Discussion of Specifying Prior Distributions in Reliability Applications

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Abstract

The paper *Specifying Prior Distributions in Reliability Applications* (Tian et al. (2023)) mainly provides an overview of methods for selecting non-informative prior distributions for parameters of basic lifetime distributions, as often used in reliability analyses. This discussion raises some related issues and comments on opportunities beyond basic Bayesian statistical methods which may be useful in reliability scenarios. The main emphasis in this discussion is on practical reliability analyses with few data available, where there is often need for informative priors rather than for non-informative priors, in order to take expert judgement into account. Furthermore, while rather abstract considerations of non-informativeness of prior distributions is of theoretic interest, in most practical scenarios one aims at decision support, and the influence of assumed priors on the final decisions should be considered, ideally with robustness of the final decision with regard to all priors which are deemed to be reasonable.

Keywords: Decision support, elicitation, expert judgement, imprecision, prior-data conflict, robustness

1. Statistical methods for reliability

The paper *Specifying Prior Distributions in Reliability Applications* (Tian et al. (2023)) mainly presents an overview of different prior distributions that are claimed to be non-informative, meaning that their influence on inferences based on the corresponding posterior distribution is small, already in case of

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small data sets used for Bayesian updating. The abstract of the paper starts with mentioning that, in reliability applications, there are often limited data available, hence the possibility to include expert judgements is an advantage of Bayesian methods, but the paper mainly considers priors that are chosen in order not to reflect expert judgements. In Section 2 we will discuss aspects of the use of expert judgements in reliability.

First, however, some common misunderstandings about statistical methods should be addressed. It is not true that frequentist statistical methods need asymptotic theory for their justification. For example, nonparametric predictive inference (Coolen et al. (2014)) only assumes a post-data version of finite exchangeability of the random quantities related to data observations and future observations, and enables reliability inferences for many scenarios without the need for substantial data. The related theory of conformal prediction enables the inclusion of parametric models for exactly calibrated frequentist inference based only on finite exchangeability assumptions (Vovk et al. (2005)). Note that the assumption of exchangeability (De Finetti (1974)) is often not carefully considered in statistical inference, but it underlies Bayesian statistics. In particular, the commonly used Bayesian approach for updating prior distributions by multiplication with the likelihood function, is typically justified by the assumption of infinite exchangeability, through De Finetti's Representation Theorem (De Finetti (1974)), which is a stronger assumption than the finite exchangeability required for the frequentist methods mentioned above. Hence, one could argue that the Bayesian inference approach is more dependent on asymptotic arguments than some frequentist statistics approaches.

Another common misunderstanding is that a small number of failures in reliability data implies that the data provide little information. That would be remarkable, as it would apparently mean that one could not learn much from experiencing extremely reliable systems over long periods of time. Of course, if systems have functioned for long periods without failures, that provides a lot of information which can be included in statistical analyses and decision support without a problem. A further common misunderstanding is that only Bayesian methods can be used to take knowledge of the physics underlying failure processes into account. Such knowledge would be included in the stochastic model for the reliability problem at hand, and the inferences based on such models can be frequentist or Bayesian, it makes no real difference. This is immediately clear from the fact that the Bayesian approach is based on the likelihood function, hence non-Bayesian statistical approaches based on the likelihood function are equally applicable. An example of such inclusion of knowledge of physics underlying failure processes is in accelerated life testing, where the Arrhenius model may be appropriate if temperature is the accelerated factor (Coolen et al. (2021)).

When it comes to study of statistical theory and methods, there is often quite a difference between academic exercises and work towards solving real-world problems. The former are often easier to perform and, perhaps as a consequence, more prominent in the literature. It is, however, important not to forget that methods are developed for solving problems, often for decision support. With regard to choice of Bayesian prior distributions, it may actually simplify things to focus on practical decision support, as one can quite easily see if different priors have noticeably different effects on resulting decisions, hence one does not have to worry too much on different concepts for non-informativeness of prior distributions. However, in many situations where statistical analyses are performed in order to support practical decision making, there is a clear wish, or even need, to take expert judgements into account. Some aspects of this are addressed in the next section.

2. Expert judgements in reliability

In many reliability applications, it is important to take expert judgements into account for uncertainty quantification due to a shortage of data. Such judgements may be needed to choose an appropriate system failure time distribution, or failure process model, and they can be further incorporated through the prior distribution in the Bayesian framework. A major challenge is the selection of a suitable prior distribution for the model parameters, such that the expert judgements well incorporated in the analysis. This process, called elicitation, is far from trivial and requires careful preparation. An obvious requirement, though often not mentioned, is that the problem considered must be important enough to the experts for them to engage fully in the process. Furthermore, the statistician leading the modelling process must realize that the topic experts are unlikely to be experts in statistics. Even more problematic, perhaps, is that even experts in statistics may struggle to carefully reflect their judgements in the form of prior probability distributions for model parameters, unless these have a clear interpretation which is meaningful to the experts; this is unfortunately rarely the case.

One way around the problem of model parameters being hard to interpret, is not to focus on such parameters in the elicitation of expert judgements.

It is often far easier for topic experts to express their judgements on observable quantities, for example time to component failure, rather than on a parameter for an assumed distribution. In the Bayesian approach, this corresponds to elicitation of aspects of the prior predictive distribution, which combines the assumed parametric model with the prior distribution. Fitting the prior distribution such that the prior predictive distribution corresponds reasonably well with expert judgements is often a suitable way to take such judgements into account. If the parametric model is such that there are conjugate prior distributions available, such that Bayesian updating is simply done by combining sufficient statistics from the likelihood with hyperparameters, which specify the prior distribution, then expert judgements can be elicited in terms of pseudo-data or the related sufficient statistics of such pseudo-data. Here, the expert reflects their judgements through an imaginary data set, or sufficient statistics for such an imaginary data set. This well-known idea can also be used for situations where experiences are mostly based on data including censored observations, for example if components have never been observed past a specific age. The pseudo-data representing prior beliefs can then also include censored data, which may simplify practical elicitation (Coolen (1996)).

Other important topics for practical applications of Bayesian methods in reliability problems include the question of who the experts are, or, if there are multiple experts whose judgements are valuable, how to combine these. Also, if one is not dealing with a one-off problem but may want to consider multiple related problems over time, using information from the same group of experts, it may be useful to learn about the expertise of the experts, and include this in the way their judgements are combined. There is substantial literature on such topics, and it goes beyond the scope of this discussion, but the main recommendation is for such issues to be considered in real-world applications, as suitable methods for one scenario may not work for other scenarios. For example, Coolen et al. (1992) and Wooff et al. (2018) describe two reliability applications where expert judgements needed to be elicited and dealt with in different ways.

3. Imprecision and prior-data conflict

For practical reasons, elicitation of expert judgements is often difficult. There may be limited time for the exercise, and experts may not feel confident to provide precise inputs, for example they may only feel confident to state that a mean failure time for a component is between 60 and 80 hours, rather than giving a precise value. It is best to work with such imprecise information in the resulting analyses, using a set of prior distributions corresponding to the imprecise expert judgements. This reflects the further uncertainties involved in the information, and this is possible using theory of imprecise probabilities (Augustin et al. (2014)). For example, one may want to choose a decision which leads to good performance for the posterior distributions corresponding to all prior distributions in such a set of prior distributions, providing robustness with regard to the experts' uncertainties.

As a further advantage of the use of imprecise probability theory, the use of sets of prior distributions in the Bayesian framework can reflect conflict between prior judgements and data (Walter et al. (2017)). This can be a useful tool to gain further insight into the uncertainties involved in a reliability study, as any discrepancies between expert judgements and process data are useful to provide more insights into the problems considered.

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