements $[\mathcal{T}_{ab}]_{N_1N_2}^{M_1M_2} = \delta_{N_1}^{M_1}\,[\mathcal{T}_{ab}]_{N_2}^{M_2} + [\mathcal{T}_{ab}]_{N_1}^{M_1}\,\delta_{N_2}^{M_2}$, where $M_1, N_1 = s_1, s_1-1, \ldots, -s_1+1, -s_1$ and $M_2, N_2 = s_2, s_2-1, \ldots, -s_2+1, -s_2$.

Under the Lorentz transformation, a general field transforms in the s_1 representation of $su(2)_L$ and s_2 representation of $su(2)_R$. As special cases, we consider fields with $s_1 = 0$ or $s_2 = 0$ such that:

$$\begin{aligned} &\boldsymbol{J} = \boldsymbol{J}^{(s)}\,, \quad \boldsymbol{K} = -i\boldsymbol{J}^{(s)} \quad \text{for type } (s,0) \text{ fields}\,,\\ &\boldsymbol{J} = \boldsymbol{J}^{(s)}\,, \quad \boldsymbol{K} = +i\boldsymbol{J}^{(s)} \quad \text{for type } (0,s) \text{ fields}\,, \end{aligned} \tag{2.6}$$

where a boldface symbol denotes a three dimensional vector. Such fields transform as $2s+1$ dimensional spinors and are denoted by Φ_L for $(s,0)$ and Φ_R for $(0,s)$ such that $\Phi_{L,R}^{(s)} \equiv (\Phi_1, \Phi_2, \ldots, \Phi_{2s+1})^T$. They are known as the *general causal fields* [22] or Weinberg-type fields [20, 21]. Under a general proper Lorentz transformation Λ (we can always take the boost to be along the z-direction without loss of generality) these fields transform as

$$\begin{aligned} &\Phi_L^{(s)} \to D^{(s)}[\Lambda]\Phi_L^{(s)} = D^{(s)}[R]e^{-i\phi K_3^{(s)}}\Phi_L^{(s)} \text{ for type } (s,0) \text{ fields}\,,\\ &\Phi_R^{(s)} \to \bar{D}^{(s)}[\Lambda]\Phi_R^{(s)} = D^{(s)}[R]e^{-i\phi K_3^{(s)}}\Phi_R^{(s)} \text{ for type } (0,s) \text{ fields}\,, \end{aligned} \tag{2.7}$$

where ϕ is the boost parameter (rapidity) along the z-direction and $D^{(s)}[R]$ is the $so(3)$ part of the Lorentz transformation matrix.

Now consider a $(s,0)$ massless field moving along the z-direction with its spin directing along the positive z-axis: $\Phi_L^{(s)} = \left(0, 0, 0, \ldots, \underbrace{1}_{2s+1}\right)^T$. Then, it is trivial to show that such field satisfies

$$J_3^{(s)}\Phi_L^{(s)} = -s\Phi_L^{(s)}\,. \tag{2.8}$$

By applying a general rotation about the z-axis we can write the above equation as

$$\boldsymbol{p}\cdot\boldsymbol{J}^{(s)}\Phi_L^{(s)} = -s|\boldsymbol{p}|\Phi_L^{(s)}\,, \quad \text{for } (s,0) \text{ fields}\,, \tag{2.9}$$

where $\boldsymbol{p}$ is the field momentum. Thus, the $(s,0)$ fields describe left-handed particles with helicity $-s$. By the same token, the $(0,s)$ fields satisfy

$$\boldsymbol{p}\cdot\boldsymbol{J}^{(s)}\Phi_R^{(s)} = +s|\boldsymbol{p}|\Phi_R^{(s)}\,, \quad \text{for } (0,s) \text{ fields}\,, \tag{2.10}$$

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and they describe right-handed particles with helicity $+s$. Now, it is not difficult to see that (2.9) and (2.10) can be rewritten as first order differential equations:

$$\begin{aligned} \boldsymbol{J}^{(s)} \cdot \nabla \Phi_L^{(s)} - s\partial_t \Phi_L^{(s)} = 0\,, \quad \text{for } (s,0) \text{ fields}\,, \\ \boldsymbol{J}^{(s)} \cdot \nabla \Phi_R^{(s)} + s\partial_t \Phi_R^{(s)} = 0\,, \quad \text{for } (0,s) \text{ fields}\,. \end{aligned} \tag{2.11}$$

Every equation describes the propagation of a free single degree of freedom. In addition, both types of fields satisfy the second order Klein-Gordon equation $\Box \Phi_{L,R} = 0$. In fact, Equations (2.11) are the Weyl equations for $s = 1/2$. With a bit of work, one can show that the case $s = 1$ corresponds to Maxwell's equations for the left and right circularly polarized radiation in free space:[2]

$$\begin{aligned} \nabla \times (\boldsymbol{E} - i\boldsymbol{B}) + i\frac{\partial}{\partial t}(\boldsymbol{E} - i\boldsymbol{B}) = 0\,, \quad \text{for } (1,0) \text{ fields}\,, \\ \nabla \times (\boldsymbol{E} + i\boldsymbol{B}) - i\frac{\partial}{\partial t}(\boldsymbol{E} + i\boldsymbol{B}) = 0\,, \quad \text{for } (0,1) \text{ fields}\,. \end{aligned} \tag{2.13}$$

Moreover, applying the divergence operator on the above equations we obtain the Bianchi identity $\nabla \cdot \boldsymbol{E} = \nabla \cdot \boldsymbol{B} = 0$. Since the equations of motion are first order, there are no pathologies associated with the higher spins when we use the general causal construction. Notice also that here we work directly with the field strengths (the physical fields) rather than the potential fields, as is evident from $s = 1/2$ and $s = 1$ examples.

The massless general causal fields are chiral by construction. Thus, under the parity transformation $\mathcal{P} : \boldsymbol{r} \to -\boldsymbol{r}$ we have

$$\boldsymbol{K} \xrightarrow{\mathcal{P}} -\boldsymbol{K}\,, \quad \boldsymbol{J} \xrightarrow{\mathcal{P}} \boldsymbol{J}\,, \tag{2.14}$$

and hence,

$$\Phi_L^{(s)} \xrightarrow{\mathcal{P}} \Phi_R^{(s)}\,. \tag{2.15}$$

For massless fields chirality and helicity coincide, and both work as a good Lorentz invariant quantum number. We will also limit our analysis to the left-handed fields $(s, 0)$ since the right-handed ones follow the exact same construction. The left handed fields will be denoted by Φ, with no L subscript, when no confusion can arise.

We can also use the Dirac's representation $(s, 0) \oplus (0, s)$. In this case one combines the left and right fields in a single field $\Psi^{(s)}$ as

$$\Psi^{(s)} = \begin{bmatrix} \Phi_L^{(s)} \\ \Phi_R^{(s)} \end{bmatrix}, \tag{2.16}$$

such that $\Psi^{(s)}$ respects the parity, charge conjugation, and time reversal symmetries. This construction is particularly important when we deal with massive fields.

[2] For example, using the assignment $\boldsymbol{\psi} = \boldsymbol{E} - i\boldsymbol{B}$, one can show the equivalence between the first equation in (2.13) and the first equation in (2.11) via the transformation (Φ denotes the left-handed field):

$$\psi_1 = \frac{-i}{\sqrt{2}}(\Phi_1 - \Phi_3)\,, \quad \psi_2 = \frac{1}{\sqrt{2}}(\Phi_1 + \Phi_3)\,, \quad \psi_3 = i\Phi_2\,. \tag{2.12}$$

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2.2 Quantization of the general causal fields in flat space: massless fields

After discussing the classical aspects of the general causal fields, now we turn into their canonical quantization. As we mentioned above, a free left-handed particle moving along the z-direction, with reference momentum κ, is described by the spinor $\xi^{(s)} = \left(0, 0, \ldots, 0, \underbrace{1}_{2s+1}\right)^T$. One can obtain the general form of the spinor by applying a general proper Lorentz transformation Λ (boost and rotation):

$$\xi^{(s)} \to \xi^{(s)}(\boldsymbol{p}) = D^{(s)}[\Lambda]\xi^{(s)} \equiv D^{(s)}[R(\boldsymbol{p})]e^{-i\phi(|\boldsymbol{p}|)K_3}\xi^{(s)}\,, \tag{2.17}$$

where the matrix $D^{(s)}[R(\boldsymbol{p})]$ is the rotational part of the Lorentz transformation,[3] while $e^{-i\phi(|\boldsymbol{p}|)K_3}$ is the boost. One can show easily that the rapidity ϕ is given by $\phi(|\boldsymbol{p}|) = \log\left[|\boldsymbol{p}|/\kappa\right]$, where $\boldsymbol{p}$ is the particle's momentum after applying the boost.

Now, the quantization of the general causal fields can be achieved by carrying out the exact same steps used in the quantization of a spin 1/2 field. One expands the classical field in terms of a complete set of orthonormal states with complex coefficients. Then, we promote these coefficients to creation and annihilation operators. In this work we are interested in the production of on-shell higher spin particles, and hence, all our fields and their Green's functions will be assumed to satisfy the on-shell condition. The quantum field $\Phi^{(s)}$, then, takes the following canonical form in the Fourier space

$$\Phi^{(s)}_M(x) = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3\sqrt{2p^0}}\left[\frac{|\boldsymbol{p}|}{\kappa}\right]^s\left[a_{\boldsymbol{p}}e^{i\boldsymbol{p}\cdot\boldsymbol{x}-ip^0t} + b^\dagger_{\boldsymbol{p}}e^{-i\boldsymbol{p}\cdot\boldsymbol{x}+ip^0t}\right]D^{(s)}_{M,-s}[R(\boldsymbol{p})]\,, \tag{2.19}$$

where the on-shell condition, $p^0 = |\boldsymbol{p}|$, has been assumed and we have used (2.17) and (2.6). The annihilation and creation operators $a_{\boldsymbol{p}}, a^\dagger_{\boldsymbol{p}}, b_{\boldsymbol{p}}$, and $b^\dagger_{\boldsymbol{p}}$ satisfy the (anti)commutation relations

$$\left[a_{\boldsymbol{p}}, a^\dagger_{\boldsymbol{p}'}\right]_\pm = (2\pi)^3\delta^3(\boldsymbol{p}-\boldsymbol{p}')\,, \quad \left[b_{\boldsymbol{p}}, b^\dagger_{\boldsymbol{p}'}\right]_\pm = (2\pi)^3\delta^3(\boldsymbol{p}-\boldsymbol{p}')\,, \tag{2.20}$$

where the commutator (the $-$ sign) is used for integer spin fields and the anti-commutator (the $+$ sign) is used for the half-integer spin fields. Then, it can be easily shown that these fields satisfy the microcausality condition

$$\left[\Phi^{(s)}_M(x), \Phi^{\dagger(s)}_N(x')\right]_\pm = -\Pi_{MN}(-i\partial)\Delta(x-x')\,, \tag{2.21}$$

and the matrices Π_{MN} are defined by (2.38) below and $\Delta(x-x')$ is the scalar propagator. In fact, the (anti)commutator relations (2.21) establish the connection between spin and statistics. It is customly to redefine the normalization of the massless field Φ by replacing the factor κ^{-s} with 2^s, which we will do in the rest of this work.

[3] The matrix $D^{(s)}[R(\boldsymbol{p})]$ is an $so(3)$ rotation that takes the form

$$D^{(s)}[R(\boldsymbol{p})] = e^{-i\hat{\boldsymbol{n}}\cdot\boldsymbol{J}^{(s)}\theta}\,, \quad \hat{\boldsymbol{n}} = \frac{(-p_y, p_x, 0)}{\sqrt{p_x^2+p_y^2}}\,, \quad \theta = \cos^{-1}\left(\frac{p_z}{|\boldsymbol{p}|}\right). \tag{2.18}$$

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Notice that in writing the quantum field $\Phi^{(s)}_M(x)$, which describes a single degree of freedom, we used the annihilation and creation operators of the physical particles, which are described by the spinor $D^{(s)}_{M,-s}[R(\boldsymbol{p})]$. This spared us from the need to introduce any spurious degrees of freedom that could lead to unphysical negative energy states or inconsistencies.

Structure of the massless Green's functions. As we show in section 3 the Green's functions are indispensable tool to couple general causal fields to a fixed background. The time-ordered Green's function for the left-handed fields is defined as

$$G^{(s)}_{0\,MN}(\boldsymbol{x},\boldsymbol{x}',t,t') = \langle 0|\mathcal{T}\Phi^{(s)}_M(\boldsymbol{x},t)\Phi^{(s)\dagger}_N(\boldsymbol{x}',t')|0\rangle\,, \tag{2.22}$$

where the time-order operator $\mathcal{T}$ is

$$\mathcal{T}\Phi^{(s)}_M(\boldsymbol{x},t)\Phi^{(s)\dagger}_N(\boldsymbol{x}',t') = \begin{cases} \Phi^{(s)}_M(\boldsymbol{x},t)\Phi^{(s)\dagger}_N(\boldsymbol{x}',t') & t>t' \\ (-1)^{2s}\Phi^{(s)\dagger}_N(\boldsymbol{x}',t')\Phi^{(s)}_M(\boldsymbol{x},t) & t<t' \end{cases}. \tag{2.23}$$

Now, using (2.19) and $a|0\rangle = b|0\rangle = 0$ we can write the Green's function in the form

$$G^{(s)}_{0\,MN}(\boldsymbol{x},\boldsymbol{x}',t,t') = \int \frac{d^3p}{(2\pi)^3} e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{x}')}\mathcal{G}^{(s)}_{0\,MN}(\boldsymbol{p},\boldsymbol{p}',t,t')\,, \tag{2.24}$$

where the momentum-space Green's function is

$$\mathcal{G}^{(s)}_{0\,MN}(\boldsymbol{p},\boldsymbol{p}',t,t') = \frac{|\boldsymbol{p}|^{2s}}{2|\boldsymbol{p}|}\begin{cases} \widehat{\Pi_{MN}}(p^0,\boldsymbol{p})e^{i|\boldsymbol{p}|(t-t')}\,, & t>t' \\ (-1)^{2s}\widehat{\Pi_{MN}}(p^0,-\boldsymbol{p})e^{i|\boldsymbol{p}|(t'-t)}\,, & t<t' \end{cases}, \tag{2.25}$$

and $\widehat{\Pi_{MN}}(p^0,\boldsymbol{p}) = D^{(s)}_{M,-s}[R(\boldsymbol{p})]D^{*(s)}_{N,-s}[R(\boldsymbol{p})]$. The matrices $\widehat{\Pi_{MN}}$ can be put in a nicer form by using the formula (which can easily be proved by induction):

$$\delta_{M,-s}\delta_{N,-s} = \frac{1}{(2s)!}\left[\prod_{\lambda=-s+1}^{s}(\lambda\mathbb{I}-J_3)\right]_{MN}, \tag{2.26}$$

and then applying the rotation matrix $D^s[R(\boldsymbol{p})]$ to find:

$$\widehat{\Pi_{MN}}(p^0,\boldsymbol{p}) = \frac{1}{(2s)!|\boldsymbol{p}|^{2s}}\left[\prod_{\lambda=-s+1}^{s}\left(\lambda p^0\mathbb{I}-\boldsymbol{p}\cdot\boldsymbol{J}\right)\right]_{MN}. \tag{2.27}$$

In section 2.4 we show that one can write $\widehat{\Pi_{MN}}(p^0,\boldsymbol{p})$ in the Lorentz covariant form

$$\widehat{\Pi_{MN}}(p^0,\boldsymbol{p}) = \frac{(-1)^{2s}}{2^{2s}}t^{a_1a_2\ldots a_{2s}}_{MN}n_{a_1}(\boldsymbol{p})n_{a_2}(\boldsymbol{p})\ldots n_{a_{2s}}(\boldsymbol{p})\,, \tag{2.28}$$

where $n_a \equiv (1,\frac{\boldsymbol{p}}{|\boldsymbol{p}|})$ are light-like vectors and $t^{a_1a_2\ldots a_{2s}}_{MN}$ are the generalized Pauli matrices as we explain in section 2.4.

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2.3 Quantization of the general causal fields in flat space: massive fields

In the massive case it is more convenient to work with the Dirac representation $(s,0)\oplus(0,s)$, where one combines the left and right fields in a single field $\Psi^{(s)}$ as defined in (2.16). The field $\Psi^{(s)}$ satisfies the massive Klein-Gordon equation

$$(\Box - m^2)\Psi^{(s)}(x) = 0\,. \tag{2.29}$$

In addition, one can show that $\Psi^{(s)}(x)$ satisfies the generalized Dirac's equation

$$\left[-i^{2s}\gamma^{a_1\ldots a_{2s}}\partial_{a_1}\ldots\partial_{a_{2s}} + m^{2s}\right]\Psi^{(s)}(x) = 0\,, \tag{2.30}$$

where the generalized γ matrices, $\gamma^{a_1\ldots a_{2s}}$, are given by

$$\gamma^{a_1\ldots a_{2s}} = \begin{bmatrix} 0 & t^{a_1\ldots a_{2s}} \\ \bar{t}^{a_1\ldots a_{2s}} & 0 \end{bmatrix}, \tag{2.31}$$

where the matrices t and $\bar{t}$ will be introduced in section 2.4. Unlike the massless case, where the left- or right- handed fields describe the propagation of a single degree of freedom, massive fields on the other hand describe $(2s+1)$ massive degrees of freedom for $(s,0)$ or $(0,s)$ fields. This is obvious since the dimension of SO(3) group (which is the little group in the massive case)[4] has dimension $(2s+1)$.

Next, as usual, we expand the left and right fields in terms of creation and annihilation operators as follows:

$$\begin{aligned}
\Phi^{(s)}_{L,M} &= \int \frac{d^3p}{(2\pi)^3\sqrt{2\omega(\boldsymbol{p})}} \sum_N \left\{ D^{(s)}_{MN}(\boldsymbol{p}) a_{\boldsymbol{p},N} e^{ip\cdot x} + D^{(s)}_{MN}(\boldsymbol{p}) b^\dagger_{\boldsymbol{p},N} e^{-ip\cdot x} \right\}, \\
\Phi^{(s)}_{R,M} &= \int \frac{d^3p}{(2\pi)^3\sqrt{2\omega(\boldsymbol{p})}} \sum_N \left\{ D^{(s)}_{MN}(-\boldsymbol{p}) a_{\boldsymbol{p},N} e^{ip\cdot x} + (-1)^{2s} D^{(s)}_{MN}(-\boldsymbol{p}) b^\dagger_{\boldsymbol{p},N} e^{-ip\cdot x} \right\},
\end{aligned} \tag{2.32}$$

where $\omega(\boldsymbol{p}) = \sqrt{m^2 + |\boldsymbol{p}|^2}$ is the on-shell condition and the sum over N goes from s to $-s$, which are all the $2s+1$ physical states. One then defines the time-ordered Green's function as

$$\mathbb{G}^{(s)}_{0\,MN}(\boldsymbol{x},\boldsymbol{x}',t,t') = \langle 0|\mathcal{T}\bar{\Psi}^{(s)}(\boldsymbol{x},t)\Psi^{(s)}(\boldsymbol{x}',t')|0\rangle\,, \tag{2.33}$$

where we have defined $\bar{\Psi}^{(s)} \equiv \beta\Psi^{(s)\dagger}$ and β is given by

$$\beta = \begin{bmatrix} 0 & \mathbb{I}_{(2s+1)\times(2s+1)} \\ \mathbb{I}_{(2s+1)\times(2s+1)} & 0 \end{bmatrix}. \tag{2.34}$$

Using (2.32) and the commutation relations

$$\left[a_{\boldsymbol{p},M}, a^\dagger_{\boldsymbol{p}',M'}\right]_\pm = (2\pi)^3\delta^3(\boldsymbol{p}-\boldsymbol{p}')\delta_{MM'}\,, \quad \left[b_{\boldsymbol{p},M}, b^\dagger_{\boldsymbol{p}',M'}\right]_\pm = (2\pi)^3\delta^3(\boldsymbol{p}-\boldsymbol{p}')\delta_{MM'}\,, \tag{2.35}$$

[4] Notice that the little group in the massless case is SO(2), and hence, the Dirac field has only 4 degrees of freedom.

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one can easily show

$$\mathbb{G}_0^{(s)}(\boldsymbol{x},\boldsymbol{x}',t,t') = \int \frac{d^3p}{(2\pi)^3} e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{x}')} \mathscr{G}_0(\boldsymbol{p},t,t')\,, \tag{2.36}$$

where

$$\mathscr{G}_0(\boldsymbol{p},t,t') = \frac{1}{2\omega(\boldsymbol{p})} \begin{cases} \begin{bmatrix} \mathbb{I} & (m)^{-2s}\Pi(p^0,\boldsymbol{p}) \\ (m)^{-2s}\bar{\Pi}(p^0,\boldsymbol{p}) & \mathbb{I} \end{bmatrix} e^{-i\omega(\boldsymbol{p})(t-t')}\,, & t > t' \\ (-1)^{2s} \begin{bmatrix} \mathbb{I} & (m)^{-2s}\Pi(p^0,-\boldsymbol{p}) \\ (m)^{-2s}\bar{\Pi}(p^0,-\boldsymbol{p}) & \mathbb{I} \end{bmatrix} e^{-i\omega(\boldsymbol{p})(t'-t)}\,, & t < t' \end{cases} . \tag{2.37}$$

The polarizations Π and $\bar{\Pi}$ are the time-like non-normalized version of $\widehat{\Pi}$ defined in (2.27). They were derived in [20] and we do not repeat this derivation here. They take the form

$$\Pi(p^0,\boldsymbol{p}) = (-1)^{2s} t^{a_1\ldots a_{2s}} p_{a_1}\ldots p_{a_{2s}}\,, \quad \bar{\Pi}(p^0,\boldsymbol{p}) = (-1)^{2s} \bar{t}^{a_1\ldots a_{2s}} p_{a_1}\ldots p_{a_{2s}}\,, \tag{2.38}$$

where p^μ are time-like on-shell vectors, and the tensors $t^{a_1\ldots a_{2s}}$ and $\bar{t}^{a_1\ldots a_{2s}}$ are defined in section 2.4.

2.4 Spinor calculus

In this section we pause to summarize an important piece of group theory that we have been using throughout this work. In particular, we elucidate the philosophy behind the generalized Pauli matrices $t^{a_1\ldots a_{2s}}$ and $\bar{t}^{a_1\ldots a_{2s}}$ that appeared in previous sections.

We start with the usual Pauli matrices $\boldsymbol{t} \equiv \boldsymbol{\sigma}$ and $t^0 \equiv \sigma^0 = \mathbb{I}$, which transform as four-vectors in the sense

$$D^{(1/2)}[\Lambda]\, t^a D^{(1/2)\dagger}[\Lambda] = \Lambda^a_b t^b\,, \tag{2.39}$$

where Λ is a general proper Lorentz transformation and $D^{(1/2)}[\Lambda]$ is the corresponding 2×2 Lorentz transformation matrix in the $(1/2,0)$ representation. This is the familiar construction of vectors in the basis of the defining representation $(1/2,0)$ of $su(2)$. Similarly, the matrices $\bar{\boldsymbol{t}} \equiv -\boldsymbol{\sigma}$ and $\bar{t}^0 \equiv \sigma^0 = \mathbb{I}$ transform as vectors in the $(0,1/2)$ representation. In fact, one can generalize this construction to represent tensors of rank $2s$ using the $2s+1$ dimensional representation matrices [20, 21, 24]. Then, one can prove that for irreducible representations of the Lorentz algebra there exists a set of $(2s+1)\times(2s+1)$ dimensional matrices $t^{a_1a_2\ldots a_{2s}}$ such that: (1) $t^{a_1a_2\ldots a_{2s}}$ is symmetric in $a_1,a_2,\ldots,a_{2s}$, (2) traceless in all indices, i.e. $\eta_{a_1a_2}t^{a_1a_2\ldots a_{2s}} = 0$, and that (3) $t^{a_1a_2\ldots a_{2s}}$ transforms as

$$D^{(s)}[\Lambda] t^{a_1a_2\ldots a_{2s}}_{MN} D^{(s)\dagger}[\Lambda] = \Lambda^{b_1}_{a_1}\Lambda^{b_2}_{a_2}\ldots\Lambda^{b_{2s}}_{a_{2s}} t^{b_1b_2\ldots b_{2s}}_{MN}\,, \tag{2.40}$$

where $M,N = s, s-1,\ldots,-s+1,-s$, and $D^{(s)}[\Lambda]$ is the corresponding $(2s+1)\times(2s+1)$ Lorentz transformation matrix in the $(s,0)$ representation [20, 21]. Similarly, there exists

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a set of $(2s+1)\times(2s+1)$ dimensional matrices $\bar{t}^{a_1a_2\ldots a_{2s}}$ that correspond to tensors in the $(0,s)$ representation. They transform according to

$$D^{(s)\dagger}[\Lambda^{-1}]\bar{t}^{a_1a_2\ldots a_{2s}}_{MN}D^{(s)}[\Lambda^{-1}] = \Lambda^{b_1}_{a_1}\Lambda^{b_2}_{a_2}\ldots\Lambda^{b_{2s}}_{a_{2s}}\bar{t}^{b_1b_2\ldots b_{2s}}_{MN}\,. \tag{2.41}$$

One can show that

$$\bar{t}^{a_1a_2\ldots a_{2s}} = (\pm 1)t^{a_1a_2\ldots a_{2s}}\,, \tag{2.42}$$

where the sign is $+1$ or -1 according to whether there are an even or odd number of space-like indices, respectively. Also one can show [20, 21]

$$t^{(a_1a_2\ldots a_{2s}}\bar{t}^{b_1b_2\ldots b_{2s})} = \eta^{(a_1b_1}\ldots\eta^{a_{2s}b_{2s})}\,, \tag{2.43}$$

where the parentheses denote complete symmetrization over the indicated indices. One can also prove the trace identity

$$\mathrm{tr}\left[t^{a_1a_2\ldots a_{2s}}\bar{t}^{b_1b_2\ldots b_{2s}}\right] = C_1\eta^{a_{i_1}a_{j_1}}\ldots\eta^{b_{i_2}b_{j_2}} + C_2\eta^{a_{i_1}b_{j_1}}\ldots\eta^{b_{i_2}a_{j_2}} + \ldots\,, \tag{2.44}$$

where $\{C_i\}$ are constants. For example, in the case $s=1$ we have $\mathrm{tr}\left[\bar{t}^{a_1a_2}t^{b_1b_2}\right] = -g^{a_1a_2}g^{b_1b_2}+2g^{a_1b_1}g^{a_2b_2}+2g^{a_1b_2}g^{a_2b_1}$. The trace identity will be crucial to our construction as we show in section 4.

Since the set of matrices $\{t^{a_1a_2\ldots a_{2s}}\}$ are linearly independent, we can use them to express $\widehat{\Pi_{MN}}(\hat{\boldsymbol{p}}) = D^{(s)}_{M,-s}[R(\hat{p})]D^{*(s)}_{N,-s}[R(\hat{p})]$ in a covariant form:

$$\widehat{\Pi_{MN}}(p^0,\boldsymbol{p}) = \frac{(-1)^{2s}}{|2\boldsymbol{p}|^{2s}}t^{a_1a_2\ldots a_{2s}}_{MN}p_{a_1}p_{a_2}\ldots p_{a_{2s}}\,, \tag{2.45}$$

where p is a light-like vector. Finally, defining $n_a(\boldsymbol{p}) \equiv \left(1, \frac{\boldsymbol{p}}{|\boldsymbol{p}|}\right)$ we can write $\widehat{\Pi_{MN}}(\hat{\boldsymbol{p}})$ that we defined in (2.27) as

$$\widehat{\Pi_{MN}}(p^0,\boldsymbol{p}) = \frac{(-1)^{2s}}{2^{2s}}t^{a_1a_2\ldots a_{2s}}_{MN}n_{a_1}(\boldsymbol{p})n_{a_2}(\boldsymbol{p})\ldots n_{a_{2s}}(\boldsymbol{p})\,. \tag{2.46}$$

By comparing (2.27) and (2.46) we can read off the expressions of $t^{a_1a_2\ldots a_{2s}}_{MN}$ in terms of $\{J^i\}$. For example, for the case $s=1$ we have $t^{00} = \bar{t}^{00} = \mathcal{I}$, $t^{0i} = t^{i0} = \boldsymbol{J}_i = -\bar{t}^{i0}$, $t^{ij} = \bar{t}^{ij} = \{\boldsymbol{J}_i,\boldsymbol{J}_j\} - \delta_{ij}$. We can use the same matrices to write the covariant expression of $\Pi(p^0,\boldsymbol{p})$ given in (2.38).

2.5 General causal fields in curved space: the vierbein formalism

The study of higher spin fields in curved background is of paramount importance since the production of ultra heavy higher spin resonances in expanding background could leave a distinct feature in the large scale observables, e.g., energy-momentum tensor correlators. In this section we review the *vierbein formalism*, which is necessary to construct general causal fields in curved background. Although our construction is general enough, the present work will limit the treatment to $s=1/2$ particles in curved background. Higher spin particles in curved background will be pursued elsewhere.

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Vierbeins, which are denoted by V^a_μ, are tangent vectors to a set of locally inertial coordinates ξ^a_X erected at a spacetime point X:

$$V^a_\mu \equiv \left(\frac{\partial \xi^a}{\partial x^\mu}\right)_{x=X} . \tag{2.47}$$

The Greek letters denote the curved space coordinates, while the latin letters $a, b, \ldots$ denote the flat space coordinates, which are raised and lowered with the flat space metric $\eta_{\mu\nu}$. We fix ξ^a_X at each spacetime point such that under a general non-inertial coordinate transformation we have

$$V'^a_\mu = \frac{\partial x^\nu}{\partial x'^\mu} V^a_\nu . \tag{2.48}$$

V^a_μ can be thought of 4 different covariant vectors, one for each value of a. Therefore, we refer any four vector or tensor at point x to an inertial frame at the same point by contracting it with the vierbein. For example, the curved spacetime metric tensor $g^{\mu\nu}(x)$ can be contracted with two virebians to obtain the inertial frame metric η_{ab} as follows

$$\eta^{ab} = V^a_\mu(x) V^b_\nu(x) g^{\mu\nu}(x) \tag{2.49}$$

or the inverse relation

$$g_{\mu\nu}(x) = V^a_\mu(x) V^b_\nu(x) \eta_{ab} . \tag{2.50}$$

Using this language we classify various objects as being Lorentz scalars, spinors, vectors, etc., or coordinate scalars, spinors, vectors, etc., or mixed combinations under both Lorentz and coordinate transformations. For example, V^a_μ is a Lorentz vector and coordinate vector, $g_{\mu\nu}$ is a coordinate tensor and Lorentz scalar, while η_{ab} is a coordinate scalar and Lorentz tensor.

One uses this technology to construct physical quantities (like the general causal fields or their covariant derivatives for example) at inertial frames and then refer them to general spacetime points using veirbiens. Thus, one can show that the covariant derivative $\mathcal{D}_a$ is given by [25]

$$\mathcal{D}_a = V^\mu_a \left(\partial_\mu + \Gamma_\mu\right) , \tag{2.51}$$

where the spin connection Γ_μ is

$$\Gamma_\mu = \frac{1}{2} \mathcal{T}^{ab} V^\nu_a V_{b\nu;\mu} , \tag{2.52}$$

and $\mathcal{T}^{ab}$ are the corresponding Lorentz generators of the object being differentiated. Here, one has to be carefule when dealing with a set of nested Lorentz generators. For example, the left-handed massless field $\Phi^{(s)}_L$ transforms as $(s, 0)$, while the covariant derivative $\mathcal{D}_b$ transforms as $\left(\frac{1}{2}, \frac{1}{2}\right)$. Thus, $\mathcal{D}_b \Phi^{(s)}_L$ transforms as $\left(\frac{1}{2}, \frac{1}{2}\right) \oplus (s, 0)$. If we denote the generators and spin connection of $\left(\frac{1}{2}, \frac{1}{2}\right) \oplus (s, 0)$ representation by $[\mathcal{T}_{ab}]^{cN}_{dM}$ and $[\Gamma_\mu]^{cN}_{dM}$, respectively, then we can write them in terms of the corresponding $\left(\frac{1}{2}, \frac{1}{2}\right)$ and $(s, 0)$ quantities as [26]

$$\begin{aligned} [\mathcal{T}_{ab}]^{cN}_{dM} &= \delta^N_M [\mathcal{T}_{ab}]^c_d + \delta^c_d [\mathcal{T}_{ab}]^N_M , \\ [\Gamma_\mu]^{cN}_{dM} &= \delta^N_M [\Gamma_\mu]^c_d + \delta^c_d [\Gamma_\mu]^N_M , \end{aligned} \tag{2.53}$$

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where the latin indices a, b, c, d label the $\left(\frac{1}{2}, \frac{1}{2}\right)$ representation, while the latin indices M, N label the $(s, 0)$ representation. In particular we have for the $\left(\frac{1}{2}, \frac{1}{2}\right)$ representation

$$[\mathcal{T}_{ab}]^c_d = \delta^d_a \eta_{bc} - \delta^c_b \eta_{ad} \,. \tag{2.54}$$

Using this construction, the massive Klein-Gordon equation of the generalized Dirac's field $\Psi^{(s)}$ takes the form

$$\begin{aligned}(\Box - m^2)\Psi^{(s)} &= \left[\mathcal{D}_a \mathcal{D}^a - m^2\right] \Psi^{(s)}_N \\ &= V^\mu_a \left[\delta^a_d \delta^M_N \partial_\mu + [\Gamma_\mu]^{aM}_{dN}\right] V^d_\nu \left[\delta^P_M \partial^\nu + [\Gamma^\nu]^P_M\right] \Psi^{(s)}_P - m^2 \Psi^{(s)}_N = 0 \,.\end{aligned} \tag{2.55}$$

One can also conformally couple $\Psi^{(s)}$ (in the $m = 0$ limit) by adding the term $\xi R \Psi^{(s)}$ to the Klein-Gordon equation, where $\xi = (s+1)/6$ [26].

In this work we are particularly interested in de Sitter space:

$$ds^2 = \frac{1}{H^2\tau^2}\left(-d\tau^2 + dx^2 + dy^2 + dz^2\right), \tag{2.56}$$

with the metric

$$g_{\mu\nu} = \frac{1}{H^2\tau^2}\eta_{\mu\nu} \,, \tag{2.57}$$

where H is the Hubble's parameter and τ is the conformal time. de Sitter space is conformally flat, and hence, the easiest way to calculate the covariant derivative is to apply a Weyl transformation to the flat space such that

$$g_{\mu\nu} \to \bar{g}_{\mu\nu} = \Omega^2 \eta_{\mu\nu} \,, \tag{2.58}$$

where $\Omega = \frac{1}{H\tau}$. Then, we have

$$\begin{aligned}\mathcal{D}_a &\to \bar{\mathcal{D}}_a = \bar{V}^\mu_a \left[\partial_\mu + \bar{\Gamma}_\mu\right], \\ \Gamma_\mu &\to \bar{\Gamma}_\mu = \Gamma_\mu - \Omega^{-1}\mathcal{T}^{ef} V^\nu_e V_{f\mu} \Omega_{,\nu} \,,\end{aligned} \tag{2.59}$$

where $\bar{V}^u_a = \Omega^{-1} V^\mu_a$, $V^\mu_a = \delta^\mu_a$, and $\Gamma_\mu = 0$.

With the aid of Weyl transformation one can show that the Klein-Gordon equation takes the form

$$\left[\tau^2 \partial_a \partial^a - 2\tau \frac{\partial}{\partial\tau} + 2\tau \mathcal{T}^{0a}\partial_a + 12\xi + \frac{m^2}{H^2} - s(s+1)\right] \Psi^{(s)}(x) = 0 \,. \tag{2.60}$$

de Sitter space admits SO(3) as a subgroup, and hence, one can write the Fourier transform of the three-dimensional part of the positive and negative frequency modes of $\Psi^{(s)}$ as

$$\Psi^{(s)}_\pm(\tau, \boldsymbol{x}) = \int \frac{d^3p}{(2\pi)^3} e^{\pm i \boldsymbol{p}\cdot\boldsymbol{x}} \tilde{\Psi}^{(s)}_\pm(\tau, \boldsymbol{p}). \tag{2.61}$$

Then, one substitutes (2.61) into (2.60) to find that the quantum field $\Psi^{(s)}$ can be expanded in terms of creation and annihilation operators as (see [26–28] for details)

$$\Phi^{(s)}_{L,M} = \int \frac{d^3p}{(2\pi)^3} \sum_N \left\{ \Xi^{+L}_{MN}(\boldsymbol{p},\tau) a(\boldsymbol{p},N) e^{i\boldsymbol{p}\cdot\boldsymbol{x}} + \Xi^{-L}_{MN}(\boldsymbol{p},\tau) b^\dagger(\boldsymbol{p},N) e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \right\},$$
$$\Phi^{(s)}_{R,M} = \int \frac{d^3p}{(2\pi)^3} \sum_N \left\{ \Xi^{+R}_{MN}(\boldsymbol{p},\tau) a(\boldsymbol{p},N) e^{i\boldsymbol{p}\cdot\boldsymbol{x}} + \Xi^{-R}_{MN}(\boldsymbol{p},\tau) b^\dagger(\boldsymbol{p},N) e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \right\}, \tag{2.62}$$

where

$$\begin{bmatrix} \Xi^{+L}_{MN} \\ \Xi^{+R}_{MN} \end{bmatrix} = \frac{H\tau}{2\sqrt{|\boldsymbol{p}|}} \begin{bmatrix} e^{\frac{i\pi N}{2}} \left(\frac{m}{H}\right)^N W_{-N,\nu}\left(2i|\boldsymbol{p}|\tau\right) D^{(s)}_{M,N}\left[R(\hat{\boldsymbol{p}})\right] \\ e^{\frac{-i\pi N}{2}} \left(\frac{m}{H}\right)^{-N} W_{N,\nu}\left(2i|\boldsymbol{p}|\tau\right) D^{(s)}_{M,N}\left[R(\hat{\boldsymbol{p}})\right] \end{bmatrix},$$
$$\begin{bmatrix} \Xi^{-L}_{MN} \\ \Xi^{-R}_{MN} \end{bmatrix} = \frac{H\tau}{2\sqrt{|\boldsymbol{p}|}} \begin{bmatrix} e^{\frac{-i\pi N}{2}} \left(\frac{m}{H}\right)^N W_{-N,\nu}\left(-2i|\boldsymbol{p}|\tau\right) D^{(s)}_{M,N}\left[R(\hat{\boldsymbol{p}})\right] \\ (-1)^s e^{\frac{-i3\pi N}{2}} \left(\frac{m}{H}\right)^{-N} W_{N,\nu}\left(-2i|\boldsymbol{p}|\tau\right) D^{(s)}_{M,N}\left[R(\hat{\boldsymbol{p}})\right] \end{bmatrix}, \tag{2.63}$$

$\nu^2 = \frac{9}{4} - m^2H^2 - 12\xi + s(s+1)$, and $W_{a,b}(x)$ are the Whittaker's functions.

In this work we will only consider left-handed massless fields in the $s = 1/2$ representation. In this case the sum over N in (2.62) is restricted to a single value $N = -s$.

Next, we define the Green's functions of the left fields as (with a similar form for the right field Green's function after replacing Π with $\bar{\Pi}$)

$$G^{(s)}_{0\,MN}(\boldsymbol{x},\boldsymbol{x}',\tau,\tau') = \langle 0|\mathcal{T}\Phi^{(s)}_M(\boldsymbol{x},\tau)\Phi^{(s)\dagger}_N(\boldsymbol{x}',\tau')|0\rangle$$
$$= \int \frac{d^3p}{(2\pi)^3} e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{x}')} \begin{cases} \widehat{\Pi_{MN}}(p^0,\boldsymbol{p}) u^*(\boldsymbol{p},\tau') u(\boldsymbol{p},\tau), & \tau > \tau' \\ \widehat{\Pi_{MN}}(p^0,-\boldsymbol{p}) v^*(-\boldsymbol{p},\tau') v(-\boldsymbol{p},\tau), & \tau' > \tau \end{cases}, \tag{2.64}$$

where

$$u(\boldsymbol{p},\tau) = \frac{H\tau}{2\sqrt{|\boldsymbol{p}|}} e^{\frac{i\pi s}{2}} \left(\frac{m}{H}\right)^s W_{s,\nu}\left(-2i|\boldsymbol{p}|\tau\right),$$
$$v(\boldsymbol{p},\tau) = \frac{H\tau}{2\sqrt{|\boldsymbol{p}|}} e^{\frac{-i\pi s}{2}} \left(\frac{m}{H}\right)^s W_{s,\nu}\left(2i|\boldsymbol{p}|\tau\right). \tag{2.65}$$

One can easily show that expression (2.64) degenerates to (2.25) in the limit $H, m \to 0$.

This ends all the mathematical background needed in order to compute energy-momentum tensor correlators of higher spin fields in a fixed background. In the next sections we show how this can be achieved using general principles of covariance and analyticity.

3 Path integral, effective action, and energy momentum tensor

3.1 Motivation and strategy

The production of particles from vacuum will generally be accompanied by the generation of macroscopic energy-momentum tensor since this process puts physical states on shell. However, this process is stochastic, and hence, the average of the transverse traceless part

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of the energy-momentum tensor over all directions and/or momenta is expected to vanish. Nevertheless, we still can obtain non-trivial energy-momentum tensor correlators, which may carry distinct features of the particles produced. The lack of a Lagrangian formulation of the general causal fields makes the task of obtaining an energy-momentum tensor, not to mention its correlators, a non-trivial one.

In this section we postulate a form of the energy-momentum tensor of the general causal fields. This tensor satisfies all the conditions of an energy-momentum tensor: it is symmetric, conserved, and Lorentz covariant second rank tensor. We motivate the form of this tensor by carefully constructing the corresponding one for $s = \frac{1}{2}$ field with no reference to a Lagrangian formulation. The idea is based on the observation that the knowledge of the free-field Green's function in a fixed background can be used to define an effective action of the field. Then, by perturbing the background with appropriate operators we can obtain the energy-momentum tensor as well as its second and higher order correlators by taking a series of functional derivatives of the effective action with respect to the background metric. In flat space, we find that the set of operators can be determined exactly by requiring the existence of non-vanishing, conserved, and covariant energy-momentum tensor. The same method can in principle be used to sort out the set of allowed operators in curved background, modulo a non-trivial point regarding the non-commutativity of covariant derivatives for spins $s > \frac{1}{2}$. In both cases (flat and curved background), we find that the use of the analytical properties of the on-shell-time-ordered Greens' function, i.e., the on-shell Feynman propagator, is enough to determine the energy-momentum tensor correlators that emerge due to production of the general causal fields from vacuum.

We first apply our formalism to the massless case in flat background, where algebra is the least cumbersome. Then, we show that the same formalism can be generalized to the massive case and in curved background.

3.2 Massless fields

Given an action $S[\Phi, \Phi^\dagger]$, where Φ is any complex field, e.g., the spin $\frac{1}{2}$ field, then one can write the path integral as (in the following we keep the spin index s general for later convenience)

$$Z[J] = \int \mathcal{D}[\Phi]\mathcal{D}[\Phi^\dagger] e^{iS[\Phi,\Phi^\dagger] + i\int d^4x \sqrt{-g(x)} J(x)\Phi(x) - i\int d^4x \sqrt{-g(x)} J^\dagger(x)\Phi^\dagger(x)} , \tag{3.1}$$

in the presence of an external current J. Physically, the partition function Z is the vacuum persistence amplitude $\langle \mathrm{Out}, 0 | 0, \mathrm{In}\rangle$. The presence of an external current J can cause an instability in the initial vacuum state $|0, \mathrm{In}\rangle$ leading to particle production. In the absence of currents in flat space we have $|0, \mathrm{In}\rangle = |0, \mathrm{Out}\rangle = |0\rangle$, and hence $Z[0] = \langle 0|0\rangle = 1$ and no particle production can take place. However, particle production can occur in curved spacetime since in general $|0, \mathrm{In}\rangle \neq |0, \mathrm{Out}\rangle$ even in the absence of external current [29]. Therefore, one can mimic the effect of curved spacetime by turning on an external current in flat space and vise versa. We also define the effective action W as

$$Z[0] = \langle \mathrm{Out}, 0 | 0, \mathrm{In}\rangle \equiv e^{iW} . \tag{3.2}$$

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In fact, the effective action provides us with a valuable tool to compute the energy momentum tensor and its correlators, as we will see in this section. This is particularly important in our case because of the absence of a Lagrangian formulation for the general causal fields. To this end we set $J = 0$ and examine the variation of (3.1) to find

$$\delta Z[0] = i \int \mathcal{D}[\Phi]\mathcal{D}[\Phi^\dagger]\delta S e^{iS} = i\langle \text{Out}, 0|\delta S|0, \text{In}\rangle \,, \tag{3.3}$$

from which one obtains

$$\langle \text{Out}, 0|T_{\mu\nu}|0, \text{In}\rangle = -i\frac{2}{\sqrt{-g}}\frac{\delta Z[0]}{\delta g_{\mu\nu}} \,. \tag{3.4}$$

Finally, one can use (3.2) to find:

$$\begin{aligned} \langle T_{\mu\nu}\rangle &\equiv \frac{\langle \text{Out}, 0|T_{\mu\nu}|0, \text{In}\rangle}{\langle \text{Out}, 0|0, \text{In}\rangle} = \frac{2}{\sqrt{-g}}\frac{\delta W}{\delta g^{\mu\nu}} \,, \\ \langle T_{\mu\nu}(x)T_{\alpha\beta}(y)\rangle &= \frac{4}{\sqrt{-g(x)}\sqrt{-g(y)}}\frac{\delta^2 W}{\delta g^{\mu\nu}(x)\delta g^{\alpha\beta}(y)} \,. \end{aligned} \tag{3.5}$$

The partition function of the free field Φ in a fixed curved background is given by

$$Z[0] = \int \mathcal{D}[\Phi]\mathcal{D}[\Phi^\dagger] e^{i\int d^4x\sqrt{-g(x)}\Phi^\dagger(x)\left[G^{-1}(x,x')\right]_{x'\to x}\Phi(x)} \,. \tag{3.6}$$

Since the integral over Φ is quadratic, it can be readily performed to obtain:

$$Z[0] = \text{Det}\left[G^{-1}\right]^{(-1)^{2s+1}} \,. \tag{3.7}$$

Now, using (3.2) we find

$$\begin{aligned} W &= -i(-1)^{2s+1}\text{Tr}\log\left[G^{-1}\right] \\ &= -i(-1)^{2s+1}\int d^4x\sqrt{-g(x)}\,\text{tr}\log\left[G^{-1}(x,x')\right]_{x'\to x} \,, \end{aligned} \tag{3.8}$$

where Tr denotes the trace over spacetime and Lorentz indices, while tr denotes the trace over Lorentz indices only. Next, we consider a small perturbation in the background metric:

$$g_{\mu\nu} = g_{0\,\mu\nu} + \delta g_{\mu\nu} \,, \quad \delta\sqrt{-g} = \frac{1}{2}\sqrt{-g_0}g_0^{\mu\nu}\delta g_{\mu\nu} \,, \tag{3.9}$$

where $g_{0\,\mu\nu}$ is the unperturbed metric. The Green's function responds to the perturbation of g by acquiring an extra piece

$$G^{-1} = G_0^{-1} + \delta g_{\mu\nu}\mathcal{O}^{\mu\nu} \,, \tag{3.10}$$

where G_0 is the unperturbed Green's function and $\mathcal{O}^{\mu\nu}$ is the vertex operator that will be determined in the next section. Using the cyclic property of the trace we have $\text{tr}\log\left[G^{-1}\right] = \text{tr}\log\left[G_0^{-1}\left[I + \delta g_{\mu\nu}\mathcal{O}^{\mu\nu}G_0\right]\right]$. Thus, we can write the effective action as

$$W = -i(-1)^{2s+1}\int d^4x\sqrt{-(g_0+\delta g)}\left\{\text{tr}\log\left[G_0^{-1}\right] + \text{tr}\log\left[I + \delta g_{\mu\nu}O^{\mu\nu}G_0\right]\right\} \,. \tag{3.11}$$

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Expanding to second order in $\delta g_{\mu\nu}$ we obtain

$$
\begin{aligned}
W = -i(-1)^{2s+1} \Bigg\{ & \int d^4x \sqrt{-g_0(x)} \mathrm{tr} \log \left[G_0^{-1}(x,x') \right]_{x' \to x} \\
& + \int d^4x \sqrt{-g_0(x)} \delta g_{\mu\nu}(x) \mathrm{tr} \left[O^{\mu\nu}(x) G_0(x,x') \right]_{x' \to x} \\
& + \frac{1}{2} \int d^4x \sqrt{-g_0(x)} g_0^{\mu\nu} \delta g_{\mu\nu}(x) \left[\mathrm{tr} \log \left[G_0^{-1} \right]_{x' \to x} + \mathrm{tr} \left[\delta g_{\alpha\beta}(x) \mathcal{O}^{\alpha\beta}(x) G_0(x,x') \right]_{x' \to x} \right] \\
& - \frac{1}{2} \int d^4x d^4y \sqrt{-g_0(x)} \sqrt{-g_0(y)} \delta g_{\mu\nu}(x) \delta g_{\alpha\beta}(y) \mathrm{tr} \left[\mathcal{O}^{\mu\nu}(x) G_0(x,y) \mathcal{O}^{\alpha\beta}(y) G_0(y,x) \right] \Bigg\} .
\end{aligned}
\tag{3.12}
$$

Now, we use (3.5) to find the expectation value of the energy-momentum tensor of the field Φ

$$
\langle T^{\mu\nu} \rangle = -2i(-1)^{2s+1} \left\{ \mathrm{tr} \left[\mathcal{O}^{\mu\nu}(x) G_0(x,x') \right]_{x' \to x} - \frac{1}{2} g_0^{\mu\nu} \mathrm{tr} \log \left[G_0(x,x') \right]_{x' \to x} \right\} . \tag{3.13}
$$

The effective action (3.8) is coordinate and Lorentz scalar (in the veirbien sense), and hence, it is evident that the energy-momentum tensor is conserved; see [25] for the proof. Using the same technology, we can also calculate the connected part of the energy-momentum tensor correlator:

$$
\langle T^{\mu\nu}(x) T^{\alpha\beta}(y) \rangle_c = 2i(-1)^{2s+1} \mathrm{tr} \left[\mathcal{O}^{\mu\nu}(x) G_0(x,y) \mathcal{O}^{\alpha\beta}(y) G_0(y,x) \right] . \tag{3.14}
$$

In the following we will be interested in the momentum-space energy-momentum tensor correlator. To this end we assume that the spacetime admits SO(3) as an isometry subgroup. Then, we can write the Green's function in Fourier space as

$$
G_0(x,y) = \int \frac{d^3p}{(2\pi)^3} e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})} \mathcal{G}_0(\boldsymbol{p}, \tau_x, \tau_y) , \tag{3.15}
$$

where τ refers to either the cosmic or conformal time. We also use the fact that $\mathcal{O}^{\mu\nu}$ is a differential operator, as we will show in section 4, and take the Fourier transform of $\langle T^{\mu\nu}(x) T^{\alpha\beta}(y) \rangle_c$ to find

$$
\begin{aligned}
\langle T^{\mu\nu}(\boldsymbol{k}, \tau_x) T^{\alpha\beta}(\boldsymbol{k}', \tau_y) \rangle_c = 2i(-1)^{2s+1} \delta^3 \left(\boldsymbol{k} + \boldsymbol{k}' \right) \int d^3p \ \mathrm{tr} \Big[& \mathcal{O}^{\mu\nu}(\boldsymbol{p}, \tau_x) \mathcal{G}_0(\boldsymbol{p}, \tau_x, \tau_y) \\
& \mathcal{O}^{\alpha\beta}(\boldsymbol{p} - \boldsymbol{k}, \tau_y) \mathcal{G}_0(\boldsymbol{p} - \boldsymbol{k}, \tau_y, \tau_x) \Big] .
\end{aligned}
\tag{3.16}
$$

Equation (3.16) is one of our main results in this work.

The derivation of (3.13) and (3.14) was carried out for $s = \frac{1}{2}$ field. The second rank tensor in (3.13) satisfies all the criteria of an energy-momentum tensor: (a) it is a second rank symmetric tensor and (b) it is conserved, i.e., $\nabla_\mu T^{\mu\nu} = 0$. These criteria are also satisfied had we replaced the spin $\frac{1}{2}$ field with a field with an arbitrary spin; all we need to do is to use the Green's function of the corresponding field. Hence, in the absence of a Lagrangian formulation of the general causal fields, we postulate equation (3.13) as the definition of the energy-momentum tensor for any field with spin $s \geq \frac{1}{2}$.

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Postulate. *The energy-momentum tensor and its two-point function of the general causal fields are given by expressions (3.13) and (3.14) (or equivalently (3.16) for spaces with* SO(3) *isometry), respectively.* Generalizing (3.16) to multi-point functions is a straightforward task.

We will show that these expressions along with the analytic properties of $\mathcal{G}_0(\boldsymbol{p}-\boldsymbol{k},\tau_y,\tau_x)$ is enough to determine the large scale correlators of the energy-momentum tensor due to the production of particles from vacuum.

3.3 Massive fields

Turning on a mass for the fields does not change any of the above steps we followed to derive the energy-momentum tensor and its correlators. Repeating the above procedure in the massive case, where we use the Dirac representation $(s,0)\oplus(0,s)$, we find

$$\langle T^{\mu\nu}\rangle = -2i(-1)^{2s+1}\left\{\mathbb{TR}\left[\mathcal{O}^{\mu\nu}(x)\mathbb{G}_0(x,x')\right]_{x'\to x} - \frac{1}{2}g_0^{\mu\nu}\mathbb{TR}\log\left[\mathbb{G}_0(x,x')\right]_{x'\to x}\right\}, \quad (3.17)$$

and

$$\langle T^{\mu\nu}(\boldsymbol{k},\tau_x)T^{\alpha\beta}(\boldsymbol{k}',\tau_y)\rangle_c = 2i(-1)^{2s+1}\delta^3\left(\boldsymbol{k}+\boldsymbol{k}'\right)\int d^3p\ \mathbb{TR}\Big[\mathcal{O}^{\mu\nu}(\boldsymbol{p},\tau_x)\mathscr{G}_0(\boldsymbol{p},\tau_x,\tau_y)$$
$$\mathcal{O}^{\alpha\beta}(\boldsymbol{p}-\boldsymbol{k},\tau_y)\mathscr{G}_0(\boldsymbol{p}-\boldsymbol{k},\tau_y,\tau_x)\Big], \quad (3.18)$$

where $\mathbb{G}_0$ and $\mathscr{G}_0$ in flat space are given by (2.36) and (2.37), respectively, and $\mathbb{TR}$ is the trace over Dirac indices in the $(s,0)\oplus(0,s)$ representation.

4 Vertex operators

In the previous section we showed how one can construct the energy-momentum tensor, and its correlators, of a free general causal field in a given background. However, this construction forced us to introduce vertex operators $\mathcal{O}^{\mu\nu}$ that are yet to be determined. This section is devoted to elucidate the method we use to construct the vertex operators, which hinges on general considerations of Lorentz covariance. The general idea is that $\mathcal{O}^{\mu\nu}$ is used to write the energy-momentum tensor via (3.13), and hence, we can use the symmetries of this tensor to construct $\mathcal{O}^{\mu\nu}$. The construction of the vertex operators in flat space follows immediately from symmetry considerations of $T^{\mu\nu}$. The construction, however, is more involved in curved background.

4.1 Flat space vertices: massless fields

The energy-momentum tensor has to obey the following obvious criteria: (1) it transforms covariantly under Lorentz transformations, (2) it is conserved, and (3) it is symmetric in its two indices. Therefore, the momentum-space expression of the energy-momentum tensor of any on-shell massless particle in flat space has to take the form

$$\langle T^{\mu\nu}(p^2)\rangle = f_1(p^2)p^\mu p^\nu - f_2(p^2)g^{\mu\nu}\,. \quad (4.1)$$

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In this section we use both the Greek μ, ν and Latin a, b letters to denote flat space coordinates. Now, since we are dealing with on-shell massless particles we have $p^2 = 0$, and hence, $f_1(p^2)$ and $f_2(p^2)$ are at most constants: $f_1(p^2) = f_1, f_2(p^2) = f_2$. However, conservation of $T^{\mu\nu}$, i.e., $p_\mu T^{\mu\nu} = 0$, demands the vanishing of f_2. Therefore, the energy-momentum tensor, which takes the simple form,

$$\langle T^{\mu\nu}(p^2)\rangle = f p^\mu p^\nu \tag{4.2}$$

or its inverse Fourier transform

$$\langle T^{\mu\nu}(x)\rangle = f \int \frac{d^3p}{(2\pi)^3 2|\boldsymbol{p}|} e^{i\boldsymbol{p}\cdot\boldsymbol{x} - i|\boldsymbol{p}|t} p^\mu p^\nu \tag{4.3}$$

will guide us in our search for the vertex operators in flat space.

The energy-momentum tensor defined via (3.13) should coincide with (4.3). Therefore, we first demand that $\mathrm{tr}\log G_0 = 0$, which can be achieved through appropriate renormalization. Now, the trace of $\mathcal{O}^{\mu\nu}$ operating on G_0 should reproduce $\sim p^\mu p^\nu$ after applying Fourier transform. Recalling that $G_0 \sim t^{a_1 a_2 \ldots a_{2s}}$ and the fact $\eta_{a_1 a_2} t^{a_1 a_2 \ldots a_{2s}} = 0$, etc. (see section 2.4) forces the vertex to be an object that carries spinoral indices such that its contraction with $t^{a_1 a_2 \ldots a_{2s}}$ gives a non-zero result. Also recalling the transformation laws of $t^{a_1 a_2 \ldots a_{2s}}$ and $\bar{t}^{a_1 a_2 \ldots a_{2s}}$, see (2.40) and (2.41), along with the trace identity (2.44) leaves us with the only choice that the vertex $\mathcal{O}^{\mu\nu}$ has to be proportional to the tensor $\bar{t}^{\mu a_2 \ldots a_{2s}}$ if we want the trace of $\mathcal{O}^{\mu\nu} G_0$ to be covariant under the Lorentz transformation. This completely fixes the form of the vertex operators modulo a p-dependent coefficient as we show below. In order to elucidate the construction of $\mathcal{O}^{\mu\nu}$ we work out $s = \frac{1}{2}$ and $s = 1$ as two examples and then give the general form of $\mathcal{O}_{\mu\nu}$.

First we consider $s = \frac{1}{2}$. The vertex operator takes the form[5]

$$\mathcal{O}^{(\frac{1}{2})\ \mu\nu} = -i\frac{\alpha_{\frac{1}{2}}}{2}\left(\bar{t}^\mu \partial^\nu + \bar{t}^\nu \partial^\mu\right). \tag{4.4}$$

Then using the identity $\mathrm{tr}\left[\bar{t}^\mu t^\nu\right] = -g^{\mu\nu}$, we find from (3.13)

$$\langle T^{\mu\nu}(x)\rangle = 2i\alpha_{\frac{1}{2}} \int \frac{d^3p}{(2\pi)^3} \frac{e^{i\boldsymbol{p}\cdot\boldsymbol{x} - ix^0|\boldsymbol{p}|}}{2|\boldsymbol{p}|} p^\mu p^\nu\,, \tag{4.5}$$

which takes the general form of the energy-momentum tensor for a massless field (4.3).

The vertex for $s = 1$ is

$$\mathcal{O}^{(1)\ \mu\nu} = -\frac{\alpha_1}{2C_1(\Box)}\left[\bar{t}^{\mu\rho}\partial_\rho\partial^\nu + \bar{t}^{\nu\rho}\partial_\rho\partial^\mu\right]. \tag{4.6}$$

Using the $(1,0)$ Green's function along with the identity $\mathrm{tr}\left[\bar{t}^{\mu\nu} t^{\alpha\beta}\right] = -g^{\mu\nu}g^{\alpha\beta} + 2g^{\alpha\mu}g^{\nu\beta} + 2g^{\mu\beta}g^{\nu\alpha}$ we find

$$\langle T^{\mu\nu}(x)\rangle = -6i\alpha_1 \int \frac{d^3p}{(2\pi)^3} \frac{p^2 p^\mu p^\nu}{2|\boldsymbol{p}| C_1(\boldsymbol{p}^2)}\,. \tag{4.7}$$

[5]One might also want to add the term $\bar{t}^a \partial_a \eta^{\mu\nu}$ to $\mathcal{O}^{\mu\nu}$. However, upon taking the trace $\mathrm{tr} O^{\mu\nu} G_0$ we find that this term gives $p^2 = 0$. The same behavior occurs for higher s.

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This expression is zero when considered on-shell. Hence, we set $C_1(p^2) = -p^2$ or $C_1(\Box) = \Box$ in order for the energy-momentum tensor to take its canonical form.

One can easily work out the general case to find

$$C_s(p^2) = \left(-p^2\right)^{2s-1} , \quad C_s(\Box) = \Box^{2s-1} . \tag{4.8}$$

Therefore, the vertex takes the general form

$$\mathcal{O}^{(s)\ \mu\nu}(x) = \frac{(-i)^{2s}\alpha_s}{2C_s(\Box)} \left[\eta^{\mu}_{a_1}\eta^{\nu}_{c}\bar{t}^{a_1 a_2 \ldots a_{2s}}\partial_{a_2}\partial_{a_3}\ldots\partial_{a_{2s}}\partial^{c} + \eta^{\nu}_{a_1}\eta^{\mu}_{c}\bar{t}^{a_1 a_2 \ldots a_{2s}}\partial_{a_2}\partial_{a_3}\ldots\partial_{a_{2s}}\partial^{c}\right] , \tag{4.9}$$

or in momentum space

$$\mathcal{O}^{(s)\ \mu\nu}(p) = \frac{\alpha_s}{2\left(-p^2\right)^{2s-1}} \left[\eta^{\mu}_{a_1}\eta^{\nu}_{c}\bar{t}^{a_1 a_2 \ldots a_{2s}}p_{a_2}p_{a_3}\ldots p_{a_{2s}}p^{c} + \eta^{\nu}_{a_1}\eta^{\mu}_{c}\bar{t}^{a_1 a_2 \ldots a_{2s}}p_{a_2}p_{a_3}\ldots p_{a_{2s}}p^{c}\right] , \tag{4.10}$$

where α_s is a numerical coefficient that depends on the physics details. The appearance of the non-locality $C_s(\Box)$ for any $s > \frac{1}{2}$ is signaling a singular behavior of the massless particles, which is going to hunt us back in calculating the energy-momentum tensor correlators.

4.2 Flat space vertices: massive fields

We follow the same procedure to construct the vertex operators in the massive case. In particular, the most general on-shell energy-momentum tensor takes the form

$$\langle T_{\mu\nu}(p^2)\rangle = f_1 p^{\mu}p^{\nu} + f_2\eta^{\mu\nu} , \tag{4.11}$$

and the conservation law $p_\mu T^{\mu\nu} = 0$ gives us a non-trivial relation between f_1 and f_2.

As we discussed before, we make use of the Dirac's representation to deal with the massive case. There are 5 vertex operators that can contribute to $\mathcal{O}^{\mu\nu}$. They can be grouped into diagonal and off diagonal operators:[6]

$$\begin{aligned} \text{diagonal} &\begin{cases} \mathcal{V}_1^{\mu\nu} = \eta^{\mu\nu}m^2 , \\ \mathcal{V}_2^{\mu\nu} = p^{\mu}p^{\nu} , \end{cases} \\ \text{off diagonal} &\begin{cases} \mathcal{V}_3^{\mu\nu} = \frac{(-1)^{2s}}{m^{2s-2}}p^{(\mu}\gamma^{\nu)a_2\ldots a_{2s}}p_{a_2}\ldots p_{a_{2s}} , \\ \mathcal{V}_4^{\mu\nu} = \frac{1}{m^{2s-2}}\eta^{\mu\nu}\gamma^{a_1 a_2\ldots a_{2s}}p_{a_1}p_{a_2}\ldots p_{a_{2s}} , \\ \mathcal{V}_5^{\mu\nu} = (-1)^{2s}\frac{p^{\mu}p^{\nu}}{m^{2s}}\gamma^{a_1\ldots a_{2s}}p_{a_1}\ldots p_{a_{2s}} . \end{cases} \end{aligned} \tag{4.12}$$

Among these operators, $\mathcal{V}_2$ and $\mathcal{V}_5$ are power counting suppressed and we neglect them in our treatment. In addition, one can trade $\mathcal{V}_4$ for $\mathcal{V}_1$ using the generalized Dirac's equation (2.30). Thus, the most general form of $\mathcal{O}^{\mu\nu}_{\mathcal{M}}$, where the subscript $_{\mathcal{M}}$ is used

[6] We restrict our treatment to parity conserving operators. At the end of section 5 we comment on the fact that turning on parity violating operators and/or the over production of left over right fields (or vice versa) will lead to helical structure in the energy-momentum tensor correlators.

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to indicate that these vertices correspond to massive particles, that yields a symmetric and conserved energy momentum tensor takes the form

$$\mathcal{O}_{\mathcal{M}}^{(s)\ \mu\nu}(p) = \frac{\alpha_s}{(\mathcal{S})^{2s-1}}\left[(-1)^{2s}p^{(\mu}\gamma^{\nu)a_2\ldots a_{2s}}p_{a_2}\ldots p_{a_{2s}} + \mathcal{C}_s m^{2s}\eta^{\mu\nu}\mathbb{I}_{(4s+2)\times(4s+2)}\right], \quad (4.13)$$

where $\mathcal{C}_s$ is a constant that we will determine momentarily, and $\mathcal{S}$ is some scale that has the dimension of mass square. In what follows we take $\mathcal{S} = m^2$ for simplicity. Using (3.17) we find

$$\langle T^{\mu\nu}(x)\rangle = i\frac{\alpha_s}{(m^2)^{2s-1}}\int\frac{d^3p}{(2\pi)^3 2\omega(\boldsymbol{p})}e^{i\boldsymbol{p}\cdot\boldsymbol{x}-i\omega(\boldsymbol{p})t}\mathbb{TR}\begin{bmatrix}\mathcal{K}_{11} & \mathcal{K}_{12}\\ \mathcal{K}_{21} & \mathcal{K}_{22}\end{bmatrix}, \quad (4.14)$$

where

$$\begin{aligned}\mathcal{K}_{11} &= t^{c_1\ldots c_{2s}}p_{c_1}\ldots p_{c_{2s}}p^{(\mu}\bar{t}^{\nu)d_2\ldots d_{2s}}p_{d_2}\ldots p_{d_{2s}} + m^{4s}\mathcal{C}_s\eta^{\mu\nu}\mathbb{I}_{(2s+1)\times(2s+1)}\,,\\ \mathcal{K}_{12} &= (-1)^{2s}\mathcal{C}_s m^{2s}t^{c_1\ldots c_{2s}}p_{c_1}\ldots p_{c_{2s}}\eta^{\mu\nu} + (-1)^{2s}m^{2s}p^{(\mu}\bar{t}^{\nu)d_2\ldots d_{2s}}p_{d_2}\ldots p_{d_{2s}}\,.\end{aligned} \quad (4.15)$$

The expressions of $\mathcal{K}_{22}$ and $\mathcal{K}_{21}$ are obtained from $\mathcal{K}_{11}$ and $\mathcal{K}_{12}$, receptively, by replacing t by $\bar{t}$ and exchanging the positions of t and $\bar{t}$. Then, taking the trace we find

$$\langle T^{\mu\nu}(p)\rangle = 2i\frac{\alpha_s}{(m^2)^{2s-1}}\left\{\mathrm{tr}\left[t^{c_1\ldots c_{2s}}p_{c_1}\ldots p_{c_{2s}}p^{(\mu}\bar{t}^{\nu)d_2\ldots d_{2s}}p_{d_2}\ldots p_{d_{2s}}\right] + (2s+1)m^{4s}\mathcal{C}_s\eta^{\mu\nu}\right\}. \quad (4.16)$$

Finally the value of $\mathcal{C}_s$ is determined by requiring the conservation of the energy-momentum tensor: $p_\mu T^{\mu\nu} = 0$. For example $\mathcal{C}_{1/2} = -1/2$ and $\mathcal{C}_1 = -1$. In fact, we will be interested only in the transverse traceless part of $T^{\mu\nu}$, and therefore, the term $m^{2s}\eta^{\mu\nu}\mathbb{I}_{(4s+2)\times(4s+2)}$ in $\mathcal{O}_{\mathcal{M}}^{(s)\,\mu\nu}$ can be safely neglected in our treatment.

4.3 Curved space vertices

Motivated by the vertex structure in flat space, we write the vertex of the massless fields in curved space as:

$$\mathcal{O}^{(s)\ \mu\nu} = \alpha_s\frac{(-i)^{2s}}{2C_s(\Box)}\left[V_{a_1}^{\mu}V_c^{\nu} + V_{a_1}^{\nu}V_c^{\mu}\right]\bar{t}^{a_1a_2\ldots a_{2s}}[\![D_{a_2}D_{a_3}\ldots D_{a_{2s}}D^c]\!]\,, \quad (4.17)$$

where the brackets $[\![\ ]\!]$ indicate total symmetrization of the covariant derivatives:

$$[\![(D_{a_2}D_{a_3}\ldots D_{a_{2s}}D^c]\!] = \sum_{\text{permutations}} D_{a_{p_1}}\ldots D^c\ldots D_{a_{p2_s}}\,. \quad (4.18)$$

This symmetrization is dedicated by the symmetric nature of $\bar{t}^{a_1a_2\ldots a_{2s}}$. The hope is that this symmetric definition will yield physical answers. Notice that unlike the flat background, the nonlocal operators $C_s(\Box)$ might not lead to singular behavior. The treatment of massive fields in curved space follows the same lines and we refrain from discussing them further here. The investigation of all these points will be left for a future work. In section 5.3 we use the curved space vertex of spin 1/2 to calculate the energy-momentum tensor correlator in FRW background as a check on our formalism.

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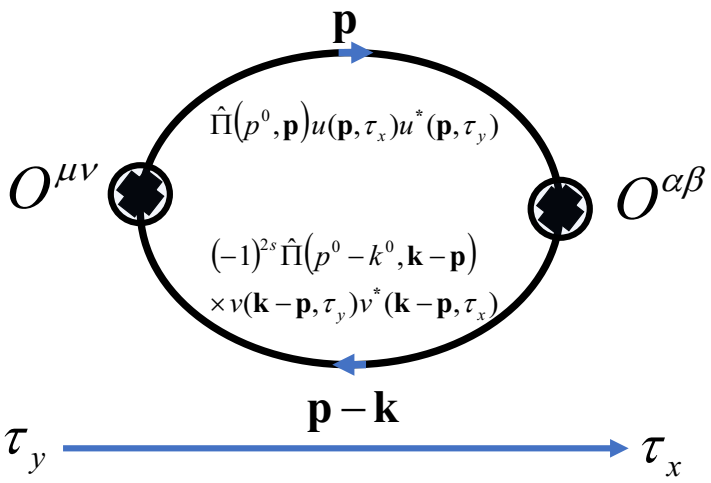

Figure 1. The structure of the two-point function of the energy-momentum tensor. The time flows from left, τ_y, to right, τ_x.

5 Structure of the energy-momentum tensor correlator

In the previous section we constructed the on-shell vertex operators required to compute the energy-momentum tensor and its correlators. Now, we are in a position to proceed to these computations. The analytic structure of the correlator is dictated by the time-ordered Green's function (2.24), i.e., the Feynman propagator.[7] In particular, one has to exercise caution in determining which part of the Green's function to pick, which depends on whether $\tau_x > \tau_y$ or vise versa, as we go from left to right in (3.16) and (3.18). Assuming that $\tau_x > \tau_y$ we pick a factor of $(-1)^{2s}$ from the second Green's function which cancels out with the same factor coming from the effective action. This intricate process is depicted in figure 1. Finally, the energy-momentum tensor correlator for the left-handed massless fields reads

$$\begin{aligned}\langle T^{\mu\nu}(\boldsymbol{k},\tau_x)T^{\alpha\beta}(\boldsymbol{k}',\tau_y)\rangle_c = -2i\delta^3\left(\boldsymbol{k}+\boldsymbol{k}'\right)\int\frac{d^3p}{4|\boldsymbol{p}||\boldsymbol{p}-\boldsymbol{k}|}\\ \mathrm{tr}\Big[\mathcal{O}^{(s)\ \mu\nu}(\boldsymbol{p},\tau_x)\widehat{\Pi}(p^0,\boldsymbol{p})u(\boldsymbol{p},\tau_x)u^*(\boldsymbol{p},\tau_y)\\ \mathcal{O}^{(s)\ \alpha\beta}(\boldsymbol{p}-\boldsymbol{k},\tau_y)\widehat{\Pi}(p^0-k^0,\boldsymbol{k}-\boldsymbol{p})v^*(\boldsymbol{k}-\boldsymbol{p},\tau_x)v(\boldsymbol{k}-\boldsymbol{p},\tau_y)\Big],\end{aligned} \tag{5.1}$$

where $\widehat{\Pi}(p^0-k^0,\boldsymbol{k}-\boldsymbol{p})$ should be understood as the polarization tensor obtained by a rotation by some angle θ, about a reference axis, to the light-like vector $\boldsymbol{p}-\boldsymbol{k}$ (as usual, we take the reference axis to be the z-axis). The correlator of the massless right-handed fields can be obtained by replacing Π with $\bar{\Pi}$ and $\mathcal{O}^{(s)\ \mu\nu}$ with $\bar{\mathcal{O}}^{(s)\ \mu\nu}$ (this demands replacing

[7] Once again, we stress that all our computations are done on-shell as we are interested in the on-shell contribution of the particles produced from vacuum to the energy-momentum tensor.

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$\bar{t}$ with t in $\mathcal{O}^{(s)\ \mu\nu}$). A similar procedure leads to the correlator of massive fields:

$$\begin{aligned}\langle T^{\mu\nu}(\boldsymbol{k},\tau_x)T^{\alpha\beta}(\boldsymbol{k}',\tau_y)\rangle_c = -2i\delta^3(\boldsymbol{k}+\boldsymbol{k}')\int\frac{d^3p}{4\omega(\boldsymbol{p})\omega(\boldsymbol{p}-\boldsymbol{k})}\\ \mathbb{TR}\left\{\mathcal{O}^{(s)\ \mu\nu}_{\mathcal{M}}(\boldsymbol{p},\tau_x)\begin{bmatrix} m^{2s}\mathbb{I} & \Pi(p^0,\boldsymbol{p})\\ \bar{\Pi}(p^0,\boldsymbol{p}) & m^{2s}\mathbb{I}\end{bmatrix}\mathcal{U}(\boldsymbol{p},\tau_x)\mathcal{U}^*(\boldsymbol{p},\tau_y)\right.\\ \times\ \mathcal{O}^{(s)\ \mu\nu}_{\mathcal{M}}(\boldsymbol{p}-\boldsymbol{k},\tau_y)\begin{bmatrix} m^{2s}\mathbb{I} & \Pi((k-p)^0,\boldsymbol{k}-\boldsymbol{p})\\ \bar{\Pi}((k-p)^0,\boldsymbol{k}-\boldsymbol{p}) & m^{2s}\mathbb{I}\end{bmatrix}\\ \left.\times\ \mathcal{V}^*(\boldsymbol{k}-\boldsymbol{p},\tau_x)\mathcal{V}(\boldsymbol{k}-\boldsymbol{p},\tau_y)\right\},\end{aligned}\tag{5.2}$$

where we have used a condensed notation such that $\mathcal{U}$ and $\mathcal{V}$ can denote any of the Ξ functions defined in (2.63). The explicit form of $\mathcal{U}$ and $\mathcal{V}$ will not concern us in the present work since we limit our examination to the flat space correlators (massive and massless), in addition to $s=\frac{1}{2}$ massless case in FRW background. Both correlators (5.1) and (5.2) are valid in a general background, keeping in mind that the construction of $\mathcal{O}^{(s)\ \mu\nu}$ in curved space is yet to be resolved for $s>\frac{1}{2}$. It is remarkable that the structure of (5.1) and (5.2) is solely dependent on the principles of covariance and analyticity, bearing no reference to a specific particle physics model.

Now, we can project the energy-momentum tensor correlator along the helicity-2 eigenbasis defined as [30]

$$\epsilon^{\lambda}_{\mu\nu}(\boldsymbol{k}) = \epsilon^{\lambda}_{\mu}(\boldsymbol{k})\epsilon^{\lambda}_{\nu}(\boldsymbol{k})\,,\tag{5.3}$$

where $\lambda=\pm$, $\epsilon^{\pm}_0(\boldsymbol{k})=0$, and

$$\boldsymbol{\epsilon}^{\pm}_{\mu}(\boldsymbol{k}) = \frac{\hat{\theta}\pm i\hat{\phi}}{\sqrt{2}}\,,\quad \hat{\theta}=(\cos\theta\cos\phi,\cos\theta\sin\phi,-\sin\theta)\,,\quad \hat{\phi}=(-\sin\phi,\cos\phi,0)\,.\tag{5.4}$$

This projection has two-fold importance. First, it enables us to find out whether the energy-momentum tensor correlators are parity violating. This can happen if the particle production process prefers one field type (left or right) over the other. Second, it can provide a succinct method to check the unitarity of the theory as we explain below. To this end, we define the amplitude $\mathcal{M}^{\lambda,\lambda'}$ as

$$\mathcal{M}^{\lambda,\lambda'}(\tau_x,\tau_y) \equiv i\langle T^{\lambda}(\boldsymbol{k},\tau_x)T^{\lambda'}(-\boldsymbol{k},\tau_y)\rangle_c\,,\tag{5.5}$$

where

$$T^{\lambda}(\boldsymbol{k},\tau_x) = \epsilon^{-\lambda}_{\mu\nu}(\boldsymbol{k})T^{\mu\nu}(\boldsymbol{k},\tau_x)\,,\tag{5.6}$$

which measures the amount of correlation projected along the two polarization directions.

By definition the quantities $T^{+}(\boldsymbol{k},\tau_x)T^{+}(-\boldsymbol{k},\tau_x)$ and $T^{-}(\boldsymbol{k},\tau_x)T^{-}(-\boldsymbol{k},\tau_x)$ (computed at the same time point) are positive definite, which can be easily shown using the reality of $T_{\mu\nu}(\boldsymbol{x},\tau_x)$ and properties of $\epsilon_{\mu}(\boldsymbol{k})$. Therefore, a unitary theory has to yield positive definite correlators $\langle T^{+}(\boldsymbol{k},\tau_x)T^{+}(-\boldsymbol{k},\tau_x)\rangle_c$ and $\langle T_{-}(\boldsymbol{k},\tau_x)T_{-}(-\boldsymbol{k},\tau_x)\rangle_c$, which can readily be checked from (5.1) or (5.2) along with (5.5).

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5.1 Flat space energy-momentum tensor correlators: massless fields

The calculations of the energy-momentum tensor correlator in flat space proceed by starting from the Green's function in flat space, keeping in mind that we are interested in the contribution to the correlator from on-shell left handed particles $(s, 0)$. The on-shell condition means that we have to replace every p^0 by $|\boldsymbol{p}|$ since we are dealing with massless particles. This is particularly important when we operate with $\mathcal{O}^{\mu\nu}$ on the Green's functions in (3.16). The flat space Green's function takes the form

$$G_0(x,x') = \int \frac{d^3p}{(2\pi)^3} \left(2|\boldsymbol{p}|\right)^{2s-1} e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{x}')-i|\boldsymbol{p}||t-t'|}\mathcal{G}(\boldsymbol{p},t,t')\,, \tag{5.7}$$

where

$$\mathcal{G}(\boldsymbol{p},t,t') = \begin{cases} \widehat{\Pi}(\hat{\boldsymbol{p}})\,, & t>t' \\ (-1)^{2s}\widehat{\Pi}(-\hat{\boldsymbol{p}})\,, & t<t' \end{cases}\,. \tag{5.8}$$

Operating with $\mathcal{O}^{\mu\nu}$ we find

$$\begin{aligned}\mathcal{O}^{\mu\nu}(x)G_0(x,x') = (-1)^{2s-1}\alpha_s \int \frac{d^3p}{(2\pi)^3}\frac{2^{2s-1}|\boldsymbol{p}|}{\left[n^2(\boldsymbol{p})\right]^{2s-1}} n^{(\nu}(\boldsymbol{p})\bar{t}^{\mu)a_2\ldots a_{2s}} n_{a_2}(\boldsymbol{p})\ldots n_{a_{2s}}(\boldsymbol{p}) \\ \times e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{x}')-i|\boldsymbol{p}||t-t'|}\mathcal{G}(\boldsymbol{p},t,t')\,,\end{aligned} \tag{5.9}$$

where $n(\boldsymbol{p}) \equiv \left(1, \frac{\boldsymbol{p}}{|\boldsymbol{p}|}\right)$ is a null vector.

Collecting everything and substituting into (5.1) we obtain

$$\begin{aligned}\langle T^{\mu\nu}(\boldsymbol{k},t_x)T^{\alpha\beta}(\boldsymbol{k}',t_y)\rangle_c = \frac{-i}{2}\alpha_s^2\left(\boldsymbol{k}+\boldsymbol{k}'\right)\int d^3p\; e^{i\vartheta(\boldsymbol{p},\boldsymbol{k},t_x,t_y)}\frac{|\boldsymbol{p}||\boldsymbol{p}-\boldsymbol{k}|}{\left[n^2(\boldsymbol{p})\right]^{2s-1}\left[n^2(\boldsymbol{p}-\boldsymbol{k})\right]^{2s-1}} \\ \times \mathrm{tr}\Big\{n_{a_2}(\boldsymbol{p})\ldots n_{a_{2s}}(\boldsymbol{p})n^{(\mu}(\boldsymbol{p})\bar{t}^{\nu)a_2\ldots a_{2s}}n_{c_1}(\boldsymbol{p})\ldots n_{c_{2s}}(\boldsymbol{p})t^{c_1\ldots c_{2s}} \\ \times n_{b_2}(\boldsymbol{p}-\boldsymbol{k})\ldots n_{b_{2s}}(\boldsymbol{p}-\boldsymbol{k})n^{(\alpha}(\boldsymbol{p}-\boldsymbol{k})\bar{t}^{\beta)b_2\ldots b_{2s}} \\ \times n_{d_1}(\boldsymbol{k}-\boldsymbol{p})\ldots n_{d_{2s}}(\boldsymbol{k}-\boldsymbol{p})t^{d_1\ldots d_{2s}}\Big\}\,,\end{aligned} \tag{5.10}$$

where the phase ϑ is given by

$$\vartheta(\boldsymbol{p},\boldsymbol{k},t_x,t_y) = \left(|\boldsymbol{p}|+|\boldsymbol{p}-\boldsymbol{k}|\right)(t_y-t_x)\,. \tag{5.11}$$

For $s > \frac{1}{2}$ the correlator has the highly singular piece $\left[n^2(\boldsymbol{p})\right]^{2s-1}\left[n^2(\boldsymbol{p}-\boldsymbol{k})\right]^{2s-1}$ in the denominator. Thus, we expect pathologies for fields with spin $s > \frac{1}{2}$.

Case I: $s = \frac{1}{2}$. Using the explicit expressions of $n(\boldsymbol{p})$, $n(\boldsymbol{p}-\boldsymbol{k})$, $\epsilon_\mu^\lambda(\boldsymbol{k})$, and substituting into (5.5) and (5.10) we obtain for the left-handed fields

$$\begin{aligned}\mathcal{M}^{\lambda,\lambda'}(t_x,t_y) = \frac{1}{2}\alpha_{1/2}^2\delta^{\lambda\lambda'}\delta^3\left(\boldsymbol{k}+\boldsymbol{k}'\right)\int d^3p\; e^{i\vartheta(\boldsymbol{p},\boldsymbol{k},t_x,t_y)}|\boldsymbol{p}|^2\sin^2\theta \\ \times\left(1+\frac{\lambda+\lambda'}{2}\cos\theta\right)\left(1+\frac{\lambda+\lambda'}{2}\frac{|\boldsymbol{p}|\cos\theta-k}{\sqrt{k^2+|\boldsymbol{p}|^2-2k|\boldsymbol{p}|\cos\theta}}\right)\,.\end{aligned} \tag{5.12}$$

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Similarly, the contribution from the right-handed fields is

$$\mathcal{M}^{\lambda,\lambda'}(t_x,t_y)=\frac{1}{2}\alpha_{1/2}^2\delta^{\lambda\lambda'}\delta^3\left(\boldsymbol{k}+\boldsymbol{k}'\right)\int d^3p\; e^{i\vartheta(\boldsymbol{p},\boldsymbol{k},t_x,t_y)}|\boldsymbol{p}|^2\sin^2\theta \\ \times\left(1-\frac{\lambda+\lambda'}{2}\cos\theta\right)\left(1-\frac{\lambda+\lambda'}{2}\frac{|\boldsymbol{p}|\cos\theta-k}{\sqrt{k^2+|\boldsymbol{p}|^2-2k|\boldsymbol{p}|\cos\theta}}\right). \tag{5.13}$$

This exactly matches the results in [2], which were obtained using canonical quantization methods.

Case II: $s\geq 1$. In this case we find that there is a highly singular piece in the denominator that causes the integrand in (5.10) to blow up. Thus, we conclude that either the energy momentum tensor of general causal fields with spin $s\geq 1$ vanishes identically or there cannot be a consistent description of these fields in flat space. In fact, Weinberg-Witten theorem excludes massless higher spin fields with $s\geq 2$ that admit Lorentz covariant energy-momentum tensor [16]. Our finding that massless general causal fields with $s\geq 2$ are sick is consistent with Weinberg-Witten theorem since the energy-momentum tensor we postulated in (3.13) is Lorentz covariant. However, we also find that fields with $s=1$ are sick in the general causal formalism, which is attributed to the non-local nature of the energy momentum tensor, see (4.6). We could have anticipated this result since the amplitude for emitting or absorbing a massless particle with momentum $|\boldsymbol{p}|$ and spin s vanishes like $|\boldsymbol{p}|^{s-\frac{1}{2}}$, as can be seen from (2.19). In fact, massless general causal fields cannot be used to build theories that describe long-range forces, as electromagnetism and gravity [31] (see also [32]). Instead, a massless spin s particle can be described by the symmetric potential $A^{\mu_1\mu_2\cdots\mu_s}$, which transforms as $(\frac{s}{2},\frac{s}{2})$ under the Lorentz group. This is the direct generalization of the electromagnetic potential A^μ and the metric tensor $h^{\mu\nu}$ to higher spin fields. Such treatment has occupied a large portion of the literature on higher spin fields, and we refer the reader to [15] for an excellent review.

5.2 Flat space energy-momentum tensor correlators: massive fields

Despite the fact that massless fields are pathological in our formalism, massive fields are well behaved as we explain in this section. Let us define $\Pi^{\mu\nu}(p^0,\boldsymbol{p})$ and $\bar{\Pi}^{\mu\nu}(p^0,\boldsymbol{p})$ as

$$\Pi^{\mu\nu}(p^0,\boldsymbol{p})=p^{(\mu}t^{\nu)a_2\ldots a_{2s}}p_{a_2}\ldots p_{a_{2s}}\,,\quad \bar{\Pi}^{\mu\nu}(p^0,\boldsymbol{p})=p^{(\mu}\bar{t}^{\nu)a_2\ldots a_{2s}}p_{a_2}\ldots p_{a_{2s}}\,. \tag{5.14}$$

Then, using (3.18) we obtain

$$\langle T^{\mu\nu}(\boldsymbol{k},t_x)T^{\alpha\beta}(\boldsymbol{k}',t_y)\rangle_c=\frac{-i}{2}\alpha_s^2m^{4-8s}\delta^3(\boldsymbol{k}+\boldsymbol{k}')\int\frac{d^3p}{\omega(\boldsymbol{p})\omega(\boldsymbol{p}-\boldsymbol{k})}e^{i\vartheta(\boldsymbol{p},\boldsymbol{k},t_x,t_y)} \\ \mathbb{TR}\left\{\begin{bmatrix}0 & \Pi^{\mu\nu}(p^0,\boldsymbol{p})\\ \bar{\Pi}^{\mu\nu}(p^0,\boldsymbol{p}) & 0\end{bmatrix}\begin{bmatrix}m^{2s}\mathbb{I} & \Pi(p^0,\boldsymbol{p})\\ \bar{\Pi}(p^0,\boldsymbol{p}) & m^{2s}\mathbb{I}\end{bmatrix}\right. \\ \times\begin{bmatrix}0 & \Pi^{\alpha\beta}((p-k)^0,\boldsymbol{p}-\boldsymbol{k})\\ \bar{\Pi}^{\alpha\beta}((p-k)^0,\boldsymbol{p}-\boldsymbol{k}) & 0\end{bmatrix} \\ \left.\times\begin{bmatrix}m^{2s}\mathbb{I} & \Pi((k-p)^0,\boldsymbol{k}-\boldsymbol{p})\\ \bar{\Pi}((k-p)^0,\boldsymbol{k}-\boldsymbol{p}) & m^{2s}\mathbb{I}\end{bmatrix}\right\}. \tag{5.15}$$

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The trace is over the Dirac indices in the representation $(s,0)\oplus(0,s)$. Expression (5.15) is one of the main results in this work. One then can project (5.15) along the polarization tensors to obtain the amplitude $\mathcal{M}^{\lambda\lambda'}$ as in (5.5). Since both fields, the left and right handed, are present, we expect to have $\mathcal{M}^{++}=\mathcal{M}^{--}$.

We can perform the matrix multiplication and take the Dirac traces of (5.15) to find

$$\langle T^{\mu\nu}(\boldsymbol{k},t_x)T^{\alpha\beta}(\boldsymbol{k}',t_y)\rangle_c=\frac{-i}{2}\alpha_s^2 m^{4-8s}\delta^3(\boldsymbol{k}+\boldsymbol{k}')\int\frac{d^3p}{\omega(\boldsymbol{p})\omega(\boldsymbol{p}-\boldsymbol{k})}e^{i\vartheta(\boldsymbol{p},\boldsymbol{k},t_x,t_y)}\times\left(\mathcal{Z}_L^{\mu\nu;\alpha\beta}+\mathcal{Z}_R^{\mu\nu;\alpha\beta}\right),\tag{5.16}$$

where

$$\mathcal{Z}_L^{\mu\nu;\alpha\beta}=\Pi^{\mu\nu}(p^0,\boldsymbol{p})\bar{\Pi}(p^0,\boldsymbol{p})\Pi^{\alpha\beta}((p-k)^0,\boldsymbol{p}-\boldsymbol{k})\bar{\Pi}((k-p)^0,\boldsymbol{k}-\boldsymbol{p})+m^{4s}\Pi^{\mu\nu}(p^0,\boldsymbol{p})\bar{\Pi}^{\alpha\beta}(p^0-k^0,\boldsymbol{p}-\boldsymbol{k}),\tag{5.17}$$

$$\mathcal{Z}_R^{\mu\nu;\alpha\beta}=\bar{\Pi}^{\mu\nu}(p^0,\boldsymbol{p})\Pi(p^0,\boldsymbol{p})\bar{\Pi}^{\alpha\beta}((p-k)^0,\boldsymbol{p}-\boldsymbol{k})\Pi((k-p)^0,\boldsymbol{k}-\boldsymbol{p})+m^{4s}\bar{\Pi}^{\mu\nu}(p^0,\boldsymbol{p})\Pi^{\alpha\beta}(p^0-k^0,\boldsymbol{p}-\boldsymbol{k}).\tag{5.18}$$

In the following, we consider $s=\frac{1}{2}$ and $s=1$ as examples for the massive case. The computations for any $s\geq 2$ are extremely painful due to the cumbersome algebra of the t matrices, and we feel that it is necessary to develop group theoretical tricks to deal with such cases. This task will be left for a future investigation.

Case I: $s=\frac{1}{2}$. The projection of the energy-momentum tensor correlator along the polarization eigenbases yields

$$\mathcal{M}^{\lambda,\lambda'}(t_x,t_y)=\alpha_{1/2}^2\delta^{\lambda\lambda'}\delta^3(\boldsymbol{k}+\boldsymbol{k}')\int d^3p e^{i\vartheta(\boldsymbol{p},\boldsymbol{k},t_x,t_y)}\times|\boldsymbol{p}|^2\sin^2\theta\left[1+\frac{|\boldsymbol{p}|\cos\theta\,(|\boldsymbol{p}|\cos\theta-k)-m^2}{\sqrt{(m^2+|\boldsymbol{p}|^2)\,(m^2+|\boldsymbol{p}|^2+k^2-2|\boldsymbol{p}|k\cos\theta)}}\right].\tag{5.19}$$

In the $m=0$ limit, it is trivial to see that (5.19) is the sum of the contributions from left-handed (5.12) and right-handed (5.13) fields.

Case II: $s=1$. In this case we find

$$\begin{aligned}\mathcal{M}^{\lambda,\lambda'}(t_x,t_y)=&\frac{\alpha_1^2}{2m^4}\delta^{\lambda\lambda'}\delta^3(\boldsymbol{k}+\boldsymbol{k}')\int d^3p e^{i\vartheta(\boldsymbol{p},\boldsymbol{k},t_x,t_y)}\frac{m^2|\boldsymbol{p}|^2}{\Gamma}\sin^2\theta\\&\times\big\{14k^2|\boldsymbol{p}|^2+7m^2|\boldsymbol{p}|^2+3|\boldsymbol{p}|^4+8k^2\Gamma+4|\boldsymbol{p}|^2\Gamma\\&-k|\boldsymbol{p}|\cos\theta\left(8k^2+8m^2+19|\boldsymbol{p}|^2+16\Gamma\right)+|\boldsymbol{p}|^2\cos(2\theta)\left(10k^2+m^2+4|\boldsymbol{p}|^2+4\Gamma\right)\\&-5k|\boldsymbol{p}|^3\cos(3\theta)+|\boldsymbol{p}|^4\cos(4\theta)\big\},\end{aligned}\tag{5.20}$$

where

$$\Gamma=\sqrt{(m^2+|\boldsymbol{p}|^2)\,(m^2+|\boldsymbol{p}|^2+k^2-2k|\boldsymbol{p}|\cos\theta)}\,.\tag{5.21}$$

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Now a few points are in order:

1. One should refrain from comparing the result (5.20) with massive electrodynamics. First, the later theory is based on the representation $(\frac{1}{2},\frac{1}{2})$ and uses the gauge potential A_μ instead of the physical fields. The $(\frac{1}{2},\frac{1}{2})$ representation contains $s=1$ and $s=0$ spin fields and one needs to isolate the $s=1$ component by applying the constraint $\partial^\mu A_\mu = 0$. This is in contrast with the $(1,0)\oplus(0,1)$ causal field construction.[8] Second, one needs to bear in mind that the energy momentum tensor of massive electrodynamics (derived from Proca Lagrangian) is different from the one adopted in this work and given by (3.13).

2. It is easy to check that $\langle T_-(k)T_-(-k)\rangle > 0$ and $\langle T_+(k)T_+(-k)\rangle > 0$ for both $s=\frac{1}{2}$ and $s=1$, and therefore, our theory respects unitarity in the checked cases. Eventhough one could have expected the $s=\frac{1}{2}$ theory to be unitary, the higher spin case is far from obvious.

3. Since both fields $(0,1)$ and $(1,0)$ are present on equal footing, we have $\langle T_-(k)T_-(-k)\rangle = \langle T_+(k)T_+(-k)\rangle$. This is attributed to the fact that both the left (5.17) and right (5.18) amplitudes enter the calculations of $\mathcal{M}^{\lambda,\lambda'}$ with the same weight. In principle, one could imagine a mechanism[9] that prefers the left over the right component, or vice versa. In this case the sum $\left(\mathcal{Z}_L^{\mu\nu;\alpha\beta}+\mathcal{Z}_R^{\mu\nu;\alpha\beta}\right)$ in (5.16) is replaced by the weighted sum $\left(W_L\mathcal{Z}_L^{\mu\nu;\alpha\beta}+W_R\mathcal{Z}_R^{\mu\nu;\alpha\beta}\right)$, where $W_{L,R}$ is the left(right) weight. Thus, we conclude that in general $\langle T_-(k)T_-(-k)\rangle \neq \langle T_+(k)T_+(-k)\rangle$. Such signal violates the macroscopic parity symmetry [36, 37].

5.3 Energy-momentum tensor correlators in curved space: massless $s=\frac{1}{2}$ field

In the remaining of this section we calculate the energy-momentum tensor in FRW background for the massless $s=\frac{1}{2}$ field. The energy-momentum tensor correlator is

$$\langle T^{\mu\nu}(x)T^{\alpha\beta}(y)\rangle = 2i\langle \mathcal{O}^{\mu\nu}(x)G_0(x,y)\mathcal{O}^{\alpha\beta}(y)G_0(y,x)\rangle\,, \tag{5.22}$$

where $\mathcal{O}^{\mu\nu}$ is given by (4.17). To compute this correlator, we first consider the quantity D^aG^0, which appears from the action of the vertex on G^0. Recalling that the Green's function is a bispinor with indices G^0_{MN} such that $M,N=-1/2,1/2$, we have

$$D^aG^0_{NM} = \eta^{ab}V_b^\rho\left[\partial_\rho\delta^P_N + (\Gamma_\rho)^P_N\right]G^0_{PM}\,. \tag{5.23}$$

As we pointed out in section 2.5, the best strategy to compute this quantity in de Sitter background is to perform the Weyl transformation $\eta_{\mu\nu}\to\bar{g}_{\mu\nu}=\eta_{\mu\nu}\Omega^2$, $\Omega=\frac{1}{H\tau}$. After some algebra we find

$$\bar{D}^aG^0_{NM} = \Omega^{-1}\left[\partial^a\delta^P_N - (\Omega)^{-1}\left(\mathcal{T}^{0a}\right)^P_N\Omega_{,0}\right]G^0_{PM}\,, \tag{5.24}$$

[8] It was also shown in [26] that the conformal anomalies of $(1,0)$ and $(\frac{1}{2},\frac{1}{2})$ fields are different.

[9] For example, coupling to axions can lead to the preference of one component over the other [33–35]. A similar effect can be obtained by turning on parity-violating operators.

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where $\mathcal{T}^{0a} = \sigma^a$ are the Lorentz generators of $s = 1/2$ representation. Taking this into account, we obtain

$$\bar{\mathcal{O}}^{ij}_{MN} G^0_{PN} = \frac{i}{2}\Omega^{-3}\left[\left(\sigma^i_{MN}\partial^j + \sigma^j_{MN}\partial^i\right) G^0_{PN} + \frac{\delta^{ij}}{\tau} G^0_{PM}\right], \tag{5.25}$$

where we limited our computation to the spatial components of $\mathcal{O}^{ij}$ since these are the only components that survive after projecting the energy-momentum tensor along the helicity eigenstates. After straightforward algebra we find

$$\begin{aligned}\mathcal{M}^{\lambda,\lambda'}(\tau_x,\tau_y) = 2\alpha_{1/2}\Omega^{-3}(\tau_x)\Omega^{-3}(\tau_y)\delta^3(\boldsymbol{k}+\boldsymbol{k}')\int \frac{d^3p}{(2\pi)^3}\boldsymbol{p}\cdot\boldsymbol{\epsilon}(\boldsymbol{k})\boldsymbol{p}\cdot\boldsymbol{\epsilon}(-\boldsymbol{k})n_c(\boldsymbol{p})n_d(\boldsymbol{k}-\boldsymbol{p})\\ \times \epsilon^\lambda_i(\boldsymbol{k})\epsilon^\lambda_l(-\boldsymbol{k})\mathrm{tr}\left[\sigma^i\sigma^c\sigma^l\sigma^d\right] u^*(\boldsymbol{p},\tau_x)u(\boldsymbol{p},\tau_y)v^*(\boldsymbol{p}-\boldsymbol{k},\tau_y)v(\boldsymbol{p}-\boldsymbol{k},\tau_x)\,.\end{aligned}$$

After taking the trace we finally obtain

$$\begin{aligned}\mathcal{M}^{\lambda,\lambda'}(\tau_x,\tau_y) = 2\alpha_{1/2}\Omega^{-3}(\tau_x)\Omega^{-3}(\tau_y)\delta^3(\boldsymbol{k}+\boldsymbol{k}')\int \frac{d^3p}{(2\pi)^3}|\boldsymbol{p}|^2\sin^2\theta\\ \times u^*(\boldsymbol{p},\tau_x)u(\boldsymbol{p},\tau_y)v^*(\boldsymbol{p}-\boldsymbol{k},\tau_y)v(\boldsymbol{p}-\boldsymbol{k},\tau_x)\\ \times\left(1-\frac{\lambda+\lambda'}{2}\cos\theta\right)\left(1-\frac{\lambda+\lambda'}{2}\frac{|\boldsymbol{p}|\cos\theta-k}{\sqrt{k^2+|\boldsymbol{p}|^2-2k|\boldsymbol{p}|\cos\theta}}\right),\end{aligned} \tag{5.26}$$

which exactly matches the result obtained previously in [2] via canonical quantization methods.

6 Comments on the gravitational wave spectrum

In the previous sections we determined the form of the energy-momentum tensor correlator, $\mathcal{M}^{\lambda\lambda'}$, for a general causal field with spin s. Given $\mathcal{M}^{\lambda\lambda'}$, we now can calculate the metric tensor correlator, and hence, the gravitational waves (GW) power spectrum. This is a standard procedure that can be found in various places in the literature, see e.g., [2, 3]. However, we choose to review it here for the sake of completeness. To this end, we write the full metric as $g_{\mu\nu}{=}a^2(\tau)\,(\eta_{\mu\nu}+h_{\mu\nu})$, where the background is taken to be FRW and $a(\tau)$ is the scale factor. After gauge fixing, the linearized equations of motion for the tensor perturbations $h_{\mu\nu}$ are

$$\left(\partial^2_\tau + 2\mathcal{H}\partial_\tau - \nabla^2\right) h^{\mu\nu}(\boldsymbol{x},\tau) = \frac{2}{m_p^2}\,T^{\mu\nu}(\boldsymbol{x},\tau)\,, \tag{6.1}$$

where $\mathcal{H} = \frac{da}{ad\tau}$ is the conformal Hubble parameter and m_p is the reduced Planck mass. One can also take the spatial Fourier transform of the above equation to obtain

$$\left(\partial^2_\tau + 2\mathcal{H}\partial_\tau + k^2\right) h^{\mu\nu}(\boldsymbol{k},\tau) = \frac{2}{m_p^2}\,T^{\mu\nu}(\boldsymbol{k},\tau)\,. \tag{6.2}$$

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The gravitational perturbations $h^{\mu\nu}$ can be expressed in terms of the two circular helicity modes $h_\pm$ as

$$h^{\mu\nu}(\boldsymbol{k},\tau) = \sum_{\lambda=\pm} \epsilon^{\mu}_{\lambda}(\boldsymbol{k})\epsilon^{\nu}_{\lambda}(\boldsymbol{k}) h_\lambda(\boldsymbol{k},\tau)\,. \tag{6.3}$$

This decomposition is consistent with the transverse and traceless conditions $h_{00} = h_{0i} = h^i_i = h^i_{i,j} = 0$, and hence, it projects only the physical degrees of freedom. One then uses the projection operator $\Pi_{\mu\nu,\alpha\beta}(\boldsymbol{k}) = \mathcal{P}_{\mu\alpha}(\boldsymbol{k})\mathcal{P}_{\nu\beta}(\boldsymbol{k}) - \frac{1}{2}\mathcal{P}_{\mu\nu}(\boldsymbol{k})\mathcal{P}_{\alpha\beta}(\boldsymbol{k})$ along with the identity $\epsilon^{\mu\nu}_{\lambda}(\boldsymbol{k})\ \Pi_{\mu\nu}{}^{\alpha\beta}(\boldsymbol{k}) = \epsilon^{\alpha\beta}_{\lambda}(\boldsymbol{k})$ to project out the physical components of the tensor perturbations $h_{\lambda=\pm}$. Finally, we find that the equation of motion of $h_{\lambda=\pm}$ reads

$$\left(\partial^2_\tau + 2\mathcal{H}\partial_\tau + k^2\right) h_\lambda(\boldsymbol{k},\tau) = \frac{1}{m_p^2}\ \epsilon^{\mu\nu}_{-\lambda}(\boldsymbol{k})\ T_{\mu\nu}(\boldsymbol{k},\tau)\,. \tag{6.4}$$

The solution of (6.4) is given by

$$h_\lambda(\boldsymbol{k},\tau) = h^{\rm hom}_\lambda(\boldsymbol{k},\tau) + \frac{2}{m_p^2}\int d\tau' G_k(\tau,\tau')\ \epsilon^{\mu\nu}_{-\lambda}(\boldsymbol{k})\ T_{\mu\nu}(\boldsymbol{k},\tau')\,, \tag{6.5}$$

where $h^{\rm hom}_\lambda(\boldsymbol{k},\tau)$ is the homogeneous solution of (6.4) and $G_k(\tau,\tau')$ is the retarded Green's function of the differential operator on the left hand side of (6.4), i.e.,

$$\left(\partial^2_\tau + 2\mathcal{H}\partial_\tau + k^2\right) G_k(\tau,\tau') = \delta(\tau-\tau')\,. \tag{6.6}$$

Finally, with the aid of definition (5.5), the correlator $\langle h_\lambda(\boldsymbol{k},\tau) h_{\lambda'}(\boldsymbol{k},\tau)\rangle$ is given by the expression:

$$\langle h_\lambda(\boldsymbol{k},\tau) h_{\lambda'}(\boldsymbol{k}',\tau)\rangle = -i\frac{4}{m_p^4}\delta(\boldsymbol{k}-\boldsymbol{k}')\int d\tau'\tau'' G_k(\tau,\tau') G_{k'}(\tau,\tau'')\ \mathcal{M}^{\lambda\lambda'}(\tau',\tau'')\,. \tag{6.7}$$

Expression (6.7) gives the GW spectrum in terms of a complicated function $\mathcal{M}^{\lambda\lambda'}(\tau',\tau'')$. Irrespective of these complications, the conformal symmetry of de Sitter space is sufficient to determine the GW correlators. For a Hubble parameter much larger than the particle mass, $H \gg m$, the general form of the GW correlators take the form

$$\begin{aligned}
\langle h_-(\boldsymbol{k},\tau) h_-(\boldsymbol{k},\tau)\rangle &\cong \frac{H^2}{\pi^2 m_p^2}\left[1 + \mathcal{N}_1 \frac{H^2}{m_p^2}\left(\frac{k}{k_0}\right)^{n_-}\right],\\
\langle h_+(\boldsymbol{k},\tau) h_+(\boldsymbol{k},\tau)\rangle &\cong \frac{H^2}{\pi^2 m_p^2}\left[1 + \mathcal{N}_2 \frac{H^2}{m_p^2}\left(\frac{k}{k_0}\right)^{n_+}\right],
\end{aligned} \tag{6.8}$$

where the first term is the vacuum contribution (the homogeneous term), $\mathcal{N}_{1(2)}$ are left (right) coefficients, k_0 is a reference wavevector, and $n_\pm$ are exponents that quantify the deviation from an exact scale invariance. In a quasi de Sitter space we expect to have $|n_\pm| \ll 1$. In addition, any imbalance between the production of $(s,0)$ and $(0,s)$ fields translates into $\mathcal{N}_1 \neq \mathcal{N}_2$. Therefore, an observation of parity violation in the primordial GW power spectrum, i.e., $\langle h_-(\boldsymbol{k},\tau) h_-(\boldsymbol{k},\tau) \neq \langle h_+(\boldsymbol{k},\tau) h_+(\boldsymbol{k},\tau)\rangle$, will point to parity violating physics in the early Universe. The coefficients $\mathcal{N}_{1,2}$ are proportional to α_s that

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appears in the vertex operator (4.13). Although in this work we do not give a detailed particle physics model that explains the origin of α_s, we expect on general grounds that a sufficient mechanism can produce higher spin bosonic fields from vacuum with an effective $\alpha_s \gg 1$. On the other hand, the production of fermionic higher spin fields from vacuum yields $\alpha_s \sim 1$, which is a result of the Pauli exclusion principle.

7 Discussion

Higher spin fields remains one of the most challenging topics in physics. In particular, their coupling to gravity or electromagnetism has proven to be a highly non-trivial task. Over the past few years there has been a noticeable progress toward a better understanding of higher spin fields, their interactions, as well as their behavior in external background fields, see [38–45] for a non-comprehensive list of references. In this work we argued that irrespective of these difficulties, general principles of Poincaré covrainace and analaticity of the Green's functions are enough to dictate the form of the energy-momentum tensor correlators and to conclude that the production of massive higher spin fields from vacuum is accompanied by large-scale signals that can be printed in the Cosmic Microwave Background (CMB) or detected in terrestrial or space-based gravitational wave interferometry (GWI) [46].

The explicit form of the energy-momentum tensor correlator depends on the formalism used to describe the higher spin particles. There are two main methods in the literature that are used to describe fields with spin $s \geq \frac{1}{2}$: the Fronsdal construction, which is based on the (s, s) symmetric representation, and the general causal (Weinberg-type) fields that transform under $(s, 0)$, $(0, s)$, or $(s, 0) \oplus (0, s)$. In this work we chose to use the latter formalism as it provides a shortcut way to make our points explicit and avoid the hassle of introducing spurious degrees of freedom and/or auxiliary fields. Weinberg-type fields are physical, and therefore, no question of inconsistencies or ghosts can appear. The lack of a Lagrangian formulation of these fields forced us to postulate an energy-momentum tensor. Its form is inspired by the transformation properties under the Poincaré group.

The fact that we can infer most information about the energy-momentum tensor correlators from general considerations of Poincaré group and analytic structure of Green's function may sound puzzling. Here, we provide an argument that elucidates this observation. The traditional method of computing field theory correlators in cosmology is to start from the Bunch-Davies vacuum $|0\rangle_a$ and then evolve the field theory operators using the Bogoliubov transformation. Let $\{a_i, a_i^\dagger\}$ be a set of annihilation and creation operators, such that the operators $\{a_i\}$ annihilate the vacuum: $a_i|0\rangle_a = 0$. In the presence of a time varying background the set $\{a_i, a_i^\dagger\}$ evolves into a new set of operators $\{b_i, b_i^\dagger\}$. The latter ones are linear combinations of $\{a_i, a_i^\dagger\}$. Therefore, an energy-momentum tensor correlator of the form $\langle 0| \sum_{ijkl} \left(\mathcal{C}_i a_i + \mathcal{C}_i^* a_i^\dagger\right) \left(\mathcal{C}_j a_j + \mathcal{C}_j^* a_j^\dagger\right) \left(\mathcal{C}_k a_k + \mathcal{C}_k^* a_k^\dagger\right) \left(\mathcal{C}_l a_l + \mathcal{C}_l^* a_l^\dagger\right) |0\rangle_a$, which is vanishing in the Bunch-Davies vacuum, will evolve into the correlator $\alpha \langle 0| \sum_{ijkl} \left(\mathcal{C}_i b_i + \mathcal{C}_i^* b_i^\dagger\right) \left(\mathcal{C}_j b_j + \mathcal{C}_j^* b_j^\dagger\right) \left(\mathcal{C}_k b_k + \mathcal{C}_k^* b_k^\dagger\right) \left(\mathcal{C}_l b_l + \mathcal{C}_l^* b_l^\dagger\right) |0\rangle_a$, which is not zero. Now, one can immediately see that the final product is a collection of correlators of the form $\langle 0| b_i^\dagger b_j |0\rangle_a$, which are nothing but Green's functions of on-shell particles. Therefore,

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all information about the energy-momentum tensor correlators are encoded in the analytic structure of Green's functions. The coefficients $\{\mathcal{C}_i\}$ are group theoretical constants that depend on the specific representation at hand, while α is some number that depends on the specifics of the physical process that lead to the particle production, e.g., parametric resonance. This is $\mathcal{O}(1)$ number for fermions, thanks to the Pauli-blocking. However, one can have $\alpha \gg 1$ for bosonic fields. Thus, one expects that an abundant production of higher spin bosons can leave an "in principle detectable" imprint on the CMB or GWI.

Another question concerns the parity symmetry of the Universe. We have shown in this work that any mechanism that causes an imbalance between the on-shell production of $(s, 0)$ and $(0, s)$ fields leads to imbalance between the left- and right-handed energy momentum correlators, $\langle T_-(k)T_-(-k)\rangle \neq \langle T_+(k)T_+(-k)\rangle$, and hence, a violation of the macroscopic parity symmetry. This generalizes the previous observation that the production of fermions with a definite helicity is accompanied by the generation of chiral gravitational waves [2].

General causal fields is useful as a proof of concept for a few ideas that we discussed in this work. However, we found that this formalism is not suitable to study the production of massless fields in flat space. In addition, one needs to deal with cumbersome algebra for $s > 1$. There are also difficulties regarding the ordering of the covariant derivatives in curved background. All these questions call for more investigations of the production of higher spin fields in a cosmological context, either using the general causal field recipe or by other means.

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References

[1] Y. Shtanov, J.H. Traschen and R.H. Brandenberger, *Universe reheating after inflation*, *Phys. Rev.* **D 51** (1995) 5438 [hep-ph/9407247] [INSPIRE].

[2] M.M. Anber and E. Sabancilar, *Chiral Gravitational Waves from Chiral Fermions*, *Phys. Rev.* **D 96** (2017) 023501 [arXiv:1607.03916] [INSPIRE].

[3] L. Sorbo, *Parity violation in the Cosmic Microwave Background from a pseudoscalar inflaton*, *JCAP* **06** (2011) 003 [arXiv:1101.1525] [INSPIRE].

JHEP06(2018)154

[4] N. Arkani-Hamed and J. Maldacena, *Cosmological Collider Physics*, arXiv:1503.08043 [INSPIRE].

[5] D. Baumann, G. Goon, H. Lee and G.L. Pimentel, *Partially Massless Fields During Inflation*, *JHEP* **04** (2018) 140 [arXiv:1712.06624] [INSPIRE].

[6] G. Franciolini, A. Kehagias and A. Riotto, *Imprints of Spinning Particles on Primordial Cosmological Perturbations*, *JCAP* **02** (2018) 023 [arXiv:1712.06626] [INSPIRE].

[7] M. Fierz and W. Pauli, *On relativistic wave equations for particles of arbitrary spin in an electromagnetic field*, *Proc. Roy. Soc. Lond.* **A 173** (1939) 211 [INSPIRE].

[8] J. Fang and C. Fronsdal, *Massless Fields with Half Integral Spin*, *Phys. Rev.* **D 18** (1978) 3630 [INSPIRE].

[9] C. Fronsdal, *Singletons and Massless, Integral Spin Fields on de Sitter Space (Elementary Particles in a Curved Space. 7.*, *Phys. Rev.* **D 20** (1979) 848 [INSPIRE].

[10] C. Fronsdal, *Massless Fields with Integer Spin*, *Phys. Rev.* **D 18** (1978) 3624 [INSPIRE].

[11] L.P.S. Singh and C.R. Hagen, *Lagrangian formulation for arbitrary spin. 1. The boson case*, *Phys. Rev.* **D 9** (1974) 898 [INSPIRE].

[12] L.P.S. Singh and C.R. Hagen, *Lagrangian formulation for arbitrary spin. 2. The fermion case*, *Phys. Rev.* **D 9** (1974) 910 [INSPIRE].

[13] M.A. Vasiliev, *Higher spin gauge theories: Star product and AdS space*, hep-th/9910096 [INSPIRE].

[14] N. Bouatta, G. Compere and A. Sagnotti, *An Introduction to free higher-spin fields*, in *Higher spin gauge theories: Proceedings, 1st Solvay Workshop*, Brussels, Belgium, 12–14 May 2004, pp. 79–99 [hep-th/0409068] [INSPIRE].

[15] R. Rahman and M. Taronna, *From Higher Spins to Strings: A Primer*, arXiv:1512.07932 [INSPIRE].

[16] S. Weinberg and E. Witten, *Limits on Massless Particles*, *Phys. Lett.* **B 96** (1980) 59 [INSPIRE].

[17] M. Porrati, *Massive spin* 5/2 *fields coupled to gravity: Tree level unitarity versus the equivalence principle*, *Phys. Lett.* **B 304** (1993) 77 [gr-qc/9301012] [INSPIRE].

[18] E.S. Fradkin and M.A. Vasiliev, *On the Gravitational Interaction of Massless Higher Spin Fields*, *Phys. Lett.* **B 189** (1987) 89 [INSPIRE].

[19] V. Bargmann and E.P. Wigner, *Group Theoretical Discussion of Relativistic Wave Equations*, *Proc. Nat. Acad. Sci.* **34** (1948) 211 [INSPIRE].

[20] S. Weinberg, *Feynman Rules for Any Spin*, *Phys. Rev.* **133** (1964) B1318 [INSPIRE].

[21] S. Weinberg, *Feynman Rules for Any Spin. 2. Massless Particles*, *Phys. Rev.* **134** (1964) B882 [INSPIRE].

[22] S. Weinberg, *The Quantum theory of fields. Vol. 1: Foundations*, Cambridge University Press (2005) [INSPIRE].

[23] W.K. Tung, *Group theory in physics*, World Scientific (1985).

[24] A.O. Barut, I. Muzinich and D.N. Williams, *Construction of Invariant Scattering Amplitudes for Arbitrary Spins and Analytic Continuation in Total Angular Momentum*, *Phys. Rev.* **130** (1963) 442 [INSPIRE].

[25] S. Weinberg, *Gravitation and Cosmology*, John Wiley and Sons, New York (1972) [INSPIRE].

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[26] N.D. Birrell, *Stress tensor conformal anomaly for Weinberg type fields in curved space-times*, *J. Phys.* **A 12** (1979) 337 [INSPIRE].

[27] M.A. Castagnino, D.D. Harari and J.P. Paz, *De Sitter selfconsistent cosmology for Weinberg type fields*, *Class. Quant. Grav.* **3** (1986) 569 [INSPIRE].

[28] G. Grensing, *Quantized Fields over de Sitter Space*, *J. Phys.* **A 10** (1977) 1687 [INSPIRE].

[29] N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space*, Cambridge Monographs on Mathematical Physics, Cambridge University Press, Cambridge, U.K. (1984) [INSPIRE].

[30] S. Weinberg, *Cosmology*, Oxford University Press (2008) [INSPIRE].

[31] S. Weinberg, *Photons and Gravitons in s Matrix Theory: Derivation of Charge Conservation and Equality of Gravitational and Inertial Mass*, *Phys. Rev.* **135** (1964) B1049 [INSPIRE].

[32] S. Weinberg, *Photons and gravitons in perturbation theory: Derivation of Maxwell's and Einstein's equations*, *Phys. Rev.* **138** (1965) B988 [INSPIRE].

[33] N. Bartolo, S. Matarrese, M. Peloso and M. Shiraishi, *Parity-violating and anisotropic correlations in pseudoscalar inflation*, *JCAP* **01** (2015) 027 [arXiv:1411.2521] [INSPIRE].

[34] P. Adshead and E.I. Sfakianakis, *Fermion production during and after axion inflation*, *JCAP* **11** (2015) 021 [arXiv:1508.00891] [INSPIRE].

[35] M.M. Anber and L. Sorbo, *Non-Gaussianities and chiral gravitational waves in natural steep inflation*, *Phys. Rev.* **D 85** (2012) 123537 [arXiv:1203.5849] [INSPIRE].

[36] A. Lue, L.-M. Wang and M. Kamionkowski, *Cosmological signature of new parity violating interactions*, *Phys. Rev. Lett.* **83** (1999) 1506 [astro-ph/9812088] [INSPIRE].

[37] S. Saito, K. Ichiki and A. Taruya, *Probing polarization states of primordial gravitational waves with CMB anisotropies*, *JCAP* **09** (2007) 002 [arXiv:0705.3701] [INSPIRE].

[38] N. Boulanger and S. Leclercq, *Consistent couplings between spin-2 and spin-3 massless fields*, *JHEP* **11** (2006) 034 [hep-th/0609221] [INSPIRE].

[39] M. Porrati and R. Rahman, *A Model Independent Ultraviolet Cutoff for Theories with Charged Massive Higher Spin Fields*, *Nucl. Phys.* **B 814** (2009) 370 [arXiv:0812.4254] [INSPIRE].

[40] V.E. Didenko and E.D. Skvortsov, *Elements of Vasiliev theory*, arXiv:1401.2975 [INSPIRE].

[41] S. Gómez-Ávila and M. Napsuciale, *Covariant basis induced by parity for the* $(j,0)\oplus(0,j)$ *representation*, *Phys. Rev.* **D 88** (2013) 096012 [arXiv:1307.4711] [INSPIRE].

[42] I.L. Buchbinder and A. Reshetnyak, *General Lagrangian Formulation for Higher Spin Fields with Arbitrary Index Symmetry. I. Bosonic fields*, *Nucl. Phys.* **B 862** (2012) 270 [arXiv:1110.5044] [INSPIRE].

[43] A. Reshetnyak, *General Lagrangian Formulation for Higher Spin Fields with Arbitrary Index Symmetry. 2. Fermionic fields*, *Nucl. Phys.* **B 869** (2013) 523 [arXiv:1211.1273] [INSPIRE].

[44] M. Henneaux, G. Lucena Gómez and R. Rahman, *Gravitational Interactions of Higher-Spin Fermions*, *JHEP* **01** (2014) 087 [arXiv:1310.5152] [INSPIRE].

[45] I. Cortese, R. Rahman and M. Sivakumar, *Consistent Non-Minimal Couplings of Massive Higher-Spin Particles*, *Nucl. Phys.* **B 879** (2014) 143 [arXiv:1307.7710] [INSPIRE].

[46] J.L. Cook and L. Sorbo, *Particle production during inflation and gravitational waves detectable by ground-based interferometers*, *Phys. Rev.* **D 85** (2012) 023534 [*Erratum ibid.* **D 86** (2012) 069901] [arXiv:1109.0022] [INSPIRE].

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