Integrating interactions between target users and opinion leaders for better recommendations: An opinion dynamics approach

Lijuan Weng^a, Qishan Zhang^a, Zhibin Lin^b, Ling Wu^{c,*}, Jin-Hua Zhang^a

^a School of Economics and Management, Fuzhou University, Fuzhou, China

^b Durham University Business School, Mill Hill Lane, Durham DH1 3LB, United Kingdom

^c College of Computer and Data Science, Fuzhou University, Fuzhou, China

Abstract: The social recommender system can accurately recommend information to users, according to their interests based on the characteristics of their social network, however, the interaction between users has not been fully captured in the existing social recommender systems. This study contributes to the literature by proposing a social recommendation method on the basis of opinion dynamics, which captures the information on the interactions between target users and opinion leaders. In our model, the impact of opinion leaders and the evolutionary opinion dynamics between opinion leaders and the target user are integrated to make a recommendation. Experiments based on two real rating datasets, Epinions and FilmTrust were conducted to test the proposed model. The results show that our proposed method can effectively solve the cold-start problem and outperforms the baseline models.

E-mail address: M180710006@fzu.edu.cn (L. Weng), zhangqs136@163.com (Q. Zhang), zhibin.lin@durham.ac.uk (Z. Lin), wuling1985@fzu.edu.cn (L. Wu), zhangjin hua@qq.com (J.-H. Zhang).

^{*} Corresponding author.

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1. Introduction

As an information filtering technology, the recommender system has been widely adopted on various digital platforms in recent years because it can improve service quality and customer satisfaction [1-4]. First proposed by Resnick and Varian [5], the recommendation methods have continuously improved and become increasingly sophisticated, because there is so much auxiliary information that needs to be considered in order to solve the problems faced by the traditional recommender systems [6-8]. Users often ask their friends on social media for information or opinions about products and services [4,5], which means that users' social network relationships can help them to filter information. Therefore, scholars have attempted to integrate users' social relationship information into the traditional recommendation framework in order to better recommend items to the target user who want to find items [9-14].

Various social recommendation algorithms based on the matrix factorization model have been proposed [15-17], including Soreg [18], Sorec [19], SocialMF [20], RSTE [21]. These algorithms factorize the social relationship matrix between users, so as to use friends' preference information for the recommendation. They are usually applied in recommendation scenarios that are based on items' rating prediction (such as on the movies, music, and e-commerce platforms). However, they have largely ignored the impact of the opinion leader on the target user [22-25]. Recent opinion leaders-based studies (e.g., OLSR) [26] hold that users' ratings of items are largely influenced

by opinion leaders. Despite that some recent studies have reviewed opinion leadersbased algorithms [27-29], most of the existing works have neglected the opinion interaction behavior between users in online social networks [30-33]. Intuitively, users are more likely to consult and communicate with trusted friends when selecting items, rather than searching for information independently. According to the opinion fusion rule [34], each user communicates with other users and considers the opinions of other users with a certain weight in the opinions update process. Recently, some scholars have considered the interactive behavior between users on the basis of traditional recommender systems [35,36], yet studies on a dynamic recommendation method based on interactive behavior between the target users and opinion leaders remain scarce.

This study proposes a novel method for Social Recommendation based on Opinion Dynamics (ODSR), by integrating the information on the interactive behavior between target users and opinion leaders into the original OLSR method for better recommendations. Opinion dynamics [37] is a problem of many dynamic processes in complex networks, with a focus on the evolution of opinion interaction between individuals in social networks, which include three core elements: opinion expression formula, opinion dynamic environment, and fusion rules. In the process of evolutionary opinion interaction, individuals update and merge each other's opinions on an issue according to a certain rule, and eventually reach a stable structure, either consensus, polarization, or division. Specifically, users express their opinions on a certain issue through some expression formula and then update their opinions repeatedly according to the fusion rules. Finally, all users' opinions form a stable structure. The bounded confidence model takes into account the influence of psychological factors, and considers that the opinions of individuals are only influenced by the opinion of others whose difference is smaller than a particular level of confidence. The Deffuant-Weisbuch (DW) [38] is a typical one, which assumes that every two individuals interact when the interaction threshold is met. Considering this kind of social interaction behavior in the recommendation process can effectively depict the real activities of users in the real world. Therefore, we propose to integrate the idea of the DW model within recommender systems.

The innovation of the proposed method is that we consider the interactive behavior between target users and opinion leaders may potentially affect target users' decisionmaking on the item, and the interactive behavior is modeled by the opinion dynamics. In addition, this paper emphasizes the role of the interaction between target users and opinion leaders in enhancing recommendation performance, which is verified by experiments. In the experimental process of this study, two real datasets containing rating and trust information, FilmTrust and Epinions, are adopted to assess the recommendation performance of ODSR. The results show that ODSR can improve the entire quality by overcoming cold-start users and sparsity issues and offering a highly accurate recommendation result, where the cold-start users refer to new users cannot be accurately recommended due to lack of basic information and historical records.

The rest of this paper is structured as follows. Section 2 reviews the related works. Section 3 describes the method proposed in this paper. Following this, we present the experiment and its results in Section 4. Finally, we summarize the study and offer future research directions in Section 5.

2. Related works

2.1. Recommendation based on social information

Most of the exiting social recommendation methods predict the users' ratings of items by adopting the preferences of users' friends, thereby incorporating social relationship information into the traditional recommendation model [39]. Li et al. [10] designed a novel strategy for mining the implicit relationships of users, and proposed a method of using implicit relationships to make recommendations. Hsu et al. [40] proposed a unified model to combine the explicit and implicit social relationship, and optimized it to learn social relevance and rating prediction together, thus promoting each other's performance. Li et al. [41] designed a new recommendation framework to overcome the cold-start and long-tail problems. For the cold-start problem of new users, they use auxiliary information of user attributes, user social relations, and others. The long-tail problem refers to the issue that most users are only interested in popular items, with only a small number of users interested in unpopular items, and resulting in an impact on the overall click rate. To solve this problem, the authors decomposed all items of interest into two parts: the low-rank part and the sparse part, which are displayed separately during the training phase and transformed into recommendations for the new users. Noh et al. [42] proposed a new approach based on the clusters of social trust relationships to enhance the recommendation performance.

2.2. Recommendation based on opinion leaders

Although the above recommendation algorithms based on social information can

improve the recommendation performance to a certain extent, they do not consider the influence of opinion leaders on recommendation results. Opinion leaders play a key role in information dissemination and user decision-making, and they can influence and shape the opinions of users [27, 28]. Therefore, scholars have further advanced social recommendation methods based on social roles, particularly the influence of opinion leaders [29, 43-45]. Turcotte et al. [29] examined the impact of opinion leaders on news recommendations and showed that opinion leaders improved users' desire for news information. Mohammadi et al. [43] used opinion leaders to overcome the cold-start problem (we named this method SNOL and used it as one of the baselines in our experiment). The opinion leader refers to people whose opinions have a significant influence on other users on the social network. In this way, when new users log in and the rating matrix is sparse, the opinion leader can be utilized to provide the new users with accurate recommendations. Wang et al. [44] proposed a graph-based end-to-end neural model-GoRec, to model the diffusion process of key opinions and to improve the recommendation result. Pasricha et al. [45] used the user's interest, preference, age, and attributes available online to identify opinion leaders and improve the recommendations.

2.3. Recommendation based on opinion dynamics

The above recommendation algorithms largely ignore the interactive behavior among users. Yet the interactive behavior between users is ubiquitous in real life, and the dynamic opinions between users are of great significance. Jiang et al. [30] modeled the dynamic diffusion process of information by using the theoretical framework of the evolutionary game. Das et al. [31] proposed a nuanced model-Biased Voter method to model how users modify their opinions in response to the opinions of neighbors and the framework of the whole opinion network.

Considering that users communicate with other users for a specific item and update each other's opinions based on a social influence framework, some scholars introduced opinion dynamics into the recommender system to enhance the recommendation performance. Xiong et al. [35] creatively integrated evolutionary opinion dynamics into the social recommender system, and then proposed an evolutionary game model to describe opinion interaction between users. Castro et al. [36] developed a group recommender system by considering the group members' relationships and opinion dynamics.

The above three types of social recommendation algorithms have both advantages and disadvantages, which are summarized in Table 1.

Algorithm types	Advantages	Disadvantages		
Recommendations based on social information [10, 40- 42]	Make full use of social information, such as explicit information and implicit information, to enhance the accuracy of recommendations to a certain extent.	The influence of opinion leaders on recommendation results is not considered.		
Recommendations based on opinion leaders [29, 43-45]	Differentiate the influence between users by identifying opinion leaders, and consider their impact on recommendation performance.	The interactive behavior among users is largely ignored.		
Recommendations based on opinion dynamics [35, 36]	Effectively depict the real activities of users in the real world by considering the interactive behavior among uses in the recommendation process, which is more in line with the actual recommendation and can effectively improve the recommendation performance.	There is still a lack of research on integrating opinion interactions into the recommender system, especially the interaction between the target user and opinion leaders.		

Table 1 Comparison of social recommendation algorithms

3. Proposed method

Our proposed method, the ODSR, takes into consideration the interactive behavior between the target user and opinion leaders. Fig 1 shows the structure of ODSR. First, the opinion leaders are identified based on users' social relationships [26]. Then, the idea of DW [38] on the opinion dynamics is introduced to model the interactive behavior between target users and opinion leaders. Finally, building upon the Probabilistic Matrix Factorization method (PMF) [46], the influence of opinion leaders and their evolving opinion dynamics with the target user are integrated to make recommendations.



Fig. 1. The structure of ODSR

3.1. Identification of opinion leaders

It is assumed that the relationship between users is represented by a graph G = (U, E), where $U = \{u_i\}_{i=1}^n$ denotes a set of users and E denotes an association relationship between users, n is the number of users. As shown in Fig. 2, the number of users and connections between users are six and eight, respectively.



Fig. 2 Association diagram

Firstly, we obtain the user's adjacency matrix A by transforming the user's social relationship network in graph G, see Equation (1). Based on matrix A, the network topology is analyzed to determine the top l influential users in the group, that is, l opinion leaders.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \text{ where } a_{if} = \begin{cases} 1, \text{ if } user \ i \text{ and } user \ f \text{ are not connected} \\ 0, \text{ if } user \ i \text{ and } user \ f \text{ are not connected} \end{cases}$$
(1)

According to the adjacency matrix A, we can get the state transition probabilistic matrix C, see Equation (2). If the user i is associated with more (fewer) users, the transition probability between them is smaller (larger). That is, if the user i is only associated with the user f and is no longer associated with other users, the user f is an important user for the user i, so the value of the transition probability c_{if} is larger. For example, user u_6 only connects to the user u_3 in Fig. 2, so the transition probability c_{63} is larger than other values.

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix}, \text{ where } c_{if} = \frac{a_{if}}{\sum_{k=1}^{n} a_{ik}}.$$
 (2)

The initial influence score of the user on other users is equal to 1, and the user's

influence matrix S is shown in Equation (3).

$$S = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & & \vdots \\ s_{n1} & s_{n2} & \cdots & s_{nn} \end{bmatrix}, \text{ where the initial value } s_{if} = 1$$
(3)

Then, we can get the following user's influence limiting matrix S^* based on the

matrix C.

$$S^{*} = \lim_{d \to \infty} S \times C^{d}$$

$$= \lim_{d \to \infty} \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & \vdots \\ s_{n1} & s_{n2} & \cdots & s_{nn} \end{bmatrix} \times \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix}^{d}$$

$$= \begin{bmatrix} s_{11}^{*} & s_{12}^{*} & \cdots & s_{1n}^{*} \\ s_{21}^{*} & s_{22}^{*} & \cdots & s_{2n}^{*} \\ \vdots & \vdots & \vdots & \vdots \\ s_{n1}^{*} & s_{n2}^{*} & \cdots & s_{nn}^{*} \end{bmatrix}$$
(4)

where d denotes the number of iterations.

Finally, the top l users who have the greatest impact on the target users are identified as opinion leaders based on the matrix S^* .

3.2. Opinion dynamics

The opinion dynamics model studies the influence of interactions among users in a group on the evolution process of opinion, it provides a unique perspective for understanding users' behavior patterns. Considering the various hypotheses in the process of opinion evolution, scholars have proposed various methods to simulate the changes in these opinions. In this section, we adopt the idea of the Deffuant-Weisbuch model on opinion dynamics to describe the opinion interaction behavior between target users and opinion leaders.

The method we propose considers that users communicate with each other for a certain item and update their opinions based on social influence [47]. The Deffuant-

Weisbuch model is consistent with this view, since in this model, users who have similar opinions communicate with each other in pairs, and users will be influenced by each other and update their opinions. It is assumed that the group size is N, i and j are two random individuals in the group. Their opinions at time t are $x_i(t)$ and $x_j(t)$ respectively, and $x_i(t)$, $x_j(t) \in [0,1]$, given the threshold $\varepsilon \in [0,1]$, ε is a constant, which is the tolerance of opinion, if $|x_i(t) - x_j(t)| \le \varepsilon$, we can get the following equation,

$$\begin{cases} x_i(t+1) = x_i(t) + \mu_i(x_j(t) - x_i(t)) \\ x_j(t+1) = x_j(t) + \mu_j(x_i(t) - x_j(t)) \end{cases}$$
(5)

Otherwise,

$$\begin{cases} x_i(t+1) = x_i(t) \\ x_j(t+1) = x_j(t) \end{cases}$$
(6)

where μ_i and μ_j are constants, indicating the convergence parameter of each opinion movement. The value of threshold ε has an important influence on the evolution of group opinions. When ε is very large, the group tends to form a consensus, that is, all individuals in the group ultimately hold the same opinion on a given issue. When ε is small, each individual keeps their opinions unchanged, the group gradually divides into two or more opinion groups, and the members of each opinion group share the same views.

According to the idea of the Deffuant-Weisbuch model, if the user f is an opinion leader, in each update event, the target user i and opinion leader f start a conversation, then the target user will update his/her opinions according to the following formula:

$$U_i \leftarrow U_i + \mu(U_f - U_i) \tag{7}$$

where U_i and U_f represent the views of target user *i* and opinion leader *f* on latent

factors respectively, and the trust parameter μ controls the degree to which the target user moves to opinion leaders. By adjusting μ , different groups can be defined. Since the target user is likely to adopt the opinion leaders' view, we set the value of μ in the interval [0,1]. When μ =0, there will be no change in the opinions of the target user. When μ =0.5, the target user will get the average of the views of both parties. When μ =1, the views of the target user will be updated to the views of opinion leaders. These situations respectively indicate target users with different characteristics. When μ is small, and corresponds to target users with tougher strategies, it is not easy for them to change their views. When μ is large, it is easy for the target users to accept the views of the opinion leaders.

3.3. Probabilistic matrix factorization with the influence of opinion leaders and opinion dynamics

Suppose there are *n* users and *m* items to form a $n \times m$ rating matrix *R*, the element R_{ij} in the matrix *R* represents the rating of user *i* on item *j*. The number of latent features is *k*, where $k \ll min(n,m)$, the $k \times n$ matrix *U* represents the user's latent feature matrix, U_i is the user *i*'s latent feature vector, the $k \times m$ matrix *V* represents the item's latent feature matrix, V_j is the item *j*'s latent feature vector. The diagram of Probabilistic Matrix Factorization (PMF) [46] is shown below.



Fig. 3. PMF diagram

Assuming that the conditional distribution of the known rating data satisfies the Gaussian distribution:

$$p(R|U, V, \sigma^2) = \prod_{i=1}^{n} \prod_{j=1}^{m} [N(R_{ij}|g(U_i^T V_j), \sigma^2)]^{I_{ij}}$$
(8)

where I_{ij} is the indicator function that can only be 1 or 0, with 1 indicating that the user *i* has rated the item *j*, and 0 indicating that there is no rating. Then, we place a spherical Gaussian priori with a mean of 0 on U_i and V_j :

$$p(U|\sigma_{U}^{2}) = \prod_{i=1}^{n} N(U_{i}|0, \sigma_{U}^{2}I)$$

$$p(V|\sigma_{V}^{2}) = \prod_{i=1}^{m} N(V_{i}|0, \sigma_{V}^{2}I)$$
(9)

Note that *I* in equation (9) is not an indicator function, it represents a diagonal matrix. Hence, through a simple Bayesian inference, we can get the following formula:

$$p(U,V|R,\sigma^2,\sigma_U^2,\sigma_V^2) \propto p(R|U,V,\sigma^2) \, p(U|\sigma_U^2) \, p(V|\sigma_V^2)$$

$$= \prod_{i=1}^{n} \prod_{j=1}^{m} [N(R_{ij}|g(U_i^T V_j), \sigma^2)]^{I_{ij}} \times \prod_{i=1}^{n} N(U_i|0, \sigma_U^2 I) \times \prod_{j=1}^{m} N(V_j|0, \sigma_V^2 I)$$
(10)

Since there are interactive behaviors between target users and opinion leaders, the characteristics of target users are not only affected by themselves, but also by opinion leaders. Therefore, the influence of opinion leaders and their evolving opinion dynamics with the target user are integrated into the PMF:

$$p(U|S^*, \sigma_U^2, \sigma_{S^*}^2) \propto p(U|\sigma_U^2) \, p(U|S^*, \sigma_{S^*}^2) = \prod_{i=1}^n N(U_i|0, \sigma_U^2 I) \times \prod_{i=1}^n N(U_i|\sum_{f \in o_i} s_{if}^* U_f, \sigma_{S^*}^2 I)$$
(11)

Then the posterior probability of U and V can be obtained by using Bayesian formula:

$$p(U, V|R, S^*, \sigma^2, \sigma_{S^*}^2, \sigma_U^2, \sigma_V^2) \propto p(R|U, V, S^*, \sigma^2) \, p(U|\sigma_U^2) \, p(V|\sigma_V^2) p(U|S^*, \sigma_{S^*}^2)$$
(12)

Taking the logarithm of equation (12), we can get:

$$\ln p(U, V|R, S^{*}, \sigma^{2}, \sigma_{S^{*}}^{2}, \sigma_{U}^{2}, \sigma_{V}^{2}) = \ln p(R|U, V, S^{*}, \sigma^{2}) + \ln p(U|\sigma_{U}^{2}) + \ln p(V|\sigma_{V}^{2}) + \ln p(U|S^{*}, \sigma_{S^{*}}^{2})$$

$$= -\frac{1}{2\sigma^{2}} \sum_{i=1}^{n} \sum_{j=1}^{m} I_{ij} \left(R_{ij} - U_{i}^{T}V_{j}\right)^{2} - \frac{1}{2\sigma_{U}^{2}} \sum_{i=1}^{n} U_{i}^{T}U_{i} - \frac{1}{2\sigma_{V}^{2}} \sum_{j=1}^{m} V_{j}^{T}V_{j} - \frac{1}{2\sigma_{S^{*}}^{2}} \sum_{i=1}^{n} \left(U_{i} - \sum_{f \in o_{i}} s_{if}^{*} U_{f}\right)^{T} \left(U_{i} - \sum_{f \in o_{i}} s_{if}^{*} U_{f}\right) - \frac{1}{2} \left(\left(\sum_{i=1}^{n} \sum_{j=1}^{m} I_{ij}\right) \ln \sigma^{2} + nk \ln \sigma_{U}^{2} + mk \ln \sigma_{S^{*}}^{2} + nk \ln \sigma_{S^{*}}^{2} \right) + C \quad (13)$$

where C is a constant that does not depend on parameters, U_f represents the latent feature vector of identified opinion leaders, o_i represents the set of opinion leaders of user *i*. The maximization of Formula (13) is equal to the minimization of the following objective function with quadratic regularization:

$$\mathcal{L} = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{m} I_{ij} \left(R_{ij} - U_i^T V_j \right)^2 + \frac{\lambda_U}{2} \sum_{i=1}^{n} ||U_i||_F^2 + \frac{\lambda_V}{2} \sum_{j=1}^{m} ||V_j||_F^2 + \frac{\lambda_{S^*}}{2} \sum_{i=1}^{n} ||U_i - \sum_{f \in o_i} S_{if}^* U_f||_F^2$$
(14)
where $\lambda_U = \frac{\sigma^2}{\sigma_U^2}$, $\lambda_V = \frac{\sigma^2}{\sigma_V^2}$, $\lambda_{S^*} = \frac{\sigma^2}{\sigma_{S^*}^2}$, and $||.||_F^2$ denotes the Frobenius norm. It should
be noted that the specific opinion leaders influence the user's feature vectors through
the last term of the above equation.

Anyone other than the target user is likely to be an opinion leader, so we can get the derivatives of U_i and V_j respectively:

$$\frac{\partial \mathcal{L}}{\partial U_{i}} = -\sum_{j} \left(R_{ij} - U_{i}^{T} V_{j} \right) V_{j} + \lambda_{U} U_{i} + \lambda_{S^{*}} \left(U_{i} - \sum_{f \in o_{i}} s_{if}^{*} U_{f} \right) - \lambda_{S^{*}} \sum_{\{k \mid i \in o_{k}\}} s_{ki}^{*} \left(U_{k} - \sum_{w \in o_{k}} s_{kw}^{*} U_{w} \right) \quad (15)$$

$$\frac{\partial \mathcal{L}}{\partial V_{j}} = -\sum_{i} \left(R_{ij} - U_{i}^{T} V_{j} \right) U_{i} + \lambda_{V} V_{j} \quad (16)$$

where o_k indicates the set of opinion leaders of user $k, k \neq i$ and $k \neq w$.

Then U_i and V_j are updated by Stochastic Gradient Descent (SGD):

$$U_{i} \leftarrow U_{i} - \gamma \frac{\partial \mathcal{L}}{\partial U_{i}} \qquad (17)$$
$$V_{j} \leftarrow V_{j} - \gamma \frac{\partial \mathcal{L}}{\partial V_{j}} \qquad (18)$$

where γ is the step size, or called the learning rate.

4. Experiments

In this section, we first describe the datasets and evaluation indicators used in the experiment; Then, we introduce the baseline methods in comparison with our proposed model; Finally, we presented and analyzed the experimental results.

4.1. Data description and experimental setup

Datasets: We chose two commonly used, real datasets for our experiment, i.e., Filmtrust[48] and Epinions[49]. The datasets record the users' rating information, which can help other users make decisions. On the Filmtrust dataset, users' rating ranges from 0.5~4, while on the Epinions dataset the range is 1~5. In addition, each user can maintain a trust relationship list to indicate their views on other users, and then establish a trust relationship network with their trusted users. The trust relationship between two users is binary (1 or 0), that is, either trust or distrust. Table 2 presents the basic feature information of these two datasets.

Table 2 Basic Features of the Two Datasets

Datasets	_	Rati	Social information			
	#Users	#Items	#Ratings	Sparsity/%	#Users	#Edges
FilmTrust	1508	2071	35497	98.86	1642	1853
Epinions	40163	139738	664824	99.99	49289	487183

Metrics: Two benchmark prediction error evaluation metrics are adopted to assess the ODSR's effectiveness, namely mean absolute error (MAE) and root mean squared error (RMSE). The smaller the value indicates the better the prediction effect. Their calculation formulas are as follows:

$$MAE = \frac{1}{N} \sum_{(i,j) \in I} |\hat{R}_{ij} - R_{ij}|$$
(19)
$$RMSE = \sqrt{\frac{1}{N} \sum_{(i,j) \in I} (\hat{R}_{ij} - R_{ij})^{2}}$$
(20)

where R_{ij} and \hat{R}_{ij} respectively represent the true value and predicted value of user *i*'s rating on item *j*, *I* represents the collection of all users and items and *N* is the number of ratings in the test set.

We then introduced four N-dependent accuracy metrics: Precision@N (Pre@N), Recall@N (Rec@N), F1@N and NDCG@N. These four indicators reflect the hit rate performance and emphasize the importance of Top-N recommendations, which can be used to judge whether the Top-N items recommended by the method are really of interest to users. The larger the four indicators, the higher the recommendation accuracy. We used $L_{(u)}$ to represent the item recommendation sequence of user u, $T_{(u)}$ to represent the item set that the user u likes, which is defined according to R_{ij} in the test set, and U_{test} to represent the user set to be tested, and the following calculation equations can be obtained:

$$\operatorname{Pre}(@N = \frac{\sum_{u \in U_{test}} |L_{(u)} \cap T_{(u)}|}{\sum_{u \in U_{test}} |L_{(u)}|} \quad (21)$$
$$\operatorname{Rec}(@N = \frac{\sum_{u \in U_{test}} |L_{(u)} \cap T_{(u)}|}{\sum_{u \in U_{test}} |T_{(u)}|} \quad (22)$$
$$\operatorname{F1}(@N = \frac{2 \times \operatorname{Pre}(@N \times \operatorname{Rec}(@N)}{\operatorname{Pre}(@N + \operatorname{Rec}(@N))} \quad (23)$$

The Normalized Discounted Cumulative Gain (NDCG for short) is a rankingbased measurement method widely used in information retrieval. We utilized it to judge the consistency between the ranking of items recommended by the algorithm and the actual situation, and further evaluate the performance of different recommender systems applied to Top-N recommendation tasks. Its value represents the ratio between the recommended list ranking and the ideal ranking, and its calculation formula is as follows:

$$NDCG@N = \frac{DCG@N}{IDCG@N} \quad (24)$$
$$DCG@N = \frac{1}{|U_{test}|} \sum_{u \in U_{test}} \sum_{i=1}^{N} \frac{2^{t_i} - 1}{log_2(i+1)} \quad (25)$$

where *IDCG@N* is the ideal *DCG@N*, that is, the value of *DCG@N* when all the recommended items are ranked by the user's preference. When the recommended *i*-th item belongs to the item in $T_{(u)}$, t_i is 1, otherwise, it is 0. The value range of *IDCG@N* is $0\sim1$.

Baselines: The following methods are adopted as the baselines:

- PMF [47]: This method adopts the user's rating information for the item, without considering social information, and the latent factors of users and items are modeled by Gaussian distribution.
- TrustPMF [15]: This method uses the PMF to separately model the situation when the user is the trustor or the trustee, and then integrates them to obtain the recommendation result.
- REOD [35]: This method uses opinion dynamics to model the interactive behavior among users, and integrates the influence of users into the recommendation model.
- SNOL [43]: This method uses the views of opinion leaders to provide new users with appropriate recommendations when new users log in and the rating matrix is sparse.

• OLSR [26]: This method integrates the information of opinion leaders according to the PMF, and combines the user's own preferences and opinion leaders' preferences in a weighted combination to generate the user's rating of an item.

Parameter settings: For each algorithm, the optimal parameters are selected as the final results for comparison, by using the grid search method for multiple parameter tuning, according to the recommended parameter values in the literature. Specifically, for PMF and OLSR in both datasets, the step size γ =0.001 and the regularization coefficient $\lambda_U = \lambda_V = 0.1$. For TrustPMF, the regularization parameters are $\lambda = 0.1$, $\beta_1 = \beta_2 = 5$ in FilmTrust, and $\lambda = 0.1$, $\beta_1 = \beta_2 = 10$ in Epinions. The parameters of REOD are as follows: $\lambda = 0.1$, $\mu = 0.12$ and the payoff parameter $\beta = 0.05$ in both datasets. For SNOL, the similarity threshold parameter is set to 0.3 in both datasets. For ODSR, we set the threshold value l = 20 and the trust parameter $\mu = 0.7$ in both datasets.

4.2. Experimental results

For each dataset, the experiment randomly selects 80% of it as the training set, with the remaining 20% as the test set. The average performance is given after five independent experiments.

4.2.1. The effect of threshold value l and trust parameter μ

The recommendation performance is studied under the condition of changes in parameters l and μ , so as to determine the optimal values of l and μ . The influence of parameters' variation on MAE in the FilmTrust and Epinions is shown in Fig. 4. We do not analyze RMSE here because its change trend is similar to that of MAE.



Fig. 4. Influence of parameters' variation on MAE

We can clearly see that for the same parameter l, with the increase of the trust parameter μ in both datasets, the prediction accuracy of ODSR increases first and then decreases. When μ =0.7, the value of MAE is the smallest. That may be because during the opinion interaction, a smaller μ leads to small changes in user opinions and the opinion interaction does not work in the social recommendation. And a larger μ causes the target user to be seriously affected by opinion leaders, thus ignoring their own characteristics. For the same parameter μ , with the increase of the threshold value l in both datasets, the recommendation performance of ODSR increases first and then decreases. When l=20, the value of MAE is the smallest. This is due to a smaller l leads to the failure to fully utilize the opinion leaders' information, which reduces the recommendation effect. And a larger l causes those who have little influence on the target users to be identified as opinion leaders, so there is exit noise in the process of recommendation, which affects the recommendation performance. Therefore, the threshold value l and the trust parameter μ should be set to 20 and 0.7 respectively in both datasets.

4.2.2. The recommendation performance for all users

We set the dimensions of the feature vector to 5, 10, and 20, to compare the effectiveness of ODSR with that of the relevant algorithms presented in Section 4.1, The results are shown in Table 3.

	Table 5 Comparison of accuracy for an users							
Datasets	Dimension	Metrics	PMF	TrustPMF	REOD	SNOL	OLSR	ODSR
	5	MAE	0.9225	0.7462	0.6978	0.7282	0.6718	0.6489
		Improve	29.66%	13.04%	7.01%	10.89%	3.41%	
		RMSE	1.0805	0.8935	0.8703	0.8806	0.8574	0.8396
		Improve	22.30%	6.03%	3.53%	4.66%	2.08%	
FilmTrust	10	MAE	0.9224	0.7461	0.6975	0.7283	0.6726	0.6515
		Improve	29.37%	12.68%	6.59%	10.55%	3.14%	
		RMSE	1.0802	0.8933	0.8698	0.8810	0.8596	0.8410
		Improve	22.14%	5.85%	3.31%	4.54%	2.16%	
	20	MAE	0.9225	0.7459	0.6972	0.7291	0.6734	0.6519
		Improve	29.33%	12.60%	6.50%	10.59%	3.19%	
		RMSE	1.0802	0.8932	0.8696	0.8814	0.8617	0.8421
		Improve	22.04%	5.72%	3.16%	4.46%	2.27%	
	5	MAE	1.0986	0.9298	0.9075	0.9164	0.8943	0.8798
		Improve	19.92%	5.38%	3.05%	3.99%	1.62%	
		RMSE	1.2849	1.1536	1.1149	1.1357	1.1076	1.0672
		Improve	16.94%	7.49%	4.28%	6.03%	3.65%	
Epinions	10	MAE	1.0988	0.9294	0.9072	0.9169	0.8947	0.8802
-		Improve	19.89%	5.29%	2.98%	4.00%	1.62%	
		RMSE	1.2852	1.1532	1.1145	1.1361	1.1079	1.0675
		Improve	16.94%	7.43%	4.22%	6.04%	3.65%	
	20	MAE	1.0989	0.9291	0.9068	0.9172	0.8950	0.8804
		Improve	19.88%	5.24%	2.91%	4.01%	1.63%	
		RMSE	1.2853	1.1530	1.1141	1.1363	1.1081	1.0679
		Improve	16.91%	7.38%	4.15%	6.02%	3.63%	

Table 3 Comparison of accuracy for all users

As shown in the table, the performance of ODSR is superior to the other methods on the two datasets. In addition, social recommendation algorithms (such as TrustPMF and REOD) significantly outperform the rating-only recommendation algorithm (PMF). This is due to the fact that social relationship among users was taken into account in generating the recommendation. Moreover, ODSR can enhance the accuracy of recommendation in two datasets with different sizes, which indicates that ODSR is not biased towards a specific dataset. Furthermore, ODSR performs better than other opinion leaders-based recommendation methods (SNOL and OLSR), which indicates the advantage of considering the opinion interactions between the target user and opinion leaders.

4.2.3. Performance on different rating sparsity

This section evaluates the performance of rating prediction of each algorithm for cold-start users and users with different numbers of ratings (a rating less than 5 indicates that the user is a cold-start user [50]). The results are shown in Fig. 5.



(a) FilmTrust



(b) Epinions

Fig. 5. The predictive performance for users with different numbers of ratings

The results show that ODSR has the lowest MAE for all groups on FilmTrust and Epinions, confirming the robustness of ODSR in enhancing the rating prediction accuracy, through the utilization of opinion dynamics to model the interactive behavior between target users and opinion leaders. We can also see that the MAE of ODSR is obviously lower than that of other methods for cold-start users, which demonstrates that modeling the interaction between the target user and opinion leaders can effectively solve the cold-start problem. Notably, the MAE and RMSE of opinion leaders-based recommendation methods (such as SNOL and OLSR) are lower than the rating-only recommendation method (PMF) thanks to the utilization of the opinion leaders'

information.

4.2.4. Validation of Top-N recommendation

The ability of different methods for Top-N items recommendation is examined in this section. Table 4 shows the experimental results.

Table 4 Top-N recommendation test in FilmTrust and Epinions dataset									
Datasets	Methods	Pre@5	Rec@5	F1@5	NDCG@5	Pre@10	Rec@10	F1@10	NDCG@10
FilmTrust	PMF	0.0213	0.0018	0.0033	0.0281	0.0134	0.0020	0.0035	0.0192
	TrustPMF	0.0335	0.0039	0.0070	0.0378	0.0289	0.0041	0.0072	0.0309
	REOD	0.0536	0.0054	0.0098	0.0552	0.0497	0.0058	0.0104	0.0539
	SNOL	0.0420	0.0050	0.0089	0.0457	0.0375	0.0052	0.0091	0.0412
	OLSR	0.0719	0.0101	0.0177	0.0752	0.0698	0.0105	0.0183	0.0706
	ODSR	0.0791	0.0132	0.0226	0.0803	0.0719	0.0158	0.0259	0.0763
Epinions	PMF	0.0010	0.0002	0.0003	0.0015	0.0009	0.0003	0.0005	0.0012
	TrustPMF	0.0028	0.0005	0.0008	0.0032	0.0023	0.0006	0.0010	0.0027
	REOD	0.0043	0.0009	0.0015	0.0049	0.0037	0.0011	0.0017	0.0042
	SNOL	0.0035	0.0007	0.0012	0.0040	0.0030	0.0008	0.0013	0.0035
	OLSR	0.0055	0.0011	0.0018	0.0059	0.0047	0.0016	0.0024	0.0051
	ODSR	0.0093	0.0018	0.0030	0.0098	0.0076	0.0021	0.0033	0.0087

It is easy to observe that each method performs better on Filmtrust than on Epinions, since the number of candidate items in Filmtrust is much smaller than that in Epinions (see Table 2). It should be noted that the rating-only recommendation algorithm (PMF) is still inferior to the five social recommendation methods in terms of four N-dependent accuracy metrics (with N=5,10) on the two datasets, which indicates that the ranking precision can be effectively improved with the help of other auxiliary information. In addition, the ODSR outperforms other methods on both datasets, and the level of improvement is greater on Epinions with large sparsity, because the interactions between the target user and opinion leaders can effectively alleviate the problem of data sparsity and help to increase the accuracy of Top-N items recommendation.

4.2.5. Ablation test

Ablation experiments are carried out to test the influence of each component of

ODSR on the final results. The following are the methods used in the experiments that eliminate the influence of opinion interaction behavior or opinion leaders.

- ODSR\OL: This stands for the method to eliminate the influence of opinion leaders, that is, the opinion interactions between users are considered, while the last member of the equation (14) is removed.
- ODSR\OI: This represents the method to eliminate the influence of opinion interactions, that is, the influence of opinion leaders is considered, while the opinion interactions are removed from each iteration of SGD training.
- ODSR\OL&OI: This represents the method to eliminate the influence of opinion leaders and opinion interaction.

Fig. 6 shows the accuracy of these methods when the feature vector dimension is 10. Compared with opinion interactions, opinion leaders have a greater impact on rating prediction in the social recommendation. But the opinion interactions are also of great significance, which can also improve the recommendation performance. Therefore, each component of ODSR can significantly boost the recommendation quality, while neglecting the impact of opinion leaders or opinion interactions degrades the performance. We can also see that the improvement of the performance under the influence of opinion leaders in the FilmTrust dataset is more obvious than that in the Epinions dataset, this is partly due to the fact that the user relationships in FilmTrust are denser than that in Epinions.





5. Conclusions and future work

There are interactive behaviors among users on social networks, users usually communicate with each other about a certain item and their opinions influence each other. The influence of opinion leaders on the target user can be powerful. The target user is willing to consult opinion leaders and interact with them. In this paper, opinion dynamics is utilized to model the interactive behavior between the target user and opinion leaders. Building upon the probabilistic matrix factorization method, the impact of opinion leaders and their evolving opinion dynamics with the target user are integrated to generate recommendations. The results of several experiments show that ODSR has a better recommendation performance than the benchmark methods, and can lessen the problems of cold-start users and data sparseness. This study thus advances the literature by offering new insights into improving the social recommender systems.

This work only utilizes the explicit social relationship among users but does not consider implicit social relationships (such as implicit friends, implicit classmates, etc.). In addition, this study only considers the interaction between the target user and opinion leaders to increase the quality of the recommendation method but does not consider the nature of temporal dynamics, which can be very useful to increase the quality of prediction. Therefore, in our future research, we may introduce the implicit social information among users and the temporal dynamic information of users' preferences into the recommendation algorithmic studies.

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