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‘Allowing them to dream’: fostering creativity in mathematics undergraduates

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ABSTRACT

With the rapid development of artificial intelligence, work and leisure is expected to change, and human creative competence is expected to be increasingly important, partly for the workplace and the economy, but also for thriving and well-being. There is some interest in fostering creative competence in mathematics, where it is seen as relating largely to mathematical problem solving, when solutions are not matters of routine, at least for the student. Early studies, however, indicate little impact on teaching, which has tended to adhere to past practices without creative competence being an explicit goal or provision for achieving it. A belief that such a competence will grow undirected and unaided, or that it is only for the gifted student goes against current views of creative potential. Such beliefs could present obstacles to change, yet seem to have received little explicit exploration. This study aimed to identify some of the obstacles to fostering mathematics undergraduates’ creative thinking arising from tutors’ notions of mathematical creativity and its place in education. The views of twenty-two UK university mathematics tutors were collected and collated by phenomenographic analysis, a qualitative method designed to identify categories of belief. This showed latent potential problems arising from differences in the tutors’ beliefs about creativity (e.g. creativity being in the process or the product), and about its origin (e.g. creativity arising from nature or nurture). Adding these to students’ beliefs increases reservations about a readiness to foster students’ creative competences in mathematics. Ways of overcoming these obstacles are suggested. We felt it essential that mathematicians have a common understanding of mathematical creativity, and that certain myths about creative abilities are corrected. These understandings need to be shared with students, and provision made for fostering their creative thinking at a macro level (e.g. courses and workshops) and a micro level (e.g. rubrics to guide thinking). Collaborative creative thinking is also valued in the workplace. It would add to students’ assets if this provision included opportunities to practise it.

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Introduction

A changing world

The capabilities of digital devices have not just increased in capacity, but changed in nature (see e.g. Fletcher and Webb 2017). Such devices now have an adaptive, artificial intelligence (AI) that lends itself well to automation. AI-enabled devices (systems that analyse ‘the environment and hence takes actions with some autonomy to achieve a particular goal’) ‘learn’ from the data they collect,

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extrapolate from them, improve their responses, make decisions and act on them (Krafft et al. 2020; Ramge 2019). They are well-suited to carry out tasks which have been the preserve of people. Their introduction has been described as the Fourth Industrial Revolution, a period of time during which work and leisure will change markedly (Maynard 2015; Prisecaru 2016). Governments worldwide believe that this digital technology offers different and major economic opportunities which could support prosperity and security (e.g. Trajtenberg 2018; Yang 2019). Some believe new kinds of employment will evolve so the loss of jobs will be small (e.g. Clark 2020). Others think that a half of current occupations are at risk (e.g. Bakshi, Frey, and Osborne 2015). Future-proof occupations are those which need human creative thought (possibly supported by AI); beyond that, much could be automated. Players in this new world would benefit from some competence in analysis, creative thinking, problem solving, and evaluation (e.g. Sternberg and Lubart 1995; Häkkinen et al. 2017), that is, in higher order thinking skills (see, e.g. Krathwohl 2002). This has implications for education and provision for fostering such competences (James et al. 2019; OECD 2019; Seldon 2018). Mathematics could make a significant contribution, not least in fostering a creative disposition, supportive attitudes, imaginative thinking, and the acquisition of effective heuristics and strategies.

There is a general agreement that the mathematics endeavour is about solving problems with imagination and creative thinking. It has been several decades since Osborn and his followers described the general process of problem solving as (in brief) problem finding, generating potential ideas for solutions (now often called ideation), developing and trying ideas, finding a solution, and convincing others (see Widya et al. (2020) for an historical overview). While there has been some interest in fostering such creative thinking in all stages of education (e.g. Kozlowski, Chamberlain, and Mann 2019), teaching has tended to be impervious to it (Goos and Kaya 2020). Teaching at all stages is often largely through convergent activities that lead to expected answers (e.g. Mursalin et al. 2018; Nadjafikhah and Yaftian 2013; Sanders 2016). Consequently, mathematics is popularly seen as uncreative (Budd 2019; Newton 2012b; Pehkonen 1997; Sheffield 2017; Silver 1997). In Higher Education some may claim that undergraduates are expected to be imaginative and creative, but this is often an unspoken expectation, not an explicit goal with a deliberate attempt to develop it (e.g. DeHaan 2017; Papaleontiou-Louca et al. 2014). On other occasions, it may be reserved for 'gifted' students, presumably in the belief that the others cannot benefit from it (e.g. Mann 2006; Ngiamsunthorn 2020).

We do not say that developing computational skills, algorithmic facility, and practising a well-oiled recall of procedures and formulae is not worthwhile: mathematical fluency of this nature is an asset that saves time and supports other mathematical activities. But an exclusive diet of such exercises can leave creative thinking unpractised and produce undergraduates with 'static beliefs' about mathematics as 'a finished system of rules, facts and formulae' (e.g. Lithner 2011; Geisler and Rolka 2021). The European Union, for instance, sees universities as being one of the places where students should engage in creative thinking and learning in preparation for the demands of the 21st Century (EU 2009).

In a rapidly changing digital world, we concur with the belief in the value of fostering higher order thinking skills, like creative thinking. In mathematics, some competence in problem solving, exemplified by creating solutions in the absence of algorithms, could prove to be an asset. What might be done to support this objective in mathematics classes? We begin by outlining the notion of creative thinking, followed by some mathematicians' beliefs, and their implications for the fostering of creative thinking in mathematics undergraduates, and finally suggest some actions.

Creative thinking and mathematics

A synthesis of definitions of creativity by Acer, Burnett, and Cabra (2017) describes creative thinking as that which produces something more or less novel or original and appropriate or fit for purpose. Although not essential, when it is also somehow satisfying, elegant, or parsimonious, it adds appeal. In mathematics, this behaviour is seen as stemming from an ability to solve and model problems, construct new concepts, and notice new relationships which extend the body of knowledge and reveal new questions or see old ones in new ways (Ervynck 2002; Kanderir and Gur 2007; Levenson, Swisa, and Tabach 2016; Liljedahl and Sriraman 2006).

Some might argue that students are not capable of producing something new in mathematics – it is simply beyond their level of knowledge, expertise, and experience. There are, however, kinds of creative thinking they can experience, and through which they may develop some competence. Beghetto and Kaufman (2009) describe creative thinking in:

- the students' constructions of personal meanings;
- the solutions to ill-defined problems produced by novices that are at least new to them;
- the professional kind of creative thinking which produces something new to the world, as seen in the work of mathematicians, and to which students might contribute,
- the rare creative act which has a major impact on some area of mathematics.

These are not exclusive categories, but extend though what some have called little-c to Big-C creativity. An undergraduate's creative thinking is more likely to be in the former (that is, skewed towards the earlier items in the list, but there can be notable exceptions). To the extent that students can imagine, conjecture, and prove in mathematics, they can be creative at some level, and the capacity may be nurtured (Gray et al. 1999; Leikin 2009). To that end, Burton (2001) has argued that there should be less teaching of 'the basics' and more replication of how professional mathematicians themselves learn. But if students arrive with notions and expectations that are out of alignment with such views, this risks alienating them further. How might it be achieved? To some extent, this depends on the notions of creativity in mathematics and mathematics education held by tutors.

Accordingly, our aims were to identify amongst some professional mathematicians who are familiar with undergraduate mathematics:

- notions of mathematical creativity;
- notions of the origins of mathematical creativity in people;
- some obstacles these might present fostering creative thinking in undergraduate mathematics; and
- should they be needed, some reflections and suggestions for overcoming these obstacles.

To satisfy these aims and inform our thinking, we collected data as described in what follows.

Method

The approach

The method used to collect notions and beliefs was a qualitative one based on Marton's phenomenographic approach (Marton 1981). This entailed asking largely open-ended questions of university mathematicians to elicit their beliefs about mathematical creativity, its place in education, and the origin and fostering of such thought. Using a questionnaire (see Appendix), we asked, for example, 'What does creativity in mathematics mean to you?' and 'Are any special intellectual abilities needed to be creative in mathematics?' Later we asked, 'Please list what you think might help to promote creative thinking in the learning of mathematics'. We had made provision for interviews to clarify written responses, but these were found to be clear and unambiguous. The responses to each question were treated as data pools of statements. The process of constructing categories involves an iterative sort of the pools into coherent groups of notions of the phenomenon of interest until no further changes are made. Additional questions, (such as, 'Can you give an example of creativity in mathematics?') were used to check our interpretation of the meaning of responses to earlier questions, (such as, 'What does creativity in mathematics mean to you?'). The authors collaborated in the sort and agreed the categories. In accordance with recommended practice, each category was given a descriptive label, its attributes were listed, and examples provided from the data pools. The approach does not claim to collect every notion or belief that exists but, in practice, as the number of participants increases, there is a diminishing return of new responses.

Participants

A list of academic staff in a large mathematics department of a research active university in the UK was compiled from the department's publicly accessible website of staff profiles. Over one hundred mathematicians of diverse mathematical interests conversant with the teaching of undergraduates were invited to participate anonymously and twenty-two did so. This level of participation compares favourably to that of other surveys. The minimum number of participants generally recommended in phenomenographic analysis is sixteen. There was no indication that, collectively, the online profiles of those who did not respond to the invitation were different from those who did. In particular, the gender balance, academic qualifications, the diversity of stated interests, and the length of service (indicated by dated publications) appeared to be similar. With twenty-two respondents, the diminishing return of new responses was clearly observed. Taking all together, this satisfied the requirements of the method.

Results

Mathematicians' notions of creativity in mathematics

All respondents agreed that mathematical problems provide the opportunity for creative thinking in mathematics. However, 18 of the 22 focused on the product – the outcome of the creative thinking. The rest focused on the process – the exercising of creative thought regardless of the outcome. We present these notions as categories with sub-categories.

Category 1: creativity is in the furthering of mathematical knowledge

This category is about adding created products in the form of knowledge and know-how to the existing body of mathematical knowledge. For example, one respondent summed this up as, 'to develop new mathematical concepts/ideas, and/or to synthesise existing concepts and ideas in new ways to provide insight into mathematics or other areas of application'. There are two sub-categories of created products:

1a: Answers to problems

One respondent wrote, 'problems are the motivation of mathematics'. Another was of the view that creativity is evident in 'finding new mathematical concepts, structures or proofs, [or] detecting hidden patterns', and constructing conjectures and algorithms. Mathematicians are also conscious of the role they play in adding to mathematics knowledge, and of being involved in 'a more diverse and active community of mathematicians, who are able to develop interesting theories and produce solutions to previously unsolved problems'.

1b: Ways of finding answers to problems

As one mathematician put it, 'we *create* a solution to a problem, where previously there was none'. Here, creativity in mathematics is evident in the discovery of new ways of representing or thinking about a mathematical question, or constructing a novel route to its solution. This shows a creative approach to the problem, and the generation of a solution.

In this way, Category 1 notions focus on products of mathematical problem solving. These products include the solutions to problems themselves, and also the method, procedure, or way constructed to find such solutions. These products add directly to the body of mathematical knowledge.

Category 2: creativity is in the mental process of mathematical problem solving

This category focuses on the mental activity which generates mathematical products. For example, four respondents believed that that even the attempt to solve a problem can be creative, regardless of its outcome. The emphasis is on the process of 'attempting' to solve a problem and not on the 'something' it may generate (the product). As one put it, creativity 'means playing around: just trying stuff. Maybe it's trying to combine techniques to do something else, or to try to combine problems to see what interesting questions are out there'. It was noted that it 'doesn't have to be correct, and it is important to recognise this and not shoot people down because their ideas don't work; the key thing is to give people space to have ideas and give recognition when they do'. To illustrate this, one wrote that, '[I generate] hundreds of ideas, to try them out, to realise that [almost all of them] don't lead anywhere; rinse and repeat, trying a new angle'. In addition, as a process, creativity can be shared: 'Creativity in maths is often collaborative, throwing out ideas over coffee'. Here, algorithms are tools that oil the process: 'you cannot do any maths without algorithms or formulae'. Algorithms are also 'ways of representing ideas and concepts' that 'can expose simplicity, and inspire new connections and different ways of thinking'.

These categories reflect the dual aspect of creativity: it is a mental process (to create) which can generate a product (the construction or creation). ('To understand' and 'an understanding' reflect a similar and related duality (Newton 2012a).)

Mathematicians' notions of mathematical creativity in education

If and how mathematical creativity can be fostered in undergraduate students depends partly on what it means to those who teach them and partly on what they think education is for and can achieve. All of the mathematicians offered views on aptitudes required for creativity in mathematics, and whether they can be cultivated, fostered or shaped. Two views were found.

Category 1 mathematical predispositions are innate

This notion reflects the view that the attributes that underpin and promote mathematical creativity are talents some people are born with. Nine of the 22 respondents expressed versions of this belief. For example, one wrote, 'You need a feeling for the mathematical concepts, the tools and methods, and an innate ability to manipulate them'. It may also include 'having intuition and imagination while able to distinguish correct and incorrect' and a predilection for logic, mental manipulation, 'an interest in solving puzzles', and abstract thought. (To illustrate, one respondent quoted Hilbert's satisfaction when a student dropped out to study poetry: 'Good, he did not have enough imagination to become a mathematician'.[1])

Category 2 mathematical aptitudes are developed

This notion reflects the view that experience, instruction, and learning all play a significant part in developing attributes that underpin and promote mathematical creativity. Mathematical aptitudes and abilities, including mathematical creative thinking competence, stem from formal and informal learning, and develop over time. Reflecting this, it was suggested that mathematical creative activity should start with pre-school children, 'to broaden the scope of the maths they do'. Fourteen respondents were concerned that mathematics is seen as uncreative, boring, and algorithmic, and they saw these views as coming from experience of the teaching and learning process. For example, one wrote, '[mathematics is] perceived, and often taught, even from early years, in a very rote-learning, fixed type of way, without much thought to wider or more creative applications'. Consequently, it 'seems like there is a set way to approach a problem or an idea'.

These two categories reflect the age-old nature-nurture conflict in education, and are discussed later.

Table 1. Respondents' differences between learning mathematics in school and in university.

	School mathematics	University mathematics
Cognitive approaches	Being clever in problem solving and coming up with solutions as 'right' answers that are already known.	Asking good questions and thinking about fundamental matters, at least some of which are new.
The ethos	An implied 'box' of knowledge (presented as facts and procedures to be learned) from which you will not need to move.	Knowledge not clear, far more open-ended.
Computational skills, algorithmic facility	The most important part of the learning, enabling the finding of 'right' answers.	A part of learning, possibly a foundation for further study/research.

School to university

Undergraduates' prior experience in UK schools is generally moulded by the requirements of examination boards, and shapes conceptions of the nature of mathematics. Respondents' views were also sought about perceived alignments and misalignments between school and university in the teaching of mathematics. In conjunction with respondents notions already described, this may contribute to reflection on what might be done to prepare mathematics students for a changing world.

Fourteen of the participants were of the view that the main difference between school and university mathematics is what was referred to as, 'unknownness'. In school mathematics, 'you know a problem is solvable, otherwise you wouldn't be seeing it . . . the path of research is often a lot less clear than learning', and, 'one has to develop their own goals, which is a form of creativity in itself'. The respondents pointed to three aspects reflecting such differences (Table 1).

The cognitive approaches pointed to school mathematics' emphasis on how to employ more effective strategies (for example, 'via pictures and diagrams'), while at the university level, there is more 'discerning [of] general abstract structures' and uncertainty. The ethos in school mathematics fosters the following of a straight and narrow path to master 'an implied "box" of knowledge' that provides guidance or hints about the right answer. In contrast, university mathematics can be more 'chaotic' and lacking in order. This more open-ended ethos when solving problems carries a challenge and a need for a willingness to expand current understanding. Nevertheless, twenty-one of the respondents pointed to the essential role of algorithms in mathematics. For example, one wrote that, 'you cannot do any maths without algorithms or formulae'. For them, algorithms 'are ways of representing ideas and concepts . . . they can expose simplicity, and inspire new connections and different ways of thinking'. In this way, algorithms are foundations or tools to 'help articulate an argument'. Acquiring them may enable routine computation, but they also have a role in the process of creative thinking: 'Fluency with algorithms and formulae is important for experimenting with mathematical objects to generate conjectures etc., as well as for being able to develop creative ideas into finished mathematics'.

Discussion

The phenomenographic approach collects notions and conceptions of some aspect of the world. It does not claim to collect every notion, nor can it reliably indicate a notion's prevalence. But it can highlight beliefs which can govern behaviour and benefit or impede change. Here, we found that mathematicians can have different views of creativity in mathematics, and also of its basis in people. Such differences, if general, could present obstacles to the fostering of creative thinking in undergraduates. On the assumption that this is a desirable goal, we discuss these differences and suggest how they might be overcome through direct teaching strategies which require the thoughtful involvement of mathematics tutors. It should be acknowledged, however, that tutors' beliefs are just one factor, albeit an important one, which determines goals and practices (Kozlowski, Chamberlain, and Mann 2019). The students also hold beliefs and expectations. The transition to university is widely recognised as presenting students with difficulties, both general and discipline specific. For instance, Van Rooij (2018) has pointed to students' general lack of competence in self-regulated learning, whilst in mathematics, Lithner (2011) found deficiencies in algorithmic proficiency. Of course, differences between individuals, cultures, and education systems can generate dissimilar problems (Luk 2005), but one found in many countries stems from the difference between the essentially utilitarian mathematics of the school and the abstract, formally-defined mathematics of the university (Rach and Heinze 2017). We do not suggest that mathematics courses should be only about being creative, and these mathematicians agreed with that. There is much that must be acquired that is useful in other domains, and much that serves as a body of knowledge on which to build.

Not entirely unexpected, these professional mathematicians see mathematical creativity as being about problem solving. But, within that popular and broad notion, their views are nuanced. Some see it as the successful creation of mathematical products, like solutions to problems and ways of finding solutions that are judged 'correct' or useful. Others emphasise the mental activity or process which may lead to these products, even when they are unsuccessful. They also accept that creative thinking can be wrong, and that being wrong is normal in the creative process, as was indicated in: 'I generate hundreds of ideas … [almost all of them] don't lead anywhere'. It is generally acknowledged that most undergraduates have school backgrounds in mathematics that favour skills in applying algorithms in solving well-defined 'problems' with known, 'correct' answers. Consequently, they could find the fostering of mathematical creativity contradictory and anxiety-producing, and, if unprepared for it, the lack of certainty disturbing.

On this basis, simply increasing opportunities to be creative in undergraduate mathematics could be unproductive; it may even generate discontent with the experience. It could be argued that schools should show mathematics to be creative and 'dynamic' so that there is less of a misalignment between school and university experience, and would-be undergraduates would choose the subject with open eyes. But universities must also attend to the problem: leaving the development of such competences to chance is likely to be inefficient, ineffective, unsystematic, unproductive, and potentially unfair. More deeply, this does not allow mathematics students to flourish in a meaningful way, in the sense that Su (2017) understands when proposing that mathematics is fundamentally embedded into the five human desires of play, beauty, truth, justice, and love. One role that fostering creativity could play is to afford more students the pleasures of viewing the goals of mathematics in a different way. What could making this explicit and the teaching deliberate entail?

One approach would be to introduce new students to creative mathematical thinking directly and explicitly. There is evidence that a short course can give students a new perspective, as demonstrated by Witzke et al. (2018) in their 'intensive' seminars used to make new students aware that geometry is not a once-and-for-all-time, fixed subject. Accordingly, mathematics tutors might provide short, introductory courses which, for example:

- make the goal of creative competence explicit,
- provide and practise a vocabulary to think and communicate with,
- describe and exemplify creative thinking in problem solving (using, for instance, historical and contemporary examples that are meaningful to the students),
- provide and demonstrate the use of heuristics,
- exercise students' creative thinking (sometimes collaboratively, as is common in the workplace), and
- depict unsuccessful ideas as a normal part of the risk taking in creative thinking.

In accordance with Beghetto and Kaufman's (2009) general framework, this would include the construction of personal meanings, finding solutions to ill-defined problems, solving problems with several possible solutions, and solving mathematical problems or puzzles noted by the students themselves. There should then be opportunities for creative thinking to be seen in and fostered across the course as a whole, so that what is introduced is not the end of it. One way of doing this may be through a version of inquiry-based learning. For instance, a question-driven approach which demands an in-depth exploration of a topic could include student-generated problems and their solution. Archer-Kuhn and MacKinnon (2020) point out that inquiry-based learning is 'very effective [in] enhancing students' ability to inquire, proficiently research, and solve problems, the top three characteristics that employers seek'. Hence, they add, it equips students with 'lifelong learning skills' like collaborative working which can support problem solving. Kynigos et al. (2014) put this into practice with collaborative opportunities for problem posing and solving using digital technology. Interested in 'the making of mathematics', George Pólya asked 'how is it possible to invent a solution' (Pólya 1973). His advice appears simple: understand the problem, plan an approach, try it, and review the outcome. The difficulty is that these steps are underspecified (for instance, how do we apply our minds to construct an approach?). Later followers have explored strategies for supporting the steps, such as having students work in pairs, one as the solver and the other as a monitor of thinking, asking for clarification and reasons for actions. Others have focused on using analogies drawn from other areas of mathematics as a source of an approach (Herron 1996; for a review, see Voskoglou 2011). Specific activities to practise and exemplify creative thinking may also be helpful. For instance, asking students to find an alternative way to solve a problem can exercise creative thinking (Leikin and Elgrably 2020), and providing students with a rubric describing what counts as creative thinking in a given context can help them. Savic et al. (2016), for example, provided their students with a rubric which described making connections between definitions and theories, representations, and examples, and, amongst other things, what is meant by posing questions, flexible thinking, and proof evaluation. Rowlett et al. (2019) have tested innovative ways of incorporating recreational mathematics in an undergraduate module with Game Theory and Recreational Mathematics. Recreational mathematics is a type of play that implies interaction with 'serious' mathematics. In their module, 40% of the marks required students to write about historical developments in the area, and to present a puzzle for a professional journal. Although the underlying intention is to focus on problem-solving skills, it also develops space for creativity in the course. But, this assumes that course tutors have reflected on and agreed these.

Some mathematicians' notions of creative thinking and mathematics education, however, could present obstacles. For instance, some might emphasise products while others focus on processes. In reality, both are relevant: creativity is a product and a process, and if feedback on students' work is to be given, knowing about both can inform it. Some mathematicians may need to adjust their notions, recognise potential relevance in others' views, and take a wider perspective (e.g. Barraza_Garcia, Ramo-Vazques, and Roa-Fuentes 2020). This is important for consistency in course design and teaching, but also in the assessment of students' competence, otherwise the same piece of work could be judged unwittingly with different criteria.

Notions of the source of mathematical ability may present greater obstacles. These respondents favoured nature *or* nurture as sources of mathematical ability, as do many lay people (Sasaki and Kim 2017). If this is a common misconception, it would indicate outmoded beliefs which see sources simplistically as due to either nature (arising from innate, genetic and brain physiology) or nurture (arising from learning and culture) are not helpful or reflect current understanding. In particular, a belief that mathematical abilities are largely or simply innate risks a casual attitude to creative competence that leaves it to develop unaided. What is perceived as talent may, at least, be partly due to a deep and prolonged immersion in a subject from an early age (Colvin 2016), and it could be argued that a 'creative play' immersion at any age may be worthwhile (Budd 2019). Whatever mathematical ability a student has is 'most often the result of an interaction between genetic and environmental influences' (Sasaki and Kim 2017, 5; Sherry 2004). Allowing that there may be students who have been favoured in the genetics stakes, the interaction between nature and nurture is complex and likely to produce a majority of students with a variety of thinking strengths and weaknesses, not just those who can and those who can't be mathematically creative. In short, undergraduates may have shown some kind of aptitude for mathematics but the nature of this aptitude can vary. Students could benefit from explicit goals, structured support, and graded practice. It may be that these views allow for some interaction between the two, but the interaction is more multifarious than is popularly recognised. As Sasaki and Kim wrote in figurative terms, 'it is more accurate to say that nature and nurture each contribute 100% to the equation' (2017, 6). This should shape course design and teaching intended to support creative thinking.

A final note of realism should be added. Those who are persuaded that fostering students' creative thinking is worthwhile will be aware that making space for creative thinking implies that something must be removed or curtailed to make that space. But fostering creative thinking is more about the approach to teaching and learning than to its content. Nevertheless, it must be acknowledged that creative thinking takes time and is not always productive when under pressure. Supporting it directly through a taught element will need to allow for this.

Some concluding thoughts

The beliefs of our participants about creativity in mathematics and mathematical creativity in people showed there were significant discrepancies. For some, mathematical creativity was the successful solving of a problem or the mental process of solving a problem. For others it was in the thinking behind the attempt to solve the problem, whether or not it led to a solution. Similarly, some participants believed mathematical creativity to be innate while others thought it could be developed. Seeing creativity only in the outcome, and seeing it as innate, particularly should such beliefs occur fairly frequently, are misconceptions which could obstruct the fostering and assessing creative thinking. Such beliefs are not unusual at other levels of mathematics education (Newton 2012b). With the Fourth Industrial Revolution, there is a strong case for deliberately fostering some creative competence in mathematics as an asset graduates take with them. Given this, the findings here suggest that there is a need to explore mathematicians' beliefs further as they could support or undermine provision. Suggestions for that provision are made here, but their success depends on a shared, well-founded perspective of creative thinking. This includes the product-process nature of creative thinking, something that will be particularly important when assessing and grading students' work. Some may also need to see mathematical ability in less simplistic ways or they may neglect or impede its development. This may need someone to coordinate what colleagues contribute to the venture and ensure variety but with an overall coherence in the approach.

Variables in education are numerous and generalisation to other contexts is not simple (for a mathematician's scrutiny, see Simpson 2020). However, for practical purposes, Bassey (2001) has introduced the useful concept of relatability. This is when contexts are compared, and findings adapted to fit a new context. To that end, it would be useful to reflect on mathematics tutors' notions in other contexts, and use or adapt the suggestions made here to promote students' creative thinking in those contexts. However, this needs a caveat: few things in life are simple, and tutors' beliefs are unlikely to be the only factor which determines students' experience of creative thinking in mathematics.

None of this is to suggest that other kinds of thinking and knowledge in mathematics do not matter (Newton and Newton 2018). Mathematics is not mathematics without them. But nor is it mathematics without creative thinking. We leave the last words, with thanks, to one of our respondents: 'in teaching and learning mathematics, it is about allowing them to dream; even if we think they contradict currently accepted common sense'.

Note

1. David Hilbert (1862–1943) worked on invariant theory, amongst other things, and prepared the way for a new, algebraic geometry. He compiled a list of mathematical problems, some of which remain unsolved.

Disclosure statement

No potential competing interest was reported by the authors.

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Douglas Newton PhD DSc SFHEA is a Professor in the School of Education with an interest in factors shaping learning, particularly those to do with fostering valued kinds of thinking, engagement in learning, and the easing of the process of learning. His work includes many articles on diverse aspects of education and several very influential books, including *Teaching for Understanding* and *Thinking with Feeling* (Routledge, London), which have attracted widespread attention, and appeared in several editions in different languages. His current interest focuses on creative thinking, particularly in the teaching of STEM subjects of which he has a long and direct experience, and on which he has published widely.

Yuqian (Linda) Wang PhD FHEA is Assistant Professor in the School of Education specialising in mathematics education, an area of her particular responsibility. Her experience of mathematics at various levels of education began first in Shanghai, and developed further in the UK, leading to a research interest in curriculum design and how it develops (or does not develop) thinking skills like deductive reasoning. With a growing concern for future-orientated mathematics education, Dr Wang became interested in the topic of mathematical creativity following various challenges to her own notions of the nature of mathematics, an interest which is being actively developed and extended.

Lynn Newton PhD FHEA is Professor of Education and Head of the School of Education in Durham University. She specialises in the fostering of higher level thinking such as creative thinking, and effective strategies for developing them, such as questioning and cognitive engagement. She has published over 100 academic papers, reports and monographs (such as, *Making Purposeful Thought Productive*, ICIE, Ulm) as well as professional articles on matters of education. Most recently, she played a major role in researching for and co-authoring the ground-breaking *Durham Commission on Creativity in Education* (Durham University & Arts Council England).

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Data sharing

Data available on request due to privacy/ethical restrictions.

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to restrictions e.g. their containing information that could compromise the privacy of research participants.

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Appendix

This is a compacted version of the survey, omitting demographic questions. The original was electronic with expandable spaces for responses, and it progressed without permitting a return to earlier questions or responses.

Creativity in mathematics survey

If you prefer, you may give you answers in bullet points or in other forms.

(1) What does creativity in mathematics mean to you?
(2) Can you give an example of creativity in mathematics? If so, describe it here.
(3) Are any special intellectual abilities needed to be creative in mathematics?
If you think they are, what are they?
(4) Do algorithms and formulae have role in being creative in mathematics?
If so, in what way(s)?
(5) Can students' mathematical creativity be promoted by some kind of teaching?
If so, please list what you think might help to promote it.
(6) Do you see obstacles or challenges to promoting students' creative thinking in mathematics?
If so, what are they?
(7) Are there differences between being creative in learning mathematics and being creative in researching mathematics?
If so, what are they?
(8) During the process of mathematics research, when do you feel creativity is most needed, if at all?
(9) Do you include creative activities for the learners in your mathematics teaching? If so, please give examples of what you might do, or have done?
(10) Please list what you think might help to promote creative thinking in the learning of mathematics, if anything.
(1) ....
(2) ....
(3) ....