Engaging non-mathematics students in mathematics learning through collaborative teaching

JINHUA MATHIAS † AND CHRISTOPHER SAVILLE ‡ STEVE LEECH †

†*Foundation Programme, Durham Centre for Academic Development, Durham University, Durham, DH1 3DE, UK*

‡*Department of Earth Science, Durham University, Durham, DH1 3DE, UK*

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Abstract

Mathematics is a common component of many science and social science degrees, but frequently not considered a prerequisite for their study. This can lead to some students studying mathematics components at undergraduate level with little or no previous mathematical learning and varying levels of motivation, or desire, to study mathematics. This situation has consequences for student experience and teaching practice, including the potential for lower levels of student engagement, higher levels of student anxiety, and lack of student motivation for studying a subject that is not what they chose as their degree subject.

This paper details an intervention designed to address issues experienced in the teaching of mathematics to undergraduate students, with varying motivation for the study of mathematics and high levels of mathematics anxiety, in their 1st year of study in a department of Earth Sciences department at a UK university. The intervention provided a bespoke mathematics module, designed and taught collaboratively by a teaching specialist in mathematics and a subject specialist in Earth Sciences. The mathematics specialist was able to provide extensive experience in teaching students from widening participation backgrounds and those with mathematics anxiety, making the content more accessible to the students, particularly those with mathematics anxiety. The Earth sciences specialist was able to contextualise the mathematics content, showing students how the content was applicable to the rest of the degree and helping locate the mathematical content within the broader scope of the discipline. Previous approaches to this module were taught either, solely by an Earth Sciences specialist, or solely by a mathematics specialist. Compared to those approaches the collaborative teaching intervention improved student attainment by 10% in the average module mark and moved the module from having the lowest student satisfaction scores in the department to the highest. This innovation also benefited both teachers with respect to professional development, which is discussed in the paper.

1. Introduction

There has been a growing demand for advanced quantitative skills in UK workplaces over the last twenty years due to an increasing need for monitoring and interpreting data within businesses (Mason et al., 2015). In response to this, many UK universities have increased the amount of mathematical content in their courses, particularly in disciplines in Social Science, Business and some physical sciences such as Psychology, Biology, Geography and Earth Sciences. Whilst higher education recruitment practices (specifically, course entry requirements) often indicate a belief that students with a GCSE (the General Certificate of Secondary/Middle School Education) at C (4/5) grade or equivalent have sufficient mathematical knowledge to cope with these subjects, students that performed at this minimum level tend to experience significant challenges during their undergraduate studies (Harris et al., 2013, Williams et al., 2008, Watters et al., 2020). Consequently, many universities have created undergraduate level 1 curricula that include bespoke mathematics modules for non-mathematicians, to build the necessary knowledge and skills required for their degree programmes.

Often such mathematics modules are taught as a service subject by a lecturer who has mathematical knowledge in the degree discipline, but who may not have experience in teaching mathematics to non-mathematicians. In these circumstances, they can face many challenges with engaging students in mathematics (Nardi, 2016). Often these students did not expect to study mathematics during their university course; they may not have chosen to study A-level mathematics (the pre-university entry level in the UK, International Baccalaureate equivalent), perhaps due to previous, unsuccessful learning experiences (Brown et al., 2008). This situation can lead to students, who may be successful in every other aspect of their study, struggling with mathematics. Such circumstances may lead to negative attitudes and expectations in regard to mathematics, reducing motivation and increasing anxiety about studying mathematics (Metje et al., 2007). It may also lead to a student failing their academic course as they cannot pass the mathematics component. Although mathematics support such as mathematics and statistics support centres within a university may improve student achievement in mathematics, not all students actively engage with this type of service, perhaps due to a fear of embarrassment, feeling of intimidation, or due to feeling demoralised by their circumstances (Lawson et al., 2020).

Mathematics anxiety and its impact on student attainment have been widely recognised (Ashcraft & Krause, 2007, Nunez-Pena et al., 2013, Zhang, et al., 2019). It describes a situation where one has increased physiological reactivity when dealing with mathematics or a situation connected to mathematical problems (Luttenberger, et al., 2018). Researchers have sought to measure mathematics anxiety (Hunt et al., 2011), but little consensus exists and the signs of anxiety and apprehension in the classroom, or variation in attitude between studying mathematics and other subjects may not be recognised; teachers often assume students have the same attitude to their mathematics studies as other parts of their course (Metje et al., 2007). It is also common for a specialist in mathematics to consider the taught content as being very basic and to have never experienced (or to have forgotten) how difficult it can be for some students to learn mathematics, let alone apply it in a practical context associated with a particular discipline, such as geology, or biology (Metje et al., 2007).

However, *two* specialists, one a specialist in teaching mathematics and the other a specialist in employing mathematics within a disciplinary context, can work together to combine their experiences and develop practice that successfully combines recognition of the barriers some students experience in studying mathematics, together with the specific discipline application of mathematics necessary for successful undergraduate study. This type of collaborative teaching is not novel. For instance, it has been used in the training of school teachers (Weilbacher & Tilford, 2015, Sebald et al., 2021) where studies have demonstrated that co-teaching teams, comprising one mentor and one or two trainee teachers, can be successful as a way of tuning trainees' skills to suit specific teaching contexts. Co- or collaborative teaching is also often used in Higher Education to develop novice lecturers' pedagogical skills and confidence, and bring together different areas of knowledge to present a bigger perspective (Burns & Mintzberg, 2019, Scherer et al., 2020).

This case study asks if co-teaching undergraduate mathematics can successfully 'tune' mathematics teaching to simultaneously counter students' negative emotions, anxiety and reduced motivation and bring together mathematics and the broader discipline perspective within the study of Earth Sciences. The case study has a novel focus on collaborative teaching to improve the engagement, confidence, enjoyment and attainment of students when they must study a potentially unwelcome support subject (here, mathematics) alongside their main studies (here, Earth Science).

2. Background

2.1. Reducing mathematics anxiety

A dislike of mathematics may have many causes, such as perceived difficulty, lack of confidence and lack of motivation, or a lack of perceived relevance (Brown et al., 2008). Tobias and Weissbrod (1980) define "the panic, helplessness, paralysis, and mental disorganisation that arises among some people when they are required to solve a mathematical problem" whether it is in a classroom environment or daily life, as mathematics anxiety. Most students who experience mathematics anxiety report historic negative experiences in studying mathematics, often in a school environment with uninspiring teaching methods, ineffective learning practices and non-engagement of students (Whyte & Anthony, 2012, Finlayson, 2014).

Mathematics anxiety often leads to failure in mathematics and frequent experience of failure and/or poor performance in mathematics tasks. Associated assessments lead to anxiety, reduced effort and persistence or even avoidance of mathematics activities which in turn causes further failure and further anxiety, giving rise to a student failure cycle (Fig. 1) (Ernest, 2013). Such negative experiences are often the reason why students avoided studying mathematics at A-level to start with. However, this failure cycle can be broken with intervention from teachers. For example, Boaler (2017) suggests that teachers should encourage students, by praising what students have done and learned, to develop a growth mindset; that "everyone can learn mathematics to the highest levels". This approach gives rise to a success cycle (Fig. 2), where success in mathematics tasks, no matter how small they are, gives students pleasure and confidence so that they are motivated to approach the next challenge more positively (Ernest, 2013).

Moreover, instead of pretending that mathematics is easy, teachers should encourage students to tackle the challenges that mathematics presents with appropriate support so as to help develop their resilience (Lee & Johnston-Wilder, 2014). This positive approach to mathematical study is further enhanced when students are taught at an appropriate level such that they can engage in collaborative and discursive learning experiences.

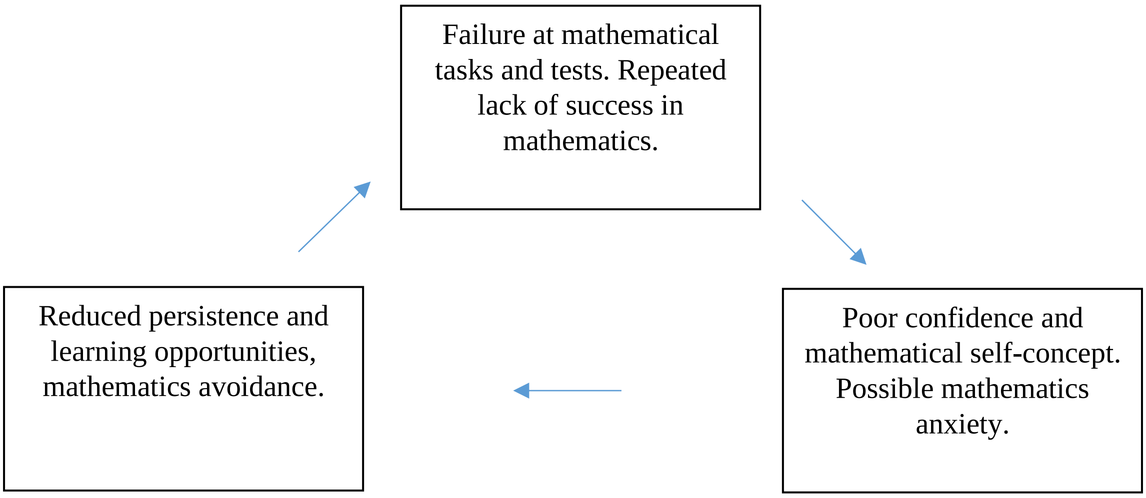

FIG. 1. The failure cycle (Source: Ernest, 2013)

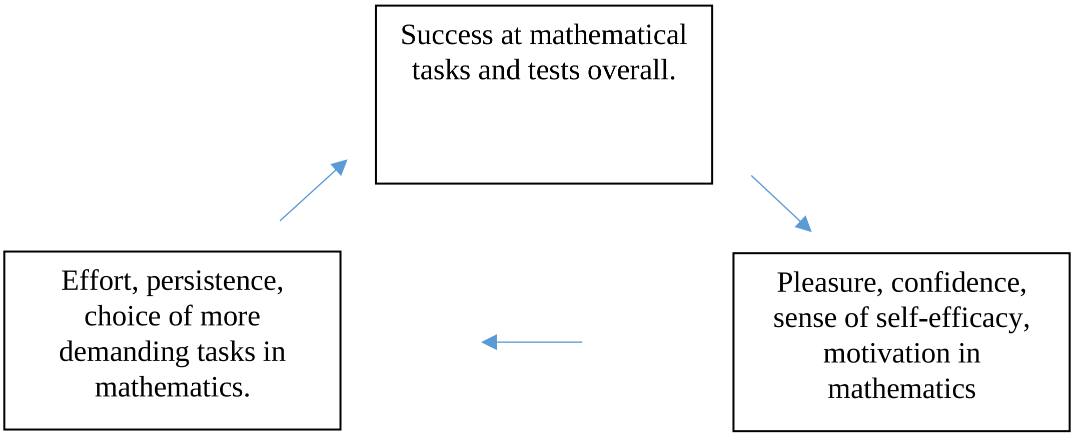

FIG. 2. The success cycle (Source: Ernest, 2013)

2.2. Motivation to learn

Motivation is a significant factor in academic learning and success and may be intrinsic or extrinsic. Intrinsic motivation refers to an internal force that motivates students to engage in academic activities, often because they are interested in learning, they enjoy the learning process, and the learning supports the attainment of personal goals (Schiefele, 1991). Extrinsic motivation refers to an external force that motivates academic engagement and many aspects such as parental expectations, expectations of other trusted role models, financial reward, etc. are thought to lead to this type of motivation (Adamma et al., 2018). Students who are extrinsically motivated tend to exhibit short-lived effort and action. Once the rewards are removed, extrinsically motivated students lose their motivation and cease to study further (DeLong & Winter, 2002). Students who are intrinsically motivated persist longer when challenged and demonstrate more accomplishments in their academic endeavours than those who are extrinsically motivated (Pintrich & Garcia, 1991).

One strategy for developing intrinsic motivation among students is to provide more practical and relevant tasks (Bobis et al., 2011). Students often engage better with mathematical aspects if the exercises can build on students' existing interests and contextual knowledge. Explicitly emphasising usefulness can significantly enhance the motivation of students to study a mathematical topic (Posamentier, 2013). Moreover, university students, as adult learners, are expected to be problem-centred, interested in immediate applications of knowledge and motivated to learn by internal rather than external factors (Merriam, 2001). Demonstrating the relevance of a subject to a degree is therefore very important for undergraduate study. Demonstrating how mathematics has been used as a tool in their subject area, such as in geology, can motivate them to learn and apply the tool.

2.3. Subject specialist or teaching specialist

Most university lecturers are specialists in their discipline and often highly knowledgeable in their specific field. The enthusiasm for the subject that a mathematics specialist can apply, may persuade students that mathematics can be an enjoyable subject (Metje et al., 2007), which may be advantageous in promoting student motivation. However, many university lecturers have not been specifically trained to teach and may not understand or have encountered mathematics anxiety personally and/or may not be able to recognise it in students. A study of 121 UK universities found that the percentage of faculty who held a teaching qualification (including Higher Education Academy (HEA) qualification and other teaching qualifications) ranges from 3% to 90% (Nurunnabi et al., 2019). Moreover, academics who have progressed to teach in mathematics intensive subjects at university level are likely to excel in mathematics and are less likely to understand how hard some students find mathematical studies. In these circumstances, it may be beneficial, if not vital, to introduce trained teaching staff to support mathematics teaching.

There is a difference between teaching as a subject specialist and as a teaching specialist. A subject specialist can apply their knowledge in their discipline but may have a poor understanding about their students' starting level. When instructing students, subject specialists often observe each new cohort of students to be less capable than the previous one, not recognising how this is relative to the broadening and deepening of their own knowledge as time goes by (Kruppa et al., 2021). Due to the "curse" of knowledge (Camerer et al., 1989), a specialist may unconsciously assume that their intuitive understanding of mathematics is readily accessible to the students and quickly misjudge the level of understanding of students and consequently hold them to unreasonable expectations (Persky & Robinson, 2017, Birch & Bloom, 2007). For example, specialists often make assumptions about student knowledge concerning notation, symbols, abbreviations and terminology, and therefore neglect to provide relevant explanations. These different perspectives between teachers and students and top-down teaching strategies can lead to frustration for both learners and lecturers (Kruppa et al., 2021).

Teaching specialists, however, are likely to use bottom-up teaching strategies, starting off by determining the current state of their students' knowledge and supporting student's learning through a series of scaffolded learning engagements (Vygotsky, 1978). They are also more likely to recognise the importance of situated cognition, i.e., contextualising learning within the student's chosen discipline

(Brown, et al., 1989). The REACT (Relating, Experiencing, Applying, Cooperating and Transferring) strategy focuses on teaching and learning in context (Crawford, 2001), a core aspect of so-called constructivism principles (Hein, 1991).

Relating refers to introducing a novice to knowledge in the context of one's life experiences or pre-existing knowledge and represents a powerful contextual teaching strategy. The strategy involves providing a variety of learning scenarios that aim to bring students' relevant memories or prior knowledge to a new learning situation. Rather than making assumptions about students' prior knowledge, the aim is to probe and identify current knowledge, often starting from one-step back to bring older knowledge to a place where students can easily relate to the new knowledge of concern. Experiencing refers to where students are guided by the teacher when working on questions in order to foster strong positive experiences. Applying is by reinforcing learning by applying learned concepts to real life context, which motivates students to learn further. This may also involve solving problems presented in their subject area with learned mathematical skills to develop students' enthusiasm in mathematics. Cooperating is learning by encouraging or creating the environment for students to work together and share knowledge. Transferring is learning that encourages students to use the knowledge they have learned and consciously apply it to new contexts or situations. Many science and mathematics teachers have implemented REACT strategies and achieved positive impact on students learning outcomes particularly in conceptual understanding (Ultay, 2012, Utami, 2016, Jelatu & Ardana, 2018, Putri & Saputro, 2019).

3. The study

3.1. The context

Our case study concerns a 20-credit module, called Mathematical Methods in Geoscience, set within a Department of Earth Sciences in a British university. The contact time between staff and students comprises of a two-hour lecture followed by one-hour seminar every week for 20 weeks. This module was previously taught by an Earth Sciences specialist for many years until 2015-16 when it was taught by a specialist mathematics teacher. Neither teaching was fully successful; the students criticised the module as being too difficult when it was taught by an Earth Scientist and not sufficiently interesting when taught by a mathematics teacher. Fig. 3 and 4 show the final assessment marks and the scores from Module Evaluation Questionnaire (MEQ) between 2012 and 2016. Both results were lower than the departmental average. The module had been considered, by the department, to be problematic for many years.

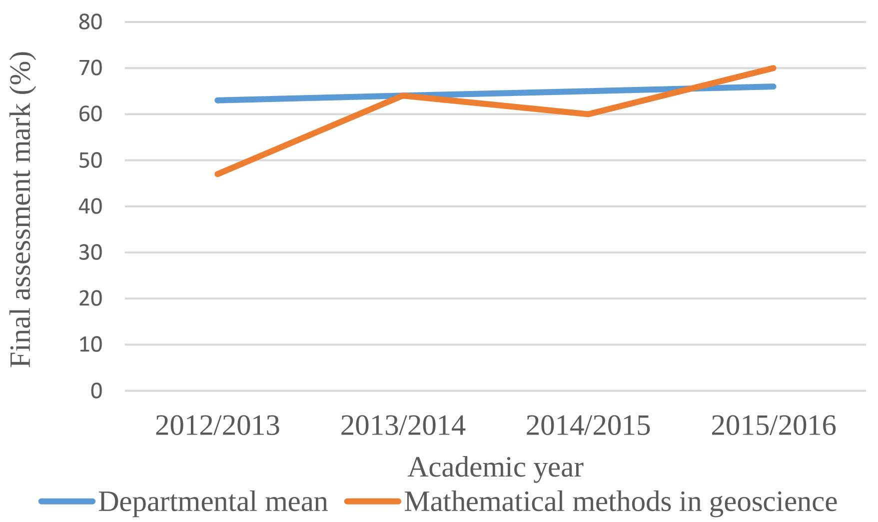

FIG. 3. Final assessment marks 2012-2016 (%)

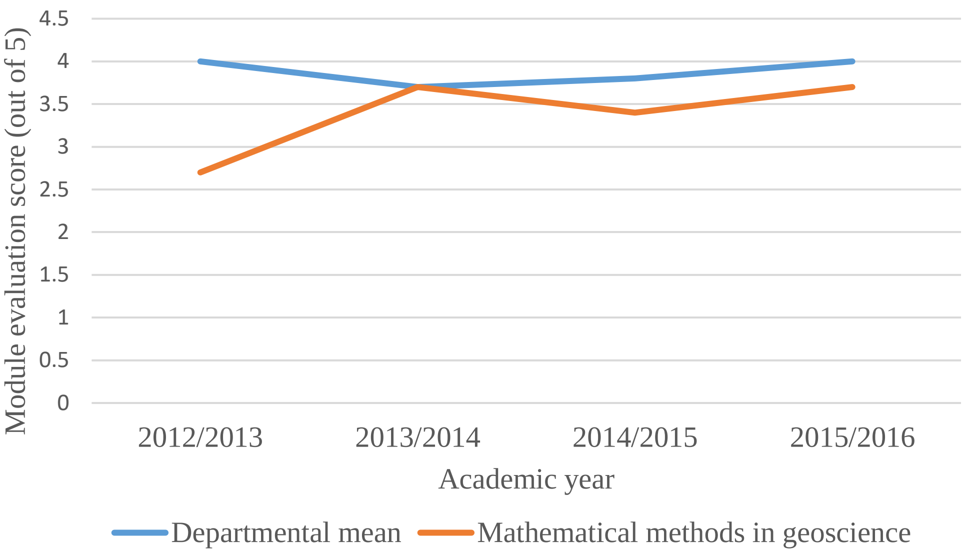

FIG. 4. Module evaluation scores 2012-2016 (out of 5)

In 2016, the department invited a teaching specialist, who had many years of experience of teaching students with mathematics anxiety, to collaborate with a subject specialist from Earth Sciences. The student cohort consisted of 31 UK home students and one international student. None of the home students had previously studied A-level mathematics and their GCSE grades in mathematics ranged from C to A*.

3.2. Teaching strategy

Lessons were planned, coordinated, and delivered jointly by the teaching specialist and subject specialist. The aim of the module was to ensure that students could confidently apply mathematics skills in other subjects and the REACT teaching strategy was used for the delivery, as shown in the example in the Table 1. The teaching specialist led the two-hour session to deliver the mathematics. The subject specialist designed the problem sheets, led the one-hour seminar sessions. The problem sheets were designed to cover a range of different Earth Science scenarios. Both specialists were present in each other's classrooms to observe teaching and support students during learning activities. Continuous reflection permitted some flexibility to adapt the module in response to perceived student needs. Both the teaching specialist and subject specialist developed better understanding for teaching mathematics to non-mathematicians in a subject based context, which is discussed in more detail later in the article.

TABLE 1. Examples of applying REACT teaching strategy in various students learning stage

REACT	Examples
Relating	At the start of a session, the teaching specialist checks students' understanding of the prior knowledge of a new topic, for example, checking students' knowledge of the law of indices before teaching logarithms. This ensures the appropriate starting level for a teaching session which reduces student anxiety. In the first session, the teaching specialist gave a diagnostic test to find out their mathematics level.
Experiencing	Each topic is broken into small steps and the students see an example and practice on similar questions. The students felt this "hands on" experience was very reassuring. Both teachers walk around the classroom to provide help.
Applying	The students are given an Earth Science problem to solve using the mathematics learned in previous sessions. They discuss their solutions in the one-hour seminar session, led by the subject specialist. This activity was intended to help the

	students to see how mathematical skills were applied in their subject area, and therefore was likely to improve their motivation.
Cooperating	In both the two-hour teaching and one-hour seminar sessions, the students sit in self-selected groups. They responded to and helped each other when doing questions. The classroom environment was very relaxed, the students focused on learning, not competing with each other or working in isolation, which helped to reduce anxiety.
Transferring	Students transferred the mathematical skills they studied in other modules such as Geophysical Methods for Geoscientists and Structural Geology & Tectonics. This built their confidence in using mathematics in their subjects.

3.3. Interventions to break the failure cycle

Based on the teaching specialist's experience of teaching students who have mathematics anxiety the first intervention used to break the failure cycle was to conduct a diagnostic test using UK GCSE level content. This was designed to provide an understanding of prior student attainment and promote learning confidence. The diagnostic test revealed that the study cohort could comfortably solve simple linear equations and simultaneous equations but struggled with equations involving fractions. Therefore, the teaching specialist revised solving linear equations and number fractions with the students in the first lecture, and then directed them to solve linear equations containing simple fractions.

When taking the diagnostic tests, the students were also asked how they felt about studying mathematics (Fig. 5). The students were asked to tick the boxes if they agreed with any of seven statements designed to establish their confidence in studying mathematics. More than 50% of the class were anxious about mathematics and over 40% were not looking forward to studying the module. This attitude can affect students' performance because it leads to mathematics being perceived as a barrier to success in their degree (Taylor & Galligan, 2006). After a few weeks, the teaching specialist observed some students were showing signs of anxiety; manifestations included students sitting by themselves, sitting at the back of class, hiding among other students, not participating, not answering questions, and focusing on copying examples from the board rather than practising questions as the teacher had instructed. It was noted that these students commonly had lower GCSE scores than others. In these cases, the teaching specialist provided a targeted "out of session" support, as the second intervention.

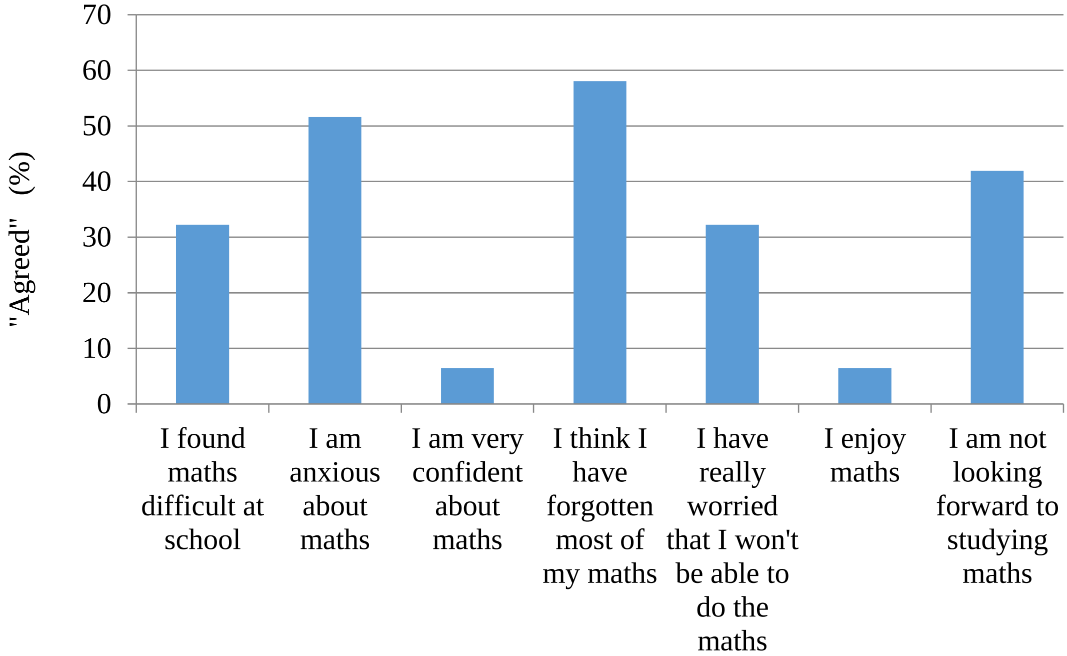

FIG. 5. Student attitude towards studying maths (N=31)

Felten and Lambert (2020) illustrate how human connections can drive students' success by building relationship-rich education. The relationship can improve students' sense of belonging, hence confidence. For the third intervention the teaching specialist created a supportive learning environment in the classroom by building a good rapport with students, ensuring they remembered all the names of the students within the first two weeks of teaching. They also encouraged students to accept mistakes and errors as being normal, repeatedly stating that everyone makes mistakes and ensuring that common mistakes were addressed in a way that included the whole class.

A key feature of classroom practice was the provision of extensive, iterative, and reinforcing practice opportunities (i.e., experiencing the learning). Students were provided with practice questions and both teachers walked around the class providing immediate feedback on student progress via ticks applied directly to answers papers. Moving around the classroom also helped to identify struggling students quickly.

Every week a problem sheet was designed by the subject specialist based on the mathematics knowledge provided in class and broader subject knowledge obtained through other modules on the students' programme of studies. The students were required to solve the problems and bring them to seminar sessions for discussion. Appendix A is an example of a problem sheet that was used for trigonometry to calculate the true thickness of a geological feature after measuring the apparent thickness at the Earth's surface. Although these questions seemed very hard at first sight, with additional guidance provided by the subject specialist and peer-peer support, progress was made, and students were able to appreciate the relevance of their newfound skills. This generated the student's interest in mathematics and enhanced their motivation of students to study mathematics.

3.4. Development of Teaching specialist and Subject Specialist

The success of this team-teaching intervention was not observed solely in the improvement of student outcomes, but also in the development opportunities provided by working closely with a professional with a different background and expertise. While previous studies have sometimes identified the variance in practitioner background as a challenge to be overcome when teaching collaboratively (Bryant et al., 2014), the differences in approach and priorities of the two practitioners in this case formed a basis for improvement of their own approaches, by exposing each other to new content and teaching techniques.

The two staff members were able to observe different ways of understanding how the subject knowledge should be sequenced over the construction of the module. The Earth sciences specialist had a model of mathematics as a tool, so initially wanted the course to be designed around when mathematical principles would be useful for the teaching of the other Earth Science modules the students were undertaking. In contrast, the mathematics specialist had much more focus on how mathematical concepts could be best sequenced to help students build up a conceptual model of how the taught components were inter-related.

This contrast between conceiving of mathematics as a tool versus mathematics as a discipline, in its own right, was also seen in the way the approach to terminology changed during the team-teaching. Initially, only terminology used in other Earth Science modules was being taught. However, working on the module with the mathematics specialist changed the approach the Earth sciences specialist used. Subsequently, students were introduced to a wider range of terminologies enabling them to better deal with underlying concepts and be more flexible about accepting the notion that different notation are used in different contexts.

4. Outcomes

4.1 Quantitative measures

The intervention was measured qualitatively in terms of cohort satisfaction and overall module mark. Cohort satisfaction was assessed using an end of module evaluation questionnaire (MEQ). These measures were compared to identical measures from the four cohorts preceding the intervention. Across two years of intervention, the results showed an increase in both student satisfaction and attainment. In terms of student satisfaction, the student rating out of 5 increased from an average of 3.3 for the four years preceding the intervention, to 4.4 as an average across the two years of intervention. In terms of student attainment, student overall module marks increased from an average of 60% for the four years preceding the intervention, to an average of 71% across the two years of intervention.

TABLE 2. Final assessment marks and MEQ scores

	2012 – 16 (Four years average) (n=141)	2016 – 18 (Two years average) (n=57)
Final assessment marks (%)	60	71
MEQ	3.3	4.4

Both specialists received positive student comments regarding the module and the director of education reported no more complaints about the course.

"xx is a great teacher and almost always had a good grasp on what people were struggling with she would then break down the subject to help people."

"xx's geological problems, whilst often challenging, are highly useful for applying the content we learn in lectures to our degree subject, something which will prove invaluable over the next few years."

"Both lecturers were passionate about their subject but from different angles."

"They complemented each other well."

4.2 Qualitative reflections

In addition to the quantitative outcomes, a large number of observations and reflections were made by the teaching and subject specialists. These provide insights relevant to fellow practitioners and future research. Primary amongst these reflections are those relating to instructor growth.

Both specialists reported learning from the intervention and each other. This was facilitated to a greater degree by observing each other's sessions every week and taking time to reflect on each session. The teaching specialist reported that building Earth Science problems into a mathematics model was particularly challenging and offered insight into the confusion that students experience when solving problems in different disciplinary contexts and the problems incurred by exposure to alternative notation conventions. They also observed the increased engagement that occurs when students used mathematics in real life problems (Herrington, et al., 2014) such as in Earth Sciences.

The subject specialist reported increased awareness of mathematics anxiety as well as increased ability to identify mathematics anxiety amongst students. Overall, it was noted by both subject and mathematics specialists that those students displaying anxiety were not problematic or weak, but displayed different approaches to learning mathematics, and a mentality which sometimes requires alternative teaching strategies. It was also reported that the presence of the subject specialist in the classroom reduced student anxiety. Initially the students were uncomfortable that an Earth Science subject was taught by an "outsider". However, the presence of the subject specialist and their interactions with students eased the situation. Students often asked the subject specialist how some topics applied in Earth Science or other subject issues they were concerned with. Overall, both specialists observed that student confidence appeared to increase and that the volume of interactions (questions, comments, etc) increased.

Following the intervention, the module is now taught solely by the subject specialist with the important aspects of the teaching specialist's approach maintained in their design and delivery. With the benefit of the initial team-teaching intervention, the module has continued to deliver the highest scores for student satisfaction in the departmental MEQs. This suggests that the intervention has made a significant and sustained change that has lasted beyond just the years it was implemented.

5. Discussion and conclusion

Mathematics is becoming ever more important in science and social science yet many students entering these degrees lack the necessary prerequisite mathematics knowledge and skills required and are therefore required to study a remedial mathematics module in their first year. In this case study, more than 50% of the class was anxious about mathematics, and over 40% was not looking forward to studying a mathematics module. Moreover, mathematics modules are often taught by a specialist who may not understand mathematics anxiety or have strategies which might overcome the problem.

The authors recognise that their study has limitations. The approach to assessing mathematics anxiety was qualitative and certainly an alternative quantitative approach could have been applied. Although it was observed that mathematics anxiety was apparent among the cohort of students studied, and they were asked about their feelings about studying mathematics again, their mathematics anxiety was not measured using a formally validated anxiety scale (Hunt et al., 2011), nor was the reduction of mathematics anxiety measured. The overarching purpose of this article was, however, to draw colleagues' attention to the impediment presented by anxiety to mathematics learning among the students who are competent in every other aspect of their study. Perhaps more than that, it showed that the challenge this presents is not insurmountable.

As universities strive to improve teaching quality by providing training courses for educational theory, curriculum design and assessments and feedback, the study demonstrates a practical and effective strategy for collaborative teaching that can help to overcome negative perceptions and bring clear relevance to those aspects of learning that are necessary, but not always palatable for the students. It provides support to increase opportunities for academics to share teaching experience and learn good practice from each other. Improving teaching quality is not something that can happen instantly; it often takes a long period of trial and error. Collaborative teaching over a long period of time can provide unique opportunities for both teaching and subject specialists to learn from each other and improve teaching quality. The strategy presented in this article, if adopted by other departments for similar subjects, may raise the quality and status of teaching in universities to the benefit of the core ambition of the Teaching Excellence Framework in UK (Hubble et al., 2016).

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Author's biographies

Jinhua Mathias

Dr Jinhua Mathias is an associate professor at Durham University. She has a PhD degree in Civil Engineering from University College London and a PGCE qualification from the Institute of Education. She specialises in teaching foundation level mathematics for non-traditional students seeking to pursue undergraduate studies in both STEM and non-STEM subjects. She is a senior fellow of HEA and an active researcher in non-traditional student learning in mathematics. She is also serves on the organising committee of the North East Three Rivers Consortium.

Christopher Saville

Dr Christopher Saville is an assistant professor teaching undergraduate students in the Department of Earth Sciences at Durham University. He is also an active researcher concerning interactions and feedbacks between tectonics, climate and the geomorphology that they create. He is especially focused on orogenic plateaux (specifically the Turkish-Iranian and Tibetan plateaux) as these allow the study of systems (mostly fluvial) that cross the boundary between two differing tectonic and climatic settings.

Steve Leech

Steve is Head of Transitional Education for Durham University, which includes Durham's Foundation Programmes and Graduate Teaching Assistant Training Fellowship programme (Foundation Fellows). He is an Anthropologist and educator and has been teaching in HE since 2002. He is Chair of the Foundation Year Network, the UK's national practitioner network for Foundation Level provision and an Executive Board member and Trustee of Advance Learning Partnership, a North-East Multi-Academy Trust providing outstanding primary and secondary education to some of the country's most disadvantaged children. Steve provides expertise in the fields of Fair Access and Social Mobility and developing strategy for the engagement of under-represented groups within Higher Education.

APPENDIX

Estimating crustal extension from dyke swarms

Aim:

Use geological field data to calculate the proportion of crustal extension along a particular vector.

Simply, you must finish today's exercise with a number showing how much crustal extension the region has undergone in a particular direction.

Learning objectives:

- **Use local measurements to solve a regional problem:** You will say something about the overall deformation in an area from collating specific targeted data.

- **Map for a purpose:** You will make a geological map that shows a particular arrangement of rock types.

- **Geometrical relationships:** You will consider how the orientation of features affects their relative impact on an area, and do appropriate corrections.

- **Time management:** You will balance the time spent collecting the measurements needed to calculate crustal extension with time spent taking more in depth geological observations and with the time needed to process your data and summarise your conclusions.

Exercise:

The area you are collecting data from is composed of Permo-Triassic country rocks intruded by Tertiary basalt and dolerite dykes. These dyke swarms are associated with the opening of the Atlantic ocean between Scotland and Greenland ≈60 Ma. ***Your job is to see how much crustal extension can be attributed to the intrusion of these dykes***.

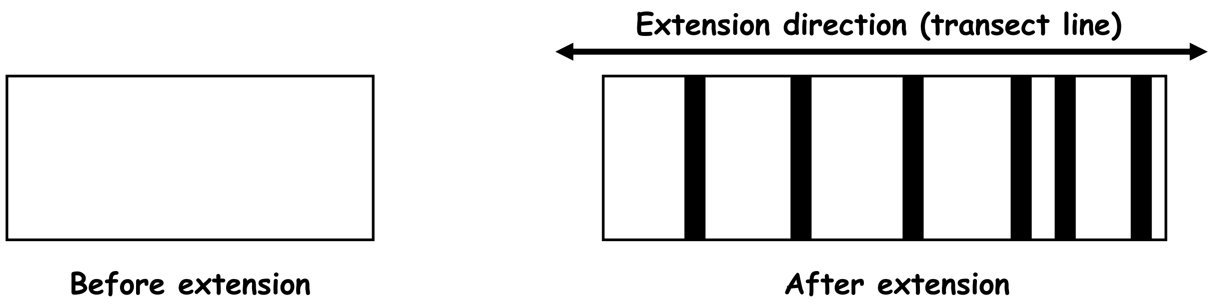

Figure1) Schematic diagram of an area after extension and intrusion of dykes.

This area has a seal colony and nesting birds. ***Please be careful to avoid disturbing these animals.***

Workflow (including maths!):

1) **Measure the dykes:** The measurements you need to take are:

> Width of the dyke – Take this perpendicular to the dyke margins

> Strike of the dyke – The sides of the dyke are very unlikely to be completely parallel (this being real life!) so take a representative strike along the dyke, NOT a strike for each side.

> Dip of the dyke - The sides of the dyke are very unlikely to be completely parallel (this being real life!) so take a representative dip along the dyke, NOT a dip for each side

2) **Decide what other geological data you should collect about the area:** By now you will have had enough experience of mapping to decide what you need to record for yourself.

3) **Decide on a transect line:** After collecting all the data you can start to analyse it. Make sure you have left enough time for this at the end of the exercise!

The amount of extension an area undergoes depends on the direction of extension you consider. If you look at figure 1, intrusion of the dykes has extended that area of crust left to right across the page, but it shows no extension up or down the page.

You will spend the day looking at the dykes intruded into the region. While doing this you should be thinking about what direction the dykes have extended the crust (i.e. perpendicular to the dyke strike). This is the bearing your transect line should go.

To help you figure out what bearing is perpendicular to the dyke strike, this is a good point to do your rose diagram of dyke orientation as a way of displaying the distribution of dyke orientations.

Draw a line at this bearing that is long enough to account for all the dykes that have extended the region. Measure the length of this line (L). It is unlikely that the transect line will go through every dyke you map. Why is this OK?

4) **Calculate difference between the transect bearing and the dykes strike:** Take the strike of the transect line away from the strike of the dykes. If the number is negative just take the positive value of the same magnitude. This number is needed for the next step.

5) **Correct for the orientation of each dyke:** You've measured the width of the dykes, so surely that's the amount the dykes has extended the crust? - Unfortunately not.

The diagrams below should show why the amount the crust has been extended by a dyke is different from the width of the dyke.

Elevation (side) view:

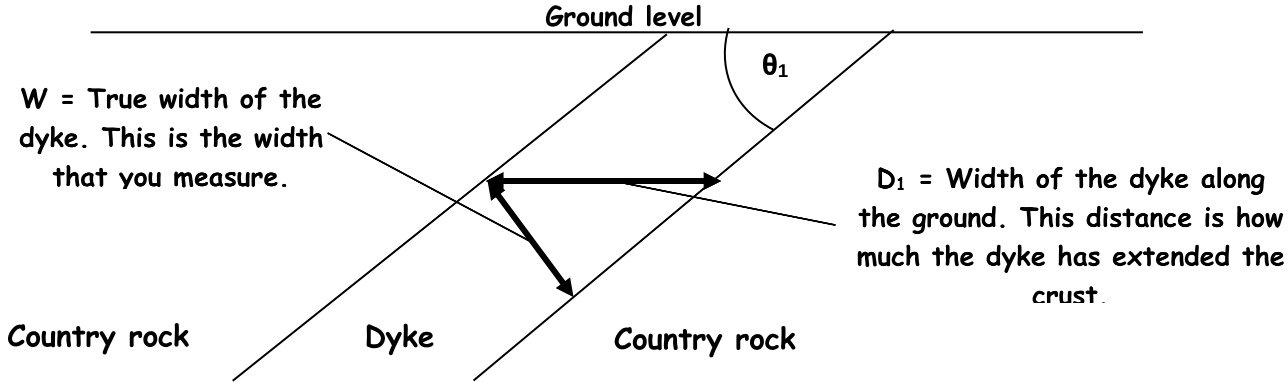

Figure 2) *Schematic diagram showing how the dip of a dyke makes the apparent width (D_1) larger than the true width (W). Unless the dip of the dyke is 90^o D_1 will be larger than W. θ_1 is the dip of the dyke.*

Plan view:

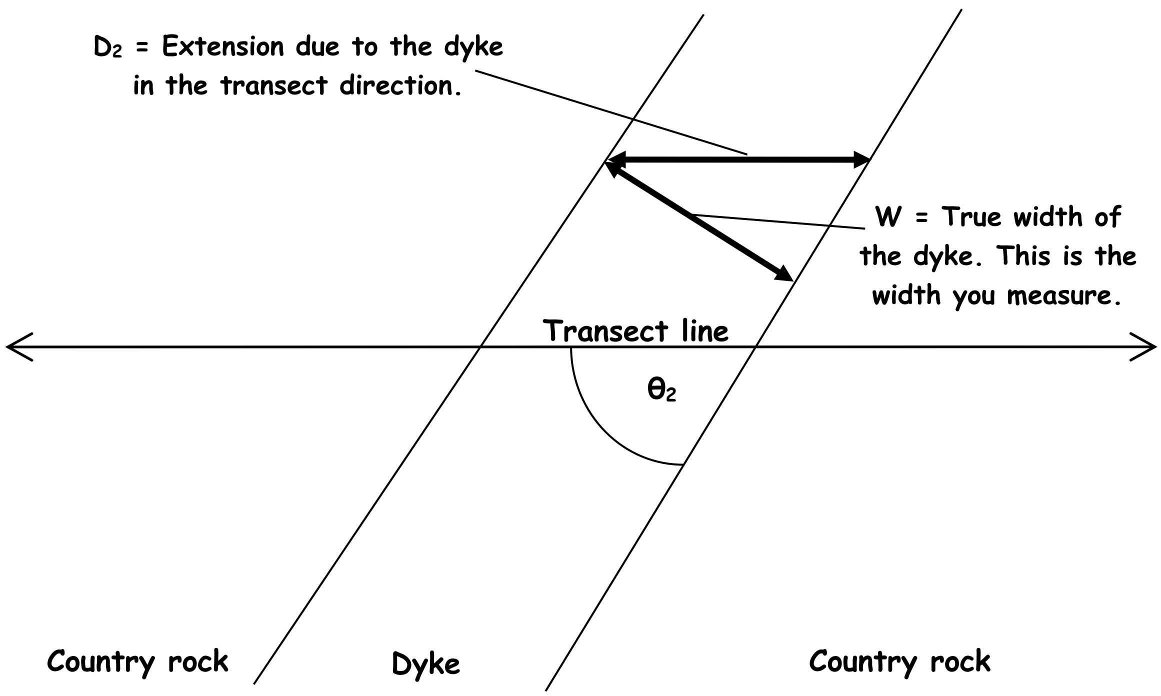

Figure 3) *Schematic diagram showing how the strike of a dyke relative to the bearing of the transect line makes the apparent width (D_2) larger than the true width (W). θ_2 is the angle between the transect strike and the strike of the dyke. Unless this angle is 90^o, D_2 will be larger than W.*

Now you know why the dip and strike of the dyke effect the amount of extension along the transect, you need to correct for this and turn the true dyke width (the measured value), into the amount of extension along the transect.