Going round in circles: Geometry in the early years

Rachel H. Oughton, Dan M. Wheadon, David S. Bolden, Kathryn Nichols, Sam Fearn, Sophia Darwin, Sarah Dixon-Jones, Mrita Mistry, Norbert Peyerimhoff and Adam Townsend discuss how they extended the geometrical thinking of learners in the early years.

The research described here came from a collaboration between university-based mathematicians and early years (EY) educators. The project emerged naturally, driven by the felt need of the EY educators to develop a broader understanding and appreciation of mathematics so that they were able to expose the children to new, challenging and exciting ideas, and to engage with their mathematical curiosities.

Every few weeks, the group of three EY educators and five academic mathematicians met over Zoom. Discussions in the meetings were wide-ranging, encompassing particular mathematical ideas or theorems, and more general concepts such as 'When are two things the same?' or 'What is a theorem?'. The EY educators would give updates about any mathematical ideas or activities in nursery, or themes the children were exploring. The goal of the academics was to explore and extend the mathematical thinking the children were already doing, rather than to follow a particular schedule, or to teach the EY educators maths teaching methods or activities. The mathematicians were led by the EY staff, as they in turn were led by the children in their setting. The expertise of the EY educators is crucial, as they determine how best to use the content of the meetings with their particular children in their day-to-day work.

The circles project

One of the geometrical concepts that really engaged the children was 'circles'. This started when one child found a tree slice and became fascinated by its pattern of concentric circles. These excerpts show some of the exchanges that took place within the setting as the children and practitioners explored circles together. This exploration of circles went on for several months, giving the children plenty of time to revisit ideas and explore them in different ways.

The conversations in the excerpts were transcribed in almost real-time by the teachers and then written up later. The children and staff have been given pseudonyms. The EY educator Nicola's thoughts and rationale are in italics to distinguish them from her speech.

Excerpt 1: Thinking about a definition

Following lots of interest in concentric circles, the children seemed adamant that the "circles always get bigger and bigger" as they grow. Nicola decided to provoke a debate with the group and see how children would react to the idea that the circles got smaller and smaller in size. In preparation, Nicola placed one large circle using string on black paper.

Nicola (practitioner): What can you see?

Eva (child): It's huge!

Rose (child): It's like a footprint.

Harrison (child): It's like a giant footprint.

Eva (child): It's got a bottom and a top.

Rose (child): Like a concentric circle.

Roisin (child): Like a ball.

Everyone: Circle!

Harrison (child): Separate circles. It looks like a big turnip. *(This is a connection to a previous story the children learned).*

Rose (child): It's a concentric circle because it's big.

Harrison (child): No, only one circle.

Abbie (child): A big one!

Patrick (child): A huge circle!

Simon (child): It can't (*be a concentric circle*) because we don't have any more circles.

Patrick (child): We need more string to make it bigger.

Henry (child): You have to have a tiny one in the middle. *(Retrieving earlier learning about the tree slice and the centre point).*

Rose (child): No, because there will only be one circle in the middle of that one. We need more! *(Speaking with confidence and assertiveness).*

Harrison (child): But one more is enough to make it a concentric circle.

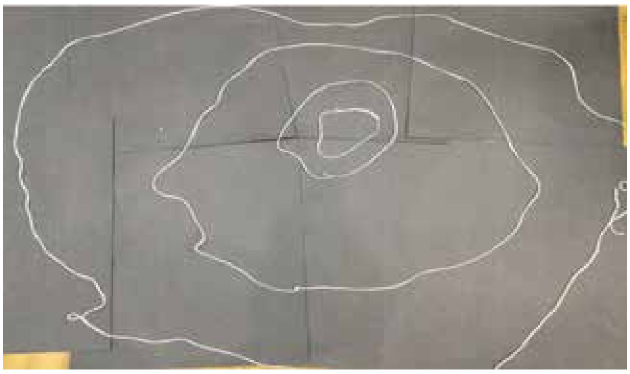

Figure 1: String 'circles'. The outer 'circle' is the initial one placed by Nicola, the inner 'circles' were added during the conversation.

In this exchange, the children display a range of deep geometrical thinking, as well as the ability to debate with one another and challenge ideas. The children are applying what they know about concentric circles to determine whether this first large circle is indeed a concentric circle. There is some misunderstanding of the necessary properties (for example, being big), but this is quickly challenged by other children who remember that with concentric circles, there are always multiple circles. Henry further adds that the new circles need to be in the middle, bringing in more information about how the circles must be arranged. Rose's suggestion that more than one extra circle is needed is quickly challenged by Harrison, showing a deeper level of understanding of the necessary and sufficient properties of concentric circles: more than one extra circle would be allowed, but only one extra circle is strictly necessary. The children debate the exact definition of 'concentric circles' in much the same way as the academic mathematicians among us would do in our research.

A notable feature is the gentle but intentional way Nicola introduces the idea. Her opening question could lead to lots of different responses, and the children are given time to reflect on the knowledge they have gained so far through the class project and to consider one another's points of view. Nicola finds ways to guide the children into seeing concentric circles with a new level of thought.

Excerpt 2: Spirals vs. Circles

During the project, Henry brought a picture of a spiral to Nicola, as something that looked like concentric circles, but was somehow different. As Nicola introduced this new word 'spiral', more conversation was sparked between the group.

Simon (child): I love spirals, them are my favourite. They go fast, round and round.

Abbie (child): I don't love them.

Rose (child): The spiral gets bigger. The concentric circles is getting bigger, but they are separate.

Henry (child): The spiral is not separate - all one line.

(Thinking further about things)

Nicola (practitioner): How do we know if we have concentric circles?

Simon (child): They are all inside each other.

Rose (child): They are all separate circles.

Hayley (child): The little ones sit in the big ones.

Nicola (practitioner): What can you tell me about a spiral?

Henry (child): It curls around.

Hayley (child): It's a curved line.

Patrick (child): It's a long line.

In this brief exchange, the children have pinpointed one of the key points of difference between a set of concentric circles and a spiral: The spiral is made of one single line, whereas the concentric circles comprise two or more separate circles that are nested in a particular way. This links to an area of maths called 'Topology', where geometric objects are considered to be the same if you can change one into the other by squeezing, stretching, or bending (but without breaking, merging or crossing any lines or surfaces). As the children focus together on drawing concentric circles and spirals, they see that unlike concentric circles, a spiral can be drawn in one fluid motion. As others have found before us, the act of drawing can cause children to become aware of geometric concepts. Explorations with string show that the spiral can be 'unravelled' and rearranged to form other, apparently very different, shapes, whereas the concentric circles are a fairly inflexible structure, remaining nested in their formation unless circles are broken, or lines are crossed. Once again, the children have moved beyond the relatively simple act of naming the two figures and are considering the properties that make them different. They use drawing, verbal and logical skills to explore these geometrical ideas.

Figure 2: Using drawing to compare concentric circles and spirals.

The children were then given sheets of acetate, so that their drawings could be projected onto the wall. They began to explore properties like orientation, size, medium, colour and so on, and to talk about one another's drawings and ideas as they worked. This touches on an idea that came up frequently in the discussions between the mathematicians and the teachers: sameness. Determining in which ways two objects, numbers, shapes, statements etc. are the same, or under what conditions they can be considered the same (or logically equivalent), is an important area of mathematical thought that is often lacking in school aged children. Is a spiral drawn starting anti-clockwise 'the same' as one drawn starting clockwise, for example? Does it matter whether the shape is seen on paper, on acetate or on the wall? Here, the children are thinking about how a particular shape is defined, and what are its necessary and sufficient properties.

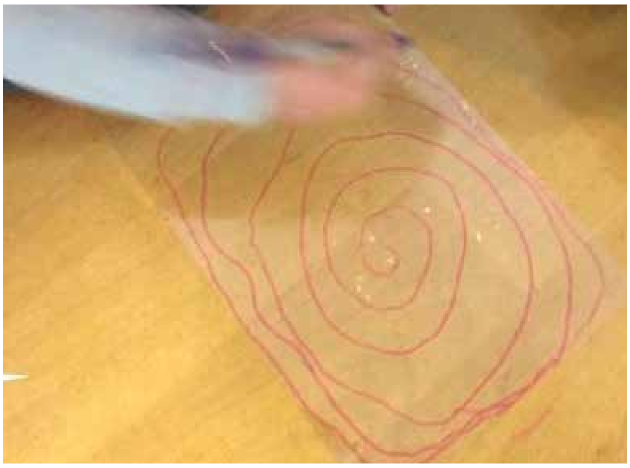

Figure 3: Drawing a spiral on acetate, ready to be projected.

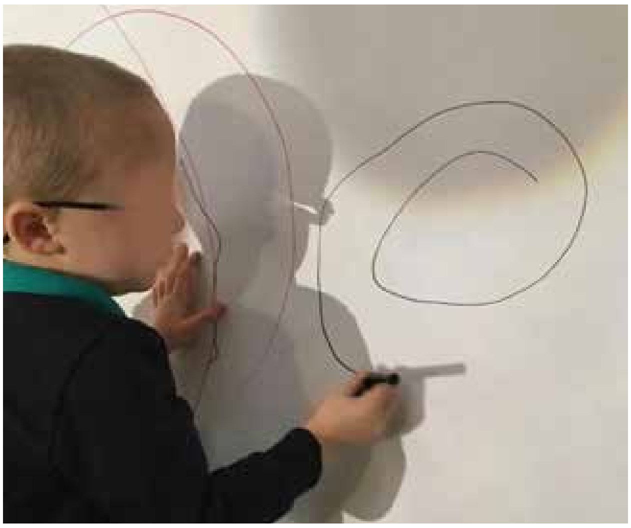

Figure 4: Experimenting with drawing a spiral 'the other way around' (starting anti-clockwise).

Excerpt 3: Measurements and theorems

The goal for this session was to revisit some earlier concepts about circles, and to move on to thinking about the centre point of the circle. Nicola had prepared a collection of circular objects in advance of the children's arrival ready to support the exploratory conversation.

Figure 5: Using the 'square' method to find the centre of the circle.

Everyone: Circles.

Henry (child): (*Pointing*) That's bigger than all the rest. That medium and that the biggest.

Nicola (practitioner): How do you know? How can you tell?

Henry (child): (*Placing objects on top of one another to compare*) So when you put that on it bigger. Look, goes all around, see?

Nicola (practitioner): *Henry's ability to visualise the* *difference in size was clear, but the other children remained unsure.*

Nicola (practitioner): What could we do to check they are bigger?

Jacob (child): That measuring tape there.

Simon (child): Measure things

Petra (child): It's got numbers on.

Nicola (practitioner): *Henry then layered up the circles and measured straight across the top circle. Everyone then had the opportunity to play with the measuring tools with a natural flow of expressive language being used, from this the words 'diameter' and 'width' emerged.*

Nicola (practitioner): How do we find the centre point?

Henry instantly pointed to the middle of the tree log slice.

Nicola (practitioner): How do you know this is the exact middle?

Henry (child): (*still pointing*) Because that's the littlest circle in the middle.

Nicola (practitioner): *I showed the children how to find the middle by creating a square around the circle then using diagonal lines crossing straight through the middle. Children were given time to explore how to do this, testing their problem-solving skills. During this process we were able to introduce language such as vertical, horizontal and diagonal lines. Once they placed the diagonal lines the children could see the centre of the circle. This was quite a rewarding discovery for them. With the centre mapped out the children wanted to measure again to see what the diameter was using the widest part of the circle. More discussion arose as the children discovered that whichever point they go from on the edge of the circle, as long as they measured through the middle they measured the same diameter. Henry was genuinely fascinated by this remarkable finding.*

Henry (child): Can I stay in here and do this?

Nicola (practitioner): *This question was a powerful one. Henry was seeking permission to continue in his own discoveries, displaying real commitment.*

A theorem tells us something that, given certain conditions, is always true. In this exchange, there are several theorems encountered by the children. Firstly, Henry has his 'theorem' that if you put one circle on top of another, and the bottom circle sticks out all around, the bottom one is the bigger of the two. Secondly, Nicola introduces a procedure to use a square to find the centre of a circle. That this works for any circle is also a theorem. As Nicola notes, the children revelled in the results of their hard work and their new knowledge that they could apply to any circle. The third 'theorem' stemmed from the children's discovery: as long as you measure through the circle's centre, the distance from one edge to the other will be the same in any direction. In this, the children were touching upon exactly what distinguishes a circle from any other shape.

Figure 6: Measuring the diameter through the centre.

Figure 7: Henry's collection of circles to measure.

Reflections from nursery staff

EY educator:

> By working together, the mathematicians and practitioners within school achieved an exciting mathematical pedagogy that brought depth to mathematical learning opportunities. As well as depth to mathematical concepts explored encompassing a broad and varied range of concepts, language and theories, children had high levels of involvement and well-being, they held a sense of drive and thirst to extend their initial ideas with a mathematical inquiry approach to learning. The children and practitioners had a strong sense of belonging within a learning community developing confident mathematical thinkers, and young mathematicians.

As a school we have audited the learning environment to ensure that it is a mathematics friendly environment consisting of provocations and continuous provision that promotes mathematical explorations, stirs curiosity and gives children time - time to think things through, ask their own questions, create hypotheses and debate their own points of views with peers and adults whilst approaching their learning in a multimodal way, to make sense of the mathematical world that is around us every day of life.

Head teacher:

I very much see staff as being the golden thread in school – they hold the children's thoughts, ideas and wonders in mind and cleverly use them as a starting point to build knowledge skills and understanding. As a result, I am always looking at how we can provide staff with high-quality CPD that extends their own knowledge and impacts the children. Maths has always been an area where staff have felt less confident. That was until we began working with the university mathematicians. This project has had a huge impact on the staff and children within school. The children embraced very different mathematical concepts compared with what would usually be happening and the staff had access to valuable knowledge and information that allowed them to work with the children in a different way – you could visibly see that the staff had a 'buzz' about maths and a new confidence to talk about 'maths things'. I think ultimately there has been a cultural change in the way we as a hard federation approach and 'see' maths.

EY educator:

The whole process of exploring these concepts and the thinking around them was invaluable to us as practitioners; it was almost like being given a set of keys to help us unlock children's thinking into different spaces of deeper/higher level thinking.

Conclusion

Young Minds Big Maths did not start with the explicit aim of exploring geometry, but we found that many of the children had a great natural appetite for geometrical ideas: appreciating abstract concepts, making spontaneous links to geometry in 'non-maths' activities, spotting patterns in new environments, and choosing to spend more time carrying out their geometrical investigations. The collaboration helped equip the EY educators to respond to their curiosities and to extend their explorations, and this led to richer mathematical experiences throughout the setting.

We noticed that as the children were exploring geometrical ideas (or indeed mechanics / physics related ideas, which proved another big theme often overlapping with geometry), numbers were used very little at first. However, the children often arrived at a point where they needed numbers to measure, order or count something. The numbers at this point held much more meaning to them than if they had been the starting point.

Based on our project, we found that a more teacher-led approach to professional development leads to increased teacher confidence and mathematical ability, for which, ultimately, the beneficiaries are the children and their mathematical learning experiences.

References

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Mrita Mistry and Kathryn Nichols are EY educators from Houghton Community Nursery School, Tyne and Wear, of which Sarah Dixon-Jones is the headteacher.

Sophia Darwin, Sam Fearn, Rachel Oughton, Norbert Peyerimhoff and Adam Townsend are members of academic staff at the Department of Mathematical Sciences, Durham University, and together form the mathematics outreach committee.

David Bolden is an associate professor and Dan Wheadon is an undergraduate student, both at the School of Education, Durham University