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(Un)conditional Collection Policies on Used Products with Strategic Customers

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Abstract: For generation products characterized by frequent releases of new versions, when a new version is introduced to the market, the current version usually still has a remaining useful life. This creates a challenge for a firm to manage used product collection and upgraded product introductions at the same time, especially with the presence of a secondary market. This paper develops an analytical model to study the design and evaluation of two widely adopted collection policies for used products. Specifically, depending on whether a monetary reward for returning used products is associated with further purchases, we examine both unconditional (buyback) and conditional (trade-in) collection policies that take place in practice. We find that, in the absence of a secondary market, a conditional collection policy can outperform an unconditional one when the base product is durable and the residual value that the manufacturer can obtain after collection is intermediate. However, when an independent secondary market exists, allowing customers to trade used products with each other, any conditional policy cannot outperform the optimal unconditional policy. In particular, the two policies generate the same profit when the residual value is low; otherwise, the unconditional policy dominates as it effectively mitigates the cannibalization of the upgraded product sales by collecting more used products and reducing the supply to the secondary market. We also discuss the environmental impacts of these two collection policies. Our study helps to understand the impact of strategic customer behavior and the secondary market on the choice of used product collection policies, and provides manufacturers with guidance on the design of the optimal collection policy.

Key words: used product collection, buyback, trade-in, secondary market, strategic customer behavior
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1 Introduction

Because of the significant economic value of product recycling and stricter regulatory enforcement of environmental sustainability (e.g., Europa-Environment 2012), used product collection is prevalent in many industries for a variety of products, including ink cartridges, furniture, vehicle components, and consumer electronics. In practice, companies have adopted various strategies to collect used products from customers, especially for the generation products characterized by frequent releases of new versions. More importantly, rapid technological advancement has significantly sped up upgraded product developments such that when a new generation is introduced to the market, the current-version product still functions well subject to normal wear and tear. Hence, there is a need for the manufacturer to provide the customers with proper incentives to encourage both used product returns and upgraded product purchases.

Broadly speaking, the firms' collection policies can be divided into two categories based on whether or not a monetary reward for returning used products is associated with further purchases of the upgraded products. When the reward for the used product collection does not require a replacement purchase, the policy is called an unconditional collection policy, also referred to as a buyback policy in practice. For example, Hewlett-Packard offers Buyback and Planet Partners Recycling Programs under which customers who return their used computers and peripherals can obtain a monetary reward (Hewlett-Packard 2015). Similarly, IKEA launches a pilot program to buy back used furniture from customers (Ikea 2017). When the collection reward for the used product is subject to a replacement purchase, the policy is called a conditional collection policy, also referred to as a trade-in policy. In practice, some firms grant a voucher/rebate that can only be redeemed for further purchases of selected items. For instance, Apple offers both in-store and online upgrade programs which encourage customers to exchange used iPhones, iPads, and Macs for rebates that can be used to purchase new Apple products of the same category (Apple 2015). Similarly, other electronics producers, such as Dell and Fujitsu, also allow customers to trade in their old products for new ones at discounted prices (Dell 2013, Fujitsu 2015). Although collection policies with different constraints exist, it is not well understood how these policies affect the customer purchase decisions and why firms choose different collection policies in practice.

Moreover, owing to the fast-growing information technologies, an increasing number of customers start to buy and resell used products online. For example, some of the best-known online platforms such as Craigslist, eBay, and Facebook Marketplace, also serve as peer-to-peer marketplaces for used goods. The presence of a secondary market expands the choices available to both existing customers who own used products and new customers. Therefore, it becomes more challenging to manage the collection of used products. When designing its collection policy, the firm must take into account the independent secondary market as another channel for used product transactions.

Despite various applications in business practice, the impact of the above-mentioned two collection policies on customers' purchase behavior and the manufacturer's profit with and without the secondary market, to the best of our knowledge, has rarely been studied thoroughly. In addition, the existing research concentrates mainly on mature products with stable demands, leaving open the question of designing the optimal collection policy with the introduction of generation products. For instance, technology conglomerates recently rushed into the wearable device market and developed a variety of electronic gadgets (smart watches, VR headsets, etc.). As a result, a growing size of customers with substantial valuation uncertainty join the market for these newly launched products. Consequently, directly following the established policies designed for the mature markets may lead to substantial profit loss in new product introductions.

Motivated by challenges faced by a manufacturer who sells generation products, we develop an analytical model that incorporates key features of real-world markets for generation products: (i) the subsequent introductions of the base and upgraded versions of a generation product, (ii) the coexistence of the manufacturer's product collection channel and the customers' reselling channel for used products, (iii) strategic customers with uncertain valuations prior to their purchases arriving in each period. We focus on a monopoly manufacturer's choice of the unconditional and conditional collection policies for used products to maximize its total profit. In particular, we address the following main research questions: How will customers respond strategically to the unconditional (buyback) and conditional (trade-in) collection policies? How should the manufacturer determine the optimal collection policy when selling generation products? How does the existence of a secondary market affect the manufacturer's choice of the collection policy as well as its profit? How do the two collection policies perform in terms of the environmental impact with and without the secondary market?

By analyzing the customers' decisions regarding the used and the upgraded products, we find that the trade-in policy equips the manufacturer with the discriminating capability to target only high-valuation customers for returns and replacement purchases. Specifically, as the trade-in policy couples product collection with a replacement purchase, the manufacturer collects used products solely from customers who have higher valuations and will purchase the upgraded product after returning the used one. By contrast, the buyback policy allows the manufacturer to collect used products from customers regardless of whether they will further purchase the upgraded products. This implies that the manufacturer can also attract the low-valuation customers to return purely for a refund under the buyback policy.

The characterization of customer decisions paves the way for the design of the optimal collection policy. Our analysis shows that, the discriminating capability of the trade-in policy does not necessarily benefit the manufacturer, and in fact, the policy choice depends on the durability of the base product and the residual value the manufacturer can obtain from the returned used product. Specifically, the trade-in policy is optimal when the base product is more durable and its residual value is intermediate. In this case, it is more profitable for the manufacturer to target the high-valuation customers and induce replacement purchases from them. When either the base product is less durable or its residual value is high, it becomes relatively more profitable to collect used products than to induce replacement purchases only from high-valuation customers, and thus the buyback policy outperforms as it collects used products from both high- and low-valuation customers.

However, with the presence of the secondary market, the trade-in policy never outperforms the buyback policy. Note that the secondary market serves as an alternative channel for the repeat customers to offload their used products. Meanwhile it also provides the new customers an additional purchase option, which, in turn, cannibalizes the demand for the upgraded products. When the residual value of the used product is higher, the manufacturer can use a higher buyback reward to restrict the supply of used products to the secondary market, thereby mitigating cannibalization of the upgraded product sales. But this mechanism is less effective under the trade-in policy because it cannot prevent the repeat customers with low valuations from reselling used products to the secondary market, leading to severe demand cannibalization for the upgraded products.

One may expect that the existence of a secondary market would result in profit loss to the manufacturer because it competes with the manufacturer for used product collection and upgraded product sales. Despite this negative impact, the secondary market grants customers a higher future utility for an immediate purchase in the first period as they anticipate a better reselling opportunity in the future and thus are willing to pay more for the base product at the beginning. We find that the positive effect of price enhancement in the first period becomes more prominent than the negative effect of demand cannibalization in the second period when customers are more strategic. As a result, the manufacturer may benefit from the existence of a secondary market when customers are highly strategic. We also study the potential benefit of the collection policy commitment, under which the manufacturer announces the used product collection policy upon the base product introduction, and it can credibly commit to a pre-announced collection policy when the upgraded product is launched. We find that the optimal collection policy under policy commitment is qualitatively the same as the base model where the collection policy is determined when the upgraded product is released. It is always weakly better off for the manufacturer to commit to the collection policy upfront, preventing it from choosing a policy that myopically maximizes the profit in the second period.

From the environmental perspective, the performance of the two collection policies boils down to an investigation of the upgraded product sales and the used product collection. Although a higher residual value induces more used product collection which in term benefits the environment, it may also increase the production quantity of the upgraded product and thus harm the environment. Such effect is more pronounced under the trade-in policy as additional production of new products is always accompanied by product returns. Our results call for a comprehensive consideration of the environmental impacts when designing the used product collection policies.

The rest of this paper is organized as follows. In Section 2, we discuss the current literature and the contributions of our work. Section 3 provides the conceptualization and formulation of the model. The optimal collection policy without a secondary market is analyzed in Section 4 and that with the presence of a secondary market in Section 5. In Section 6, we discuss the impact of the secondary market on the choice of collection policy and the manufacturer's total profit. The benefit of collection policy commitment is studied in Section 7, and the environmental performances of the two collection policies are examined in Section 8. We summarize the main results and discuss future works in Section 9. All the proofs are provided in Appendix.

2 Literature Review

The industrial practice of used product collection is driven mainly by the increasing benefit of product remanufacturing and recycling. There is a rapidly growing stream of literature considering decisions relating to remanufactured products and their interactions with the new product sales. A comprehensive review is provided by Guide Jr and Van Wassenhove (2009). Note that most prior research adopts simplified assumptions about the collection of used products, modeling the collection rate to be either an exogenous parameter or a reduced-form function of the level of effort chosen by the firm. However, when collecting used durable products, the firm should provide substantial monetary incentive as the used product is still valuable to the customer (Zhang and Zhang 2018, Alev et al. 2020). Focusing on the durable generation products, our research explicitly evaluates and compares the performance of both the conditional (trade-in) and unconditional (buyback) collection policies.

Trade-in programs have been widely studied as a price discrimination device (e.g., van Ackere and Reyniers 1995, Ray et al. 2005). With regard to the generation product setting, recent papers discuss how an optimal trade-in program is affected by different market specific features such as user upgrade costs (Bala and Carr 2009), up-front fees (Yin and Tang 2014), upgrade uncertainties (Yin et al. 2015), and upgraded product returns (Cao and Choi 2022). Xiao and Zhou (2020) further consider the firm selling the refurbished used product and show that the optimal trade-in depends on its inventory level. In comparison, we uncover that the manufacturer may, conversely, suffer from the discriminating power embedded in the trade-in policy.

With secondary markets, the coexistence of new and used goods makes these products imperfect substitutes, raising concerns about the cannibalization problem that reduces firms' new product sales and profits. Using data from the automobile industry, Chen et al. (2013) show a drop of 35% in retailers' profitability caused by secondary trading of used goods. Levinthal and Purohit (1989) suggest to use buyback programs deal with the cannibalization problem caused by the secondary market. Fudenberg and Tirole (1998) study the monopoly pricing of overlapping generations of a durable good with and without a secondary market. Rao et al. (2009) consider the trade-in program as the firm's intervention on the secondary market to resolve the adverse selection problem. In this stream of literature on durable goods, most studies focus on the mature market. However, we offer complement insights on the introduction of new series of generation products, where the demand side is usually characterized by market growth and valuation uncertainty.

The impact of secondary markets on operational issues have also been studied in the literature, but under different settings. For instance, Yin et al. (2010) consider the retailer controlled platform. Gümüş et al. (2013) explore the use of the channel return to coordinate the supply chain. Agrawal et al. (2015) examine the exclusivity-seeking customer behavior. Jiang et al. (2017) study the profit-maximizing marketplace's optimal decision of transaction fees. Vedantam et al. (2021) compare profitability and environmental impacts of the two strategies: when the firm offers a trade-in program and when the firm establishes a resale marketplace. Our work investigates both the buyback and trade-in policies, and highlights how the existence of the secondary market affects the choice of the used products collection policy.

Our paper is also closely related to the studies of strategic customer behavior. Strategic customers are forward-looking and strategize over the timing of their purchases to maximize their individual utilities. Following the seminal paper by Besanko and Winston (1990), several works in operations management examine the implications of strategic customer behavior from various aspects. Examples include decisions on pricing and inventory (Aviv and Pazgal 2008), capacity rationing (Liu and Van Ryzin 2008), channel coordination (Su and Zhang 2008, Kabul and Parlaktürk 2019), inventory display formats (Yin et al. 2009), product rollovers (Liang et al. 2014), product variety choices (Parlaktürk 2012), and customer endogenous strategic level (Aflaki et al. 2020). We contribute to this stream of literature by investigating how strategic customer behavior moderates the impact of an independent secondary market on the manufacturer's profit.

3 The Model

We consider a profit-maximizing manufacturer that launches a base product and its upgraded version to the market over a two-period selling horizon. In this section, we introduce the model setup regarding the generation product, customer preferences and decisions, and the collection program of used products.

3.1 Generation Product

The manufacturer produces the base product at a unit cost c_1 and sells it at a price p_1 in the first period. The quality level of the base product is normalized to 1 without loss of generality. After being used for one period, the quality of the base product deteriorates to $1-\beta$ in the second period, where $\beta \in [0,1)$ represents the physical and economic deterioration of the product. A small β implies that the product is more durable such that its quality remains high in the second period; as β approaches 1, the product becomes worthless in the second period.

The manufacturer offers an upgraded version of the product at a price p_2 in the second period. When an upgraded version is introduced, the base product is withdrawn from sale (e.g., Lobel et al. 2015). The sequential introduction of generation products is widely employed in businesses such as consumer electronics, home appliances, and furniture. The upgraded product has a quality level $1+\alpha$, where $\alpha \geq 0$ captures the innovation level of the upgraded product relative to the base product. For example, α may represent the number of new functions or improved features introduced in the upgraded product. The unit production cost of the upgraded product is c_2. Note that there is no restriction on the relative value between c_1 and c_2. This is because the upgraded product may incur an additional production cost compared to the base product due to newly added functions; it may also experience a reduction in the production cost due to the production learning effect.

3.2 Customers

When a new version of a generation product is released to the market, it usually draws attention from customers who do not have this product and enlarges the total market size of the manufacturer. To capture this distinguishing feature, we assume that a new cohort of unit-mass customers enter the market in each period. Customers, each owning at most one unit of product at a time, are heterogeneous in their valuations for product quality. Specifically, customers' valuation for product quality, denoted by V, is uniformly distributed on $[0,1]$. Prior to arrival, each customer knows only the common prior distribution of the valuation V but not its exact realization (e.g., Su 2009). This assumption captures customers' uncertainty about whether the product fits with their private tastes, especially when products are newly released to the market. Customers know their own valuations only by consuming the product. In particular, the customers who arrive in the first period are ex ante homogeneous when deciding whether to purchase the base product, but they are ex post heterogeneous after purchasing the base product. In the second period, there are two groups of customers in the market. One group consists of those customers who have already purchased the base product and know their valuations of the product, referred to as *repeat customers*. Specifically, if a customer with the valuation V purchases the base product in the first period, her valuation after using it for one period is $(1-\beta)V$, and the valuation if she purchases the upgraded product is $(1+\alpha)V$. The other group consists of *new customers* who arrive in the second period and do not know their exact valuations of the product.

We assume that there is an independent peer-to-peer platform that allows customers to trade used products with each other anonymously and frictionlessly (e.g., Desai and Purohit 1998). It is referred to as the secondary market, which operates at an endogenously determined market-clearing price, denoted by s, at which the demand for used products matches the supply.

Customers are forward looking and maximize their net present utilities. We use δ to denote the customers' discount rate for future utilities to measure their strategic level. A large (small) δ implies that customers value more (less) about future gains, thus being more (less) strategic. The manufacturer's discount rate is normalized to one.

3.3 Used Product Collection

The manufacturer can generate revenue by extracting materials and components from used products. Specifically, let r denote the residual value of the used product. We assume that $r < c_i$, $i = 1,2$. This assumption is reasonable when used products collected by the manufacturer are decomposed for residual value exploitation.

In order to collect used products from customers, the manufacturer can adopt different policies depending on whether the reward for returning the used product is associated with a further purchase. Specifically, the manufacturer can implement (i) an *unconditional* collection policy (referred to as the buyback policy below), where a reward w is offered to customers for each unit of the used product returned, or (ii) a *conditional* collection policy (referred to as the trade-in policy below), where the return reward w can be redeemed only at the replacement purchase of the upgraded product. We use $y \in \{B, T\}$ to denote the manufacturer's choice of the buyback or the trade-in policy, respectively.

The sequence of events starts in the first period with the manufacturer setting the base product price, p_1. A cohort of new customers arrive and choose whether to buy the base product immediately or to wait until the second period. Specifically, these uninformed customers will purchase the base product if and only if the selling price does not exceed their reservation price, denoted by z. At the beginning of the second period, a new cohort of customers enters the market. The manufacturer determines the upgraded product price p_2, as well as the collection policy y and the corresponding reward w. The customers who have purchased the based product and known their valuations are also potential buyers in the second period. The price of used products in the secondary market is determined competitively through a market-clearing mechanism, matching the total demand with the total supply.

To analyze customers' purchase decisions in the first period with uncertain valuations, we adopt the rational expectation equilibrium framework to characterize the market outcome in the first period (e.g., Su and Zhang 2008, Cachon and Swinney 2011). Note that customers who buy the base product in the first period will have the opportunity to return or resell the used product and make a replacement purchase in the second period. However, when they make purchases, they do not know the collection policy, denoted by(y, w), to be implemented in the second period by the manufacturer, nor the selling price of the upgraded product p_2. Instead, they may possess some beliefs about the manufacturer's future decisions. In particular, the customer's beliefs are captured by the triplet (ξ_y, ξ_p, ξ_w), where $\xi_y \in \{B, T\}$ is the customers' belief on the collection policy to be announced in the second period, and ξ_p and ξ_w are the beliefs on the selling price p_2 and the collection reward w, respectively. In this case, the customers' reservation price, which takes their expected utility over two periods into account, is hence a function of these beliefs, denoted by $z^* = z(\xi_y, \xi_p, \xi_w)$. On the other hand, the manufacturer sets the base product price according to its belief about customers' purchase behavior. Specifically, we let ξ_z be the manufacturer's belief about customers' reservation price z. The manufacturer chooses the price which equals the customer's reservation price, namely $p_1^* = \xi_z$, the highest price at which customers make purchases in the first period. Now we provide the definition of a rational expectation equilibrium as follows.

DEFINITION 1. A rational expectation equilibrium consists of $(p_1^*, z^*, y^*, p_2^*, w^*, \xi_z, \xi_y, \xi_p, \xi_w)$ that satisfies (i) $p_1^* = \xi_z$, (ii) $z^* = z(\xi_y, \xi_p, \xi_w)$, (iii) $(\xi_y, \xi_p, \xi_w) = (y^*, p_2^*, w^*)$, and (iv) $\xi_z = z^*$.

The four conditions in Definition 1 state that the manufacturer and customers make their respective decisions based on their beliefs about the other party's decisions, and that the beliefs are consistent with the respective optimal decisions in equilibrium.

We start with the base model in which the collection policy is determined along with the introduction of the upgraded product in the second period. We also verify the case when the manufacturer announces the collection policy in the first period and is able to commit credibly to the pre-announced policy in the second period, which is discussed in Section 7. To ensure that the manufacturer always finds it optimal to induce new customers who arrive in the second period to purchase the upgraded product, we assume $c_2 \leq (1+\alpha)/4$ throughout the paper. Nevertheless, we can still fully analyze the manufacturer's optimal pricing decision and collection policy without this assumption; see Appendix EC.1 for details.

4 Product Collection without a Secondary Market

In order to study the impact of the secondary market on the used product collection program, we start with the benchmark case of no secondary market. We solve the manufacturer's problem following the backward induction. That is, we first study the second-period problem and derive the manufacturer's optimal choice of the collection policy in Section 4.1. The manufacturer's base product price under the rational expectation framework is then solved in Section 4.2.

4.1 Second-period Problem

In this part, the two collection policies, i.e., buyback and trade-in, are analyzed separately first. Then the optimal choice of the collection policy is the one that yields a higher profit.

4.1.1 Buyback Policy

Suppose the manufacturer adopts the buyback policy in the second period. Given the posted price p_2 of the upgraded product and the collection reward w for returning the used product, repeat customers make decisions based on their realized valuations while new customers make decisions based on their expected valuations. The maximal utility gained in the second period by a repeat customer, denoted by U_r^B, is given in the following:

$$U_r^B = \max\{(1-\beta)V,\ w,\ (1+\alpha)V - p_2 + w\}.$$

A repeat customer can choose either to keep consuming the used base product on hand and gain the utility $(1-\beta)V$, or to return it to the manufacturer and obtain the reward w, or to replace the used product by the upgraded one and gain the utility $(1+\alpha)V - p_2 + w$. On the other hand, the maximal utility gained by a new customer, denoted by U_n^B, is the following:

$$U_n^B = \max\{(1+\alpha)\mathbb{E}[V] - p_2,\ 0\},$$

where $\mathbb{E}[V] = 1/2$. Specifically, each new customer chooses whether to purchase the upgraded product or not. Note that all the new customers are ex ante homogeneous and share the same belief on their valuation. We focus on the pure-strategy equilibrium, in which new customers uniformly purchase the upgraded product if $p_2 \leq (1+\alpha)/2$.

Given the optimal responses of both new and repeat customers, the manufacturer's profit in the second period, denoted by π_2^B, is formulated as

$$\pi_2^B(p_2, w) = (p_2 - c_2)D_n^B + (p_2 - w - c_2 + r)D_r^B + (r - w)R_r^B, \tag{1}$$

where $D_n^B = \mathbb{1}_{p_2 \le \frac{1}{2}(1+\alpha)}$ represents the new customers' demand for the upgraded product, $D_r^B = (1 - \max\{\frac{p_2 - w}{\alpha+\beta}, \frac{p_2}{1+\alpha}\})^+$ is the replacement purchase demand where $x^+ = \max\{x, 0\}$, and $R_r^B = \min\{\frac{w}{1-\beta}, \frac{p_2}{1+\alpha}\}$ is the amount of purely returned used products without any replacement purchase.

The manufacturer's optimal selling price of the upgraded product and buyback reward in the second period are characterized in the following lemma. The formulations of the thresholds used in the lemmas and propositions throughout the paper are listed in Appendix EC.3.

LEMMA 1. *In the absence of a secondary market, the manufacturer's optimal selling price and collection reward under the buyback policy, (p_2^B, w^B), are given as follows.*

(i) When $1 - \beta < c_2$,

$$(p_2^B, w^B) = \begin{cases} \left(\frac{1+\alpha}{2}, 0\right), & \text{if } r \le \underline{r}^b \\ \left(\frac{1+\alpha}{2}, \frac{r}{2} + \frac{(1-\beta)(1-\beta-c_2)}{2(1+\alpha)}\right), & \text{if } \underline{r}^b < r \le \bar{r}^b \\ \left(\frac{1+\alpha}{2}, \frac{1-\beta}{2}\right), & \text{otherwise.} \end{cases}$$

(ii) When $1 - \beta \ge c_2$,

$$(p_2^B, w^B) = \begin{cases} \left(\frac{1+\alpha}{2}, \frac{r}{2}\right), & \text{if } r \le \tilde{r}^b \\ \left(\frac{1+\alpha}{2}, \frac{r}{2} + \frac{(1-\beta)(1-\beta-c_2)}{2(1+\alpha)}\right), & \text{otherwise.} \end{cases}$$

Lemma 1 essentially shows that the optimal collection reward under the buyback policy depends on the economic values of the used products for the manufacturer and customers. As expected, the manufacturer, in general, offers a higher buyback reward, collects more used products and induces more replacement purchases as the residual value of the used product increases. Furthermore, as the product becomes more durable, the manufacturer is pushed to offer a higher refund to induce used product returns and replacement purchases.

4.1.2 Trade-in Policy

Given the posted selling price for the upgraded product p_2 and the trade-in reward w in the second period, a repeat customer's maximal utility gained in the second period under the trade-in policy, denoted by U_r^T, is as follows:

$$U_r^T = \max\{(1-\beta)V,\ (1+\alpha)V - p_2 + w\}.$$

A repeat customer can either keep consuming the used product on hand and gain the utility $(1-\beta)V$, or trade it in for an upgraded product and gain the utility $(1+\alpha)V - p_2 + w$. For new customers, they choose to purchase the upgraded products only when $p_2 \le (1+\alpha)/2$ and do not make any purchase otherwise. The maximal utility gained by a new customer is given by $U_n^T = \max\{(1+\alpha)\mathbb{E}[V] - p_2, 0\}$. Hence, the profit of the manufacturer in the second period is

$$\pi_2^T(p_2, w) = (p_2 - c_2)D_n^T + (p_2 - w - c_2 + r)D_r^T, \tag{2}$$

where $D_n^T = \mathbb{1}_{p_2 \leq \frac{1}{2}(1+\alpha)}$ is the demand for the upgraded product from new customers, and $D_r^T = (1 - \frac{p_2 - w}{\alpha+\beta})^+$ represents the demand of the replacement purchase. The manufacturer's optimal selling price and trade-in reward are characterized in the following lemma.

LEMMA 2. *In the absence of a secondary market, the manufacturer's optimal selling price and collection reward under the trade-in policy, are* $(p_2^T, w^T) = \left(\frac{1+\alpha}{2}, \frac{1-\beta-c_2+r}{2}\right)$ *if* $r \geq \max\{c_2 - \alpha - \beta, c_2 - 1 + \beta\}$*; otherwise,* $(p_2^T, w^T) = \left(\frac{1+\alpha}{2}, 0\right)$.

Similar to Lemma 1, Lemma 2 shows that the manufacturer offers a higher trade-in reward, and thereby induces more replacement purchases as the residual value of used products increases.

4.1.3 Comparison and Optimal Policy Choice

Having outlined the optimal selling prices and collection rewards under both buyback and trade-in policies, we now turn our attention to the optimal policy choice $y \in \{B, T\}$ in the second period. To evaluate the performances of the buyback and the trade-in policies, we first compare the optimal collection rewards and collection amounts under the two policies, which are summarized in the following lemma.

LEMMA 3. *In the absence of a secondary market, the manufacturer offers a higher reward under the trade-in policy if* $1 - \beta \geq c_2$ *and* $r \geq c_2 - \alpha - \beta$*; furthermore, it collects more used products under the trade-in policy if* $1 - \beta \geq c_2$ *and* $\tilde{r} \leq r \leq \tilde{r}^b$.

When the base product is more durable ($1 - \beta \geq c_2$) or, equivalently, the production cost of the upgraded product is low, the replacement purchase brings a higher profit margin. In this case, with the discriminating power of the trade-in policy, the manufacturer would offer a higher reward to precisely target the high-end customers and induce more replacement purchases as long as the residual value of the product is not small, i.e., $r \geq c_2 - \alpha - \beta$. Otherwise, the collection reward under the trade-in policy will be lower.

For the collection amounts, when the base product is less durable, the manufacturer always collects more used products under the buyback policy since it always offers a higher collection reward. But when the product is less durable, the comparison of the collection amounts depends further on the residual value. When the residual value is low, it is too costly to induce replacement purchases with a high reward but still profitable to accept pure returns. In this case, the buyback policy collects some used products but the trade-in policy collects nothing. When the residual value is intermediate, i.e., $\tilde{r} \leq r \leq \tilde{r}^b$, the manufacturer targets the high-end customers under the trade-in policy while still only induces the low-end customers to return under the buyback policy. Thus the manufacturer collects more used products under the trade-in policy since more replacement purchases from the high-end customers are prompted by the higher reward under the trade-in policy. When the residual value is even higher, the manufacturer collects more used products under the buyback policy as it collects from both high-end and low-end customers.

By comparing the manufacturer's profits under the two collection policies, the optimal collection policy is determined in the following proposition.

PROPOSITION 1. *In the absence of a secondary market, the manufacturer's optimal decisions* (y^*, p_2^*, w^*) *in the second period are given as follows.*

(i) When $1-\beta < c_2$*,*

(a) $(y^*, p_2^*, w^*) = (B \text{ or } T, \frac{1+\alpha}{2}, 0)$ *if* $r \le \underline{r}^b$*;*

(b) $(y^*, p_2^*, w^*) = (B, \frac{1+\alpha}{2}, \frac{r}{2} + \frac{(1-\beta)(1-\beta-c_2)}{2(1+\alpha)})$ *if* $\underline{r}^b < r \le \bar{r}^b$*;*

(c) *and* $(y^*, p_2^*, w^*) = (B, \frac{1+\alpha}{2}, \frac{1-\beta}{2})$ *otherwise.*

(ii) When $1-\beta \ge c_2$*,*

(a) $(y^*, p_2^*, w^*) = (B, \frac{1+\alpha}{2}, \frac{r}{2})$ *if* $r \le \underline{r}$*;*

(b) $(y^*, p_2^*, w^*) = (T, \frac{1+\alpha}{2}, \frac{1-\beta-c_2+r}{2})$ *if* $\underline{r} < r < \hat{r}$*;*

(c) *and* $(y^*, p_2^*, w^*) = (B, \frac{1+\alpha}{2}, \frac{r}{2} + \frac{(1-\beta)(1-\beta-c_2)}{2(1+\alpha)})$ *otherwise.*

As expected, when the residual value of used products is sufficiently high, the buyback policy will yield a higher profit as it can collect more used products. But the case with a lower residual value is more subtle. Specifically, when the product is less durable ($1-\beta < c_2$), inducing replacement purchases is less profitable and the manufacturer cares more about collecting used products. Hence, the trade-in policy can never outperform the buyback policy. Conversely, when the base product is more durable ($1-\beta > c_2$), the replacement purchase has a higher profit margin. Thus, the trade-in policy allows the manufacturer to selectively target the replacement purchases, yielding a higher profit when the product has moderate residual value. When the residual value is sufficiently low, it is no longer profitable to induce replacement purchases under the trade-in policy. However, the buyback policy enables the manufacturer to accept pure returns from the low-end customers, and hence the manufacturer adopts the buyback policy.

In general, as the base product becomes more durable, it is optimal to choose the buyback policy first, then the trade-in policy, and finally the buyback policy again. The results uncover the bright and dark sides of the discriminating capability on the manufacturer's design of the collection policy. To induce product returns, the collection reward must compensate the customers' consumption utility of the used product. Hence, the manufacturer focuses more on boosting the replacement purchase demand than collecting used products when it becomes more costly to collect the more durable used products. In this case, the conditional trade-in policy outperforms by selectively targeting the high-end repeat customers. However, when the product is extremely durable, it is no longer profitable to induce any replacement purchase. Hence, the unconditional buyback policy allows the manufacturer to effectively collect used products from the low-end customers, surpassing the trade-in policy.

4.2 First-period Problem

Returning to the first period, we analyze the customers' optimal purchase decision and the manufacturer's optimal price for the base product. The strategic customers who arrive in the first period take into account the potential return and replacement purchase opportunities in the second period when making the initial purchase decision.

Specifically, if customers choose to buy the new product in the first period and believe a buyback policy, (ξ_p, ξ_w), to be implemented, their expected utility gained in the second period is

$$\mathbb{E}[U_r^B(\xi_p,\xi_w)] = \xi_w \frac{\xi_w}{1-\beta} + \int_{\frac{\xi_w}{1-\beta}}^{\min\{\frac{\xi_p-\xi_w}{\alpha+\beta},1\}} (1-\beta)v dv + \int_{\min\{\frac{\xi_p-\xi_w}{\alpha+\beta},1\}}^{1} [(1+\alpha)v - \xi_p + \xi_w] dv.$$

If customers believe a trade-in policy to be implemented, the corresponding expected utility is

$$\mathbb{E}[U_r^T(\xi_p,\xi_w)] = \int_0^{\frac{\xi_p-\xi_w}{\alpha+\beta}} (1-\beta)v dv + \int_{\frac{\xi_p-\xi_w}{\alpha+\beta}}^{1} [(1+\alpha)v - \xi_p + \xi_w] dv.$$

Hence, we can write the customers' expected utility gained in the second period if a purchase is made in the first period as follows:

$$\mathbb{E}[U_r(\xi_y,\xi_p,\xi_w)] = \begin{cases} \mathbb{E}[U_r^B(\xi_p,\xi_w)], & \text{if } \xi_y = B \\ \mathbb{E}[U_r^T(\xi_p,\xi_w)], & \text{if } \xi_y = T. \end{cases}$$

On the other hand, if the customers choose to wait until the second period, they will remain uninformed about the valuation and consider whether to purchase the upgraded product based on the prior belief. The manufacturer's decision problem in the second period then degenerates to the case of selling a single upgraded product to ex ante homogeneous customers, leading to the optimal price for the upgraded product equal to $(1+\alpha)/2$ and the expected utility $\mathbb{E}[U_n] = 0$. Customers make purchases in the first period only when the corresponding utility is higher than the delayed purchase, which is $\mathbb{E}[V] + \delta\mathbb{E}[U_r(\xi_y,\xi_p,\xi_w)] - p_1 \geq \mathbb{E}[U_n]$. Hence, the customers' reservation price in the first period is $z^* = z(\xi_y,\xi_p,\xi_w) = \mathbb{E}[V] + \delta\mathbb{E}[U_r(\xi_y,\xi_p,\xi_w)]$. According to Definition 1, the manufacturer's first period price under the rational expectation equilibrium is concluded in the following proposition.

PROPOSITION 2. *In the absence of a secondary market, the manufacturer's optimal price for the base product is $p_1^* = \mathbb{E}[V] + \delta\mathbb{E}[U_r(y^*, p_2^*, w^*)]$, under which all the customers will buy the base product immediately in the first period.*

Following Proposition 2, the manufacturer's total profit is $\Pi^* = p_1^* - c_1 + \pi_2^*(y^*, p_2^*, w^*)$ in the absence of a secondary market. After characterizing the equilibrium outcomes, we can discuss how the manufacturer's profit changes with system parameters. First, it is not surprising to find that the manufacturer's profit increases with innovation level α and residual value r. Indeed, all else being equal, a higher innovation level allows the manufacturer to charge a higher price for the upgraded product, and a higher residual value implies that a higher margin can be obtained from used product collection. However, we find that the impact of the product deterioration rate β on the manufacturer's profit exhibits a non-monotone pattern numerically. Figure 1 provides a numerical example with the parameter setting $\alpha = 0.1, c_1 = 0.2, c_2 = 0.25, r = 0.1$ and $\delta = 0.9$.

Figure 1 The Impact of the Product Deterioration Rate on the Manufacturer's Profit without a Secondary Market

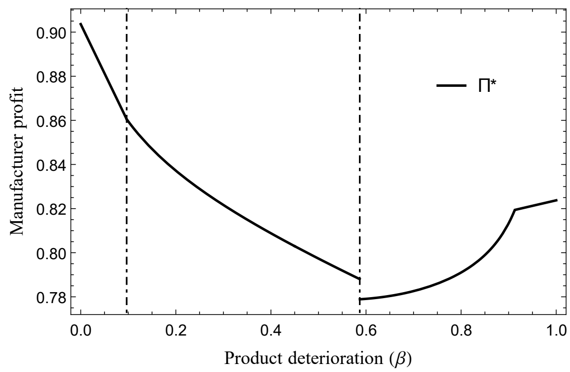


Figure 1 shows that the manufacturer's profit first decreases and then increases with β. This is because the product deterioration rate affects the manufacturer's profit in two ways. On the one hand, a higher deterioration rate (lower durability) enables the manufacturer to collect the used product back at a lower cost, which results in a higher profit in the second period. On the other hand, the customers expect the less durable product to be less valuable in the future, and hence are willing to pay less in the first period. The overall impact of the deterioration rate depends on these two competing effects. When the product deterioration rate is extremely low, the manufacturer can only collect used products from the low-end repeat customers since it is too costly to offer a high collection reward to induce replacement purchases from the high-end customers. In this case, a more deteriorated (less durable) base product brings down the manufacturer's total profit because the cost saving from offering a low collection reward to the low-end repeat customers in the second period cannot compensate for the reduced reservation price of the customers in the first period. Conversely, when the product deterioration rate is extremely high, the manufacturer would collect all the used products from the repeat customers such that no customer keeps the product. In this case, the customers' utility is not directly related to the deterioration rate as they do not keep using the products. Hence, the benefit of reducing the collection cost in the second period outweighs and the manufacturer's profit increases. In general, the collection reduction effect gradually surpasses the reservation price reduction effect as the deterioration rate increases. Therefore, the manufacturer's profit first decreases and then increases in the deterioration rate.

5 Product Collection with a Secondary Market

Now consider the case when there is an active secondary market that allows the repeat customers to resell used base products to the new customers in the second period, denoted by the superscript S. We investigate customers' behavior in the presence of a secondary market, and examine its impact on the optimal collection policy as well as the manufacturer's profit.

5.1 Second-period Problem

5.1.1 Buyback Policy

Compared to the case without a secondary market, now each repeat customer has an extra choice to resell the used product to the secondary market at the market-clearing price s, at which the supply of used products (resold by the repeat customers) is equal to the demand of used products (required by new customers in the second period). Then the maximal utility gained by a repeat customer in the second period, denoted by U_r^{SB}, is given by

$$U_r^{SB} = \max\{(1-\beta)V,\ s,\ (1+\alpha)V - p_2 + s,\ w,\ (1+\alpha)V - p_2 + w\}.$$

Each new customer also has an additional option to buy the used product from the secondary market, resulting in the following maximal utility in the second period,

$$U_n^{SB} = \max\{(1+\alpha)\mathbb{E}[V] - p_2,\ (1-\beta)\mathbb{E}[V] - s,\ 0\}.$$

The manufacturer competes directly with the secondary market for used products. The repeat customers prefer to return the used product when $w \geq s$ and to resell it otherwise. Here, we assume that the repeat customers return used products to the manufacturer at $w = s$ without loss of generality. Specifically, when $w \geq s$, the secondary market vanishes as the repeat customers unanimously prefer to return the used products to the manufacturer rather than to resell on the secondary market. The problem reduces to the case with no secondary market, and the expected profit of the manufacturer in the second period is given by

$$\pi_2^{SB}(p_2, w) = (p_2 - c_2)D_n^{SB} + (p_2 - w - c_2 + r)D_r^{SB} + (r - w)R_r^{SB}, \tag{3}$$

where $D_n^{SB} = \mathbb{1}_{p_2 \leq \frac{1}{2}(1+\alpha)}$ represents the demand for the upgraded product from new customers, $D_r^{SB} = (1 - \max\{\frac{p_2 - w}{\alpha+\beta}, \frac{p_2}{1+\alpha}\})^+$ is the replacement purchase demand of the upgraded product, and $R_r^{SB} = \min\{\frac{w}{1-\beta}, \frac{p_2}{1+\alpha}\}$ is the amount of purely returned used products.

When $w < s$, the repeat customers never return their used products to the manufacturer and only consider reselling them to the secondary market. Hence, the expected profit of the manufacturer in the second period can be characterized as

$$\pi_2^{SB}(p_2, w) = (p_2 - c_2)D_n^{SB} + (p_2 - c_2)D_r^{SB}, \tag{4}$$

where $D_n^{SB} = (\frac{p_2-s}{\alpha+\beta} - \frac{s}{1-\beta})^+\mathbb{1}_{p_2 \le \frac{1}{2}(1+\alpha)}$ and $D_r^{SB} = (1 - \max\{\frac{p_2-s}{\alpha+\beta}, \frac{p_2}{1+\alpha}\})^+$ are the demands for the upgraded products from new and repeat customers, respectively. Note that in equilibrium, the new customers are indifferent between purchasing the upgraded product from the manufacturer and the used product from the secondary market. The amount of the new customers who purchase the used products on the secondary market must be equal to the amount of used products resold by the repeat customers such that the secondary market is cleared. The rest new customers who cannot get any used ones will buy upgraded products. In other words, the used products that are resold to the secondary market cannibalize new customers' demand for the upgraded product.

Combining the two cases discussed above, the optimal selling price and the buyback reward when there exists a secondary market are presented in the following lemma.

LEMMA 4. *With the presence of the secondary market, the manufacturer's optimal selling price and collection reward under the buyback policy, (p_2^{SB}, w^{SB}), are given as follows.*

(i) When $r < c_2 - \frac{\alpha+\beta}{2}$, $(p_2^{SB}, w^{SB}) = \left(\frac{1+\alpha}{2}, 0\right)$, where the secondary market prices at $s^{SB} = \frac{1-\beta}{2}$.

(ii) When $r \ge c_2 - \frac{\alpha+\beta}{2}$, $(p_2^{SB}, w^{SB}) = \left(\frac{1+\alpha}{2}, \frac{1-\beta}{2}\right)$, where the secondary market has no transaction.

The optimal buyback reward follows a simple threshold rule. When the residual value is low, i.e., $r < c_2 - (\alpha+\beta)/2$, it is too costly for the manufacturer to set a buyback reward that is higher than the resell price. In this case, the active secondary market allows potential used product resellers to trade with the new customers, while the high-end repeat customers will further purchase the upgraded products after resale. When the residual value is high, i.e., $r \ge c_2 - (\alpha+\beta)/2$, the manufacturer offers an attractive buyback reward such that all repeat customers choose to return used products back to the manufacturer. The secondary market vanishes consequently.

5.1.2 Trade-in Policy

Under a trade-in policy, a repeat customer can choose to resell the used product unconditionally while she can only return it to the manufacturer conditionally. So the maximal utility is given by

$$U_r^{ST} = \max\{(1-\beta)V,\ s,\ (1+\alpha)V - p_2 + s,\ (1+\alpha)V - p_2 + w\}.$$

A new customer decides whether to purchase the upgraded product from the manufacturer or the used product from the secondary market, and her maximal utility is

$$U_n^{ST} = \max\{(1+\alpha)\mathbb{E}[V] - p_2, (1-\beta)\mathbb{E}[V] - s,\ 0\}.$$

Similar to the arguments for the buyback policy, when $w \ge s$, the high-end repeat customers with a replacement purchase plan will choose the trade-in option, but the low-end customers will resell the used products to the secondary market. The expected profit of the manufacturer in the second period is characterized as

$$\pi_2^{ST}(p_2, w) = (p_2 - c_2)D_n^{ST} + (p_2 - w - c_2 + r)D_r^{ST}, \tag{5}$$

where $D_n^{ST} = (1 - \min\{\frac{s}{1-\beta}, \frac{p_2-w+s}{1+\alpha}\})^+ \mathbb{1}_{p_2 \le \frac{1}{2}(1+\alpha)}$ represents the demand for the upgraded products from the new customers, and $D_r^{ST} = (1 - \max\{\frac{p_2-w}{\alpha+\beta}, \frac{p_2-w+s}{1+\alpha}\})^+$ is the replacement purchase demand of the upgraded products from the repeat customers.

When $w < s$, the repeat customers prefer to resell the used products to the secondary market. The expected profit of the manufacturer in the second period is then given by

$$\pi_2^{ST}(p_2, w) = (p_2 - c_2)D_n^{ST} + (p_2 - c_2)D_r^{ST},$$

where $D_n^{ST} = (\frac{p_2-s}{\alpha+\beta} - \frac{s}{1-\beta})^+ \mathbb{1}_{p_2 \le \frac{1}{2}(1+\alpha)}$ represents the demand for the upgraded products from new customers, and $D_r^{ST} = (1 - \max\{\frac{p_2-s}{\alpha+\beta}, \frac{p_2}{1+\alpha}\})^+$ is the replacement purchase demand of the upgraded products from repeat customers.

Combining the two cases discussed above, the manufacturer's optimal price for the upgraded product and collection reward under the trade-in policy is established in Lemma 5 below.

LEMMA 5. *With the presence of a secondary market, the manufacturer's optimal selling price and collection reward under the trade-in policy, (p_2^{ST}, w^{ST}), are given as follows.*

(i) When $r < c_2 - \frac{\alpha+\beta}{2}$, $(p_2^{ST}, w^{ST}) = \left(\frac{1+\alpha}{2}, 0\right)$.

(ii) When $r \ge c_2 - \frac{\alpha+\beta}{2}$,

$$(p_2^{ST}, w^{ST}) = \begin{cases} \left(\frac{1+\alpha}{2}, \frac{1-\beta}{2}\right), & \text{if } c_2 > \frac{\alpha+\beta}{2} \\ \left(\frac{2+\alpha-\beta+2c_2}{4}, \frac{2-\alpha-3\beta+2c_2}{4}\right), & \text{if } \frac{\alpha+3\beta-2}{2} < c_2 \le \frac{\alpha+\beta}{2} \\ \left(\frac{\alpha+\beta}{2}, 0\right), & \text{otherwise.} \end{cases}$$

The secondary market has an equilibrium price $s^{ST} = p_2^{ST} - \frac{\alpha+\beta}{2}$.

In general, a similar threshold policy is adopted under the trade-in policy compared to the buyback policy. When the residual value is low, i.e., $r < c_2 - (\alpha+\beta)/2$, the manufacturer will not offer trade-in, which is the same as the buyback policy. When the residual value is relatively high, i.e., $r \ge c_2 - (\alpha+\beta)/2$, the manufacturer sets the trade-in reward equal to the resell price on the secondary market to collect used products from the high-end customers. However, the manufacturer cannot prevent the low-end repeat customers from reselling their used products to the new customers by simply offering a high collection reward under the conditional trade-in policy. In response to the demand cannibalization from the secondary market, the manufacturer has to reduce the selling price of the upgraded product to lower down the resale price, thereby diminishing the amount of resold products.

5.1.3 Comparison and Optimal Policy Choice

Now we examine the manufacturer's optimal choice of collection policy in the presence of a secondary market. The comparison of the optimal collection reward and collection amount under the two policies is summarized in the following lemma.

LEMMA 6. *With the presence of a secondary market, the manufacturer always offers a higher reward, and collects more used products under the buyback policy.*

With the existence of the secondary market, the manufacturer under the buyback policy always offers a higher reward and collects more used products. This is because the manufacturer can offer a sufficiently high reward to collect all the potentially resold products from the repeat customers and completely eliminate the demand cannibalization under the buyback policy, while this cannot happen under the trade-in policy.

The manufacturer's optimal choice of the collection policy and the corresponding upgraded product price and collection reward when the secondary market exists are established below.

PROPOSITION 3. *With the presence of a secondary market, the manufacturer's optimal decisions* $(y^{S*}, p_2^{S*}, w^{S*})$ *in the second period are given as follows.*

(i) $(y^{S*}, p_2^{S*}, w^{S*}) = (B \text{ or } T, \frac{1+\alpha}{2}, 0)$ *if* $r < c_2 - \frac{\alpha+\beta}{2}$*;*

(ii) and $(y^{S*}, p_2^{S*}, w^{S*}) = (B, \frac{1+\alpha}{2}, \frac{1-\beta}{2})$ *otherwise.*

The existence of the secondary market changes the manufacturer's preference between the two collection policies, making the trade-in policy weakly dominated. This is because the used product collection becomes more important for the manufacturer when facing the demand cannibalization from the secondary market. When the residual value is low, due to the high cost of collecting used products, a competitive collection reward is never offered under either policy and thus customers resell the used products on the secondary market. In this case, the two collection policies are equivalent effectively. On the contrary, when the residual value is high enough, the manufacturer prefers to choose the buyback policy. This is because the unconditional buyback reward allows the manufacturer to collect all the used products and completely eliminate the supply to the secondary market, whereas under the conditional trade-in policy, there exists a fraction of the low-end repeat customers who always choose to resell used products as they are not eligible to simply return used products without further purchases. Since the used products that are sold by the repeat customers cannibalize the demand for the upgraded products from the new customers, the manufacturer strictly prefers the buyback policy as long as the base product has a sufficient residual value.

5.2 First-period Problem

Returning to the first period, strategic customers in the first period anticipate the additional option of reselling the used products in the secondary market in the second period. Correspondingly, let $\hat{w} =$

$\max\{\xi_w, s^*(\xi_y, \xi_p, \xi_w)\}$ be the virtual collection reward, where $s^*(\xi_y, \xi_p, \xi_w)$ is the equilibrium resale price in the secondary market under customers' beliefs (ξ_y, ξ_p, ξ_w). Hence, $\hat{w}$ is the effective reward that repeat customers can get from either the manufacturer or the secondary market in exchange for the used product.

According to Proposition 3, a trade-in policy with a positive collection reward is never adopted in the second period with the existence of a secondary market. Thus the first-period customers only expect a buyback policy to be implemented in equilibrium. Their expected utility gained in the second period is, hence, given by

$$\mathbb{E}[U_r^S(\xi_y, \xi_p, \xi_w)] = \hat{w}\frac{\hat{w}}{1-\beta} + \int_{\frac{\hat{w}}{1-\beta}}^{\frac{\xi_p - \hat{w}}{\alpha+\beta}} (1-\beta)v dv + \int_{\frac{\xi_p - \hat{w}}{\alpha+\beta}}^{1} [(1+\alpha)v - \xi_p + \hat{w}] dv,$$

where $\xi_y = B$. On the other hand, if the customers who arrive in the first period all choose to wait, there will be no sales of the base product and thus no supply of the used product in the second period. Then, the secondary market vanishes. Moreover, these customers are still homogeneously uninformed about their own valuations just like the new customers who arrive in the second period. Using the same arguments as the case with no secondary market, the expected utility of waiting is zero, i.e., $\mathbb{E}[U_n] = 0$. Thus, the optimal reservation price in the first period is $z^{S*} = z(\xi_y, \xi_p, \xi_w) = \mathbb{E}[V] + \delta\mathbb{E}[U_r^S(\xi_y, \xi_p, \xi_w)]$.

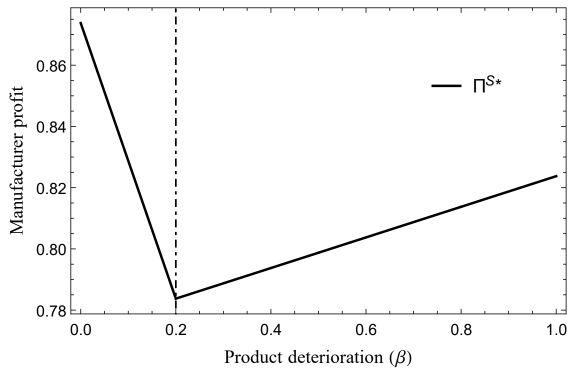
The manufacturer's first-period price in equilibrium with a secondary market is presented in the following proposition.

PROPOSITION 4. *With the presence of a secondary market, the manufacturer's optimal price for the base product is $p_1^{S*} = \mathbb{E}[V] + \delta\mathbb{E}[U_r^S(y^{S*}, p_1^{S*}, w^{S*})]$, at which all the customers buy the base product immediately in the first period.*

Immediately following the rational expectation equilibrium, the manufacturer's total profit with the presence of a secondary market is $\Pi^{S*} = p_1^{S*} - c_1 + \pi_2^{S*}(y^{S*}, p_2^{S*}, w^{S*})$. Consistent with the previous discussion about the case without a secondary market, the manufacturer's profit increases with the innovation level α and the residual value r. Regarding the impact of the product deterioration rate, we also find a similar non-monotone pattern, shown in Figure 2 below.

With the presence of a secondary market, the manufacturer either collects all the used products from the repeat customers or gives up offering any collection reward such that customers resell the used products via the secondary market. When the used product is more durable, the manufacturer forgoes the used product collection as it is too costly to match the market-clearing resale price. Thus a higher deterioration rate brings down the resale price and the customers' first-period reservation price, which, in turn, harms the manufacturer's total profit. When the used product is less durable, the manufacturer collects all the used products from the repeat customers through the buyback policy. A higher deterioration rate reduces the manufacturer's collection cost and thereby increases its profit. Therefore, the manufacturer's profit first decreases and then increases with the deterioration rate β.

Figure 2 The Impact of the Product Deterioration Rate on the Manufacturer's Profit with a Secondary Market



6 Impact of the Secondary Market

In this section, we investigate the impact of the secondary market on the manufacturer's optimal decisions and total profit. The next proposition shows how the existence of the secondary market changes the manufacturer's pricing decisions under the optimal collection policy.

PROPOSITION 5. *The presence of a secondary market makes the manufacturer inclined to implement the buyback policy. Its impacts on the manufacturer's pricing decisions are as follows.*

(i) The collection reward: $w^{S*} < w^*$ *if* $r < c_2 - \frac{\alpha+\beta}{2}$ *and* $w^{S*} \geq w^*$ *otherwise;*

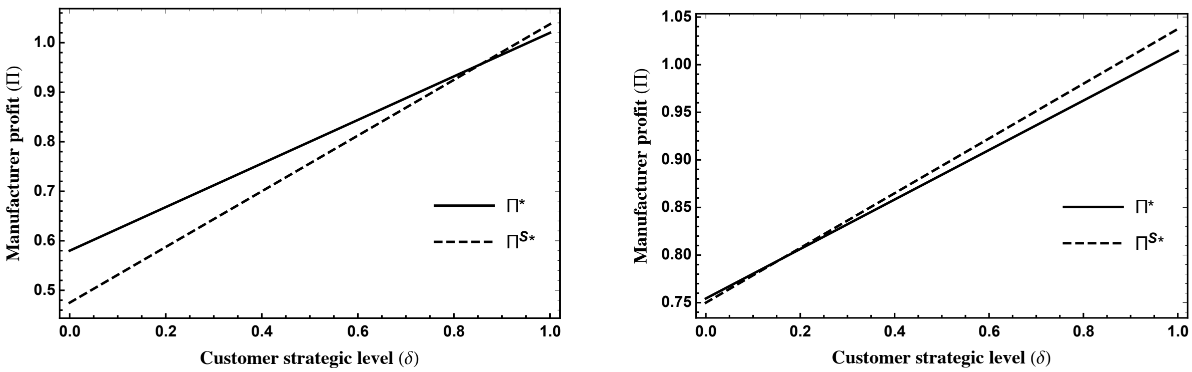
(ii) the upgraded product price: $p_2^{S*} = p_2^*$*;*

(iii) the base product price: $p_1^{S*} \geq p_1^*$.

Firstly, when the residual value of the used product is low, the manufacturer gives up offering a competitive collection reward to match the resale price and the repeat customers only resell via the secondary market. Thus the collection reward is zero with a secondary market, lower than that without a secondary market in this case. However, as long as the used product has a sufficient residual value, the manufacturer offers a higher collection reward to collect more used products to mitigate the demand cannibalization caused by the secondary market. Secondly, the price of the upgraded product remains $(1+\alpha)/2$ even when the secondary market exists. This is because the highest price that the manufacturer can charge to induce purchases from the new customers in the second period is the same. Lastly, anticipating the extra option of reselling the used product with the existence of a secondary market, the customers in the first period would be willing to pay more for the base product, leading to a higher price for the base product,

Next, we present the numerical results regarding the impact of the secondary market on the manufacturer's choice of collection policy. Recall that the secondary market cannibalizes the demand for the upgraded products by allowing the new customers to purchase the used products from the repeated customers, which leads to a profit loss for the manufacturer in the second period. However, the customers also take the extra resale option into account and thereby are willing to pay more for the base product in the first period. Therefore, the customers' strategic level, captured by δ, is the key to understanding the impact of the secondary market on the manufacturer's total profit. Figure 3 shows how the manufacturer's profit changes with δ when a secondary market exists (Π^{S*}) and when it does not (Π^{*}). The parameter setting is the following: $\alpha = 0.3, c_1 = 0.2, c_2 = 0.3, r = 0.2$, and $\beta = 0.75$ for Figure 3(a) and $\beta = 0.2$ for Figure 3(b).

Figure 3 The Impact of the Secondary Market on the Manufacturer's Profit



(a) Profit comparison when $1-\beta < c_2$ (b) Profit comparison when $1-\beta \geq c_2$

Collectively, we have two observations as shown in Figure 3. First, there is a general pattern that the manufacturer's profit increases with the customers' strategic level. Indeed, a higher strategic level means the customers discount the future utility less, allowing the manufacturer to set a higher price for the base product. Second, the existence of a secondary market harms the manufacturer's total profit when the customers' strategic level is low, but it benefits the manufacturer when the customers' strategic level is high. Note that the secondary market not only cannibalizes the sales for the upgraded product but also brings a higher surplus to the repeated customers. When customers are not highly strategic, the increased reservation price in the first period cannot compensate for the profit loss in the second period, and hence the manufacturer suffers from the existence of the secondary market. As customers become more strategic, the benefit of enhancing the customers' reservation price in the first period outweighs the cost of demand cannibalization in the second period, and thus the manufacturer benefits from the existence of the secondary market.

7 Benefit of Policy Commitment

Thus far, the base model assumes that the manufacturer determines its collection policy at the beginning of the second period when the upgraded product is introduced. What if the manufacturer announces the collection policy at the beginning of the first period when it sells the base product, and it is able to commit credibly to the collection policy in the second period when the upgraded product is released to the market? In practice, we do observe that some firms commit to a particular type of collection policy. For example, Apple initiated its iPhone trade-in program in 2013 (Sherr 2013), but the trade-in reward for each generation is set until the next generation is introduced. Moreover, the collection format (e.g., whether to adopt a buyback policy or a trade-in policy) is commonly a strategic and long-term decision that will be applied to future generations of products, whereas the specific collection reward (the buyback or the trade-in reward) can vary across product generations. Hence, it is important to examine the case of policy commitment and quantify the potential benefit of the collection policy commitment if there is.

The sequence of events with the policy commitment is similar to the base model. The only difference is that the decision on the collection policy type $y \in \{B, T\}$ is made at the beginning of the first period under the policy commitment model. Correspondingly, the customers know the collection policy type when making purchase decisions and they need to form beliefs only about the upgraded product price ξ_p and the collection reward ξ_w.

The manufacturer's optimal pricing decisions in the second period for a given collection policy have been presented in Sections 4.1 and 5.1. Compared to the case when the collection policy is determined in the second period, the manufacturer should choose the collection policy that maximizes its total profit over two periods under the policy commitment. We can derive the manufacturer's optimal collection policy commitment with and without a secondary market, and the results are provided in Lemmas EC.5 and EC.6 in Appendix EC.3. We then compare the manufacturer's profits under the base model and the policy commitment model, and find that setting the collection policy in the second period is weakly dominated by committing to the collection policy in the first period.

PROPOSITION 6. *In the absence of a secondary market, committing to a buyback or a trade-in policy in the first period weakly dominates announcing a buyback or a trade-in policy in the second period.*

(i) When $1-\beta \leq \frac{1+\alpha}{2}$ and $\hat{r} < r < \bar{r}$, the manufacturer obtains a higher profit by committing to a trade-in policy than no commitment where a buyback policy is chosen in the second period.

(ii) When $1-\beta > \frac{1+\alpha}{2}$ and $\max\{\bar{r}, \tilde{r}^b\} < r < \hat{r}$, the manufacturer obtains a higher profit by committing to a buyback policy than no commitment where a trade-in policy is chosen in the second period.

(iii) Otherwise, the manufacturer obtains the same profit as it chooses the same policy in equilibrium with or without commitment.

Note that the manufacturer's optimal collection policy choice maximizes its profit in the second period without collection policy commitment, whereas it maximizes its total profit over both periods when there is policy commitment. Therefore, making a policy commitment can benefit the manufacturer when it is tempted to myopically choose a different policy in the second period other than the one maximizing total profit. Proposition 6 illustrates the detailed comparison when there does not exist a secondary market. In the presence of a secondary market, because the buyback policy always weakly dominates the trade-in policy with and without policy commitment, it is straightforward to see that the policy commitment achieves the same profit as no commitment.

8 Environmental Impact

When the manufacturer makes a collection policy decision, it should also take into account other factors beyond profitability. Particularly, nowadays the prevalence of the used product collection is partly driven by the stricter environmental regulations and the firms' environmental responsibilities. Thus it is valuable to evaluate the environmental performance of the buyback and trade-in policies.

In our model, the total environmental impact of the product is generated in both the production and recycling processes (e.g., Zhang and Zhang 2018). Specifically, let $k_i > 0 (i = 1, 2)$ denote the unit environmental impact of producing the base product and upgraded product, respectively. Such impact may refer to the use of natural resources and hazard substance emissions during the production process. Let $k_r > 0$ be the unit environmental benefit of collecting the used product, which usually comes from components recycling and reusing. We assume the environmental benefit of collecting one unit never exceeds the negative impact of producing it, that is, $k_r < \min\{k_1, k_2\}$. The total environmental impact is thereby the environmental impact of the production (including the base and the upgraded products) subtracted by the environmental benefit of the used product collection. We do not explicitly model the direct environmental impact of reselling on the secondary market. But it indirectly reduces the manufacturer's total environmental impact by cannibalizing the upgraded product sales.

We consider the policy commitment model. The total environmental impact under both the buyback and the trade-in policies is expressed as follows:

$$I = k_1 - k_r(D_r + R_r)\mathbb{1}_{w \geq s} + k_2 D_r + k_2 D_n.$$

The first three terms denote the environmental impacts of serving the repeat customers. Each of them purchases a base product once arrives, creating the environmental impact of production k_1. In the second period, given the realized valuation, some repeat customers may simply return the used products, captured by the amount of pure returns R_r, and some may make further replacement purchases, captured by the replacement purchase demand D_r. Each of the returned used products generates the environmental benefit k_r while the replacement purchase incurs an additional environmental impact of the production k_2. The last term is the environmental impact of producing D_n amount of upgraded products that are sold in the second period.

The following proposition characterizes the total environmental impact of the two collection policies in the absence of a secondary market.

PROPOSITION 7. *In the absence of a secondary market,*

(i) when $r \leq \underline{r}^b$*, the buyback and trade-in policies have the same total environmental impact;*

(ii) when $\underline{r}^b < r < \tilde{r}^e$ *and* $\frac{(1+\alpha)k_r}{k_2} < 1 - \beta < c_2$*, the trade-in policy is more environmentally friendly;*

(iii) otherwise, the buyback policy is more environmentally friendly.

When the residual value is low, both policies only collect used products from high-end customers who make replacement purchases. In this case, the two policies yield the same profit and environmental impact. For a higher residual value, the manufacturer offers a larger collection reward under the buyback policy when the base product is less durable ($1-\beta < c_2$). In this case, the buyback policy not only collects more used products but also induces more replacement purchases. Consequently, the buyback policy is likely to be more detrimental to the environment if the production of upgraded products has a stronger environmental effect than the collection of used products, captured by $\frac{(1+\alpha)k_r}{k_2} < 1-\beta$. When the residual value is sufficiently high, the buyback policy collects many more used products and the effect of used product collection dominates the effect of induced replacement purchases, making it more environmentally friendly.

We next examine the environmental impact of the two collection policies with the presence of a secondary market.

PROPOSITION 8. *With the presence of a secondary market,*

1. *when $r \leq c_2 - \frac{\alpha+\beta}{2}$, the buyback and trade-in policies have the same total environmental impact;*
2. *when $r > c_2 - \frac{\alpha+\beta}{2}$, the buyback policy has a smaller environmental impact if $c_2 \leq \tilde{c}_2^e$ and the trade-in policy has a smaller total environmental impact otherwise.*

With the existence of a secondary market, when the residual value is low, the two policies yield the same total environmental impact as the manufacturer cannot afford a competitive collection reward under either policy. When the residual value is higher, the manufacturer collects more used products and induces all the new customers to purchase upgraded products under the buyback policy. By contrast, under the trade-in policy, the low-end customers can only resell the used products to the secondary market, which cannibalizes the demand for the upgraded products. Briefly, the trade-in policy leads to less used product collection and less upgraded product sales. When the production cost of the upgraded product is low, the environmental benefit from the reduction in new product sales cannot compensate the additional environmental impact from less product collection, resulting in a higher total environmental impact under the trade-in policy. Otherwise the trade-in policy is more environmental friendly.

9 Conclusions

Many firms today introduce new generations of products at a blistering pace to meet the customer's ever-increasing demand for product technology upgrades. However, the well-functioning old-version products stand as obstacles to selling the upgraded version. As the buyback and trade-in policies work differently in collecting used products, it is worthwhile to investigate the proper design of these collection policies and gain a deep understanding of the roles these collection policies have in used product recycling and upgraded product promotion.

In this paper, we develop analytical models to study the performances of the unconditional buyback policy and the conditional trade-in policy and the impact of a secondary market on these two collection policies. We find that the discriminating capability of the trade-in policy allows the manufacturer to effectively encourage replacement purchases, which is more profitable when the used product is durable and its residual value is intermediate. When either of the two conditions does not hold, the discriminating capability can backfire as the unconditional buyback policy can collect additionally from the low-end repeat customers who would not make a further purchase.

However, with the presence of a secondary market that allows repeat customers to resell their used products to new customers, the trade-in policy never outperforms the buyback policy. The trade-in policy restricts the manufacturer from collecting used products from the customers with no replacement demand, leading to cannibalization of the upgraded product sales from the new customers. Moreover, although the secondary market competes with the manufacturer for used product collection and upgraded product sales, it may conversely benefit the manufacturer since customers anticipate a higher future utility from the resale opportunity and thus are willing to pay more for the base product at the beginning.

In addition, we find that the manufacturer always weakly benefits from committing to a collection policy in the first period since it can prevent the manufacturer from the opportunistic behavior that only maximizes the profit in the second period. We also examine the environmental impacts of the two collection policies. Contrary to the conventional wisdom, collecting more used products may not always be environmentally friendly because more product returns may lead to more replacement purchases of new products, which can cause substantial environmental damage.

We conclude with a discussion of the limitations of our research and directions for future research. First, we account only for the used product collection of a monopoly manufacturer. Future work might examine the effect of market competition on the firms' strategies in managing used product collection. In such a setting, the trade-in policy would have an additional role to lock customers' replacement purchases to the firm's own upgraded product. Second, we assume the qualities of the used products are observable to the manufacturer and the potential buyers. However, it is well known that secondary market transactions are plagued by severe adverse selection problems. A future research direction is to study a manufacturer's choice of collection policy and how it influences the adverse selection in the secondary market. Third, although we provide the theoretical evaluation of different collection policies, a future research direction is to empirically study how firms adjust their collection policies when market conditions change. For example, the legislation may impose regulations regarding recycling standards and collection targets, which can be regarded as exogenous shocks on the residual value of the used product. Empirical research that examines firms' response to the shocks would complement our understanding on the firms' choices of collection policies.

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E-Companion for (Un)conditional Collection Policies on Used Products with Strategic Customers

EC.1 Derivations of the Assumption on c_2

We claim that the manufacturer chooses a price $p_2 \le \frac{1+\alpha}{2}$ when $c_2 \le \frac{1+\alpha}{4}$. To show the sufficiency of this assumption, we fully analyze the second-period problem without this constraint. The decisions under buy-back and trade-in policies without and with a secondary market are summarized in Lemmas EC.1, EC.2, EC.3 and EC.4, respectively. Note that it is reasonable to assume $c_2 \le \frac{1+\alpha}{2}$ in all the cases; otherwise, the manufacturer cannot make profit from selling upgraded products to new customers.

LEMMA EC.1. *In the absence of a secondary market, the manufacturer's optimal selling price and collection reward under the buyback policy, (p_2^B, w^B), are given as follows.*

1. *When $1-\beta < \frac{1+\alpha}{2}$ and $r < \tilde{r}^{bx}$, the manufacturer chooses a price $p_2 > \frac{1+\alpha}{2}$.*
   (a) *When $r \le \bar{r}^b$, $p_2^B = p_2^{bx(0)} = \frac{\alpha+\beta+c_2}{2} > \frac{1+\alpha}{2}$ and $w^B = w^{bx(0)} = \frac{r}{2}$.*
   (b) *When $\bar{r}^b < r < c_2$, $p_2^B = \bar{p}_2^{bx} = \frac{\alpha+\beta+c_2}{2} > \frac{1+\alpha}{2}$ and $w^B = \bar{w}^{bx} = \frac{(1-\beta)(\alpha+\beta+c_2)}{2(1+\alpha)}$ such that $\bar{w}^{bx} = \frac{(1-\beta)\bar{p}_2^{bx}}{1+\alpha}$ and $D_r^B + R_r^B = 1$.*
2. *When $1-\beta \ge \frac{1+\alpha}{2}$ or $r \ge \tilde{r}^{bx}$, the manufacturer chooses a price $p_2 \le \frac{1+\alpha}{2}$.*
   (a) *When $1-\beta < \frac{1+\alpha}{2}$ (or equivalently $\beta > \frac{1-\alpha}{2}$),*
      i. *when $r \le \underline{r}^b$, $p_2^B = \underline{p}_2^b = \frac{1+\alpha}{2}$ and $w^B = \underline{w}^b = 0$ such that $D_r^B < 1$ and $R_r^B = 0$.*
      ii. *When $\underline{r}^b < r \le \bar{r}^b$, $p_2^B = p_2^{b(0)} = \frac{1+\alpha}{2}$ and $w^B = w^{b(0)} = \frac{(1-\beta)(1-\beta-c_2)+(1+\alpha)r}{2(1+\alpha)}$.*
      iii. *When $\bar{r}^b < r < c_2$, $p_2^B = \bar{p}_2^b = \frac{1+\alpha}{2}$ and $w^B = \bar{w}^b = \frac{1-\beta}{2}$ such that $\bar{w}^b = \frac{(1-\beta)\bar{p}_2^b}{1+\alpha}$ and $D_r^B + R_r^B = 1$.*
   (b) *When $1-\beta \ge \frac{1+\alpha}{2}$ (or equivalently $\beta \le \frac{1-\alpha}{2}$),*
      i. *when $r \le \tilde{r}^b$, $p_2^B = \tilde{p}_2^b = \frac{1+\alpha}{2}$ and $w^B = \tilde{w}^b = \frac{r}{2}$ such that $\tilde{w}^b < \tilde{p}_2^b - \alpha - \beta$ and $D_r^B = 0$.*
      ii. *When $\tilde{r}^b < r < c_2$, $p_2^B = p_2^{b(0)} = \frac{1+\alpha}{2}$ and $w^B = w^{b(0)} = \frac{(1-\beta)(1-\beta-c_2)+(1+\alpha)r}{2(1+\alpha)}$.*

Moreover, $\tilde{r}^{bx} = 0$ when $1-\beta < \frac{1+\alpha}{2}$ and $c_2 \le \frac{1+\alpha}{4}$.

By Lemma EC.1, we can see that the case $1-\beta < \frac{1+\alpha}{2}$ and $r < \tilde{r}^{bx}$ is not feasible when $c_2 \le \frac{1+\alpha}{4}$ as $\tilde{r}^{bx} = 0$. Hence, the manufacturer always chooses $p_2 \le \frac{1+\alpha}{2}$ when $c_2 \le \frac{1+\alpha}{4}$.

LEMMA EC.2. *In the absence of a secondary market, the manufacturer's optimal selling price and collection reward under the trade-in policy, (p_2^T, w^T), are given as follows.*

1. *When $r < \tilde{r}^{tx}$, $p_2^T = p_2^{tx(0)}$ and $w^T = w^{tx(0)}$ which satisfies $p_2^{tx(0)} - w^{tx(0)} = \frac{\alpha+\beta+c_2-r}{2}$, $p_2^{tx(0)} > \frac{1+\alpha}{2}$ and $w^{tx(0)} \ge 0$.*
2. *When $r \ge \tilde{r}^{tx}$, the manufacturer chooses a price $p_2 \le \frac{1+\alpha}{2}$.*
   (a) i. *When $1-\beta < \frac{1+\alpha}{2}$ (or equivalently $\beta > \frac{1-\alpha}{2}$) and $r \le c_2 - (1-\beta)$, $p_2^T = \underline{p}_2^t = \frac{1+\alpha}{2}$ and $w^T = \underline{w}^t = 0$.*

ii. *When* $1-\beta \geq \frac{1+\alpha}{2}$ *(or equivalently* $\beta \leq \frac{1-\alpha}{2}$*) and* $r \leq c_2-(\alpha+\beta)$*,* $p_2^T = \tilde{p}_2^t = \frac{1+\alpha}{2}$ *and* $w^T = \tilde{w}^t = 0 < \frac{1-\alpha-2\beta}{2}$ *such that* $\tilde{w}^t < \tilde{p}_2^t - \alpha - \beta$ *and* $D_r = 0$*.*

(b) *When* $\max\{c_2-(\alpha+\beta), c_2-(1-\beta)\} < r < c_2$*,* $p_2^T = p_2^{t(0)} = \frac{1+\alpha}{2}$ *and* $w^T = w^{t(0)} = \frac{1-\beta-c_2+r}{2}$*.*

Moreover, $\tilde{r}^{tx} = 0$ *when* $c_2 \leq \frac{1+\alpha}{4}$*.*

According to Lemma EC.2, $r < \tilde{r}^{tx}$ is not feasible when $c_2 \leq \frac{1+\alpha}{4}$ as $\tilde{r}^{tx} = 0$. This implies that the manufacturer always chooses $p_2 \leq \frac{1+\alpha}{2}$ when $c_2 \leq \frac{1+\alpha}{4}$.

LEMMA EC.3. *With the presence of a secondary market, the manufacturer's optimal selling price and collection reward under the buyback policy,* (p_2^{SB}, w^{SB})*, are given as follows.*

1. *When* $\beta > 1-2c_2$ *and* $r < \tilde{r}^{sbx}$*, the manufacturer chooses a price* $p_2 > \frac{1+\alpha}{2}$*.*
   (a) *When* $r < \frac{1-\beta}{2}$*,* $p_2^{SB} = p_2^{sbxl} = \frac{1+2\alpha+\beta+2c_2}{4}$ *and* $w^{SB} = 0$ *such that* $D_r^{SB} > 0$ *and* $D_r^{SB} + R_r^{SB} < 1$*.*
   (b) *When* $\frac{1-\beta}{2} \leq r < \min\{1-\beta, c_2\}$*,* $p_2^{SB} = \underline{p}_2^{sbxh} = \frac{1+\alpha+c_2-r}{2}$ *and* $w^{SB} = \frac{1-\beta}{2}$ *such that* $D_r^{SB} > 0$ *and* $D_r^{SB} + R_r^{SB} < 1$*.*
   (c) *When* $1-\beta \leq r \leq \bar{r}^b$*,* $p_2^{SB} = p_2^{bx(0)} = \frac{\alpha+\beta+c_2}{2}$ *and* $w^{SB} = w^{bx(0)} = \frac{r}{2} \geq \frac{1-\beta}{2}$*.*
   (d) *When* $\bar{r}^b < r < c_2$*,* $p_2^{SB} = \bar{p}_2^{bx} = \frac{\alpha+\beta+c_2}{2}$ *and* $w^{SB} = \bar{w}^{bx} = \frac{(1-\beta)(\alpha+\beta+c_2)}{2(1+\alpha)}$ *such that* $\bar{w}^{bx} = \frac{(1-\beta)\bar{p}_2^{bx}}{1+\alpha} \geq \frac{1-\beta}{2}$ *and* $D_r^{SB} + R_r^{SB} = 1$*.*
2. *When* $\beta \leq 1-2c_2$ *or* $\tilde{r}^{sbx} \leq r < c_2$*, the manufacturer chooses a price* $p_2 \leq \frac{1+\alpha}{2}$*.*
   (a) *When* $r < \tilde{r}^s$*, the manufacturer does not offer a buyback, i.e.,* $w^{SB} = w^{sbl} = 0$*, and* $p_2^{SB} = \bar{p}_2^{sbl} = \frac{1+\alpha}{2}$ *such that* $D_r^{SB} + R_r^{SB} = 1$*.*
   (b) *When* $r \geq \tilde{r}^s$*, the manufacturer offers a buyback, which is* $p_2^{SB} = p_2^{sbh} = \frac{1+\alpha}{2}$ *and* $w^{SB} = w^{sbh} = \frac{1-\beta}{2} = s(p_2^{sbh})$ *such that* $D_r^{SB} + R_r^{SB} = 1$*.*

Moreover, $\tilde{r}^{sbx} = 0$ *when* $\beta > 1-2c_2$ *and* $c_2 \leq \frac{1+\alpha}{4}$*.*

In Lemma EC.3, when $w < s(p_2)$, $R_r^{SB} = \min\{\frac{s(p_2)}{1-\beta}, \frac{p_2}{1+\alpha}\}$ represents the amount of purely resold products by the repeat customers. Similarly, $\beta > 1-2c_2$ and $r < \tilde{r}^{sbx}$ is not feasible when $c_2 \leq \frac{1+\alpha}{4}$ as $\tilde{r}^{sbx} = 0$ in this case. Therefore, the manufacturer always chooses $p_2 \leq \frac{1+\alpha}{2}$ when $c_2 \leq \frac{1+\alpha}{4}$.

LEMMA EC.4. *With the presence of a secondary market, the manufacturer's optimal selling price and collection reward under the trade-in policy,* (p_2^{ST}, w^{ST})*, are given as follows.*

1. *When* $r < \tilde{r}^{stx}$*, the manufacturer chooses a price* $p_2 > \frac{1+\alpha}{2}$*.*
   (a) *When* $r < \frac{1-\beta}{2}$*,* $p_2^{ST} = p_2^{sbxl} = \frac{1+2\alpha+\beta+2c_2}{4}$ *and* $w^{ST} = 0$*.*
   (b) *When* $\frac{1-\beta}{2} \leq r < c_2$*,* $p_2^{ST} = p_2^{tx(0)}$ *and* $w^{ST} = w^{tx(0)}$ *which satisfies* $p_2^{tx(0)} - w^{tx(0)} = \frac{\alpha+\beta+c_2-r}{2}$*,* $p_2^{tx(0)} > \frac{1+\alpha}{2}$ *and* $w^{tx(0)} \geq \frac{1-\beta}{2}$*.*
2. *When* $\tilde{r}^{stx} \leq r < c_2$*, the manufacturer chooses a price* $p_2 \leq \frac{1+\alpha}{2}$*.*
   (a) *When* $r < \tilde{r}^s$*, the manufacturer does not offer a trade-in policy, i.e.,* $w^{ST} = w^{sbl} = 0$ *and* $p_2^{ST} = \bar{p}_2^{sbl} = \frac{1+\alpha}{2}$ *such that* $D_r^{ST} + R_r^{ST} = 1$*.*

(b) *When $\tilde{r}^s \leq r < c_2$, the manufacturer has the incentive to offer a trade-in policy.*

i. *When $c_2 \leq \frac{\alpha+3\beta-2}{2}$, $p_2^{ST} = \underline{p}_2^{sbl} = \frac{\alpha+\beta}{2}$ and $w^{ST} = s(\underline{p}_2^{sbl}) = 0$ such that $R_r^{ST} = 0$.*

ii. *When $\frac{\alpha+3\beta-2}{2} < c_2 \leq \frac{\alpha+\beta}{2}$, $p_2^{ST} = p_2^{sbl(0)} = \frac{2+\alpha-\beta+2c_2}{4}$ and $w^{ST} = s(p_2^{sbl(0)}) = \frac{2-\alpha-3\beta+2c_2}{4} > 0$.*

iii. *When $\frac{\alpha+\beta}{2} < c_2 \leq \frac{1+\alpha}{2}$, $p_2^{ST} = \bar{p}_2^{sbl} = \frac{1+\alpha}{2}$ and $w^{ST} = s(\bar{p}_2^{sbl}) = \frac{1-\beta}{2} > 0$ such that $D_r^{ST} + R_r^{ST} = 1$.*

Moreover, $\tilde{r}^{stx} = 0$ when $c_2 \leq \frac{1+\alpha}{4}$.

In Lemma EC.4, $R_r^{ST} = \min\{\frac{s(p_2)}{1-\beta}, \frac{p_2-w+s(p_2)}{1+\alpha}\}$ represents the amount of purely resold products by the repeat customers when $w \geq s(p_2)$ under a trade-in policy. Recall that the amount of purely resold products by the repeat customers is $R_r^{ST} = \min\{\frac{s(p_2)}{1-\beta}, \frac{p_2}{1+\alpha}\}$ when $w < s(p_2)$. Again, the manufacturer never chooses $p_2 > \frac{1+\alpha}{2}$ when $c_2 \leq \frac{1+\alpha}{4}$ as $\tilde{r}^{stx} = 0$ in this case.

EC.2 Proofs of Lemmas EC.1, EC.2, EC.3 and EC.4

Proof of Lemma EC.1 Lemma EC.1 is proved by the following three steps. The case when $p_2 \le \frac{1+\alpha}{2}$ is analyzed first. Then the case when $p_2 > \frac{1+\alpha}{2}$. Finally, the proof is completed by comparing and summarizing the results of these two cases.

With upgraded product sales for the new customers when $p_2 \le \frac{1+\alpha}{2}$. Let's first analyze the manufacturer's problem with offering $p_2 \le \frac{1+\alpha}{2}$ such that the second period new customers buy upgraded products. The manufacturer's profit is

$$\pi_2^B(p_2, w) = (p_2 - c_2) + (p_2 - w - c_2 + r)D_r^B + (r - w)R_r^B$$

by (1), where $D_n^B = 1$ when $p_2 \le \frac{1+\alpha}{2}$. The 2nd case of Lemma EC.1 presents the manufacturer's decisions when $p_2 \le \frac{1+\alpha}{2}$, which also corresponds to the Lemma 1. The detail proof is as follows.

Recall $D_r^B = (1 - \max\{\frac{p_2-w}{\alpha+\beta}, \frac{p_2}{1+\alpha}\})^+$ and $R_r^B = \min\{\frac{w}{1-\beta}, \frac{p_2}{1+\alpha}\}$ from Section 4.1.1. Note $\frac{p_2-w}{\alpha+\beta} \ge \frac{p_2}{1+\alpha} \ge \frac{w}{1-\beta}$ when $w \le \frac{(1-\beta)p_2}{1+\alpha}$; and $\frac{p_2-w}{\alpha+\beta} < \frac{p_2}{1+\alpha} < \frac{w}{1-\beta}$ and $D_r^B + R_r^B = 1$ when $w > \frac{(1-\beta)p_2}{1+\alpha}$. $1 - \frac{p_2-w}{\alpha+\beta} \ge 0$ if and only if (iff) $w \ge p_2 - \alpha - \beta$. And $p_2 - \alpha - \beta \le \frac{(1-\beta)p_2}{1+\alpha}$ as $p_2 \le 1+\alpha$. Moreover, $\pi_2^B(p_2, w)$ is unimodal in (p_2, w) when $p_2 \le \frac{1+\alpha}{2}$ and $p_2 - \alpha - \beta \le 0$.

Let's start from the interior case, which is $p_2 \le \frac{1+\alpha}{2}$ and $\max\{p_2 - \alpha - \beta, 0\} \le w \le \frac{(1-\beta)p_2}{1+\alpha}$. Then $\pi_2^B(p_2, w) = (p_2 - c_2) + (p_2 - w - c_2 + r)D_r^B + (r - w)R_r^B$ is jointly concave in p_2 and w. The solution to $(\frac{\partial}{\partial p_2}\pi_2^B(p_2, w), \frac{\partial}{\partial w}\pi_2^B(p_2, w)) = (0, 0)$ satisfies $p_2 > \frac{1+\alpha}{2}$. Thus the interior solution is $p_2^B = p_2^{b(0)} = \frac{1+\alpha}{2}$ and $w^B = w^{b(0)} = \frac{(1-\beta)(1-\beta-c_2)+(1+\alpha)r}{2(1+\alpha)}$, where $w^{b(0)}$ is the solution to $\frac{d}{dw}\pi_2^B(\frac{1+\alpha}{2}, w) = 0$.

Here $w^{b(0)} \ge p_2^{b(0)} - \alpha - \beta$ iff $r \ge \underline{r}_2^b$, where $\underline{r}_2^b = \max\{\frac{(1-\beta)c_2-(\alpha+\beta)^2}{1+\alpha}, 0\}$. $w^{b(0)} \ge 0$ iff $r \ge \underline{r}^b$, where $\underline{r}^b = \max\{\frac{(1-\beta)[c_2-(1-\beta)]}{1+\alpha}, 0\}$. $w^{b(0)} \le \frac{(1-\beta)p_2^{b(0)}}{1+\alpha}$ iff $r \le \bar{r}^b$, where $\bar{r}^b = \frac{(1-\beta)(\alpha+\beta+c_2)}{1+\alpha}$. $\underline{r}^b \le \bar{r}^b$ and $\underline{r}_2^b \le \bar{r}^b$; $\underline{r}^b \le \underline{r}_2^b$ iff $\beta \le \frac{1-\alpha}{2}$; and $\bar{r}^b \ge c_2$ when $\beta \le \frac{1-\alpha}{2}$ and $c_2 \le \frac{1+\alpha}{2}$.

When $p_2 \le \frac{1+\alpha}{2}$ and $w = 0$, the optimal decision is $p_2^B = \underline{p}_2^b = \frac{1+\alpha}{2}$ as $\pi_2^B(p_2, 0)$ is concave in p_2 and the solution to $\frac{d}{dp_2}\pi_2^B(p_2, 0) = 0$ satisfies $p_2 > \frac{1+\alpha}{2}$. Note here $w^B = \underline{w}^b = 0$ implies $R_r^B = 0$. $D_r^B = 1 - \frac{p_2^b - w^b}{\alpha+\beta} > 0$ iff $\beta > \frac{1-\alpha}{2}$. Thus it is only necessary to consider this case when $\beta > \frac{1-\alpha}{2}$, which is equivalent to $\underline{r}^b > \underline{r}_2^b$. And it is easy to show that $\pi_2^B(p_2^{b(0)}, w^{b(0)}) > \pi_2^B(\underline{p}_2^b, 0)$ iff $r > \underline{r}^b$.

When $p_2 \le \frac{1+\alpha}{2}$ and $0 < w < p_2 - \alpha - \beta$, $D_r^B = 0$. Similarly, we can show the optimal decision is $p_2^B = \tilde{p}_2^b = \frac{1+\alpha}{2}$ and $w^B = \tilde{w}^b = \frac{r}{2}$, where $\tilde{w}^b < \tilde{p}_2^b - \alpha - \beta$ iff $r < 1 - \alpha - 2\beta$. Thus it is only necessary to consider this case when $\beta \le \frac{1-\alpha}{2}$ such that $1 - \alpha - 2\beta \ge 0$, which also implies $\underline{r}^b \le \underline{r}_2^b$. $\pi_2^B(p_2^{b(0)}, w^{b(0)}) > \pi_2^B(\tilde{p}_2^b, \tilde{w}^b)$ iff $r > \tilde{r}^b$, where $\tilde{r}^b = \max\{c_2 - (\alpha+\beta)(1 - \frac{|1-bt-c_2|}{\sqrt{(1+\alpha)(\alpha+\beta)}}), 0\}$. It can be proved that $\underline{r}_2^b \le \tilde{r}^b \le 1 - \alpha - 2\beta$ when $\beta \le \frac{1-\alpha}{2}$ and $c_2 \le \frac{1+\alpha}{2}$. And $\tilde{r}^b \ge \underline{r}^b$ when $\beta \le \frac{1-\alpha}{2}$ and $\tilde{r}^b \le \underline{r}^b$ when $\beta > \frac{1-\alpha}{2}$.

When $p_2 \le \frac{1+\alpha}{2}$ and $w > \frac{(1-\beta)p_2}{1+\alpha}$, $D_r^B + R_r^B = 1$ and $\frac{\partial}{\partial w}\pi_2^B(p_2, w) < 0$. Thus $w = \frac{(1-\beta)p_2}{1+\alpha}$. $p_2^B = \bar{p}_2^b = \frac{1+\alpha}{2}$ as the solution to $\frac{d}{dp_2}\pi_2^B(p_2, \frac{(1-\beta)p_2}{1+\alpha}) = 0$ satisfies $p_2 > \frac{1+\alpha}{2}$. Correspondingly, $w^B = \bar{w}^b = \frac{1-\beta}{2}$. And it is easy to show that $\pi_2^B(p_2^{b(0)}, w^{b(0)}) \ge \pi_2^B(\bar{p}_2^b, \bar{w}^b)$ iff $r \le \bar{r}^b$.

By comparing the cases above, the optimal decisions are summarized as the second case of Lemma EC.1 when $p_2 \leq \frac{1+\alpha}{2}$. Recall that $\bar{r}^b \geq c_2$ when $\beta \leq \frac{1-\alpha}{2}$. Thus the case when $\bar{r}^b < r < c_2$ only exists when $\beta \leq \frac{1-\alpha}{2}$.

Without upgraded product sales for the new customers when $p_2 > \frac{1+\alpha}{2}$. When the manufacturer's chooses a $p_2 > \frac{1+\alpha}{2}$ such that the second period new customers do not buy upgraded products, the manufacturer's profit is

$$\pi_2^B(p_2, w) = (p_2 - w - c_2 + r)D_r^B + (r - w)R_r^B$$

by (1), where $D_n^B = 0$ when $p_2 > \frac{1+\alpha}{2}$. Similarly, $\pi_2^B(p_2, w)$ is unimodal in (p_2, w) when $p_2 > \frac{1+\alpha}{2}$ and $p_2 - \alpha - \beta \leq 0$.

Again, we start from the interior case, which is $p_2 > \frac{1+\alpha}{2}$ and $\max\{p_2 - \alpha - \beta, 0\} \leq w \leq \frac{(1-\beta)p_2}{1+\alpha}$. By solving $(\frac{\partial}{\partial p_2}\pi_2^B(p_2, w), \frac{\partial}{\partial w}\pi_2^B(p_2, w)) = (0, 0)$, we have $p_2^B = p_2^{bx(0)} = \frac{\alpha+\beta+c_2}{2}$ and $w^B = w^{bx(0)} = \frac{r}{2}$. Here $p_2^{bx(0)} > \frac{1+\alpha}{2}$ iff $\beta > 1 - c_2$, and $w^{bx(0)} \leq \frac{(1-\beta)p_2^{bx(0)}}{1+\alpha}$ iff $r \leq \bar{r}^b$. Note $w^{bx(0)} \geq p_2^{bx(0)} - \alpha - \beta$ when $\beta > 1 - c_2$ and $c_2 \leq \frac{1+\alpha}{2}$, and $w^{bx(0)} = \frac{r}{2} \geq 0$.

As $w^{bx(0)} = \frac{r}{2} \geq 0$, the manufacturer does not choose $w = 0$ by the unimodality of the profit function. Then consider the case when $p_2 > \frac{1+\alpha}{2}$ and $0 \leq w < p_2 - \alpha - \beta$ such that $D_r^B = 0$. In this case, $\pi_2^B(p_2, w) = (r - w)R_r^B$, where $R_r^B = \frac{w}{1-\beta}$. Then $w = \frac{r}{2}$ and the p_2 could be any value which satisfies $p_2 - \alpha - \beta > \frac{r}{2}$. And we simply assume the manufacturer chooses $p_2 = \alpha + \beta + \frac{r}{2}$ such that $w = p_2 - \alpha - \beta$. As $\pi_2^B(p_2^{bx(0)}, w^{bx(0)}) \geq \pi_2^B(\alpha + \beta + \frac{r}{2}, \frac{r}{2})$, it is unnecessary to consider this case.

When $r > \bar{r}^b$, which indicates $w > \frac{(1-\beta)p_2}{1+\alpha}$ and $D_r^B + R_r^B = 1$, it is easy to derive that $p_2^B = \bar{p}_2^{bx} = \frac{\alpha+\beta+c_2}{2}$ and $w^B = \bar{w}^{bx} = \frac{(1-\beta)(\alpha+\beta+c_2)}{2(1+\alpha)}$, which satisfies $\bar{w}^{bx} = \frac{(1-\beta)\bar{p}_2^{bx}}{1+\alpha}$.

The manufacturer's optimal decision when $p_2 > \frac{1+\alpha}{2}$ under a buyback policy is summarized as follows: *(1) when* $\beta > 1 - c_2$ *and* $r \leq \bar{r}^b$, $p_2^B = p_2^{bx(0)} = \frac{\alpha+\beta+c_2}{2}$ *and* $w^B = w^{bx(0)} = \frac{r}{2}$*; (2) when* $\beta > 1 - c_2$ *and* $r > \bar{r}^b$, $p_2^B = \bar{p}_2^{bx} = \frac{\alpha+\beta+c_2}{2}$ *and* $w^B = \bar{w}^{bx} = \frac{(1-\beta)(\alpha+\beta+c_2)}{2(1+\alpha)}$ *such that* $\bar{w}^{bx} = \frac{(1-\beta)\bar{p}_2^{bx}}{1+\alpha}$ *and* $D_r^B + R_r^B = 1$.

Summary of the manufacturer's buyback policy without a secondary market. As the optimal decision when $p_2 > \frac{1+\alpha}{2}$ requires $\beta > 1 - c_2$, and $1 - c_2 \geq \frac{1-\alpha}{2}$ when $c_2 \leq \frac{1+\alpha}{2}$. Thus it is only possible for the manufacturer to choose $p_2 > \frac{1+\alpha}{2}$ when $\beta > \frac{1-\alpha}{2}$.

We can show that $\pi_2^B(\underline{p}_2^b, \underline{w}^b) \geq \pi_2^B(p_2^{bx(0)}, w^{bx(0)})$ iff $r \geq \tilde{r}^{bx0}$, where $\tilde{r}^{bx0} = \frac{(1-\beta)[c_2-(1-\beta)]-\sqrt{(1-\beta)(\alpha+\beta)[2(1+\alpha)(1+\alpha-2c_2)-(1-\beta-c_2)^2]}}{1+\alpha}$. Furthermore, $\pi_2^B(p_2^{b(0)}, w^{b(0)}) - \pi_2^B(p_2^{bx(0)}, w^{bx(0)})$ (when $r \leq \bar{r}^b$) and $\pi_2^B(\bar{p}_2^b, \bar{w}^b) - \pi_2^B(\bar{p}_2^{bx}, \bar{w}^{bx})$ (when $r > \bar{r}^b$) are independent of r. And $\pi_2^B(p_2^{b(0)}, w^{b(0)}) \geq \pi_2^B(p_2^{bx(0)}, w^{bx(0)})$ (and $\pi_2^B(\bar{p}_2^b, \bar{w}^b) \geq \pi_2^B(\bar{p}_2^{bx}, \bar{w}^{bx})$) iff $c_2 \leq \tilde{c}_2^b$, where $\tilde{c}_2^b = \begin{cases} \frac{1+\alpha}{2} & \text{, when } \beta \leq \frac{1-\alpha}{2} \\ \sqrt{2(1+\alpha)(1+3\alpha+2\beta)} - (1+2\alpha+\beta) & \text{, when } \beta > \frac{1-\alpha}{2} \end{cases}$. Note that $\beta > 1 - \tilde{c}_2^b$ when $\beta > \frac{1-\alpha}{2}$; and $\tilde{r}^{bx0} \leq \underline{r}^b$ iff $c_2 \leq \tilde{c}_2^b$. Let $\tilde{r}^{bx} = \begin{cases} \max\{\tilde{r}^{bx0}, 0\} & \text{, when } c_2 \leq \tilde{c}_2^b \\ c_2 & \text{, otherwise} \end{cases}$. Therefore, it is easy to show that the manufacturer chooses $p_2 > \frac{1+\alpha}{2}$ when $\beta > \frac{1-\alpha}{2}$ and $r < \tilde{r}^{bx}$, and chooses $p_2 \leq \frac{1+\alpha}{2}$ otherwise. Finally, Lemma EC.1 is proved.

Here $\tilde{r}^{bx}$ is increasing in c_2 and $\tilde{r}^{bx} > 0$ iff $c_2 > \sqrt{(1+\alpha)^2+(\alpha+2\beta-1)(1+5\alpha+4\beta)}-(2\alpha+3\beta-1)$. It is easy to verify that $\sqrt{(1+\alpha)^2+(\alpha+2\beta-1)(1+5\alpha+4\beta)}-(2\alpha+3\beta-1) > (\sqrt{6}-2)(1+\alpha)$ when $\beta > \frac{1-\alpha}{2}$. It implies $\tilde{r}^{bx} = 0$ when $\beta > \frac{1-\alpha}{2}$ and $c_2 \leq (\sqrt{6}-2)(1+\alpha)$. Note $(\sqrt{6}-2)(1+\alpha) > \frac{1+\alpha}{4}$. Therefore, the manufacturer always chooses $p_2 \leq \frac{1+\alpha}{2}$ when $c_2 \leq \frac{1+\alpha}{4}$. ■

Proof of Lemma EC.2 Similar to the Proof of Lemma EC.1, Lemma EC.2 is proved by three steps.

With upgraded product sales for the new customers when $p_2 \leq \frac{1+\alpha}{2}$. According to (2), when $p_2 \leq \frac{1+\alpha}{2}$, the manufacturer's profit is

$$\pi_2^T(p_2,w) = (p_2-c_2)+(p_2-w-c_2+r)D_r^T.$$

Recall $D_r^T = (1-\frac{p_2-w}{\alpha+\beta})^+$ from Section 4.1.2. $D_r^T > 0$ iff $w > p_2-\alpha-\beta$, and $D_r^T \leq 1$ as $w \leq p_2$. It is easy to show that $\pi_2^T(p_2,w)$ is unimodal in (p_2,w). By the similar approaches presented in the Proof of Lemma EC.1, the interior solution for $p_2 \leq \frac{1+\alpha}{2}$ and $\max\{p_2-\alpha-\beta,0\} \leq w \leq p_2$ is $p_2^T = p_2^{t(0)} = \frac{1+\alpha}{2}$ and $w^T = w^{t(0)} = \frac{1-\beta-c_2+r}{2}$. $w^{t(0)} \geq 0$ iff $r \geq c_2-(1-\beta)$, $w^{t(0)} \geq p_2^{t(0)}-\alpha-\beta$ iff $r \geq c_2-(\alpha+\beta)$, and $w^{t(0)} \leq p_2^{t(0)}$ as $r < c_2$. Note $c_2-(1-\beta) < 0$ when $\beta \leq \frac{1-\alpha}{2}$, and $c_2-(\alpha+\beta) < 0$ when $\beta > \frac{1-\alpha}{2}$, where $c_2 \leq \frac{1+\alpha}{2}$.

When $\beta \leq \frac{1-\alpha}{2}$ and $r < c_2-(\alpha+\beta)$, which implies $p_2 \leq \frac{1+\alpha}{2}$ and $0 < w < p_2-\alpha-\beta$ by the unimodality. In this case, $D_r^T = 0$ and $\pi_2^T(p_2,w) = (p_2-c_2)$. The decision is $p_2^T = \tilde{p}_2^t = \frac{1+\alpha}{2}$ and $w^T = \tilde{w}^t = 0$. Indeed, $\tilde{w}^t$ could be any value satisfies $w < \tilde{p}_2^t-\alpha-\beta = \frac{1-\alpha-2\beta}{2}$ and simply choose $\tilde{w}^t = 0$. Here $\frac{1-\alpha-2\beta}{2} \geq 0$ iff $\beta \leq \frac{1-\alpha}{2}$.

When $\beta > \frac{1-\alpha}{2}$ and $r < c_2-(1-\beta)$, which implies $p_2 \leq \frac{1+\alpha}{2}$ and $w = 0$, the optimal decision is $p_2^T = \underline{p}_2^t = \frac{1+\alpha}{2}$. $\underline{p}_2^t-\alpha-\beta < 0$ iff $\beta > \frac{1-\alpha}{2}$.

By comparing and summarizing the results, it is easy to obtain the decisions in the second case of the Lemma EC.2 when $p_2 \leq \frac{1+\alpha}{2}$, which also corresponds to the Lemma 2.

Without upgraded product sales for the new customers when $p_2 > \frac{1+\alpha}{2}$. When $p_2 > \frac{1+\alpha}{2}$, the manufacturer's profit is

$$\pi_2^T(p_2,w) = (p_2-w-c_2+r)D_r^T,$$

and $\pi_2^T(p_2,w)$ is determined by and concave in p_2-w.

The interior solution for $p_2 > \frac{1+\alpha}{2}$ and $\max\{p_2-\alpha-\beta,0\} \leq w \leq p_2$ is $p_2^T = p_2^{tx(0)}$ and $w^T = w^{tx(0)}$ which satisfies $p_2^{tx(0)}-w^{tx(0)} = \frac{\alpha+\beta+c_2-r}{2}$, $p_2^{tx(0)} > \frac{1+\alpha}{2}$ and $w^{tx(0)} \geq 0$. $w^{tx(0)} \geq p_2^{tx(0)}-\alpha-\beta$ iff $r \geq c_2-(\alpha+\beta)$, and $w^{tx(0)} \leq p_2^{tx(0)}$ as $r < c_2$. Note that when $r < c_2-(\alpha+\beta)$, $\pi_2^T(p_2,w) = 0$ as $w < p_2-\alpha-\beta$, i.e., $D_r^T = 0$.

Summary of the manufacturer's trade-in policy without a secondary market. Note that only considers $p_2 > \frac{1+\alpha}{2}$ when $r \geq c_2-(\alpha+\beta)$, and the corresponding profit is $\pi_2^T(p_2^{tx(0)},w^{tx(0)})$. It is easy to show that $\pi_2^T(p_2^{t(0)},w^{t(0)}) \geq \pi_2^T(p_2^{tx(0)},w^{tx(0)})$ when $c_2 \leq \frac{1+\alpha}{2}$. Thus only $p_2 \leq \frac{1+\alpha}{2}$ is chooses when $\beta \leq \frac{1-\alpha}{2}$. Furthermore, $\pi_2^T(\underline{p}_2^t,\underline{w}^t) \geq \pi_2^T(p_2^{tx(0)},w^{tx(0)})$ iff $r \geq \tilde{r}^{tx}$, where $\tilde{r}^{tx} = \max\{c_2-(1-\beta)-2\sqrt{(\alpha+\beta)(\frac{1+\alpha}{2}-c_2)},0\}$. Note $\tilde{r}^{tx} < c_2-(1-\beta)$. Therefore, Lemma EC.2 is proved by summarizing the results above.

And again, $\tilde{r}^{tx} > 0$ iff $\beta > \frac{1-\alpha}{2}$ and $c_2 > \sqrt{(1+\alpha)^2 + (\alpha+2\beta-1)(1+5\alpha+4\beta)} - (2\alpha+3\beta-1)$. Recall $\sqrt{(1+\alpha)^2 + (\alpha+2\beta-1)(1+5\alpha+4\beta)} - (2\alpha+3\beta-1) > (\sqrt{6}-2)(1+\alpha)$ when $\beta > \frac{1-\alpha}{2}$. Therefore, $\tilde{r}^{tx} = 0$ and the manufacturer always chooses $p_2 \le \frac{1+\alpha}{2}$ when $c_2 \le (\sqrt{6}-2)(1+\alpha)$, where $(\sqrt{6}-2)(1+\alpha) > \frac{1+\alpha}{4}$. ■

Proof of Lemma EC.3 <u>With upgraded product sales for the new customers when $p_2 \le \frac{1+\alpha}{2}$</u>. First note that $s(p_2) = (p_2 - \frac{\alpha+\beta}{2})^+$ when $p_2 \le \frac{1+\alpha}{2}$.

When the manufacturer chooses $w > s(p_2)$, the repeat customers choose to return. The manufacturer's second period profit is

$$\pi_2^{SB}(p_2, w) = (p_2 - c) + (p_2 - w - c + r)D_r^{SB} + (r - w)R_r^{SB}$$

by (3), where $D_r^{SB} = (1 - \max\{\frac{p_2 - w}{\alpha+\beta}, \frac{p_2}{1+\alpha}\})^+$ and $R_r^{SB} = \min\{\frac{w}{1-\beta}, \frac{p_2}{1+\alpha}\}$. The profit function is the same as (1) when there is not a secondary market, but with an additional constraint $w > s(p_2)$. As $s(p_2) \ge p_2 - \alpha - \beta$, the interior solution requires $s(p_2) \le w \le \frac{(1-\beta)p_2}{1+\alpha}$. And $s(p_2) \le \frac{(1-\beta)p_2}{1+\alpha}$ iff $p_2 \le \frac{1+\alpha}{2}$. Then it is easy to show that the optimal decision is $p_2^{SB} = p_2^{sbh} = \frac{1+\alpha}{2}$ and $w^{SB} = w^{sbh} = \frac{1-\beta}{2} = s(p_2^{sbh})$ such that $D_r^{SB} + R_r^{SB} = 1$.

When $w = s(p_2)$, the repeat customers are indifferent between returning and reselling. Assume that a $k \in [0,1]$ fraction of $D_r^{SB} + R_r^{SB}$ from the repeat customers choose to resell, and the rest $1-k$ fraction choose to return. Then the manufacturer's profit when $p_2 \le \frac{1+\alpha}{2}$ is

$$\begin{aligned}\pi_2^{SB}(p_2, w) &= (p_2 - c_2)\{D_r^{SB} + [1 - k(D_r^{SB} + R_r^{SB})]\} + (r - w)(1-k)(D_r^{SB} + R_r^{SB}) \\ &= (p_2 - c) + (p_2 - w - c + r)D_r^{SB} + (r - w)R_r^{SB} - k[(p_2 - c_2) + (r - w)](D_r^{SB} + R_r^{SB}).\end{aligned}$$

Note that $1 - k(D_r^{SB} + R_r^{SB})$ new customers buy upgraded products and the rest $k(D_r^{SB} + R_r^{SB})$ buy resold products. The manufacturer prefers $k = 0$ and chooses $w > s(p_2)$ when $(p_2 - c_2) + (r - w) > 0$; and the manufacturer prefers $k = 1$ and chooses $w < s(p_2)$ when $(p_2 - c_2) + (r - w) < 0$. When $(p_2 - c_2) + (r - w) = 0$, any k yields the same profit. Thus we simply assume the manufacturer chooses $w > s(p_2)$ and $k = 0$ in this case, or equivalently we assume that customers return used products instead of reselling when $w \ge s(p_2)$.

Lastly, when $w < s(p_2)$, customers resell used products directly. The manufacturer's second period profit is

$$\pi_2^{SB}(p_2, w) = (p_2 - c)(D_n^{SB} + D_r^{SB})$$

by (4), where $D_n^{SB} = (\frac{p_2 - s(p_2)}{\alpha+\beta} - \frac{s(p_2)}{1-\beta})^+$ and $D_r^{SB} = (1 - \max\{\frac{p_2 - s(p_2)}{\alpha+\beta}, \frac{p_2}{1+\alpha}\})^+$. Note there is no profit from returns for the manufacturer and $\pi_2^{SB}(p_2, w)$ is independent of w. Thus simply set $w = 0$. And it is easy to show that $\pi_2^{SB}(p_2, w)$ is unimodal in p_2. The interior solution requires $p_2 \le \frac{1+\alpha}{2}$ and $0 < s(p_2) \le \frac{(1-\beta)p_2}{1+\alpha}$, which is equivalent to $\frac{\alpha+\beta}{2} < p_2 \le \frac{1+\alpha}{2}$. The interior solution for $\frac{\alpha+\beta}{2} < p_2 \le \frac{1+\alpha}{2}$ is $p_2^{SB} = p_2^{sbl(0)} = \frac{2+\alpha-\beta+2c_2}{4}$, where $\frac{\alpha+\beta}{2} \le p_2^{sbl(0)} \le \frac{1+\alpha}{2}$ iff $\frac{\alpha+3\beta-2}{2} \le c_2 \le \frac{\alpha+\beta}{2}$. The detail decisions when $p_2 \le \frac{1+\alpha}{2}$ and $w < s(p_2)$ are as follows: *(a) when $c_2 \le \frac{\alpha+3\beta-2}{2}$, $p_2^{SB} = \underline{p}_2^{sbl} = \frac{\alpha+\beta}{2}$ such that $s(\underline{p}_2^{sbl}) = 0$; (b) when $\frac{\alpha+3\beta-2}{2} < c_2 \le \frac{\alpha+\beta}{2}$,*

$p_2^{SB} = p_2^{sbl(0)} = \frac{2+\alpha-\beta+2c_2}{4}$*; (c) when* $\frac{\alpha+\beta}{2} < c_2 \le \frac{1+\alpha}{2}$*,* $p_2^{SB} = \bar{p}_2^{sbl} = \frac{1+\alpha}{2}$ *such that* $D_r^{SB} + R_r^{SB} = 1$*.* Note here $w^{SB} = 0$.

In the statement above, we can show that the profit when $w = 0 < s(p_2)$, namely $\pi_2^{SB}(p_2^{SB}, 0)$, is continuous decreasing in c_2. Plus, it is easy to show that $\pi_2^{SB}(p_2^{sbh}, w^{sbh}) \ge \pi_2^{SB}(\bar{p}_2^{sbl}, 0)$ iff $r \ge \tilde{r}^s$, where $\tilde{r}^s = \max\{c_2 - \frac{\alpha+\beta}{2}, 0\}$ is the threshold in r above which it is optimal for the manufacturer to collect used products when there exists a secondary market. It also implies $\pi_2^{SB}(p_2^{sbh}, w^{sbh}) \ge \pi_2^{SB}(p_2^{sbl(0)}, 0)$ and $\pi_2^{SB}(p_2^{sbh}, w^{sbh}) \ge \pi_2^{SB}(\underline{p}_2^{sbl}, 0)$ when $r \ge \tilde{r}^s$. Thus the optimal decisions which includes $w < s(p_2)$ and $w \ge s(p_2)$ when $p_2 \le \frac{1+\alpha}{2}$ are summarized as the second case of Lemma EC.3 when $p_2 \le \frac{1+\alpha}{2}$.

<u>Without upgraded product sales for the new customers when $p_2 > \frac{1+\alpha}{2}$</u>. When $p_2 > \frac{1+\alpha}{2}$, first note that $s(p_2) = \frac{1-\beta}{2}$ when $p_2 > \frac{1+\alpha}{2}$.

Similarly, when $p_2 > \frac{1+\alpha}{2}$ and $w > s(p_2)$, the manufacturer's profit is $\pi_2^{SB}(p_2, w) = (p_2 - w - c + r)D_r^{SB} + (r - w)R_r^{SB}$ by (3), which is the same as (1) without a secondary market, but with the additional constraint $w > s(p_2)$. The interior solution for $p_2 > \frac{1+\alpha}{2}$ and $\max\{\frac{1-\beta}{2}, p_2 - \alpha - \beta\} \le w \le \frac{(1-\beta)p_2}{1+\alpha}$ is $p_2^{SB} = p_2^{bx(0)} = \frac{\alpha+\beta+c_2}{2}$ and $w^{SB} = w^{bx(0)} = \frac{r}{2}$, which is the same as the interior solution to (1) when $p_2 > \frac{1+\alpha}{2}$. Recall that $p_2^{bx(0)} > \frac{1+\alpha}{2}$ iff $\beta > 1 - c_2$, and $w^{bx(0)} \le \frac{(1-\beta)p_2^{bx(0)}}{1+\alpha}$ iff $r \le \bar{r}^b$, $w^{bx(0)} \ge p_2^{bx(0)} - \alpha - \beta$ when $\beta > 1 - c_2$ and $c_2 \le \frac{1+\alpha}{2}$, and $w^{bx(0)} = \frac{r}{2} \ge 0$. Plus, $w^{bx(0)} \ge \frac{1-\beta}{2}$ iff $r \ge 1 - \beta$. Note here it is still unnecessary to consider the case when $p_2 > \frac{1+\alpha}{2}$ and $\frac{1-\beta}{2} \le w < p_2 - \alpha - \beta$ such that $D_r^{SB} = 0$ as it yields a lower profit according to the Proof of Lemma EC.1 when $p_2 > \frac{1+\alpha}{2}$. By the unimodality of the profit function, the detail decisions when $p_2 > \frac{1+\alpha}{2}$ and $w \ge s(p_2)$ are as follows: *(a) when* $r < \min\{1-\beta, c_2\}$*,* $p_2^{SB} = \underline{p}_2^{sbxh} = \frac{1+\alpha+c_2-r}{2}$ *and* $w^{SB} = \frac{1-\beta}{2}$ *such that* $D_r^{SB} > 0$ *and* $D_r^{SB} + R_r^{SB} < 1$*; (b) when* $\beta > 1 - c_2$ *and* $1 - \beta \le r \le \bar{r}^b$*,* $p_2^{SB} = p_2^{bx(0)} = \frac{\alpha+\beta+c_2}{2}$ *and* $w^{SB} = w^{bx(0)} = \frac{r}{2} \ge \frac{1-\beta}{2}$*; (c) when* $\beta > 1 - c_2$ *and* $\bar{r}^b < r < c_2$*,* $p_2^{SB} = \bar{p}_2^{bx} = \frac{\alpha+\beta+c_2}{2}$ *and* $w^{SB} = \bar{w}^{bx} = \frac{(1-\beta)(\alpha+\beta+c_2)}{2(1+\alpha)}$ *such that* $\bar{w}^{bx} = \frac{(1-\beta)\bar{p}_2^{bx}}{1+\alpha} \ge \frac{1-\beta}{2}$ *and* $D_r^{SB} + R_r^{SB} = 1$*.* Note here $\bar{r}^b < c_2$ when $\beta > 1 - c_2$.

Again, it is unnecessary to consider the case when $w = s(p_2)$ by the similar argument. And we still assume that customers return used products instead of reselling when $w \ge s(p_2)$.

Lastly, when $p_2 > \frac{1+\alpha}{2}$ and $w < s(p_2)$, $w = 0$ and the manufacturer's profit is $\pi_2^{SB}(p_2, w) = (p_2 - c_2)D_r^{SB}$ by (4). Note that $s(p_2) = \frac{1-\beta}{2} < \frac{(1-\beta)p_2}{1+\alpha}$ when $p_2 > \frac{1+\alpha}{2}$. Thus the interior solution requires $p_2 > \frac{1+\alpha}{2}$ and $s(p_2) \ge \max\{p_2 - \alpha - \beta, 0\}$, which is equivalent to $\frac{1+\alpha}{2} < p_2 \le \frac{1+2\alpha+\beta}{2}$. The interior solution for $\frac{1+\alpha}{2} < p_2 \le \frac{1+2\alpha+\beta}{2}$ is the only solution for this case, which is $p_2^{SB} = p_2^{sbxl} = \frac{1+2\alpha+\beta+2c_2}{4}$. Here $p_2^{sbxl} > \frac{1+\alpha}{2}$ iff $\beta > 1 - 2c_2$, and the manufacturer does not choose $p_2 > \frac{1+\alpha}{2}$ when $\beta \le 1 - 2c_2$. Plus, $p_2^{sbxl} \le \frac{1+2\alpha+\beta}{2}$ when $c_2 \le \frac{1+\alpha}{2}$.

We can show that $\pi_2^{SB}(p_2, w)$ is increasing in r when $p_2 > \frac{1+\alpha}{2}$ and $w \ge s(p_2)$. And $\pi_2^{SB}(\underline{p}_2^{sbxh}, \frac{1-\beta}{2}) \ge \pi_2^{SB}(p_2^{sbxl}, 0)$ iff $r \ge \frac{1-\beta}{2}$. It also implies $\pi_2^{SB}(p_2^{bx(0)}, w^{bx(0)}) \ge \pi_2^{SB}(p_2^{sbxl}, 0)$ and $\pi_2^{SB}(\bar{p}_2^{bx}, \bar{w}^{bx}) \ge \pi_2^{SB}(p_2^{sbxl}, 0)$ when $r \ge \frac{1-\beta}{2}$. Thus the optimal decision which includes $w < s(p_2)$ and $w \ge s(p_2)$ when $p_2 > \frac{1+\alpha}{2}$ is summarized as follows: *(1) when* $\beta > 1 - 2c_2$ *and* $r < \frac{1-\beta}{2}$*,* $p_2^{SB} = p_2^{sbxl} = \frac{1+2\alpha+\beta+2c_2}{4}$ *and* $w^{SB} = 0$ *such that* $D_r^{SB} > 0$ *and* $D_r^{SB} + R_r^{SB} < 1$*; (2) when* $\frac{1-\beta}{2} \le r < \min\{1-\beta, c_2\}$*,* $p_2^{SB} = \underline{p}_2^{sbxh} = \frac{1+\alpha+c_2-r}{2}$ *and* $w^{SB} = \frac{1-\beta}{2}$ *such that* $D_r^{SB} > 0$ *and* $D_r^{SB} + R_r^{SB} < 1$*; (3) when* $\beta > 1 - c_2$ *and* $1 - \beta \le r \le \bar{r}^b$*,* $p_2^{SB} = p_2^{bx(0)} = \frac{\alpha+\beta+c_2}{2}$ *and* $w^{SB} = w^{bx(0)} =$

$\frac{r}{2} \geq \frac{1-\beta}{2}$; *(4) when* $\beta > 1 - c_2$ *and* $\bar{r}^b < r < c_2$, $p_2^{SB} = \bar{p}_2^{bx} = \frac{\alpha+\beta+c_2}{2}$ *and* $w^{SB} = \bar{w}^{bx} = \frac{(1-\beta)(\alpha+\beta+c_2)}{2(1+\alpha)}$ *such that* $\bar{w}^{bx} = \frac{(1-\beta)\bar{p}_2^{bx}}{1+\alpha} \geq \frac{1-\beta}{2}$ *and* $D_r^{SB} + R_r^{SB} = 1$. Note here $\frac{1-\beta}{2} < \min\{1-\beta, c_2\}$ iff $\beta > 1 - 2c_2$.

Summary of the manufacturer's buyback policy with a secondary market. Note that $\tilde{r}^s \leq \frac{1-\beta}{2} < 1 - \beta$ and $\tilde{r}^s \leq \bar{r}^b$ when $c_2 \leq \frac{1+\alpha}{2}$. The manufacturer offers $p_2 > \frac{1+\alpha}{2}$ only when $\beta > 1 - 2c_2$, and the profit of $p_2 > \frac{1+\alpha}{2}$ is increasing in r. Moreover, we can show that $\pi_2^{SB}(\bar{p}_2^{sbl}, 0) \leq \pi_2^{SB}(p_2^{sbxl}, 0)$. Besides, there exists a $\tilde{r}^{sbx}$ such that $\pi_2^{SB}(p_2^{sbh}, w^{sbh})$ is higher than the profit of $p_2 > \frac{1+\alpha}{2}$ iff $r \geq \tilde{r}^{sbx}$. Therefore, it is straightforward to show that the manufacturer chooses $p_2 > \frac{1+\alpha}{2}$ when $\beta > 1 - 2c_2$ and $r < \tilde{r}^{sbx}$; otherwise, when $\beta \leq 1 - 2c_2$ or $r \geq \tilde{r}^{sbx}$, the manufacturer chooses $p_2 \leq \frac{1+\alpha}{2}$. Finally, Lemma EC.3 is proved.

Here we can show that $\tilde{r}^{sbx}$ is increasing in c_2, and the smallest positive value of $\tilde{r}^{sbx}$ is the solution to $\pi_2^{SB}(p_2^{sbh}, w^{sbh}) = \pi_2^{SB}(p_2^{sbxl}, 0)$ in r. And it is easy to verify that $\tilde{r}^{sbx} = 0$ when $\beta > 1 - 2c_2$ and $c_2 \leq \frac{1+\alpha}{4}$. Therefore, the manufacturer always chooses $p_2 \leq \frac{1+\alpha}{2}$, i.e., $\tilde{r}^{sbx} = 0$, when $c_2 \leq \frac{1+\alpha}{4}$. ■

Proof of Lemma EC.4 With upgraded product sales for the new customers when $p_2 \leq \frac{1+\alpha}{2}$. If the manufacturer chooses a trade-in reward $w \geq s(p_2)$, the manufacturer's profit is $\pi_2^{ST}(p_2, w) = (p_2 - c_2)D_n^{ST} + (p_2 - w - c_2 + r)D_r^{ST}$ by (5), where $D_n^{ST} = (1 - \min\{\frac{s(p_2)}{1-\beta}, \frac{p_2-w+s(p_2)}{1+\alpha}\})^+$ and $D_r^{ST} = (1 - \max\{\frac{p_2-w}{\alpha+\beta}, \frac{p_2-w+s(p_2)}{1+\alpha}\})^+$. Notice that (5) (with a secondary market and $w \geq s(p_2)$) is not the same as (2) (without a secondary market). Compared to $w < s(p_2)$, the difference of choosing $w \geq s(p_2)$ is that the high valuation customers switch from reselling to returning, which yields a unit marginal profit $p_2 - c_2 + r - w = (p_2 - w) - (c_2 - r)$. As the cost of replacement purchases $c_2 - r$ is positive when $r < c_2$, there is no incentive for the manufacturer to offer a higher refund $w > s(p_2)$. This implies $w = s(p_2)$ is optimal. In this case, customers' choice is similar to case when $p_2 \leq \frac{1+\alpha}{2}$ and $w < s(p_2)$ in the Proof of Lemma EC.3, and the only difference is that the high valuation customers exchange their used product for upgraded products by trading in, while it is not by directly reselling. Therefore, the decision on p_2 is the same as the case when $p_2 \leq \frac{1+\alpha}{2}$ and $w < s(p_2)$ in Lemma EC.3, while $w = s(p_2)$. Then the manufacturer's decisions are as follows: *(a) when* $c_2 \leq \frac{\alpha+3\beta-2}{2}$, $p_2^{ST} = \underline{p}_2^{sbl} = \frac{\alpha+\beta}{2}$ *and* $w^{ST} = s(\underline{p}_2^{sbl}) = 0$ *such that* $R_r^{ST} = 0$; *(b) when* $\frac{\alpha+3\beta-2}{2} < c_2 \leq \frac{\alpha+\beta}{2}$, $p_2^{ST} = p_2^{sbl(0)} = \frac{2+\alpha-\beta+2c_2}{4}$ *and* $w^{ST} = s(p_2^{sbl(0)}) = \frac{2-\alpha-3\beta+2c_2}{4} > 0$; *(c) when* $\frac{\alpha+\beta}{2} < c_2 \leq \frac{1+\alpha}{2}$, $p_2^{ST} = \bar{p}_2^{sbl} = \frac{1+\alpha}{2}$ *and* $w^{ST} = s(\bar{p}_2^{sbl}) = \frac{1-\beta}{2} > 0$ *such that* $D_r^{ST} + R_r^{ST} = 1$. Note here $p_2^{ST} = \underline{p}_2^{sbl} = \frac{\alpha+\beta}{2}$ means p_2^{ST} is slightly larger than $\frac{\alpha+\beta}{2}$ such that $w^{ST} = s(\underline{p}_2^{sbl})$ is slightly higher than 0.

If the manufacturer chooses a trade-in reward $w < s(p_2)$, customers resell used products directly. The problem is the same as analyzed in the Proof of Lemma EC.3 when $p_2 \leq \frac{1+\alpha}{2}$ and $w < s(p_2)$.

It is easy to verify that $\pi_2^{ST}(p_2^{ST}, s(p_2^{ST})) \geq \pi_2^{ST}(p_2^{ST}, 0)$ iff $r \geq \tilde{r}^s$. Recall that $\tilde{r}^s = \max\{c_2 - \frac{\alpha+\beta}{2}, 0\}$ defined in the Proof of Lemma EC.3 when $p_2 \leq \frac{1+\alpha}{2}$, and $\tilde{r}^s > 0$ iff $c_2 > \frac{\alpha+\beta}{2}$. Then it is straightforward to have the manufacturer's decision when $p_2 \leq \frac{1+\alpha}{2}$ which is summarized in the second case of Lemma EC.4.

Without upgraded product sales for the new customers when $p_2 > \frac{1+\alpha}{2}$. When $p_2 > \frac{1+\alpha}{2}$, the new customers do not buy upgraded products. When $w \geq s(p_2)$, note that the new customers do not buy upgraded products when $p_2 > \frac{1+\alpha}{2}$ and the profit only comes from the trade-in purchases of the repeat customers.

Hence, the profit is similar to the case in Lemma EC.2 when $p_2 > \frac{1+\alpha}{2}$ and $w \geq s(p_2)$, i.e., $\pi_2^{ST}(p_2,w) = (p_2 - w - c_2 + r)D_r^{ST}$ by (5), but with extra constraints $w > s(p_2) = \frac{1-\beta}{2}$. Note $D_r^{ST} = D_r^T$ when $w \geq s(p_2)$ and iff $w \leq p_2 - \frac{\alpha+\beta}{1-\beta}s(p_2) = p_2 - \frac{\alpha+\beta}{2}$, or equivalently $\frac{p_2-w}{\alpha+\beta} \geq \frac{p_2-w+s(p_2)}{1+\alpha}$. And recall that $D_r^T > 0$ iff $w > p_2 - \alpha - \beta$, and $D_r^T \leq 1$ as $w \leq p_2$. The interior solution for $p_2 > \frac{1+\alpha}{2}$ and $\max\{s(p_2), p_2 - \alpha - \beta\} \leq w \leq p_2 - \frac{\alpha+\beta}{2}$ is $p_2^{ST} = p_2^{tx(0)}$ and $w^{ST} = w^{tx(0)}$ which satisfies $p_2^{tx(0)} - w^{tx(0)} = \frac{\alpha+\beta+c_2-r}{2}$, $p_2^{tx(0)} > \frac{1+\alpha}{2}$ and $w^{tx(0)} \geq \frac{1-\beta}{2}$. $w^{tx(0)} \geq p_2^{tx(0)} - \alpha - \beta$ iff $r \geq c_2 - (\alpha+\beta)$ and $\pi_2^{ST}(p_2,w) = 0$ when $r < c_2 - (\alpha+\beta)$ by the unimodality; and $w^{tx(0)} \leq p_2^{tx(0)} - \frac{\alpha+\beta}{2}$ as $r < c_2$.

When $w < s(p_2)$, it is equivalent to the problem in the Proof of Lemma EC.3 when $p_2 > \frac{1+\alpha}{2}$ and $w < s(p_2)$, which is $p_2^{ST} = p_2^{sbxl} = \frac{1+2\alpha+\beta+2c_2}{4}$ and $p_2^{ST} = 0$. Recall $p_2^{sbxl} > \frac{1+\alpha}{2}$ iff $\beta > 1 - 2c_2$.

We can show that $\pi_2^{ST}(p_2^{tx(0)}, w^{tx(0)}) \geq \pi_2^{ST}(p_2^{sbxl}, 0)$ iff $r \geq \frac{1-\beta}{2}$. $\frac{1-\beta}{2} \geq c_2 - (\alpha+\beta)$ as $c_2 \leq \frac{1+\alpha}{2}$. Let $\underline{r}^{sthx} = \begin{cases} c_2 - (\alpha+\beta) & \text{, when } \beta \leq 1 - 2c_2 \\ \frac{1-\beta}{2} & \text{, when } \beta > 1 - 2c_2 \end{cases}$. Then the decision when $p_2 > \frac{1+\alpha}{2}$ is summarized as follows: *(1) when $\beta > 1 - 2c_2$ and $r < \underline{r}^{sthx}$, $p_2^{ST} = p_2^{sbxl} = \frac{1+2\alpha+\beta+2c_2}{4}$ and $w^{ST} = 0$; (2) when $\underline{r}^{sthx} \leq r < c_2$, $p_2^{ST} = p_2^{tx(0)}$ and $w^{ST} = w^{tx(0)}$ which satisfies $p_2^{tx(0)} - w^{tx(0)} = \frac{\alpha+\beta+c_2-r}{2}$, $p_2^{tx(0)} > \frac{1+\alpha}{2}$ and $w^{tx(0)} \geq \frac{1-\beta}{2}$.*

Summary of the manufacturer's trade-in policy with a secondary market. Similar to the Proof of Lemma EC.3, we can show that there exists a $\tilde{r}^{stx}$ such that the manufacturer chooses $p_2 > \frac{1+\alpha}{2}$ iff $r < \tilde{r}^{stx}$. Specifically, note that $\tilde{r}^s > c_2 - (\alpha+\beta)$ and $\tilde{r}^s \leq \frac{1-\beta}{2}$. And we can show that $\pi_2^{ST}(\bar{p}_2^{sbl}, 0) \geq \pi_2^{ST}(p_2^{tx(0)}, w^{tx(0)})$ when $\beta \leq 1 - 2c_2$ and $r \leq \tilde{r}^s$, which implies $\tilde{r}^{stx} = 0$ when $\beta \leq 1 - 2c_2$, or equivalently $c_2 \leq \frac{1-\beta}{2}$. Thus $r < \tilde{r}^{stx}$ is only feasible when $c_2 > \frac{1-\beta}{2}$, where $\underline{r}^{sthx} = \frac{1-\beta}{2}$. Therefore, Lemma EC.4 is proved.

We can show that $\tilde{r}^{stx}$ is increasing in c_2 and recall $\tilde{r}^{stx} = 0$ when $\beta \leq 1 - 2c_2$. Plus, $\pi_2^{ST}(\bar{p}_2^{sbl}, 0) < \pi_2^{ST}(p_2^{sbxl}, 0)$ when $r < \tilde{r}^s$. Note $\tilde{r}^s > 0$ iff $c_2 > \frac{\alpha+\beta}{2}$, and $\frac{1-\beta}{2} \geq \frac{\alpha+\beta}{2}$ iff $\beta \leq \frac{1-\alpha}{2}$. Hence, when $\beta \leq \frac{1-\alpha}{2}$, $\tilde{r}^{stx} = 0$ when $c_2 \leq \frac{1-\beta}{2}$. When $\beta > \frac{1-\alpha}{2}$, the lowest positive value of $\tilde{r}^{stx}$ is the solution to $\pi_2^{ST}(p_2^{sbl(0)}, s(p_2^{sbl(0)})) = \pi_2^{ST}(p_2^{sbxl}, 0)$ in r. There exists a $\bar{c}_2^{st}$ such that $\tilde{r}^{stx} > 0$ iff $c_2 > \bar{c}_2^{st}$, where $\bar{c}_2^{st}$ is increasing in β. Note $\frac{1-\beta}{2} \geq \frac{1+\alpha}{4}$ when $\beta \leq \frac{1-\alpha}{2}$, and we can show that $\bar{c}_2^{st} > \frac{1+\alpha}{4}$ when $\beta > \frac{1-\alpha}{2}$. Therefore, the manufacturer always chooses $p_2 \leq \frac{1+\alpha}{2}$, i.e., $\tilde{r}^{stx} = 0$, when $c_2 \leq \frac{1+\alpha}{4}$. ■

EC.3 Proofs of (Un)conditional Collection Policies on Used Products with Strategic Customers

Table EC.1 Definitions of Thresholds

$\tilde{r}^b = \max\{c_2 - (\alpha+\beta)(1 - \frac{\lvert 1-\beta-c_2\rvert}{\sqrt{(1+\alpha)(\alpha+\beta)}}), 0\}$
$\underline{r}^b = \max\{\frac{(1-\beta)[c_2-(1-\beta)]}{1+\alpha}, 0\}$
$\bar{r}^b = \frac{(1-\beta)(\alpha+\beta+c_2)}{1+\alpha}$
$\tilde{r} = \max\{\frac{(1-\beta)(c_2-\alpha-\beta)}{1-\alpha-2\beta}, 0\}$
$\underline{r} = \frac{[(1-\beta)+\sqrt{(1-\beta)(\alpha+\beta)}](c_2-\alpha-\beta)}{1-\alpha-2\beta}$
$\hat{r} = \sqrt{\frac{1-\beta}{1+\alpha}}(1-\beta-c_2)$
$\bar{r} = \sqrt{\frac{1-\beta}{\delta+2} \cdot \frac{(1-\beta-c_2)[2(1-\beta-c_2)+(2\alpha+3\beta-1-c_2)\delta]}{1+\alpha}}$
$\tilde{r}^e = \max\{\min\{-\frac{(1-\beta)(c_2-(1-\beta))k_2}{(1+\alpha)k_r}, \frac{c_2k_2+(\alpha+\beta+c_2)k_r}{k_2+k_r}\}, c_2-(1-\beta)\}$
$\tilde{c}_2^e = \frac{2(1-\beta)k_r-(2-\alpha-3\beta)k_2}{2k_2}$

Proof of Lemma 1 Lemma 1 is a special case of Lemma EC.1 when $c_2 \le \frac{1+\alpha}{4}$ such that $\tilde{r}^{bx} = 0$ and the manufacturer chooses $p_2 \le \frac{1+\alpha}{2}$. Note that $\underline{r}^b \le 0$ and $\bar{r}^b \ge c_2$ when $1-\beta \ge c_2$, and $\tilde{r}^b < 0$ when $1-\beta < \frac{1+\alpha}{2}$. Plus $c_2 \le \frac{1+\alpha}{2}$. Hence it is natural to change the conditions $1-\beta < \frac{1+\alpha}{2}$ and $1-\beta \ge \frac{1+\alpha}{2}$ in Lemma EC.1 to $1-\beta < c_2$ and $1-\beta \ge c_2$ in Lemma 1, respectively. Hereby, Lemma 1 is proved. ■

Proof of Lemma 2 Lemma 2 is a special case of Lemma EC.2 when $c_2 \le \frac{1+\alpha}{4}$ such that $\tilde{r}^{tx} = 0$ and the manufacturer chooses $p_2 \le \frac{1+\alpha}{2}$. By the similar reason in the Proof of Lemma 1, the conditions $1-\beta < \frac{1+\alpha}{2}$ and $1-\beta \ge \frac{1+\alpha}{2}$ in Lemma EC.2 can be revised to $1-\beta < c_2$ and $1-\beta \ge c_2$ in Lemma 2, respectively. Then Lemma 2 is proved. ■

Proof of Lemma 3 First, we can show that $c_2 - 1 + \beta \ge \underline{r}^b$ and $c_2 - \alpha - \beta \le \tilde{r}^b$. It is easy to show that $w^{t(0)} \ge w^{b(0)}$ (and $w^{t(0)} \ge \tilde{w}^b = \frac{r}{2}$) iff $1-\beta \ge c_2$. $w^B = \tilde{w}^b = \frac{r}{2}$ and $w^T = 0$ when $1-\beta \ge c_2$ and $r < c_2 - \alpha - \beta$. Based on Lemmas 1 and 2, it is easy to show that $w^T \ge w^B$ when $1-\beta \ge c_2$ and $r \ge c_2 - \alpha - \beta$, and $w^T \le w^B$ otherwise.

Note that the used products are only collected from replacement purchases under a trade-in policy, while there are additional purely returned products under a buyback policy. Then it is straightforward to see that $D_r^T < D_r^B + R_r^B$ when $r > \tilde{r}^b$. When $r \le \tilde{r}^b$, it is easy to show that $D_r^T \ge D_r^B + R_r^B$ iff $r \ge \frac{(1-\beta)(c_2-\alpha-\beta)}{1-\alpha-2\beta}$. Thus $D_r^T \ge D_r^B + R_r^B$ iff $\frac{(1-\beta)(c_2-\alpha-\beta)}{1-\alpha-2\beta} \le r \le \tilde{r}^b$. Let $\tilde{r} = \frac{(1-\beta)(c_2-\alpha-\beta)}{1-\alpha-2\beta}$, Lemma 3 is proved. ■

Proof of Proposition 1 The manufacturer chooses the collection policy which maximizes its second-period profit.

Recall that $c_2 - (1-\beta) \ge \underline{r}^b$ and $c_2 - (\alpha+\beta) \le \tilde{r}^b$ from the Proof of Lemma 3. Based on Lemmas 1 and 2, for the case when $1-\beta < c_2$, when $r \le \underline{r}^b$, no refund is offered (w = 0) and $\pi_2^B(\frac{1+\alpha}{2}, 0) = \pi_2^T(\frac{1+\alpha}{2}, 0)$; thus $(y^*, p_2^*, w^*) = (B \text{ or } T, \frac{1+\alpha}{2}, 0)$. When $\underline{r}^b < r \le \bar{r}^b$, we can show that $\pi_2^B(\frac{1+\alpha}{2}, \frac{r}{2} + \frac{(1-\beta)(1-\beta-c_2)}{2(1+\alpha)}) > \pi_2^T(p_2^T, w^T)$;

thus $(y^*, p_2^*, w^*) = (B, \frac{1+\alpha}{2}, \frac{r}{2} + \frac{(1-\beta)(1-\beta-c_2)}{2(1+\alpha)})$. When $r > \bar{r}^b$, it is still $\pi_2^B(\frac{1+\alpha}{2}, \frac{1-\beta}{2}) > \pi_2^T(p_2^T, w^T)$, which results in $(y^*, p_2^*, w^*) = (B, \frac{1+\alpha}{2}, \frac{1-\beta}{2})$.

For the case when $1-\beta \geq c_2$, when $r \leq \tilde{r}^b$, $\pi^B(\frac{1+\alpha}{2}, \frac{r}{2}) \geq \pi^T(p_2^T, w^T)$ iff $r \leq \underline{r}$. When $r > \tilde{r}^b$, $\pi^B(\frac{1+\alpha}{2}, \frac{r}{2} + \frac{(1-\beta)(1-\beta-c_2)}{2(1+\alpha)}) < \pi^T(\frac{1+\alpha}{2}, \frac{1-\beta-c_2+r}{2})$ iff $r < \hat{r}$. Moreover, we can show that there exists a threshold $\tilde{\beta}$ such that $\underline{r} \leq \tilde{r}^b$ (and $\hat{r} \geq \tilde{r}^b$) iff $\beta \geq \tilde{\beta}$. Then it is straightforward to have Proposition 1(ii).

To sum up, Proposition 1 is proved. ■

Proof of Proposition 2 Based on Definition 1 and Proposition 1, it is straightforward to have Proposition 2. ■

Proof of Lemma 4 Lemma 4 is a special case of Lemma EC.3 when $c_2 \leq \frac{1+\alpha}{4}$ such that $\tilde{r}^{sbx} = 0$ and the manufacturer chooses $p_2 \leq \frac{1+\alpha}{2}$. ■

Proof of Lemma 5 Lemma 5 is a special case of Lemma EC.4 when $c_2 \leq \frac{1+\alpha}{4}$ such that $\tilde{r}^{stx} = 0$ and the manufacturer chooses $p_2 \leq \frac{1+\alpha}{2}$. ■

Proof of Lemma 6 By Lemmas 4 and 5, it is easy to see that $w^{SB} > w^{ST}$ when $c_2 \leq \frac{\alpha+\beta}{2}$ and $w^{SB} = w^{ST}$ otherwise, which implies $w^{SB} \geq w^{ST}$. When $r < c_2 - \frac{\alpha+\beta}{2}$, $w^{SB} = w^{ST} = 0$ and there is no product returned to the manufacturer under either case. When $c_2 - \frac{\alpha+\beta}{2} < r < c_2$, it is easy to show that $D_r^{SB} + R_r^{SB} > D_r^{ST}$. Hereby, Lemma 6 is proved.

Proof of Proposition 3 By Lemmas 4 and 5, when $r < c_2 - \frac{\alpha+\beta}{2}$, no refund is offered ($w = 0$) and $\pi^{SB}(\frac{1+\alpha}{2}, 0) = \pi^{ST}(\frac{1+\alpha}{2}, 0)$; thus $(y^{S*}, p_2^{S*}, w^{S*}) = (B \text{ or } T, \frac{1+\alpha}{2}, 0)$. When $c_2 - \frac{\alpha+\beta}{2} \leq r < c_2$, we can show that $\pi^{SB}(\frac{1+\alpha}{2}, \frac{1-\beta}{2}) - \pi^{ST}(p_2^{ST}, w^{ST})$ is increasing in r and $\pi^{SB}(\frac{1+\alpha}{2}, \frac{1-\beta}{2}) = \pi^{ST}(p_2^{ST}, w^{ST})$ when $r = \max\{c_2 - \frac{\alpha+\beta}{2}, 0\}$; thus $(y^{S*}, p_2^{S*}, w^{S*}) = (B, \frac{1+\alpha}{2}, \frac{1-\beta}{2})$. ■

Proof of Proposition 4 Based on Definition 1 and Proposition 3, it is straightforward to have Proposition 4. ■

Proof of Proposition 5 Based on Proposition 3, the buyback policy weakly dominates the trade-in policy with the presence of a secondary market, while it is optimal to choose the trade-in policy when $1-\beta \geq c_2$ and $\underline{r} < r < \hat{r}$ according to Proposition 1. Thus it is straightforward to see that the manufacturer is inclined to implement the buyback policy with the presence of the secondary market.

Note that $c_2 - \frac{\alpha+\beta}{2} > 0$ iff $1-\beta > 1+\alpha-2c_2$, where $1+\alpha-2c_2 \geq c_2$ when $c_2 \leq \frac{1+\alpha}{4}$. Thus $c_2 - \frac{\alpha+\beta}{2} > 0$ implies $1-\beta > c_2$. Then by Propositions 1 and 3, it is easy to see that $w^{S*} < w^*$ when $r < c_2 - \frac{\alpha+\beta}{2}$ as $w^{S*} = 0$ while $w^* > 0$. When $r \geq c_2 - \frac{\alpha+\beta}{2}$, $w^{S*} = \frac{1-\beta}{2}$ and w^* is increasing in r; besides, $w^{S*} = w^* = \frac{1-\beta}{2}$ when $r > \bar{r}^b$. Hence, it is easy to have $w^{S*} \geq w^*$ iff $r \geq c_2 - \frac{\alpha+\beta}{2}$.

Secondly, $p_2^{S*} = p_2^* = \frac{1+\alpha}{2}$.

Lastly, p_1^{S*} is independent of r as (p_2^{S*}, w^{S*}) is independent of r, while we can show that p_1^* is increasing in r; and $p_1^{S*} = p_1^*$ when $\bar{r}^b \leq r < c_2$ as the manufacturer's decisions on (y, p_2, w) are the same regardless of the existence of the secondary market. Then it is straightforward to see that $p_1^{S*} > p_1^*$ when $r < \bar{r}^b$ and $p_1^{S*} = p_1^*$ when $\bar{r}^b \leq r < c_2$, i.e., $p_1^{S*} \geq p_1^*$. ■

To prove Proposition 6, the manufacturer's optimal collection policy commitment with and without a secondary market are presented in the following two lemmas beforehand, respectively.

LEMMA EC.5. *In the absence of a secondary market,*

(i) when $1-\beta<c_2$, the manufacturer announces a buyback policy in the first period if $r>\underline{r}^b$, and no collection policy is offered otherwise;

(ii) when $1-\beta\geq c_2$, the manufacturer announces a trade-in policy in the first period if $\underline{r}<r<\max\{\bar{r},\tilde{r}^b\}$, and a buyback policy otherwise.

Proof of Lemma EC.5 It is important to note that for any given collection policy (buyback or trade-in), the manufacturer's second-period decisions, as well as the other corresponding terms like the second-period and total profits, are the same regardless of whether the policy is announced in the first or the second period. The corresponding decisions are demonstrated in Lemmas 1, 2, 4 and 5.

Let Π^{CB} (Π^{CT}) be the manufacturer's total profit when a buyback (trade-in) policy is announced in the first period in the absence of a secondary market. The manufacturer's maximal total profit is denoted by $\Pi^{C*}=\max\{\Pi^{CB},\Pi^{CT}\}$, and also use the superscript $C*$ to denote the corresponding optimal decisions and terms in the following. It is easy to show that $\Pi^{CT}=\Pi^{CB}$ when $r\leq\underline{r}^b$ as $(p_2^*,w^*)=\left(\frac{1+\alpha}{2},0\right)$ such that no customer returns for both policies according to Lemmas 1 and 2. Moreover, we can show that when $r\leq\tilde{r}^b$, $\Pi^{CT}\geq\Pi^{CB}$ iff $r\geq\underline{r}$. When $r>\max\{\tilde{r}^b,\underline{r}^b\}$, $\Pi^{CT}\geq\Pi^{CB}$ iff $r\leq\bar{r}$. Hereby, Lemma EC.5 is proved. ■

LEMMA EC.6. *With the presence of a secondary market, the manufacturer chooses a buyback policy if $r>c_2-\frac{\alpha+\beta}{2}$, and no collection policy is offered otherwise.*

Proof of Lemma EC.6 Let Π^{CSB} (Π^{CST}) be the manufacturer's total profit when a buyback (trade-in) policy is announced in the first period with the presence of a secondary market. The manufacturer's maximal total profit is denoted by $\Pi^{CS}=\max\{\Pi^{CSB},\Pi^{CST}\}$, and also use the superscript CS to denote the corresponding optimal decisions and terms in the following. For the similar reason as explained in the Proof of Lemma EC.5, by Lemmas 4 and 5, when $r\leq c_2-\frac{\alpha+\beta}{2}$, no refund is offered ($w=0$), and the buyback and trade-in policies are equivalent and yield the same profit, i.e., $\Pi^{CSB}=\Pi^{CST}$. When $c_2-\frac{\alpha+\beta}{2}<r<c_2$, we can show that $\Pi^{CSB}-\Pi^{CST}$ is increasing in r and $\Pi^{CSB}\geq\Pi^{CST}$ when $r=\max\{c_2-\frac{\alpha+\beta}{2},0\}$. Thus $\Pi^{CSB}>\Pi^{CST}$ when $r>c_2-\frac{\alpha+\beta}{2}$. ■

Proof of Proposition 6 First of all, according to the explanation in the Proof of Lemma EC.5, it is easy to see that the manufacturer obtains the same profit when the same policy (either buyback or trade-in) is chosen in equilibrium no matter the policy is announced in the first or the second period.

Based on Proposition 1 and Lemma EC.5, when $1-\beta<c_2$, it is easy to see that $y^{C*}=y^*$, and thus $\Pi^{C*}=\Pi^*$, when $r\leq\min\{\hat{r},\bar{r},\tilde{r}^b\}$ or $r\geq\max\{\hat{r},\bar{r}\}$. Moreover, it is easy to verify that $\hat{r}\leq\bar{r}$ iff $1-\beta\leq\frac{1+\alpha}{2}$. When $1-\beta\leq\frac{1+\alpha}{2}$ and $\hat{r}<r<\bar{r}$, $y^{C*}=T$ and $y^*=B$. And we can show that $\Pi^{C*}\geq\Pi^*$. When $1-\beta>\frac{1+\alpha}{2}$ and $\max\{\bar{r},\tilde{r}^b\}<r<\hat{r}$, $y^{C*}=B$ and $y^*=T$. And it is still $\Pi^{C*}\geq\Pi^*$.

When $1-\beta \geq c_2$, it is easy to see $\Pi^{C*} = \Pi^*$ as $y^{C*} = y^*$.

To sum up, Proposition 6 is proved.

Moreover, for the case when the secondary market exists, $y^{CS} = y^{S*} = B$ or T when $r < c_2 - \frac{\alpha+\beta}{2}$ and $y^{CS} = y^{S*} = B$ otherwise by Proposition 3 and Lemma EC.6. It is easy to see that $\Pi^{CS} = \Pi^{S*}$. ■

Proof of Proposition 7 Let I^B and I^T denote the environmental impacts of the buyback and trade-in policies without a secondary market respectively. Recall that $c_2 - (1-\beta) \geq \underline{r}^b$ and $c_2 - (\alpha+\beta) \leq \tilde{r}^b$ from the Proof of Lemma 3. Then it is easy to see that $I^B = I^T$ when $r \leq \underline{r}^b$ as $D_r^B = D_r^T$, $D_n^B = D_n^T = 1$, and $R_r^B = 0$. When $\underline{r}^b < r \leq c_2 - (1-\beta)$. We can show that $I^B > I^T$ iff $1-\beta > \frac{(1+\alpha)k_r}{k_2}$, and note that $c_2 - (1-\beta) > 0$ iff $1-\beta < c_2$. When $r \leq \tilde{r}^b$, it is easy to see $I^B < I^T$ as $D_r^B = 0$ and $k_r < k_2$. When $\max\{c_2 - (1-\beta), \tilde{r}^b\} < r \leq \bar{r}^b$, we can show that $I^B > I^T$ iff $r < \frac{(1-\beta)(c_2-(1-\beta))k_2}{(1+\alpha)k_r}$. When $\max\{c_2 - (1-\beta), \bar{r}^b\} < r < c_2$, we can show that $I^B > I^T$ iff $r < \frac{c_2k_2-(\alpha+\beta+c_2)k_r}{k_2-k_r}\}, c_2 - (1-\beta)\}$. Let $\tilde{r}^e = \max\{\min\{\frac{(1-\beta)(c_2-(1-\beta))k_2}{(1+\alpha)k_r}, \frac{c_2k_2-(\alpha+\beta+c_2)k_r}{k_2-k_r}\}, c_2-(1-\beta)\}$, then Proposition 7 is proved. ■

Proof of Proposition 8 Let I^{SB} and I^{ST} denote the environmental impacts of the buyback and trade-in policies with a secondary market respectively. First, it is easy to see that $r \leq c_2 - \frac{\alpha+\beta}{2}$ as $(p_2^{SB}, w^{SB}) = (p_2^{ST}, w^{ST}) = (\frac{1+\alpha}{2}, 0)$ and customer only resell used product under both policies. When $r > c_2 - \frac{\alpha+\beta}{2}$, I^{SB} is independent of c_2 as $D_r^{SB} = \frac{1}{2}$, $R_r^{SB} = \frac{1}{2}$ and $D_n^{SB} = 1$ are all constant. Plus, it is easy to show that I^{ST} is decreasing in c_2 as $D_r^{ST} = \frac{1}{2}$ and D_n^{ST} is decreasing in c_2. Thus it is easy to show that $I^{SB} > I^{ST}$ iff $c_2 > \tilde{c}_2^e$, where $\tilde{c}_2^e = \frac{2(1-\beta)k_r-(2-\alpha-3\beta)k_2}{2k_2}$. Hereby, Proposition 8 is proved. ■