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The Theoretical status of $\bar{B} - B$ -mixing and lifetimes of heavy hadrons

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In this talk we review the theoretical status of the lifetime ratios of heavy hadrons and of the B -mixing quantities ΔM_s , $\Delta\Gamma_s$ and ϕ_s . While ΔM_s and $\Delta\Gamma_s$ suffer from large uncertainties due to the badly known decay constants, the ratio $\Delta\Gamma_s/\Delta M_s$ can be determined with almost no non-perturbative uncertainties, therefore it can be used to look for possible new physics effects.

1. Introduction

The heavy quark expansion (HQE) is the theoretical framework to describe inclusive decays (see e.g.¹ and references therein). In this approach the decay rate is expanded in inverse powers of the heavy b -quark mass: $\Gamma = \Gamma_0 + \Lambda^2/m_b^2 \Gamma_2 + \Lambda^3/m_b^3 \Gamma_3 + \dots$. Γ_0 is the decay of a free heavy b -quark, according to this contribution all b -mesons have the same lifetime. The first correction arises at order $1/m_b^2$, due to the kinetic and the chromomagnetic operator. At order $1/m_b^3$ the spectator quark gets involved for the first time. Although being suppressed by three powers of the heavy b -quark mass, this contributions are numerically enhanced by a phase space factor of $16\pi^2$. Each of the Γ_i contains perturbatively calculable Wilson coefficients and non-perturbative parameters, like decay constants or bag parameters. In the case of exclusive b -hadron decays the non-perturbative parameters are given by the meson distribution amplitudes, see e.g.² This approach clearly has to be distinguished from QCD inspired models. It is derived directly from QCD and the basic assumptions (convergence of the expansion in α_s and Λ/m_b) can be simply tested by comparing experiment and theory for different quantities (see e.g.³).

2. Lifetimes

The lifetime ratio of two heavy mesons reads

$$\frac{\tau_1}{\tau_2} = 1 + \frac{\Lambda^3}{m_b^3} \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \dots \right) + \frac{\Lambda^4}{m_b^4} \left(\Gamma_4^{(0)} + \dots \right)$$

Neglecting small isospin or $SU(3)$ violating effects one has no $1/m_b^2$ corrections^a and a deviation of the lifetime ratio from one starts at order $1/m_b^3$. For the ratio

^aIn the case of $\tau_{\Lambda_b}/\tau_{B_d}$ these effects are expected to be of the order of 5%.

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τ_{B^+}/τ_{B_d} the leading term $\Gamma_3^{(0)}$ has been determined quite some time ago e.g. in ⁴. For a quantitative treatment of the lifetime ratios NLO QCD corrections are mandatory - $\Gamma_3^{(1)}$ has been determined in ⁵. Subleading effects of $\mathcal{O}(1/m_b)$ turned out to be negligible ⁶. Using the result from ⁵ with matrix elements from ⁷ and the values $V_{cb} = 0.0415$, $m_b = 4.63$ GeV and $f_B = 216$ MeV ⁸ we obtain a value, which is in excellent agreement with the experimental number ⁹:

$$\frac{\tau(B^+)}{\tau(B_d^0)_{\text{NLO}}} = 1.063 \pm 0.027, \quad \frac{\tau(B^+)}{\tau(B_d^0)_{\text{Exp}}} = 1.076 \pm 0.008.$$

To improve the theoretical accuracy further more precise lattice values are necessary, in particular of the appearing color-suppressed operators. In the lifetime ratio τ_{B_s}/τ_{B_d} a cancellation of weak annihilation contributions arises, that differ only by small SU(3)-violation effects. One expects a number that is very close to one ^{4,5,10,11}. The experimental number ⁹ is slightly smaller

$$\frac{\tau(B_s)}{\tau(B_d)_{\text{Theo}}} = 1.00 \pm 0.01, \quad \frac{\tau(B_s)}{\tau(B_d)_{\text{Exp}}} = 0.950 \pm 0.019.$$

Here an increased experimental precision is needed to find out, whether there is discrepancy. Next we consider two hadrons, where the theoretical situation is much worse compared to the mesons discussed above. The lifetime of B_c has been investigated in ¹² in LO QCD.

$$\tau(B_c)_{\text{LO}} = 0.52_{-0.12}^{+0.18} \text{ ps}, \quad \tau(B_c)_{\text{Exp}} = 0.460 \pm 0.066 \text{ ps}.$$

In addition to the b-quark now also the c-charm quark can decay, giving rise to the biggest contribution to the total decay rate. The current experimental number is taken from ^{13,9}. In the case of the Λ_b -baryon the NLO-QCD corrections are not complete and there are only preliminary lattice studies for a part of the arising matrix elements, see e.g. ¹⁴, so the theoretical error has to be met with some skepticism. Moreover there are some discrepancies in the experimental numbers ^{9,15}.

$$\frac{\tau(\Lambda_b)}{\tau(B_d)_{\text{Theo}}} = 0.88 \pm 0.05, \quad \frac{\tau(\Lambda_b)}{\tau(B_d)_{\text{Exp}}} = 0.912 \pm 0.032.$$

3. Mixing Parameters

The mixing of the neutral B-mesons is described by the off diagonal elements Γ_{12} and M_{12} of the mixing matrix. Γ_{12} stems from the absorptive part of the box diagrams - only internal up and charm quarks contribute, while M_{12} stems from the dispersive part of the box diagram, therefore being sensitive to heavy internal particles like the top quark or heavy new physics particles (see eg. ¹⁶ or reference in ¹⁷). $|M_{12}|$, $|\Gamma_{12}|$ and $\phi = \arg(-M_{12}/\Gamma_{12})$ can be related to three physical observables (see ^{17,18} for a detailed description):

- Mass difference $\Delta M \approx 2|M_{12}|$

- Decay rate difference $\Delta\Gamma \approx 2|\Gamma_{12}|\cos\phi$
- Flavor specific or semi-leptonic CP asymmetries: $a_{fs} = \text{Im}\frac{\Gamma_{12}}{M_{12}} = \frac{\Delta\Gamma}{\Delta M}\tan\phi$.

Calculating the box diagram with internal top quarks one obtains

$$M_{12,q} = \frac{G_F^2}{12\pi^2}(V_{tq}^*V_{tb})^2 M_W^2 S_0(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B$$

The Inami-Lim function $S_0(x_t = \bar{m}_t^2/M_W^2)$ ¹⁹ is the result of the box diagram without any gluon corrections. The NLO QCD correction is parameterized by $\hat{\eta}_B \approx 0.84$ ²⁰. The non-perturbative matrix element is parameterized by the bag parameter B and the decay constant f_B . Using the conservative estimate $f_{B_s} = 240 \pm 40$ MeV¹⁷ and the bag parameter B from JLQCD²¹ we obtain in units of ps^{-1} (experiment from^{9,22,23})

$$\Delta M_s^{\text{Theo}} = 19.3 \pm 6.4 \pm 1.9, \Delta M_s^{\text{Exp}} = 17.77 \pm 0.12$$

The first error in the theory prediction stems from the uncertainty in f_{B_s} and the second error summarizes the remaining theoretical uncertainties. The determination of ΔM_d is affected by even larger uncertainties because here one has to extrapolate the decay constant to the small mass of the down-quark. The ratio $\Delta M_s/\Delta M_d$ is theoretically better under control since in the ratio of the non-perturbative parameters many systematic errors cancel, but on the other hand it is affected by large uncertainties due to $|V_{ts}|^2/|V_{td}|^2$. To be able to distinguish possible new physics contributions to ΔM_s from QCD uncertainties much more precise numbers for f_{B_s} are needed.

In order to determine the decay rate difference of the neutral B-mesons and flavor specific CP asymmetries a precise determination of Γ_{12} is needed, which can be written as

$$\Gamma_{12} = \frac{\Lambda^3}{m_b^3} \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \dots \right) + \frac{\Lambda^4}{m_b^4} \left(\Gamma_4^{(0)} + \dots \right)$$

The arising diagrams are similar to the ones of the lifetime predictions. The leading term $\Gamma_3^{(0)}$ was determined in²⁴. The numerical and conceptual important NLO-QCD corrections ($\Gamma_3^{(1)}$) were determined in^{25,18}. Subleading $1/m$ -corrections, i.e. $\Gamma_4^{(0)}$ were calculated in^{11,26} and even the Wilson coefficients of the $1/m^2$ -corrections ($\Gamma_5^{(0)}$) were calculated and found to be small^{17,27}. In¹⁷ a strategy was worked out to reduce the theoretical uncertainty in Γ_{12}/M_{12} by almost a factor of 3, see Fig. (1) for an illustration. in the new approach one gets

$$\frac{\Delta\Gamma_s}{\Delta M_s} = 10^{-4} \cdot \left[46.2 + 10.6 \frac{B'_S}{B} - 11.9 \frac{B_R}{B} \right]$$

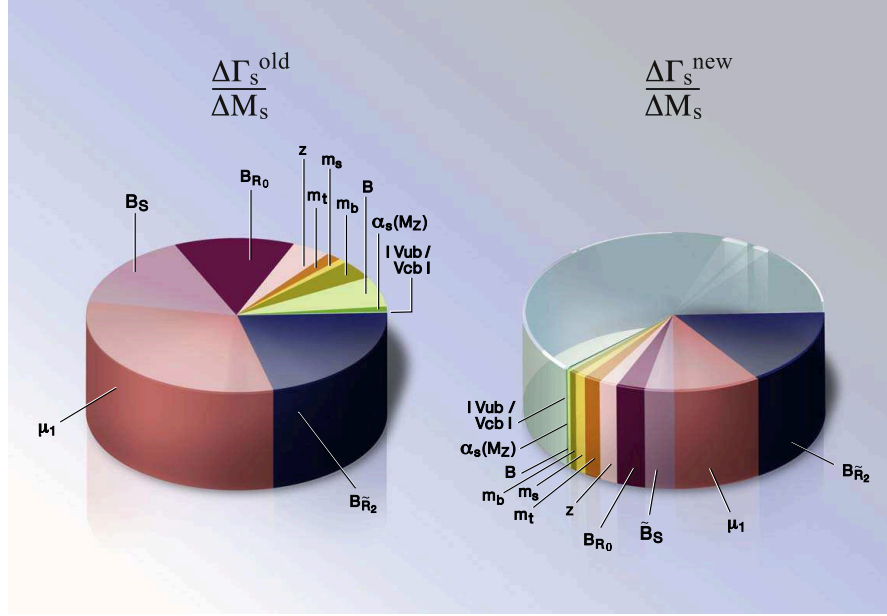


Fig. 1. Error budget for the theoretical determination of $\Delta\Gamma_s/\Delta M_s$. Compared to previous approaches (left) the new strategy lead to a reduction of the theoretical error by almost a factor of three.

The dominant part of $\Delta\Gamma/\Delta M$ can now be determined without any hadronic uncertainties and we obtained the following final numbers (see ¹⁷)

$$\begin{aligned}\Delta\Gamma_s &= (0.096 \pm 0.039) \text{ ps}^{-1}, \quad \frac{\Delta\Gamma_s}{\Gamma_s} = 0.147 \pm 0.060, \\ a_{fs}^s &= (2.06 \pm 0.57) \cdot 10^{-5}, \quad \frac{\Delta\Gamma_s}{\Delta M_s} = (49.7 \pm 9.4) 10^{-4}, \\ \phi_s &= 0.0041 \pm 0.0008 = 0.24^\circ \pm 0.04.\end{aligned}$$

New physics (see e.g. ¹⁶ and references in ¹⁷) is expected to have almost no impact on Γ_{12} , but it can change M_{12} considerably – we denote the deviation factor by the complex number Δ . Therefore one can write

$$\Gamma_{12,s} = \Gamma_{12,s}^{\text{SM}}, \quad M_{12,s} = M_{12,s}^{\text{SM}} \cdot \Delta_s; \quad \Delta_s = |\Delta_s| e^{i\phi_s^\Delta}$$

With this parameterisation the physical mixing parameters can be written as

$$\begin{aligned}\Delta M_s &= 2|M_{12,s}^{\text{SM}}| \cdot |\Delta_s|, \\ \Delta\Gamma_s &= 2|\Gamma_{12,s}| \cdot \cos(\phi_s^{\text{SM}} + \phi_s^\Delta), \\ \frac{\Delta\Gamma_s}{\Delta M_s} &= \frac{|\Gamma_{12,s}|}{|M_{12,s}^{\text{SM}}|} \cdot \frac{\cos(\phi_s^{\text{SM}} + \phi_s^\Delta)}{|\Delta_s|}, \\ a_{fs}^s &= \frac{|\Gamma_{12,s}|}{|M_{12,s}^{\text{SM}}|} \cdot \frac{\sin(\phi_s^{\text{SM}} + \phi_s^\Delta)}{|\Delta_s|}.\end{aligned}\tag{1}$$

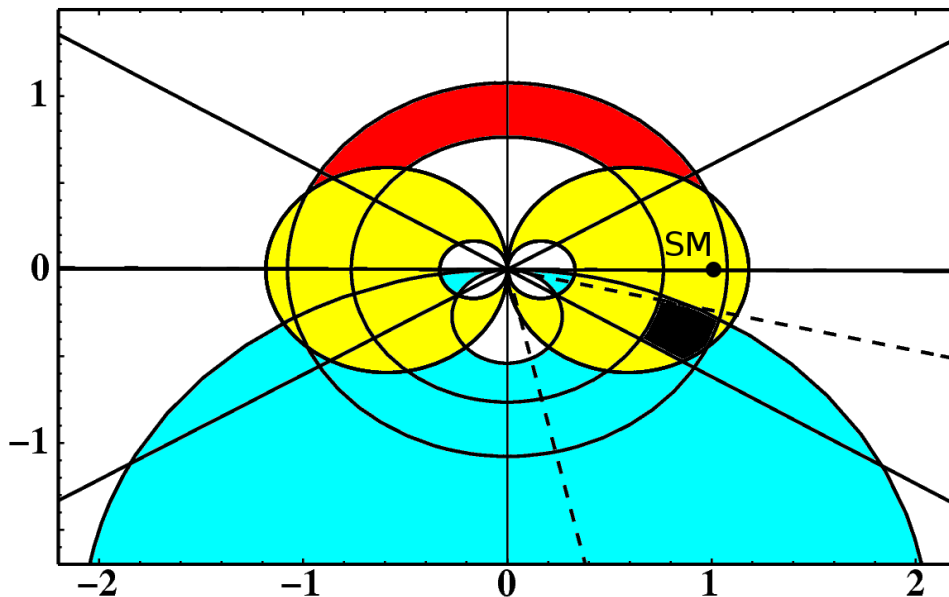


Fig. 2. Current experimental bounds in the complex Δ_s -plane. The bound from ΔM_s is given by the red (dark-grey) ring around the origin. The bound from $\Delta\Gamma_s/\Delta M_s$ is given by the yellow (light-grey) region and the bound from a_{fs}^s is given by the light-blue (grey) region. The angle ϕ_s^Δ can be extracted from $\Delta\Gamma_s$ (solid lines) with a four fold ambiguity - one bound coincides with the x-axis! - or from the angular analysis in $B_s \rightarrow J/\Psi\phi$ (dashed line). If the standard model is valid all bounds should coincide in the point (1,0). The current experimental situation shows a small deviation, which might become significant, if the experimental uncertainties in $\Delta\Gamma_s$, a_{sl}^s and ϕ_s will go down in near future.

Note that $\Gamma_{12,s}/M_{12,s}^{\text{SM}}$ is now due to the improvements in ¹⁷ theoretically very well under control. Combining the current experimental numbers with the theoretical predictions one can extract bounds in the imaginary Δ_s -plane by the use of Eqs. (1), see Fig. (2). The width difference $\Delta\Gamma_s/\Gamma_s$ was investigated in ^{29,23}. The semi-leptonic CP asymmetry in the B_s system has been determined in ^{23,30} (see ¹⁷ for more details). Therefore we use as experimental input

$$\Delta\Gamma_s = 0.17 \pm 0.09 \text{ ps}^{-1}, \quad \phi_s = -0.79 \pm 0.56.$$

$$a_{sl}^s = (-5.2 \pm 3.9) \cdot 10^{-3}.$$

4. Conclusion and outlook

We have reviewed the theoretical status of lifetimes of heavy hadrons and the measurable mixing quantities of the neutral B-mesons. Both classes of quantities can be described with the help of the HQE - a systematic expansion based simply on QCD.

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The theoretical uncertainty in the mixing parameters ΔM and $\Delta\Gamma$ is completely dominated by the decay constant. Here some progress on the non-perturbative side is mandatory. In $\Delta M_s/\Delta M_d$ the dominant uncertainty is given by $|V_{ts}/V_{td}|^2$. In $\tau(B_c)$ and $\tau(\Lambda_b)$ the important NLO-QCD are missing or are incomplete, moreover we have only preliminary lattice studies of the non-perturbative matrix elements. Theoretical predictions of τ_{B^+}/τ_{B_d} are in excellent agreement with the experimental numbers. We do not see any signal of possible duality violations in the HQE. To become even more quantitative in the prediction of τ_{B^+}/τ_{B_d} the non-perturbative estimates of the bag parameters - in particular of the color-suppressed ones - have to be improved. In ¹⁷ a method was worked out to reduce the theoretical error in $\Delta\Gamma/\Delta M$ considerably. For a further reduction of the theoretical uncertainty in the mixing quantities the unknown matrix elements of the power suppressed operators have to be determined. Here any non-perturbative estimate would be very desirable. A first step in that direction was performed in ³¹. If accurate non-perturbative parameters are available one might think about NNLO calculations (α_s/m_b - or α_s^2 -corrections) to reduce the remaining μ -dependence and the uncertainties due to the missing definition of the b-quark mass in the power corrections. The improvements for $\Delta\Gamma/\Delta M$ apply to a_{fs} and Φ_q as well.

The relatively clean standard model predictions for the mixing quantities can now be used to look for new physics effects in B_s -mixing. From the currently available experimental bounds on $\Delta\Gamma_s$ and a_{fs} one already gets some hints for deviations from the standard model. To settle this issue we are eagerly waiting for more data from TeVatron, LHCb ²³ and SUPER-B ³²!

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References

1. M. Battaglia *et al.*, arXiv:hep-ph/0304132; A. Lenz, U. Nierste and G. Ostermaier, Phys. Rev. D **56** (1997) 7228 [arXiv:hep-ph/9706501]; A. Lenz, U. Nierste and G. Ostermaier, Phys. Rev. D **59** (1999) 034008 [arXiv:hep-ph/9802202]; A. Lenz, arXiv:hep-ph/0011258.
2. P. Ball, V. M. Braun and A. Lenz, JHEP **0708** (2007) 090 [arXiv:0707.1201 [hep-ph]]; P. Ball, V. M. Braun and A. Lenz, JHEP **0605** (2006) 004 [arXiv:hep-ph/0603063]; V. M. Braun and A. Lenz, Phys. Rev. D **70** (2004) 074020 [arXiv:hep-ph/0407282].
3. A. Lenz, arXiv:hep-ph/9906317; M. Beneke and A. Lenz, J. Phys. G **27** (2001) 1219 [arXiv:hep-ph/0012222]; A. Lenz and S. Willocq, J. Phys. G **27** (2001) 1207; A. Lenz, arXiv:hep-ph/0107033; A. Lenz, arXiv:hep-ph/0412007.
4. I. I. Y. Bigi, B. Blok, M. A. Shifman, N. Uraltsev and A. I. Vainshtein, arXiv:hep-ph/9401298; I. I. Y. Bigi, arXiv:hep-ph/9508408; M. Neubert and C. T. Sachrajda, Nucl. Phys. B **483** (1997) 339 [arXiv:hep-ph/9603202].
5. M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Nucl. Phys. B **639** (2002)

- 389 [arXiv:hep-ph/0202106]; E. Franco, V. Lubicz, F. Mescia and C. Tarantino, Nucl. Phys. B **633** (2002) 212 [arXiv:hep-ph/0203089].
6. A. Lenz and U. Nierste, unpublished, e.g. talk at Academia Sinica 2003; F. Gabriani, A. I. Onishchenko and A. A. Petrov, Phys. Rev. D **70** (2004) 094031 [arXiv:hep-ph/0407004].
 7. M. Di Pierro and C. T. Sachrajda [UKQCD Collaboration], Nucl. Phys. B **534** (1998) 373 [arXiv:hep-lat/9805028]; D. Becirevic, arXiv:hep-ph/0110124.
 8. A. Gray *et al.* [HPQCD Collaboration], Phys. Rev. Lett. **95** (2005) 212001 [arXiv:hep-lat/0507015].
 9. H. F. A. Group, arXiv:0704.3575 [hep-ex].
 10. Y. Y. Keum and U. Nierste, Phys. Rev. D **57** (1998) 4282 [arXiv:hep-ph/9710512].
 11. M. Beneke, G. Buchalla and I. Dunietz, Phys. Rev. D **54** (1996) 4419 [arXiv:hep-ph/9605259].
 12. C. H. Chang, S. L. Chen, T. F. Feng and X. Q. Li, Phys. Rev. D **64** (2001) 014003 [arXiv:hep-ph/0007162]; V. V. Kiselev, A. E. Kovalsky and A. K. Likhoded, Nucl. Phys. B **585** (2000) 353 [arXiv:hep-ph/0002127]. A. Y. Anisimov, I. M. Narodetsky, C. Semay and B. Silvestre-Brac, Phys. Lett. B **452** (1999) 129 [arXiv:hep-ph/9812514]. M. Beneke and G. Buchalla, Phys. Rev. D **53** (1996) 4991 [arXiv:hep-ph/9601249].
 13. A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **97** (2006) 012002 [arXiv:hep-ex/0603027]. D0 Collaboration, D0 conference note 4539
 14. C. Tarantino, arXiv:hep-ph/0702235.
 15. A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **98** (2007) 122001 [arXiv:hep-ex/0609021]; D0-note 5263.
 16. A. Lenz, Phys. Rev. D **76** (2007) 065006 [arXiv:0707.1535 [hep-ph]]; S. Jager and U. Nierste, Eur. Phys. J. C **33**, S256 (2004) [arXiv:hep-ph/0312145];
 17. A. Lenz and U. Nierste, JHEP **0706** (2007) 072 [arXiv:hep-ph/0612167]; A. Lenz, arXiv:hep-ph/0612176; A. Lenz, arXiv:0705.3802 [hep-ph].
 18. M. Beneke, G. Buchalla, A. Lenz and U. Nierste, Phys. Lett. B **576** (2003) 173 [arXiv:hep-ph/0307344]; M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C. Tarantino, JHEP **0308** (2003) 031 [arXiv:hep-ph/0308029].
 19. T. Inami and C. S. Lim, Prog. Theor. Phys. **65** (1981) 297 [Erratum-ibid. **65** (1981) 1772].
 20. A. J. Buras, M. Jamin and P. H. Weisz, Nucl. Phys. B **347** (1990) 491.
 21. S. Aoki *et al.* [JLQCD Collaboration], Phys. Rev. Lett. **91** (2003) 212001 [arXiv:hep-ph/0307039].
 22. A. Abulencia *et al.* [CDF Collaboration], arXiv:hep-ex/0609040; A. Abulencia [CDF - Run II Collaboration], Phys. Rev. Lett. **97** (2006) 062003 [arXiv:hep-ex/0606027]; V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **97** (2006) 021802 [arXiv:hep-ex/0603029].
 23. L. Han, these proceedings; S. Farrington, these proceedings; N. Harnew, these proceedings.
 24. J.S. Hagelin, Nucl. Phys. **B193**, 123 (1981); E. Franco, M. Lusignoli and A. Pugliese, Nucl. Phys. **B194**, 403 (1982); L.L. Chau, Phys. Rep. **95**, 1 (1983); A.J. Buras, W. Slominski and H. Steger, Nucl. Phys. **B245**, 369 (1984); M.B. Voloshin, N.G. Uraltsev, V.A. Khoze and M.A. Shifman, Sov. J. Nucl. Phys. **46**, 112 (1987); A. Datta, E.A. Paschos and U. Türke, Phys. Lett. **B196**, 382 (1987); A. Datta, E.A. Paschos and Y.L. Wu, Nucl. Phys. **B311**, 35 (1988).
 25. M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Phys. Lett. B **459** (1999) 631 [arXiv:hep-ph/9808385].
 26. A. S. Dighe, T. Hurth, C. S. Kim and T. Yoshikawa, Nucl. Phys. B **624** (2002) 377

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- [arXiv:hep-ph/0109088].
27. A. Badin, F. Gabbiani and A. A. Petrov, Phys. Lett. B **653** (2007) 230 [arXiv:0707.0294 [hep-ph]].
 28. D. Becirevic, V. Gimenez, G. Martinelli, M. Papinutto and J. Reyes, JHEP **0204** (2002) 025 [arXiv:hep-lat/0110091].
 29. R. Barate *et al.* [ALEPH Collaboration], Phys. Lett. B **486** (2000) 286; CDF collaboration, conference note 7925, <http://www-cdf.fnal.gov>; V. M. Abazov *et al.* [D0 Collaboration], arXiv:hep-ex/0702049; D. Acosta *et al.* [CDF Collaboration], Phys. Rev. Lett. **94** (2005) 101803 [arXiv:hep-ex/0412057]; A. Abulencia [CDF Collaboration], arXiv:hep-ex/0607021; V. M. Abazov *et al.* [D0 Collaboration], arXiv:hep-ex/0604046. R. Van Kooten, eConf **C060409** (2006) 031 [arXiv:hep-ex/0606005]; V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **98** (2007) 121801 [arXiv:hep-ex/0701012].
 30. V. M. Abazov *et al.* [D0 Collaboration], arXiv:hep-ex/0701007; V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **74** (2006) 092001 [arXiv:hep-ex/0609014].
 31. T. Mannel, B. D. Pecjak and A. A. Pivovarov, arXiv:hep-ph/0703244.
 32. M. Bona *et al.*, arXiv:0709.0451 [hep-ex].