Some comments on the missing charm puzzle

Alexander Lenz

Universität Regensburg, D-93040 Regensburg, Germany

E-mail: alexander.lenz@physik.uni-regensburg.de

Abstract. In this talk we summarize the status of theoretical predictions for the average number of charm quarks in a B-hadron decay.

1. Introduction

Since quite a long time there exists a discrepancy between theoretical predictions and measurements of the quantity n_c, which describes the average number of charm quarks in the final state of a B-hadron decay [1]. In the last years this difference became smaller and it became a matter of taste whether one speaks of a missing charm puzzle or not. In this talk we try to summarize the theoretical results and to clarify the origin of different numbers for n_c.

One can calculate n_c in the following ways:

$$n_c = 0 + \frac{\Gamma(b \to 1c)}{\Gamma_{tot}} + 2\frac{\Gamma(b \to 2c)}{\Gamma_{tot}} \tag{1}$$

$$= 1 + \frac{\Gamma(b \to 2c)}{\Gamma_{tot}} - \frac{\Gamma(b \to 0c)}{\Gamma_{tot}} \tag{2}$$

$$= 2 - \frac{\Gamma(b \to 1c)}{\Gamma_{tot}} - 2\frac{\Gamma(b \to 0c)}{\Gamma_{tot}} \tag{3}$$

$\Gamma(b \to 0c)$ sums up all charmless decay rates like the non-leptonic channels $b \to u\bar{u}s, d$, $b \to s\bar{s}s, d$, $b \to d\bar{d}s, d$ and the semi-leptonic channels $b \to ul\nu$ and $b \to sg, gg$. $\Gamma(b \to 1c)$ sums up all decay rates with one charm quark in the final state, like the non-leptonic channels $b \to c\bar{u}s, d$, $b \to u\bar{c}s, d$ and the semi-leptonic channels $b \to cl\nu$. Finally we have $\Gamma(b \to 2c)$ with two charm quarks in the final state: $b \to c\bar{c}s, d$.

Before we compare experimental results and theoretical predictions, let us look at the calculation of these decay rates.

2. Calculation of inclusive decay rates

The Heavy Quark Expansion (HQE) (for a recent review see [2]) is the theoretical framework to handle inclusive B-decays. It allows us to expand the decay rate in the following way

$$\Gamma = \Gamma_0 + \left(\frac{\Lambda}{m_b}\right)^2 \Gamma_2 + \left(\frac{\Lambda}{m_b}\right)^3 \Gamma_3 + \cdots \tag{4}$$

Here we have an systematic expansion in the small parameter Λ/m_b. The different terms have the following physical interpretations:

- Γ_0: The leading term is described by the decay of a free quark (parton model), we have no non-perturbative corrections.
- Γ_1: In the derivation of eq. (4) we make an operator product expansion. From dimensional reasons we do not get an operator which would contribute to this order in the HQE. ‡
- Γ_2: First non-perturbative corrections arise at the second order in the expansion due to the kinetic and the chromomagnetic operator. They can be regarded as the first terms in a non-relativistic expansion.
- Γ_3: In the third order we get the so-called weak annihilation and pauli interference diagrams. Here the spectator quark is included for the first time. These diagrams give rise to different lifetimes for different B hadrons.
- The dots represent higher order terms in $1/m_b$, possible non-perturbative $1/m_c^2$ corrections (like in the decay $B \to X_s \gamma$ [3]) and unknown terms which are due to duality violation (see [4] for a nice review).

Schematically one can write the Γ_i's as products of perturbatively calculable functions (depending on couplings, masses, renormalization scale,...) and matrix elements, which have to be determined by some non-perturbative methods like lattice-QCD or sum rules. Now we may have a closer look at eq. (4). Each of the appearing terms can be expanded in a power series in the strong coupling constant

$$\Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s}{\pi} \Gamma_i^{(1)} + \cdots \; . \tag{5}$$

We start with a discussion of the perturbative part of the $\Gamma_i^{(j)}$'s and then we make some comments about the status of the non-perturbative parameters.

2.1. Leading term: Γ_0

$\Gamma_0^{(0)}$ is well known. In addition we have analytic expressions of $\Gamma_0^{(1)}$ for $b \to c l \nu$ [5] and $b \to c \bar{u} d$ [6] and a numerical value for $b \to c \bar{c} s$ [7]. The effects of the charm quark mass were found to be quite sizeable. Although suppressed by one power of α_s, penguin diagrams are dominant for $b \to$ *no charm* [8], [9]. Recently the NLO calculation for $b \to sg$ has been finished [10]. The inclusion of penguin diagrams with current-current operators for the decay $b \to c\bar{c}s$ and penguin diagrams with penguin operators for $b \to$ *no charm* is still missing, but their effects are not expected to be

‡ Strictly spoken we get one operator of the appropriate dimension, but with the equations of motion we can incorporate it in the leading term.

large. It is a remarkable feature of the HQE that in the leading term Γ_0 only the unit operator appears, so the matrix elements of this operator are trivial. Therefore we have no non-perturbative parameters in Γ_0.

2.2. Sub-leading term: Γ_2

$\Gamma_2^{(0)}$ is known for the most important operator insertions [11]. Some penguin operator insertions are still missing. It would be nice to have a result for $\Gamma_2^{(1)}$, but the calculation seems to be quite tough. One has to calculate the imaginary part of three loop diagrams with one external gluon. Here we have two matrix elements: λ_1 and λ_2. The first one is not very well known, see e.g [12], while the second number can be extracted from experiment.

2.3. Spectator effects: Γ_3

Spectator effects arise first in the third order of the expansion in $1/m_b$. $\Gamma_3^{(0)}$ is known for $\Delta\Gamma_{B_S}$[13] and for B^+, B_s and Λ_b with charm quark mass effects [14]. $\Gamma_3^{(1)}$ was calculated for $\Delta\Gamma_{B_S}$ by [15]. The calculation of $\Gamma_3^{(1)}$ for B^+, B_s and Λ_b is still missing. In Γ_3 we have the following non-perturbative parameters: decay constants f_M (depending on the decaying meson M) and Bag-Barameters B_{D_M} (depeding on the decaying meson M and the Dirac structures D of the appearing operators). For $\Delta\Gamma_{B_S}$ we have already quite stable lattice predictions for these quantities, while for B^+, B_s and Λ_b relieable numbers are still missing (see [16], [17]).

2.4. $1/m_b^4$ *corrections:* Γ_4

For $\Delta\Gamma_{B_S}$ even $\Gamma_4^{(0)}$ has been calculated by [18]; This could be done for B^+, B_s and Λ_b, too. The appearing matrix elements were estimated in vacuum insertion approximation.

3. Different normalization

In order to determine n_c we have to determine the branching ratios for b decays into 0,1 and 2 charm quarks. So one could simply calculate $\Gamma(0,1,2c)$ and Γ_{tot}. But there are several reasons, why it might be better not to calculate these quantities straightforward. First, the semi-leptonic decay rate Γ_{sl} is clearly the most reliable prediction, while Γ_{tot} is probably the least reliable prediction. By writing

$$B_{b\to X} = \frac{\Gamma_X}{\Gamma_{sl}} * \frac{\Gamma_{sl}}{\Gamma_{tot}} =: r_X * B_{sl}^{exp} \tag{6}$$

we can eliminate Γ_{tot} in favor of Γ_{sl}. In r_X we have no m_b^5- and λ_1-dependence anymore. Second, the decay $b \to c\bar{c}x$ is most sensitive to possible quark hadron duality violations. This is due to the fact that the HQE is actually not an expansion in $1/m_b$, but in $1/E$, where E is the energy release in the decay. For $b \to c\bar{c}x$ we have $E = m_b - 2m_c$, which is already quite a small number. If we use eq. (3) and the r's instead of the branching ratios, we have eliminated the decay $b \to c\bar{c}x$, as proposed in [19]. Now $r(0c)$ is an important input parameter for the determination of n_c. Possible enhancements of $r(0c)$ due to new physics would lower B_{sl}^{theory} and n_c^{theory} simultaneously. Different mechanisms for such an enhancement were studied in the literature [20].

4. Results in the literature

Now we summarize the results for the relevant decay rates from the literature and determine n_c in various ways.

4.1. Counting of one Charm Quark

The dominant decay is $b \to c\bar{u}d$. There was quite a confusion due to two different numbers in the literature: Ball et al. quote $r(c\bar{u}d) = 4.0 \pm 0.4$[6], while Neubert was showing $r(c\bar{u}d) = 4.2 \pm 0.4$[21] in Jerusalem. The difference of these numbers is an effect of second order in α_s. While the authors of [6] were calculating ratios like $(a + \alpha_s b)/(c + \alpha_s d)$ nummerically, the author of [21] expanded the ratio in α_s [22]. Unfortunateley the difference is quite sizeable. For all possible semi-leptonic decays we get $r_{cl\nu} = 2.22 \pm 0.04$ and for the Cabibbo suppressed decay modes the result is $r_{u\bar{c}s'} = 0.03 \pm 0.00$. Depending on our input for $r(c\bar{u}d)$ we get two different results:

$$r(1c) = 6.25 \pm 0.4 \text{ [6]} \qquad\qquad r(1c) = 6.45 \pm 0.4 \text{ [21]}$$

4.2. Counting of no Charm Quark

For the non-leptonic charmless b-decays it turned out, that penguin diagrams are as important as the leading contribution to these decays, although being suppressed by α_s [8]. Even α_s^2 contributions, so-called double penguins have a sizeable value [9]. One gets $r(0c) = 0.18 \pm 0.08$ [8, 9] for all charmless final states. Recently the NLO QCD calculation of $b \to sg$ and $b \to sgg$ was finished [10]. Greub and Liniger get an enhancement of more than 100% compared to the LO value

$$r(b \to sg, sgg) = \begin{cases} 0.022 \pm 0.008 & \text{LO} \\ 0.05 \pm 0.01 & \text{NLO} . \end{cases}$$

With the new result for $b \to sg$ and $b \to sgg$ at hand we get:

$$r(0c) = 0.21 \pm 0.08 \text{ [10]}$$

4.3. Counting of two Charm Quarks

For $b \to c\bar{c}s$ we have again two different results. Ball et. al quote $r(2c) = 2.0 \mp 0.5$ [7], while Neubert gets 1.89 ∓ 0.54 [21]. The difference has the same origin as in section 4.1.

4.4. Results for n_c

With the experimental value for the semi-leptonic branching ratio presented in Osaka $B_{sl}^{exp.} = 0.1059 \pm 0.0016$ [23], we can determine n_c in three different ways.

(i) Elimination of no charm: $n_c = (r(1c) + 2r(2c))\, B_{sl}^{exp} = 1.09 \pm 0.11$

(ii) Elimination of one charm: $n_c = 1 + (r(2c) - r(0c))\, B_{sl}^{exp} = 1.18 \pm 0.06$

(iii) Elimination of two charm: $n_c = 2 - (r(1c) + 2r(0c))\, B_{sl}^{exp} = 1.28 \pm 0.05$

For $r(1c)$ and $r(2c)$ we used the average of [6, 7] and [21]. Of course, all these numbers should be the same. The reason for the disagreement is found by comparing the theoretical and experimental value of the semi-leptonic branching ratio. Theory tells us

$$(r(0c) + r(1c) + r(2c))^{-1} = 0.118 \pm 0.009 = B_{sl}^{theory} .$$

The central value is quite above the experimental number for B_{sl}, but the errors are large. When we introduced r_X in eq. (6), we asummed that $B_{sl}^{theory} = B_{sl}^{exp.}$, which is not satisfied. This is the reason for the inconsistencies in the determination of n_c. If we use B_{sl}^{theory} to determine n_c, we get in all three cases the central value $n_c = 1.21$.
In Osaka $n_c = 1.16 \pm 0.05$ was given as the experimental value [23], while Kagan gets a value of $n_c = 1.085 \pm 0.05$ [24]. It is beyond the scope of this talk to clarify the origin of these two different experimental numbers .

5. Disscussion and outlook

In this talk we tried to clarify the orgin of different values for n_c on the market. First we have different numbers for $r(1c)$ and $r(2c)$ due to a different treatment of $\mathcal{O}(\alpha_s^2)$ contributions. The numbers of [21] give a slightly smaller value for n_c, than the numbers of [6, 7]. Second, we get quite different results for the three possibilities (eq. (1)-(3)) to determine n_c, if we use a normalization of the decay rates to Γ_{sl} instead of Γ_{tot}§. The reason for that is the disagreement of the theoretical number for B_{sl} with the experimental value. This problem has to be resolved in the future. Third, the experimental value of n_c seems to be not completely clear.

So we are still not in the position to say the final word about the existence of a missing charm puzzle. If we use an appropriate theoretical input and set $\mu = m_b/4$ (which means a high value for α_s) and $m_c/m_b = 0.33$‖, than experiment (the numbers shown in Osaka) and theory agree more or less. On the other hand there is still room for a deviation, which might be due to a new physics enhanced $r(0c)$ or dualtity violation in $b \to c\bar{c}s$ or.... Precise experimental values of $r(2c)$ and $r(0c)$ would help a lot, to confirm or to rule out these interesting possibilities.

§ In the determination of $\Delta\Gamma_{B_s}$ we have the same situation, that we get quite different numbers for different normalizations (see talk [16]).
‖ Here one should keep in mind, that the ratio m_c/m_b is fixed by HQET.

Acknowledgments

I want to thank the organizers of the workshop for their successful work, U. Nierste for the pleasant collaboration, V. Braun, P. Ball, C. Greub, A. Kagan, P. Liniger, M. Neubert and N. Uraltsev for useful informations and discussions and V. Braun and M. Schenk for proofreading the manuscript.

References

[1] I. Bigi, B. Blok, M. Shifman, A. Vainshtein, Phys.Lett. **B323**, (1994), 408; A. Falk, M. Wise, I. Dunietz Phys.Rev. **D51**, (1995), 1183.

[2] N. Uraltsev, hep-ph/0010328.

[3] M. Voloshin, Phys. Lett. **B397**, (1997), 275.

[4] M. Shifman, hep-ph/0009131.

[5] Y. Nir, Phys. Lett. **B221**, (1989), 184.

[6] E. Bagan, P. Ball, V. Braun und P. Gosdzinsky, Nucl. Phys. **B432**, (1994), 3.

[7] E. Bagan, P. Ball, B. Fiol und P. Gosdzinsky, Phys. Lett. **B351**, (1995), 546.

[8] A. Lenz, U. Nierste, G. Ostermaier, Phys.Rev. **D56**, (1997), 7228.

[9] A. Lenz, U. Nierste, G. Ostermaier, Phys.Rev. **D59**, (1999), 034008.

[10] C. Greub, P. Liniger, hep-ph/0008071; hep-ph/0009144; P. Liniger, hep-ph/0011093, these proceedings.

[11] I. Bigi, N. Uraltsev, A. Vainshtein, Phys. Lett. **B 293**, (1992), 430; B. Blok, M. Shifman, Nucl. Phys. **399**, (1993), 441 and 459; L. Koyrakh, Phys. Rev. **D 49**, (1994), 3379.

[12] V. Braun, hep-ph/9911206.

[13] J.S. Hagelin, Nucl. Phys. **B193**, (1981), 123; E. Franco, M. Lusignoli and A. Pugliese, Nucl. Phys. **B194**, (1982), 403; L.L. Chau, Phys. Rep. **95**, (1983), 1; A.J. Buras, W. Słominski and H. Steger, Nucl. Phys. **B245**, (1984), 369; M.B. Voloshin, N.G. Uraltsev, V.A. Khoze and M.A. Shifman, Sov. J. Nucl. Phys. **46**, (1987), 112; A. Datta, E.A. Paschos and U. Türke, Phys. Lett. **B196**, (1987), 382; A. Datta, E.A. Paschos and Y.L. Wu, Nucl. Phys. **B311**, (1988), 35.

[14] M. Neubert und C.T. Sachrajda, Nucl. Phys. **B 483**, (1997), 339.

[15] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Phys. Lett. **B 459**, (1999), 631.

[16] M. Beneke, these proceedings.

[17] J. Flynn, these proceedings.

[18] M. Beneke, G. Buchalla and I. Dunietz, Phys. Rev. **D54**, (1996), 4419.

[19] G. Buchalla, I. Dunietz, H. Yamamoto, Phys.Lett. **B364**, (1995), 188.

[20] A. Kagan, hep-ph/9806266; A. Kagan, J. Rathsman, hep-ph/9701300; A. Kagan, Phys. Rev. **D51**, (1995), 6196; Z. Xiao, C. Li, K. Chao, Phys. Rev. **D62**, (2000), 094008.

[21] M. Neubert, hep-ph/9801269.

[22] private communication with P. Ball, V. Braun and M. Neubert.

[23] G. Barker, talk in Osaka, 2000.

[24] A. Kagan, private communication.