Durham University Business School

Centre for Economic Growth and Policy

Unconditionally Optimal Ramsey policy

Tatiana Damjanovic, Vladislav Damjanovic and Charles Nolan

Working Paper 2017 (01)

Unconditionally Optimal Ramsey policy*

Tatiana Damjanovic[†], Vladislav Damjanovic[‡] and Charles Nolan[§],

January 2017.

Abstract

We discuss a time invariant policy which delivers the unconditionally optimal outcomes in purely forward-looking models and Ramsey outcomes in purely backward-looking models. This policy is a product of interaction between two institutions with distinct responsibilities. Motivated by Brendon and Ellison (2015), we think of them as arms of government. One institution is responsible for 'forward guidance', setting rules which are necessary and sufficient to determine private expectations. The second institution implements optimal policy taking expectations as given. The forward guidance rules are designed to maximise the unconditional expectation of the social objectives.

JEL Classification: E50; E60; E51.
Keywords: Time inconsistency, forward guidance, unconditional optimisation, Ramsey optimal policy.

*The paper was inspired by Brendon and Ellison (2015). We are garteful to Martin Elison and to participants of the Birkbeck Centre for Applied Macroeconomics workshop for helpful comments and discussion.

[†]Durham University
[‡]Durham University
[§]University of Glasgow

1 Introduction

Ramsey policy is time inconsistent in models with forward-looking behaviour (Kydland and Prescott, 1977). That is because the government can affect the economy via its current and future actions. Future policy influences current outcomes via an expectations channel, whilst current policy only affects the economy contemporaneously. Therefore the optimal policy at 'period zero' is different from future optimal policy. That property of optimal Ramsey rules is known as time inconsistency and typically implies that it will not be optimal for policymakers to make good on policy promises when the time arrives to deliver on those promises.

To deal with time inconsistency, two types of time invariant rules are often considered in the literature. The first analyses policies which maximise the unconditional expectation of the social objective (unconditionally optimal, or UO, policy) proposed by Taylor (1979).[1] The second analyses policies that are optimal from a Timeless Perspective (TP policy) introduced by Giannoni and Woodford (2002), which assumes commitment to Ramsey policy designed many periods ago. Commitment to these invariant rules is tantamount to assuming that institutions have been devised which deliver the outcomes associated with these rules although that assumption is rarely, if ever, made explicit. We discuss this issue in a little more detail below.

In many cases UO and TP policies perform very similarly. Indeed, Blake (2001) and Damjanovic, Damjanovic and Nolan (2009) proved that TP can be converted to UO policy if the government were accounting for all generations equally: that is if it sets the social discount rate equal to zero. Nevertheless, TP and UO policies have important theoretical and sometimes quantitative differences and neither obviously dominates the other.

On the one hand, TP policy can lead to non-stationary outcomes in models with forward-looking constraints (Blake and Kirsanova, 2004; Benigno and Woodford, 2012). In particularly, Schmitt-Grohe and Uribe (2004) demonstrate that TP policy results in non-stationary dynamics of government debt. Unlike the TP, the UO policy implies stationarity by design, since any non-stationarity would result in infinitely large expected value of the loss function[2]. Moreover, TP policy may put an unreasonably large weight on a relatively distant event in the past, which is not the case for UO policy (Jensen and McCallum, 2010).

[1] Since then, unconditional welfare optimisation has been widely used in the literature. Whiteman (1986), Rotemberg and Woodford (1997, 1998), Clarida, Gali and Gertler (1999), Erceg, Henderson and Levin (2000), Kollman (2002) and McCallum (2005) are some prominent examples.

[2] Horvath (2011) has shown that government debt dynamics are stationary when UO policy is implemented

On the other hand, TP policy has a number of attractive features in models with only backward looking constraints. Thus, it coincides with Ramsey optimal discretionary policy. That is not the case for UO policy which requires a commitment device even in purely backward looking models. And whilst it is true that the UO policy would dominate TP policy conditional on the fact that all generations had followed it in the past, since the current generation would have had a better start in terms of economic environment[3], even in this case, it is still optimal for the current generation to deviate towards TP. Therefore, there is a sense in which TP policy is more stable and could be preferable in models with backward-looking constraints.

As UO policy performs better in forward-looking models whilst TP/Ramsey policy is sometimes more desirable in backward-looking models, the question is which policy to use if an economy has both types of constraints. In this paper we discuss a time invariant policy which inherits properties of the TP policy in backward-looking and UO in forward-looking models. We will call this policy UO-Ramsey. Our paper shows an easy, intuitive and transparent way to design such a policy.

Following Brendon and Ellison (2015) we consider a little more explicitly the issue of institutional design. In particularly, policies are designed by two authorities with distinct and distinctive responsibilities–we think of these two authorities as arms of government. One arm of government ("*outer*" government in Brendon and Ellison), is responsible for forward guidance[4]. The outer government makes promises and determines private expectations about future policy outcomes. The second arm of government ("*inner*" government) implements policy taking promises and corresponding private expectations as given. In this framework, expectations are taken as exogenous and therefore cannot be changed by the inner government. As the inner government cannot use the expectations channel to affect the economy, the inner policy maker does not face any problem related to time consistency.

Our main contribution compared to Brendon and Ellison (2015) is that we propose an alternative way to design the problem of the forward-guiding outer government, which is responsible for expectations formation. In this paper we show that the outer government maximises the unconditional expectations of social objectives. Hence, although the policy outcome is the same as Brendon and Ellison (2015), our approach serves to illuminate the fundamental objectives of the outer government in

[3]According to Ramsey (1928), discounting future generations welfare is unethical. See also Pigou (1932), who thought that private discount rates are excessive. For a detailed discussion about the advantages of UO policies in backward-looking models Damjanovic, Damjanovic and Nolan (2015).

[4]In terms of monetary policy, forward guidance is defined as communication about the likely future course of monetary policy and the FOMC began using forward guidance in its post-meeting statements in the early 2000s. Since then forward guidance has attracted attention among academics and policymakers. See for example Svensson (2014).

an intuitive and transparent way.

After presenting the design of UO-Ramsey policy, we consider two interesting applications. In the first one we design UO-Ramsey policy in a linear-quadratic model with a so-called hybrid Phillips curve. This case can easily be nested to either the purely forward-looking new Keynesian model or to the purely backward-looking model where Ramsey policy is time consistent. Our policy will deliver UO policy in the first case and Ramsey policy in the second.

Then we consider UO-Ramsey fiscal policy in a model with government debt. We show that, in contrast to the TP policy, the UO-Ramsey policy implies a non-trivial relationship between real variables and government debt.

The paper is structured as follows. Section 2 describes and explains the design of UO Ramsey policy. Section 3 applies the UO-Ramsey policy to a linear-quadratic model with hybrid Phillips curve. Section 4 discusses the UO-Ramsey fiscal policy in an economy with debt. Section 5 concludes.

2 The model

Following Brendon and Ellison (2015) we introduce two arms of governments. The first makes promises about future policy outcomes which become part of the forward-looking constraints of the policy problem. This may reflect decisions by parliament or the executive branch of government determining, say, the legal framework for monetary policy (e.g., the inflation target) or of an independent committee responsible for forward policy guidance. Brendon and Ellison (2015) name that government "outer" or "promise - making" government. The second arm of government maximises social welfare choosing among the policy rules which are consistent with expectations about promises. This is the "inner" government and it takes both promises and expectations as given. Note, that without an assumption of forward-looking behaviour, there will be no scope for outer government as nothing will depend on expectations. On the other hand, if all constraints were forward looking, "inner" government would have no choices to make; it will be completely constrained by the promises made by "outer" government. The inner government takes promises as exogenous constraints which have to be respected. Within that framework, inner government has no power to alter expectations, and from its perspective, the environment is purely backward-looking and there is no problem with time inconsistency. Therefore, the inner government acts as a fully effective Ramsey policymaker whose actions are credible and time consistent.

The actions of the outer government are easy to describe: it makes promises which maximise the unconditional expectation of the social welfare function.

2.1 Formal set up

Consider a model where social welfare is the discounted stream of utilities,

$$W = E_0 \sum_{t=0}^{\infty} \gamma^t u(x_t).$$

There are certain dynamic constraints described as

$$x_t = F\left(x_{t-1}, x_t, E_t x_{t+1}, \mu_t\right), \tag{1}$$

where x_t is a vector of endogenous variables, including policy tools, and μ_t is an exogenous shock, γ is the policymaker's time discount factor. Following Brendon and Ellison (2015) we decompose constraint (1) into its forward-looking and backward-looking parts. The expectations part will be managed by the outer, promise-making, government. The promise-making government sets state-dependent promises about all variables which are included in the expectations part of (1). That promise is in the form $E_t x_{t+1} = \omega_{t+1}$. In other words, we will replace constraint (1) with

$$x_t = F\left(x_{t-1}, x_t, E_t \omega_{t+1}, \mu_t\right), \tag{2}$$
$$x_t = \omega_t. \tag{3}$$

The inner government will be responsible for setting x_t but will take promises ω_t as given. The inner government will not be able to affect the economy through the expectations channel and will treat expectations as exogenous. Therefore their optimisation problem has no forward looking part and their fully optimal Ramsey policy will be time-consistent. Up to that step, we follow Brendon and Ellison (2015). Formally, the Lagrangian for the inner government is

$$V(\omega) = \max_{x_t} E_0 \sum_{t=0}^{\infty} \gamma^t \left\{ u(x_t) + \lambda_t \left[-x_t + F\left(x_{t-1}, x_t, E_t \omega_{t+1}, \mu_t\right) \right] + \rho_t \left(x_t - \omega_t\right) \right\}. \tag{4}$$

And the first order condition is

$$\frac{\partial V}{x_t} = u'(x_t) + \lambda_t \left[-1 + \frac{\partial}{x_t} F\left(x_{t-1}, x_t, E_t \omega_{t+1}, \mu_t\right) \right] + \gamma E_t \lambda_{t+1} \frac{\partial}{x_t} F\left(x_t, x_{t+1}, E_{t+1} \omega_{t+2}, \mu_t\right) + \rho_t = 0. \tag{5}$$

Our contribution is to suggest that the promise-making government should set promises ω_t in such a way that they will maximise the unconditional expectation of the social value function. So the problem of the outer government is

$$\max_{\omega} J = E_u V(\omega), \tag{6}$$

where E_u denotes the unconditional expectation. Plugging in the value function of the outer government, we get

$$J = \max_{\omega} E_u \left(\sum_{t=0}^{\infty} \gamma^t \left\{ u(x_t) + \lambda_t \left[-x_t + F\left(x_{t-1}, x_t, E_t \omega_{t+1}, \mu_t\right) \right] + \rho_t \left(x_t - \omega_t\right) \right\} \right).$$

Applying the UO maximisation algorithm in Damjanovic, Damjanovic and Nolan (2008) we obtain the first order condition for the promise-making government

$$\frac{\partial J}{\partial \omega_t} = \lambda_{t-1} \frac{\partial}{\omega_t} F\left(x_{t-2}, x_{t-1}, \omega_t, \mu_{t-1}\right) - \rho_t = 0. \tag{7}$$

The policy we propose solves problem (6), subject to constraints (2, 3). The solution satisfies the first order conditions (5), and (7). In purely backward looking models it coincides with Ramsey-optimal policy. In purely forward-looking models it coincides with UO policy. The next section shows that result in a transparent way using a model that has been popular in applied monetary policy analysis.

3 Hybrid Phillips Curve

In this section we show how to derive the UO Ramsey policy in a model with a hybrid Phillips curve. The government has a conventional loss function which consists of output and inflation gap terms

$$\min_{\pi_t, y_t} L = \sum \gamma^t \left(\pi_t^2 + y_t^2\right). \tag{8}$$

The behaviour of the private sector is described by a hybrid Phillips curve:

$$\pi_t = y_t + (1-\phi)\beta E_t \pi_{t+1} + \phi \pi_{t-1} + \mu_t, \tag{9}$$

where β is the discount factor of the private sector and μ_t is an exogenous cost-push shock.

Let s_t denote the state of the economy which consists of the history of exogenous shocks. To design an UO Ramsey policy the outer government creates a menu of state-dependent promises about future inflation

$$\pi(s_t) = \omega(s_t). \tag{10}$$

Then, the Hybrid Phillips curve is transformed into

$$\pi_t = y_t + (1-\phi)\beta E_t \omega_{t+1} + \phi \pi_{t-1} + \mu_t, \tag{11}$$

where promise ω_t is given by the outer government and taken as an exogenous variable by the inner government. The inner government minimises loss (8) taking promises as given. The Lagrangian is

$$J = \min_{\omega_t} E\left[\min_{\pi_t, y_t} \sum_{t=0}^{+\infty} (\gamma)^t \frac{1}{2}\left(\pi_t^2 + y_t^2\right) + \rho_t\left(-\omega_t + y_t + (1-\phi)\beta E_t \omega_{t+1} + \phi \pi_{t-1} + \mu_t\right) + \eta_t\left(\omega_t - \pi_t\right)\right].$$

The first-order conditions for inner government are

$$(\gamma)^{-t} \frac{\partial J}{\partial \pi_t} = \pi_t + \phi\gamma E_t \rho_{t+1} - \eta_t = 0; \tag{12}$$

$$(\gamma)^{-t} \frac{\partial J}{\partial y_t} = y_t + \rho_t = 0. \tag{13}$$

Now the outer government needs to minimise the unconditional expectation of loss (8) subject to private behaviour and beliefs (11), and the promise-keeping condition (10). To solve it we follow Damjanovic, Damjanovic and Nolan (2008) who show that UO policy is similar to TP/Ramsey policy if the government time discount rate is set equal to zero ($\gamma = 1$). The first-order condition for outer government is

$$\frac{\partial J}{\partial \omega_t} = -\rho_t + (1-\phi)\beta\rho_{t-1} + \eta_t = 0. \tag{14}$$

The combination of (12), (13) and (14) results in the following combined policy rule

$$\pi_t = \phi\left(\gamma E_t y_{t+1} - y_t\right) + (1-\phi)\left(\beta y_{t-1} - y_t\right).$$

Table 1 compares UO, Ramsey and UO-Ramsey policies for an economy with a hybrid Phillips curve. As can be seen, UO-Ramsey policy is identical to UO policy when the Phillips curve is purely forward-looking ($\phi = 0$); and it coincides with the Ramsey policy for the backward-looking case ($\phi = 1$).

Table 1. The optimal monetary policy from different perspectives

	Backward-looking	**Forward-looking**
Phillips curve	$\pi_t = y_t + \pi_{t-1} + \mu_t$	$\pi_t = y_t + \beta E_t \pi_{t+1} + \mu_t$
Ramsey/ TP	$\pi_t = \gamma E y_{t+1} - y_t$	$\pi_t = \frac{\beta}{\gamma} y_{t-1} - y_t$
UO	$\pi_t = E y_{t+1} - y_t$	$\pi_t = \beta y_{t-1} - y_t$
UO-Ramsey	$\pi_t = \gamma E y_{t+1} - y_t$	$\pi_t = \beta y_{t-1} - y_t$
	Hybrid Phillips Curve	
Phillips curve	$\pi_t = (1-\phi)\beta E_t \pi_{t+1} + \phi\pi_{t-1} + y_t + \mu_t$	
Ramsey/ TP	$\pi_t = (1-\phi)\left(\frac{\beta}{\gamma} y_{t-1} - y_t\right) + \phi\left(\gamma E y_{t+1} - y_t\right)$	
UO	$\pi_t = (1-\phi)\left(\beta y_{t-1} - y_t\right) + \phi\left(E y_{t+1} - y_t\right)$	
UO-Ramsey	$\pi_t = (1-\phi)\left(\beta y_{t-1} - y_t\right) + \phi\left(\gamma E_t y_{t+1} - y_t\right)$	

4 UO-Ramsey policy and government debt

It is well known that the TP approach to optimal policy results in non-stationary behaviour of government debt. In contrast, debt is stationary when UO policy is adopted (Horvath, 2011). Since UO-Ramsey policy involves unconditional optimisation, it has to result in a stationary outcome as in UO case. That is simply because a non-stationary policy will result in an infinitely large value of the loss function.

In the following example we show that TP policy provides no role for public debt in the optimal consumption program whilst UO-Ramsey policy implies a negative relation between consumption and debt.

Consider the following simple model where households maximise utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(U(C_t) - V\left(N_t\right) \right). \tag{15}$$

subject to the budget constraint

$$C_t + B_t = R_{t-1} B_{t-1} + (1 - \tau_t) W_t N_t, \tag{16}$$

where E_t denotes the expectations operator at time t, β is the discount factor, C_t is consumption and N_t and W_t are labour and wage respectively. The household decides how much to save in the form of government bonds, B_t, with risk free return R_t, given wage W_t and income tax τ_t. The necessary conditions for an optimum include:

$$V'\left(N_t\right) = (1 - \tau_t) W_t U'(C_t); \tag{17}$$

and

$$R_t \beta E_t U'(C_{t+1}) = U'(C_t). \tag{18}$$

Then Government's policy problem can be written as the maximisation of (15) subject to the set of constraints

$$\begin{aligned} U'(C_t) - R_t \beta E_t U'(C_{t+1}) &= 0; & (19)\\ C_t U'(C_t) + B_t U'(C_t) &= R_{t-1} B_{t-1} U'(C_t) + V'\left(N_t\right) N_t; & (20)\\ F\left(C_t, N_t, A_t\right) &= 0. & (21) \end{aligned}$$

Here, (20) is combination of (16) and (17); formula (21) denotes all the remaining equations where A_t is a vector of all remaining variables.

In the appendix we show that the resulting first order conditions with respect to consumption are different across the TP (22) and UO-Ramsey (23)) approaches:

$$\begin{aligned} U'(C_t) - \mu \left[U''(C_t) C_t + U'(C_t) \right] + \phi_t F_c\left(C_t, N_t, A_t\right) &= 0; & (22)\\ U'(C_t) - \mu \left[U''(C_t) C_t + U'(C_t) - (1 - \beta) R_{t-1} B_{t-1} U''(C_t) \right] + \phi_t F_c\left(C_t, N_t, A_t\right) &= 0. & (23) \end{aligned}$$

TP policy implies that optimal consumption does not depend on debt and this is why debt follows the random walk as shown by Schmitt-Grohe and Uribe (2004). In contrast, the UO-Ramsey policy stabilises debt since the marginal utility of consumption is positively related to debt, which implies that consumption is smaller when government debt is larger.

5 Conclusion

We present a policy (UO Ramsey) which behaves as Ramsey optimal policy in backward-looking models, but is similar to UO policy in forward looking models. We demonstrate how that policy works by considering two examples. First we show that in a linear-quadratic model with a hybrid Phillips curve the policy delivers the Ramsey policy when price setting is purely backward-looking, while it is the same as UO policy when the Phillips curve is purely forward-looking. Second, we show that UO-Ramsey fiscal policy implies lower consumption when government debt is higher.

References

[1] Benigno, Pierpaolo and Michael Woodford, 2012, Linear-quadratic approximation of optimal policy problems, Journal of Economic Theory, Volume 147, Issue 1, January 2012, Pages 1–42.

[2] Blake, Andrew P., 2001. A "Timeless Perspective" on optimality in forward-looking rational expectations models, Papers 188, National Institute of Economic and Social Research.

[3] Blake, Andrew P. & Kirsanova, Tatiana, 2004. A note on timeless perspective policy design, Economics Letters, Elsevier, vol. 85(1), pp. 9-16, October.

[4] Brendon, Charles and Martin Ellison, 2015. Time-Consistent Institutional Design, 2015 Meeting Papers 495, Society for Economic Dynamics.

[5] Damjanovic, Tatiana, Vladislav Damjanovic and Charles Nolan, 2008, Unconditionally optimal monetary policy, The Journal of Monetary Economics, 55, 491-500.

[6] Damjanovic, Tatiana, Vladislav Damjanovic and Charles Nolan, 2015. Ordering Policy Rules with an Unconditional Welfare Measure, International Journal of Central Banking, vol. 11(1), pages 103-149, January.

[7] FED, 2015, What is forward guidance and how is it used in the Federal Reserve's monetary policy?, Board of the Governors explanatory notes.

[8] Giannoni M. P., and M. Woodford, 2002, Optimal Interest-Rate Rules 1: General Theory, NBER Working Paper No. 9419.

[9] Horvath, Michal, 2011, Alternative perspectives on optimal public debt adjustment, The B.E. Journal of Macroeconomics, Berkeley Electronic Press, vol. 11(1), pages 34.

[10] Jensen, Christian and Bennett T. McCallum, 2010, Optimal continuation versus the timeless perspective in monetary policy, working paper, Journal of Money, Credit and Banking Volume 42, Issue 6, pages 1093–1107.

[11] Kydland, F. and E. Prescott, (1977), Rules rather than discretion: The inconsistency of optimal plans, Journal of Political Economy, 85, 473-490.

[12] Kollmann, Robert, 2002, Monetary policy rules in the open economy: Effect on welfare and business cycles, Journal of Monetary Economics, 49, pp. 989-1015.

[13] McCallum, Bennett T., 2005, What is the proper perspective for monetary policy optimality? Monetary and Economic Studies, Bank of Japan, 23, 13-24.

[14] Pigou, A. C., 1932, 'The Economics of Welfare', (4 th ed), London: Macmillan.

[15] Rotemberg Julio. J. and Michael Woodford, 1998, An optimization-based econometric framework for the evaluation of monetary policy: Expanded Version, NBER Technical Working Paper No. 233.

[16] Schmitt-Grohe, Stephanie and Martin Uribe, 2004, Optimal fiscal and monetary policy under sticky prices, Journal of Economic Theory, Elsevier, vol. 114(2), pages 198-230, February.

[17] Svensson, Lars E.O. 2014. Forward Guidance, NBER Working Papers 20796.

[18] Taylor, John B., 1979, Estimation and control of a macroeconomic model with rational expectations, Econometrica, 47, 5, pp. 1267-1286.

[19] Ramsey, F.P., 1928, A mathematical theory of saving, Economic Journal, 38 (December): 543-559

[20] Whiteman, Charles H, 1986, An analytical policy design under rational expectations, Econometrica, 54(6), pp. 1387-1405, November.

[21] Woodford, Michael, 2002, Inflation stabilization and welfare, Contributions to Macroeconomics, vol. 2, issue 1, article 1.

6 Appendix: TP vs. UO-Ramsey policy in an economy with debt

The Lagrangian for the TP policy problem is:

$$
\begin{aligned}
\max_{R_{t+1},N_{t+1},C_{t+1},Y_{t+1}} J^{TP} &= E_0 \sum_{t=0}^{\infty} \beta^t \{(U(C_t) - V(N_t)) \\
&\quad -E_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(C_t U'(C_t) + B_t U'(C_t) - R_{t-1} B_{t-1} U'(C_t) + V'(N_t) N_t\right) \\
&\quad +E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left(U'(C_t) - R_t \beta E_t U'(C_{t+1})\right) \\
&\quad +E_0 \sum_{t=0}^{\infty} \beta^t \phi_t F(C_t, N_t, A_t).
\end{aligned}
$$

The first order conditions follow in a straight forward way:

$$
\begin{aligned}
\frac{\partial J}{\partial B_t} B_t &= -\mu_t U'(C_t) B_t + \mu_{t+1} \beta R_t B_t U'(C_{t+1}) = 0; \Longrightarrow \mu_t = \mu. \\
\frac{\partial J}{\partial R_t} &= -\lambda_t R_t \beta E_t U'(C_{t+1}) + \mu_{t+1} \beta B_t R_t U'(C_{t+1}) = 0; \Longrightarrow \lambda_t = \mu B_t; \\
\frac{\partial J}{\partial C_t} &= U'(C_t) - \mu \left(U''(C_t) C_t + U'(C_t) + (B_t - R_{t-1} B_{t-1}) U''(C_t)\right) \\
&\quad +\mu \left(B_t - B_{t-1} R_{t-1}\right) U''(C_t) + \phi_t F_c(C_t, N_t, Y_t); \\
\frac{\partial J}{\partial N_t} &= V'(N_t) + \mu \left(V''(N_t) N_t + V'(N_t)\right) + \beta \phi_t F_n(C_t, N_t, A_t).
\end{aligned}
$$

In turn, these can be simplified as follows:

$$\begin{aligned}
U'(C_t) - \mu\left(U''(C_t)C_t + U'(C_t)\right) + \phi_t F_c\left(C_t, N_t, A_t\right) &= 0; &(24)\\
V'(N_t) + \mu\left(V''(N_t)N_t + V'(N_t)\right) + \phi_t F_n\left(C_t, N_t, A_t\right) &= 0; &(25)\\
U'(C_t) - R_t \beta E_t U'(C_{t+1}) &= 0; &(26)\\
F_n\left(C_t, N_t, A_t\right) &= 0; &(27)\\
B_t - R_{t-1}B_{t-1} - \frac{V'\left(N_t\right)}{U'(C_t)} N_t + C_t &= 0. &(28)
\end{aligned}$$

As we asserted in the main text, system (24-27) does not depend on debt and solves for all endogenous variables. The last equation (28) is autonomous and describe the dynamics of debt, which is clearly non-stationary as $R_{t-1} > 1$.

6.1 UO-Ramsey

As proposed by Brendon and Ellison, we replace forward-looking Euler equations with promise-making and promise-keeping constraints,

$$\begin{aligned}
U'(C_t) - R_t \beta E_t U'(\omega_{t+1}) &= 0;\\
C_t &= \omega_t.
\end{aligned}$$

Now we solve the inner government's problem taking promises as exogenous variables

$$\begin{aligned}
\max_{R_{t+1}, N_{t+1}, C_{t+1}, Y_{t+1}} J^{IN} = \; & E_0 \sum_{t=0}^{\infty} \beta^t \{ (U(C_t) - V\left(N_t\right)) \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(C_t U'(C_t) + B_t U'(C_t) - R_{t-1}B_{t-1}U'(C_t) + V'\left(N_t\right) N_t\right) \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left(U'(C_t) - R_t \beta E_t U'(\omega_{t+1})\right) \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \eta_t \left(C_t - \omega_t\right)) \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \phi_t F\left(C_t, N_t, A_t\right)
\end{aligned}$$

The first-order conditions for the inner government in equilibrium imply

$$\begin{aligned}
\frac{\partial J^{IN}}{\partial B_t}B_t &= -\mu_t U'(C_t)B_t + \mu_{t+1}\beta R_t B_t U'(C_{t+1}) = 0; \Longrightarrow \mu_t = \mu; \\
\frac{\partial J^{IN}}{\partial R_t} &= -\lambda_t R_t \beta E_t U'(C_{t+1}) + \mu_{t+1}\beta B_t R_t U'(C_{t+1}) = 0; \Longrightarrow \lambda_t = \mu B_t; \\
\frac{\partial J^{IN}}{\partial C_t} &= U'(C_t) - \mu\left(U''(C_t)C_t + U'(C_t) + (B_t - R_{t-1}B_{t-1})\,U''(C_t)\right) \\
&\quad + \mu B_t U''(C_t) - \eta_t + \phi_t F_c\left(C_t, N_t, Y_t\right); \qquad (29) \\
\frac{\partial J^{IN}}{\partial N_t} &= V'(N_t) + \mu\left(V''(N_t)N_t + V'(N_t)\right) + \beta\phi_t F_n\left(C_t, N_t, A_t\right).
\end{aligned}$$

The outer government will choose promises to maximise the unconditional expectation of the inner government's problem. We use the same technique as Damjanovic, Damjanovic and Nolan (2008) to obtain the first order condition with respect ω_t.

$$\eta_t = \mu B_{t-1}R_{t-1}\beta U''(C_t).$$

Combining this with (29) we get

$$\begin{aligned}
&U'(C_t) - \mu\left(U''(C_t)C_t + U'(C_t) + (B_t - R_{t-1}B_{t-1})\,U''(C_t)\right) \\
&+\mu\left(B_t - \beta B_{t-1}R_{t-1}\right)U''(C_t) + \phi_t F_c\left(C_t, N_t, A_t\right) = 0. \qquad (30)
\end{aligned}$$

This in turn may be simplified to obtain

$$U'(C_t) - \mu\left(U''(C_t)C_t + U'(C_t) - (1-\beta)\,R_{t-1}B_{t-1}U''(C_t)\right) + \phi_t F_c\left(C_t, N_t, A_t\right) = 0. \qquad (31)$$

This expression (31) is different from the corresponding expression (24) for TP optimisation and explains the difference in debt dynamics across thee different policy programs.