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Computational Modelling of Braided Fibre for Concrete Reinforcement

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ABSTRACT

This paper presents numerical modelling of braided fibre, to be used as concrete reinforcement. Ultimately, a corrosive and fire resistant concrete will be produced.

A Cubit script (geometry and mesh software) was created to mesh braided yarns under different geometric parameters. Fibres were represented using elastic transversely isotropic materials, for which the fibre directions for every yarn were precisely determined from the gradients of the resultant stream functions of potential flow problems. Applying an elastic interfaces between yarns to preventing penetration and having free sliding, convergence studies were conducted on a coarse and fine mesh, using hierarchical higher order approximation [1] for uniform p- and hp-refinement.

Key Words: *Fibre Reinforced Concrete; Composite; Bond Strength; Interface Cohesion Elements; Hierarchical Refinement*

1. Introduction

This paper presents a numerical investigation into the modeling of braided carbon fibre ropes. These ropes can provide an alternative to steel reinforcement in concrete, whereby the braiding can provide good bond adhesion between the fibre and concrete [2] and mitigate the need for weak resin binders [3]. In order to realize this ambition and to develop an appropriate modeling framework, a number of modeling challenges need to be overcome. First, an accurate geometrical representation of the complex geometry of the braided rope needs to be achieved. Second, the fibre direction during deformation needs to be accurately determined. Third, the interface between fibre yarns needs to be modeled. Novel hierarchic approximation functions [1] are also adopted and the paper demonstrates the convergence performance of h, p and hp-refinement. The fibres are modeled as transversely isotropic, although restricted to linear elasticity in this paper.

2. Numerical Modelling Approach

2.1. Mesh Modelling

Due to the lack of tools for modelling braided geometries, a flexible Python/APERPRO script was written in Cubit to generate 12-strand plaited sinnet geometry as shown in Figure 1. The algorithm of this script is as follows: a) sets of vertices representing the centre of axis for every yarn were created following the pattern of braiding, b) axis splines were formed along every set of vertices, c) circular surfaces were extrude along the same splines to form the braiding geometry. The input parameters were: i) the diameter of the yarn, ii) permissible tolerance between the yarns, iii) pitch and iv) the number of turns required. The interfaces between the yarns were created by subtracting one yarn from the overlapping adjacent one. Two square clamps were modelled at both ends of the braided model. Tetrahedral mesh was generated using Cubit mesh generator. The respective boundary conditions and material parameters were assigned in Cubit and directly read by MoFEM (Mesh Oriented Finite Element Method, our group's open-source FEM software).

Figure 1: Type of Braided Fibre Modelled

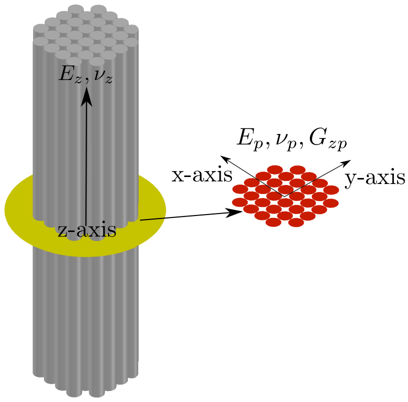

Figure 2: 5 Material Parameters for Transversely Isotropic Material (with reference to fibres)

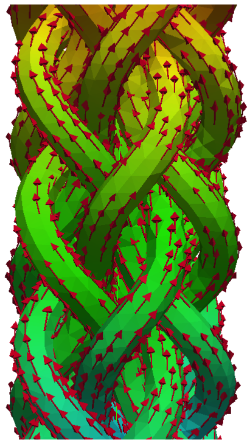

Figure 3: Flow Direction Vectors in Rope Model

2.2. *Transversely Isotropic Material*

A suitable material to represent fibre yarns was chosen to be transversely isotropic material. This material is described by 5 independent material parameters, which are the principal stiffness E_z, and poisson's ratio ν_z along the transverse direction (z-axis) of the fibre and the stiffness E_p, poisson's ratio ν_p and shear modulus G_{zp} in the plan of orthotropy (xy plane) as shown in Figure 2.

The fibre directions can be determined from the spline axes of the yarns. However, this is inaccurate when non-uniform cross-section are considered and/or rapid change of direction exists. An alternative way of obtaining the fibre directions is to solve a steady laminar incompressible potential flow problem, where the gradients of the stream function ψ are the resultant flow velocities defining the fibre directions **F** at every Gauss point as shown in Figure 3. By using a higher order approximation for the potential flow solution very accurate stream functions are computed for coarse meshes.

To transform the material response between the local fibre direction and global axes, the axis of rotation is expressed as $\mathbf{A} = \mathbf{F} \times \mathbf{Z}$, where $\mathbf{Z}$ is the unit vector $(0, 0, 1)$ representing the global z-axis and the angle of rotation is expressed by $\theta = \cos^{-1} \frac{\mathbf{F}.\mathbf{Z}}{\|\mathbf{F}\| \, \|\mathbf{Z}\|}$. This rotation matrix can be computed from $\mathbf{R} = \mathbf{I} + \sin\theta\mathbf{N} + (1 - \cos\theta)\mathbf{N}^2$, where $\mathbf{I}$ is the identity matrix, $\mathbf{N} = \frac{\Omega}{\omega}$, $\omega = +\sqrt{A_1^2 + A_2^2 + A_3^2}$ and $\Omega = \begin{bmatrix} 0 & -A_3 & A_2 \\ A_3 & 0 & -A_1 \\ -A_2 & 0 & 0 \end{bmatrix}$.

2.3. *Interface Elements*

Interface elements represented by prism elements in the FE mesh, were inserted between yarns, and an elastic cohesive model was used for its formulation [2]. Orthogonal penetration/separation of the interface was controlled using a stiffness about thousand times the principal material stiffness, while no stiffness was used to control shear movement, i.e. fibre were free to slide.

2.4. *Convergence Study*

A convergence study was conducted using p, h and hp-refinement on the problem described in Section 2.1. Hierarchical higher order (HO) approximation was used to perform uniform p-refinement up to 4^{th} order polynomial, where a minimum of 45 Gauss points were necessary for the numerical integration. Hierarchical HO shape functions for edges, faces and volumes were constructed using standard linear nodal shape functions (used in linear FEs) and legendre polynomials, as described by Ainsworth [1]. This allows local p-refinement, which is computationally cheaper that global p-refinement. However, the work presented here is restricted to global p-refinement.

Two meshes were considered: coarse mesh (49,624 elements) and a fine mesh (396,992 elements). The coarse mesh was analysed using 1^{st}, 2^{nd}, 3^{rd} and 4^{th} order approximations. The fine mesh was analysed using 1^{st}, 2^{nd} and 3^{rd} (5,823,150 DOF) order approximation.

The error was computed as $|u_z - u_z^h|$, where u_z is the displacement of a random node, and u_z^h is the displacement at the same location on the coarse mesh with 5^{th} order approximation. The 5^{th} order p-refined mesh is assumed to provide the reference solution.

3. Results

Creating the geometry and mesh using a Python/APRPRO script has proven to be an effective dynamic solution to generate braided type geometries. The flexibility of this script is not only the capability of varying the parameters of such braiding, but could be easily adopted for other types of braiding. Loose braiding was formed and hence pre-stressing using nonlinear geometry (large rotations and small strains) is required at a later stage.

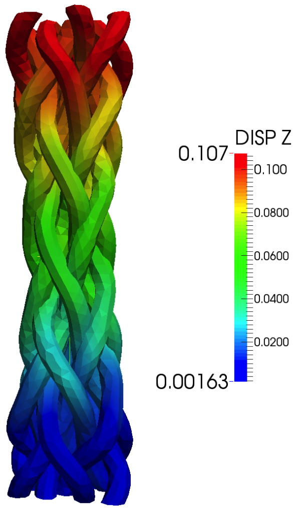

Figure 4: Linear Elastic Material with interface between yarns

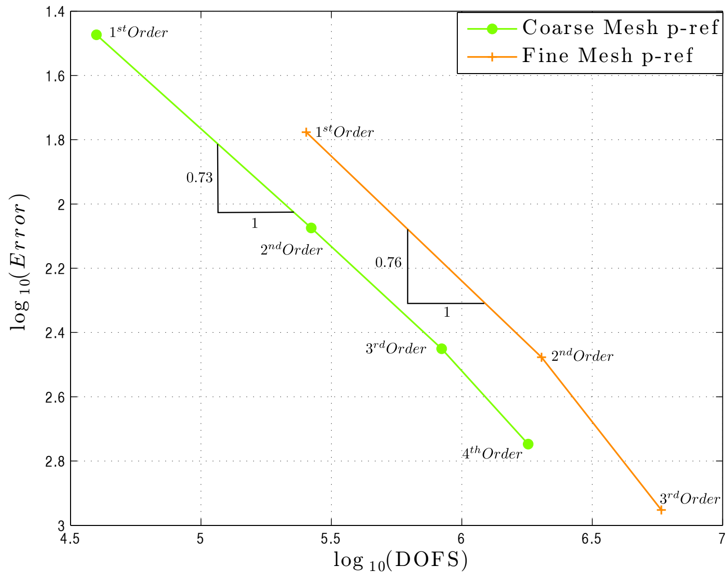

Figure 5: Convergence Results for Coarse and Fine Mesh with different order of approximation

Isotropic steel material was used for the clamps (not shown in fig. 4), and carbon fibre transverse isotropic material properties was used for the yarns. Figure 4 shows the deformed shape of the braided rope. Negligible penetration between yarns was observed.

P-refinement on the coarse and fine mesh resulted in similar rates of convergence (Figure 5). This reflects the theoretical rates of convergence for both the energy and displacement norms [4].

P-refinement on the coarse mesh is found to be the optimal refinement, achieving accurate results with the least number of DOFs. Hence uniform hp-refinement is not recommended in this case, although one might investigate the use of adaptive p and/or hp-refinement, where the theory suggest an exponential rate of convergence would be achieved.

4. Conclusions

An effective method was achieved to model and mesh complex geometries such as braided ropes. Mechanical modelling of such geometries, required the implementation of an appropriate material, i.e. transverse isotropy, that well represent the behaviour of the individual fibre yarns. Although such a material could be orientated using the centre axis of the yarns as a mean of representing the fibre directions, this would lead to problems when the model is subject to large deformations and yarn cross-sections do not remain uniform. An effective solution for this problem, was to solve a potential flow problem, where by the velocity vectors at every gauss point (later used as the fibre directions) were computed using the gradients of the resultant stream function (ψ). Furthermore, convergence studies, shows that although a local error was computed (a nodal displacement was used to compute the error), linear convergence holds with the theory, achieving about 93% of accuracy for 2^{nd} order of approximation with the coarse mesh.

After this initial study, the process to investigate the bond behaviour between the fibre rope and concrete will be as follows:

- Include a non-linear interface between the yarns.
- Pre-stress the rope model using large rotations/small strains.
- A better representation of the geometry using higher order elements.
- Implement the potential flow solution for every iteration of the nonlinear mechanical analysis, hence fibre direction will be updated every time the geometry changes.
- Encase the pre-stress ropes in concrete and use a suitable cohesive damage law for the interface
- Investigate the pull-out bond strength for plain and ribbed braided ropes
- Implement concrete fracture to investigate cracking induced by reinforcement

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References

[1] M. Ainsworth, J. Coyle, Hierarchic finite element bases on unstructured tetrahedral meshes, International Journal for Numerical Methods in Engineering 58 (14) (2003) 2103–2130.

[2] M. Cortis, L. Kaczmarczyk, C. J. Pearce, Computational modelling and experimental investigation of the bond behaviour between concrete and braided fibre ropes, in: International Conference on Computational Mechanics (CM13), no. March, 2013.

[3] A. Katz, N. Berman, Modeling the efect of high temperature on the bond of FRP reinforcing bars to concrete 22 (2000) 433–443.

[4] L. Demkowicz, Computing with hp-adaptive finite elements: Volume 1 One and Two Dimensional Elliptic and Maxwell Problems, Chapman & Hall/CRC Applied Mathematics and Nonlinear Science Series, Taylor & Francis, 2006.