Onshore Wind Farm Fast Wake Estimation Method: Critical Analysis of the Jensen Model

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***Abstract-* Wind turbine wake effects have a detrimental effect on the power output and life span of wind turbines within a wind farm. Through integration with control systems, wake estimation methods attempt to calculate and reduce the wake effects in order to increase the power output of the wind farm. The Jensen model is commercially popular but relatively untested for onshore wind farms, the primary aim of the work was to analyse the model under these conditions.**

Jensen model calculations were compared with observed data from the Brazos onshore wind farms to test model accuracy and compatibility with a control system. The model was capable of estimating the average wake speeds through the turbine arrays, even under highly turbulent conditions. Through a demonstration of the benefits provided by 'powering back' a lead turbine, the Jensen model showed its ability to be incorporated into a control system.

***Index Terms*— Jensen (Park) model, wake estimation, onshore wind farm, coordinated control system.**

1. INTRODUCTION

Wind energy is the most rapidly expanding source of renewable energy in the UK and has supplied 5% of the country's electricity requirements since 2010. In a global climate that's increasingly opposed to carbon based fuels, research and investment in wind energy continues to increase. With finite space available for future development, investors and manufacturers are constantly looking for new methods to increase the efficiency of both active and planned wind farms.

Wind turbine wake effects are a common cause of energy loss within a wind farm; capable of reducing a trailing turbine's output by up to 30%. Through the use of wake estimation models, wind farms can be designed and operated to reduce the wake effects experienced by turbines.

The Jensen model is a fast wake estimation model, derived from the momentum equation, capable of providing wind speed and wake radius values at distances downstream of a turbine. The simplicity and low computation time of the model have made it highly popular in the commercial market [1]

The majority of the research carried out using the model has involved comparisons with other wake estimation models and has been focused primarily on offshore wind farms due to the greater scale of both space and turbine size. The lack of onshore research has meant there is little understanding of the Jensen model's capabilities in locations with high roughness values or non-uniform terrain.

This paper aims to critically analyse the accuracy and appropriateness of the Jensen model for operation on an onshore wind farm. As a data driven analysis, no comparison will be made with other computational methods. Therefore other wake models [2] and turbulence models [3] are superfluous.

The secondary objective of this report is to use the analysis results to evaluate the Jensen models appropriateness for application in a control system.

2. BACKGROUND - WAKE EFFECTS

Wind turbines generate electricity by extracting kinetic energy from the wind and transforming it into electrical power. The wind flow behind the turbine contains a reduced level of energy due to its lower speed and increased turbulence; known as the "wake effects". As the distance downstream increases, the wake effects diminish until the flow has returned to ambient conditions [4]. Another turbine wholly or partially exposed to the wake will experience a reduced output of anywhere between 5% and 40% [5].

The primary objective of wind turbines is to produce the maximum possible energy at the lowest cost. In order to capitalise upon economies of scale wind turbines are collected into arrays (Wind Farms); enabling lower installation and maintenance costs while maximising output per unit land. However, due to the presence of wake effects the energy yield of the wind farm will be lower and the loads experienced by the turbines will be higher than for the same quantity of turbines all experiencing free-flow conditions [6].

Traditionally, Wind Farms operate with a "greedy" approach (each turbine operating at its individual maximum output). References [7] and [8] both state that the de-rating of individual turbines within a turbine array can generate an increase in total farm power output.

In order for such a control system to be incorporated into a wind farm a method of estimating turbine wake speeds is required. It is for this and several other applications, including wind farm site planning, that the Jensen model is most commonly used. The Jensen model is often considered for real time control systems due to its simplicity and speed; this work evaluates its accuracy and adaptability.

3. BACKGROUND – MODEL THEORY

The aim of a fast wake estimation model is to accurately calculate the wind speed inside the wake of a turbine with minimal computation time. The Jensen model, also known as the Park model, is currently one of the most popular Fast Wake Estimation models for Wind Farm programmes (*Wind Atlas Analysis and Application Programme* (WAsP) and *WindPRO*) [9].

A. Jensen Wake Model

In 1983 N.O. Jensen, [10], created a simple fast wake estimation model with the aim of calculating power losses within turbine arrays due to wake interference. The model generates a linear wake based on the momentum equation, shown in equation (1).

$$\pi {r_0}^2 v_0 + \pi(r^2 - {r_0}^2)u_0 = \pi r^2 v \qquad (1)$$

Where, r_0 [m] is the turbine radius, r [m] is the wake radius at distance x, u_0 [m/s] is the ambient wind speed, v_0 [m/s] is the immediate wind speed after the turbine, and v [m/s] is the wake wind speed at distance x.

By expressing the initial velocity deficit (change in the speed of flow as it passes through the turbine) as seen in equation (2),

$$\left(1 - \frac{v_0}{u_0}\right) = (1 - \sqrt{1 - C_T})/2, \qquad (2)$$

where C_T is the turbine thrust coefficient; and it is assumed the wake is linear ($r \propto x$) [9]. Through basic algebraic manipulation, solving for v, gives (3)

$$v = u_0\left[1 - \left(\frac{1 - \sqrt{1 - C_T}}{1 + \left(\frac{kx}{r_0}\right)^2}\right)\right], \qquad (3)$$

where k is the wake decay constant.

Equation (3) calculates a wind speed v at distance x dependent on parameters C_T, k and r_0. As the turbine radius is constant, equation (3) highlights the models dependency on the mathematical values of the thrust coefficient and wake decay constant [10]. Figure 1 shows a schematic view of the Jensen Wake model.

Assumptions made for the Jensen model:

- The wake is linear ($r \propto x$).
- The wake has constant velocity at any one distance, x, downstream.
- Wake interactions with the ground are neglected.
- The model is inaccurate within the near wake (< 2 diameter lengths); however, this is acceptable as Turbines are unlikely to be situated so close together[10].

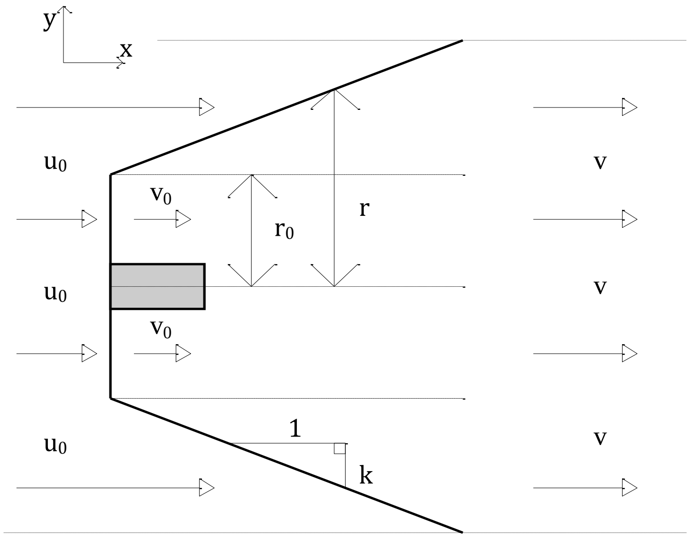

Figure 1: Top-down schematic view of Jensen wake model

B. Thrust Coefficient, C_T

A turbine's thrust coefficient is a situation specific value related to the change in wind speed through the rotor. Equation (2) highlights the importance of C_T, as the primary component for calculating the initial velocity deficit. The thrust coefficient can be easily calculated if the drop in wind speed or change in pressure is known. Unfortunately, due to the requirement for measurements to be taken immediately before and after each turbine, this information is often not available on commercial wind farms [11].

However, although different wind turbine models have unique thrust coefficient curves the majority of industry standard (3-bladed Danish style) turbines have very similar C_T curves. By assuming the rotor effects are similar between turbines it is possible to relate the thrust coefficient directly to the ambient wind speed as shown in equation (4):

$$C_T = \frac{3.5(2u_0 - 3.5)}{{u_0}^2} \approx \frac{7}{u_0} \qquad (4)$$

This relationship allows the Jensen model to be applied to situations without prior knowledge of the turbines' specific thrust coefficient curves [12].

C. Wake Decay Constant, *k*

The wake decay constant controls both the rate of wind speed deficit recovery and the rate of wake expansion; thus the lower the value of k the slower the wake recovery and consequently the greater the wake effects (i.e. wake recovery time is inversely proportional to the wake decay constant).

There is no general equation to calculate wake decay constant; instead the value entirely depends on ambient turbulence, which itself is highly dependent on surface roughness [13].

Reference [5] provides equation (5) linking the terrain roughness length, z_0, which can be found in lookup tables with the appropriate wake decay constant, if the ground conditions are known.

$$k = 0.5/\log(\frac{h}{z_0}) \tag{5}$$

Where h is the turbine hub height. Figures 2(a) and Figure 2(b) show a turbine from the horizontal viewpoint and the respective Jensen model estimation for the wind speed vs. distance curve of the turbine in ambient wind conditions of 15m/s.

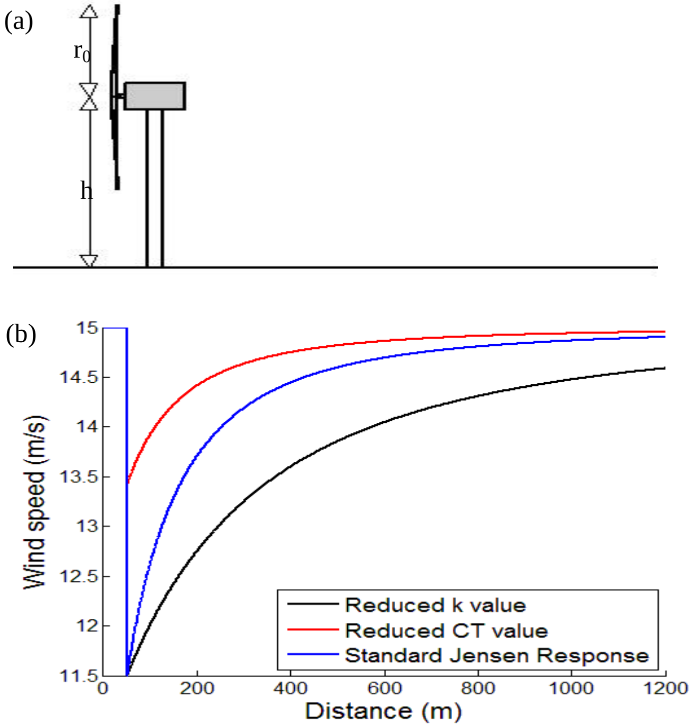

Figure 2: (a) Horizontal view of wind turbine (b) Wind speed against distance down wake. (a) and (b) are not to scale

Figure 2(b) further demonstrates both the dependence of initial speed deficit on changes in thrust coefficient and the affect the wake decay constant has on the rate of wake recovery.

Alternatively to equation (5) several commercial software packages commonly select values of *k*=0.075 for flat land and *k*=0.04 for calm seas [14]. However, in practice wake decay values are highly variable and can differ between separate rows within a wind farm, even with similar surface conditions. Therefore, it is unwise to trust a value of wake decay constant-from either a commercial programme or an equation - unless it has been compared to site specific observed data.

D. Wake accumulation

Due to the Jensen model's dependency on the ambient wind speed it cannot accurately model an individual turbine that is not experiencing free flow conditions (e.g. in the wake of another turbine). This is due to a fundamental principal of the model, shown in Figure 2(b); whereby, even with varied input parameters (k and C_T) the wake speed will always tend towards the initial/ambient wind speed. This is similar to actual turbines as the free flow air surrounding the wake is one of several factors that helps return the wake to ambient conditions. However, if the wind entering the turbine is below ambient speed the model will compute the wake speed as tending towards this lower value. This is inaccurate because the wake should eventually reach the overall free flow speed, as would be expected from the wake of the final turbine in an array.

To enable the Jensen model to compute the response of a turbine experiencing wake effects or to model conditions where two or more wakes have crossed it is assumed that the kinetic energy deficit of the combined wakes is the sum of the kinetic energy deficits of the individual wakes at the same point, as shown in equation (6) [4].

$$(1-\frac{v}{u_0})^2 = (1-\frac{v_1}{u_0})^2 + (1-\frac{v_2}{u_0})^2 + \cdots \tag{6}$$

Where v_1 and v_2 are the individual wake speeds at distance *x* related to turbines 1 and 2 respectively.

Due to the squaring of the values (6) will result in a gradual plateauing effect, whereby additional turbines or wakes further downwind will have a reduced influence on the total wind speed [4]. Equation (6) is another aspect of the Jensen model which is incorporated into both the *WAsP* and *WindPro* Programmes [15].

E. Implementation in Matlab

Matlab provides a simple yet powerful platform from which to operate the Jensen Model. The file system allows a number of codes of varying complexity to be written simultaneously. Whilst the editor enables all parameters to be adjusted through operator set values or linked to other parameters via an equation; in the way (4) links thrust coefficient with ambient wind speed. The most advanced forms of the Jensen model code are capable of assigning individual turbines bespoke parameters, to more accurately model observed conditions.

4. METHOD

The primary objective of this paper is to carry out a critical data driven analysis of the Jensen model. To achieve this, three separate Case studies (1, 2 and 3) were designed to compare the Jensen's computational results with observed data from an onshore wind farm. The focus was on the accuracy and consistency of the model, with particular regard to site specific conditions.

The secondary objective of this paper is to evaluate the appropriateness of the Jensen model for application in a coordinated control system. For this task two Case studies (4 and 5) were conceived to test the impact of a theoretical control system incorporating the model. These were theoretical experiments and therefore the models were not compared with observed data.

The turbine row formation was selected for the Jensen model analysis as it has the potential to generate the most wake interactions and accumulation effects, when compared with other array layouts.

Recorded data was supplied from two wind farms, Brazos A and B, situated in Northern Texas (see Appendix for site map). Together the wind farms consist of 160 Mitsubishi turbines each rated at one megawatt with rotor diameters of 62m.

Five turbine rows were selected from the wind farms; values for the number of turbines in each row and the distances between the turbines are shown in Table 1. Each row was selected for a specific characteristic intended to analyse the versatility of the Jensen model.

Table 1: Turbine Row Information

Row	*No. Turbines*	*Turbine Spacing (m)*	*Reason for selection*
A1	7	185	Control
A2	5	120	Shorter Spacing
A3	5	120	Rough Terrain
A4	5	185, 315, 120,120	Varied Turbine Spacing
B1	12	120	Alternate Site

In order to maximise the wake effects, for each of the following cases all observed wind speed data is flowing parallel to the length of the row. For rows A1 - A4 wind readings were selected from within 5° either side of directly East (90°). For Row B1 due to its alternative orientation the speed values are all flowing from between 90° and 100°.

Due to the constantly varying nature of wind, the model was tested in all cases against the mean wind speed data. This factor, as well as the standard deviation of the wind flow, is highlighted when necessary within the Section VI.

A. Case 1: Two Turbine Array

The objectives for this case were to analyse the model's ability to accurately portray a two turbine array, as well as to calibrate the wake decay constant for the site conditions. A comparison of the results from different rows will test the model's accuracy when using both global parameter values for a wind farm and local row specific values.

The Jensen model was set for a two turbine array and by inputting the ambient speed at the first turbine, a wake speed at the second turbine was estimated. The outputs were compared to the mean recorded wind speed at the second turbine in each row. Equation (4) was implemented to provide a value of thrust coefficient; however, the wake accumulation (6) was not incorporated as the interaction effects between the two turbine wakes were unrelated to the case's aims.

In order to calibrate the code to the localised surface roughness, varying values of *k* were inputted to find the optimum value for each turbine row. Equation (5) was also used to generate a global value of *k*. The varying accuracies of the model with local and global wake decay constants were then compared.

B. Case 2: Wake Accumulation

The aim of this case was to assess the ability of the Jensen model to accurately replicate the wind speed along a row of turbines. The focus was on the modelling of the interactions and accumulation of turbine wakes. Row A4 was of particular interest here, due to the varying distances between turbines.

Equation (6) was integrated with the model to enable the extended row analysis. The parameters for the first five turbines in each row were inputted into the code. The value of C_T for the whole row was set using (4), with a wind speed of u_0; this resulted in a faster computation time than recalculating C_T before every turbine. For each row a localised value of *k* was selected from the results of Case 1.

The computed and observed wind speeds at each turbine were compared as well as the wake response along the row.

C. Case 3: Varying Thrust Coefficient

Up to this point equation (4) has been used to calculate a thrust coefficient value for the entire row of turbines, dependent on the wind speed experienced by the lead turbine. However, as stated in Sections III and IV, each turbine experiences a different wind speed and therefore, by equation (4), should operate with an individual thrust coefficient.

The aim of this case was to test whether the Jensen model with a varying thrust coefficient was more accurate to the observed response than the Jensen model with a constant value for C_T.

The Jensen model code was adjusted to recalculate the thrust coefficient for each turbine using the localised wind speed at that point. The experiment was carried out as in Case 2 and the results were compared with the observed data and original Jensen model.

D. Case 4: Theoretical Control System, Two Turbine Array

The aim of this case was to analyse whether the Jensen model was a capable tool for use in a control system. To test this, the Jensen model would attempt to recreate a two-turbine array operating with a control system, as described in Section 2; whereby the de-rating of the first turbine increases the output of the following turbine.

To recreate the wake of the de-rated turbine, with the Jensen model, the computational turbine's thrust coefficient was reduced. The effect was to reduce the initial velocity deficit and therefore the wake effects experienced by the downstream turbine. The computed wind speed at the second turbine was recorded as the lead turbine's thrust coefficient was varied.

E. Case 5: Theoretical Control System, Wake Accumulation

The objectives for this case were to analyse the Jensen model's ability to replicate the effects of a control system along the length of a turbine row and to observe the response, focusing on any outcomes due to site specific parameters.

The Jensen code was set up as in Case 2; but the lead turbine was de-rated as in Case 4. This would highlight how capable the Jensen model was of transmitting any effects caused by the control system along the row.

The computed wind speeds along the row were compared to the response of the Jensen model without a de-rated turbine in order to directly highlight any changes.

5. WIND FARM SITE SPECIFIC OBSERVATIONS AND CONSIDERATIONS

Prior to the analysis of results, it is important to highlight any peculiarities or site specific conditions that may affect the observed data. This section highlights some important observations and briefly relates them to the theory discussed in Section IV:

***Altitude*:** Brazos A is entirely situated atop a wide, smooth peak. The altitude within the wind farm varies gradually between 860m and 875m. Brazos B covers a much larger area than its sister sight; however, Row B1 is situated in similar conditions to Brazos A, with an average height of 860m [16]. Notably, for both sites there is never more than a 2m jump between adjacent turbines. As this change in height is only 1/30 of the rotors' diameter it can be assumed, for the sake of the model, the turbines within each row are at the same altitude.

***Terrain*:** Within both Brazos A and B the terrain is open (flat terrain with grass or low vegetation) and therefore has an associated surface roughness of 0.03m [17]. Surrounding the wind farms the ground drops away on all sides. To the north and west a continuous cliff edge causes a rapid drop in altitude, whilst to the south and east the decline is more gradual but interspersed with long canyons and crevasses along the entire length [16]. These observations suggest that although the farm's internal values of roughness are low the wind, even before the first turbine, is likely to be highly turbulent.

***Turbine Spacing*:** As shown in Table 1 the turbine rows under observation are situated with spacing of only two (120m) or three (185m) turbine diameters separation. This suggests the wake flow will have very little time for recovery and is likely to rapidly drop in speed along the row. Furthermore, the Jensen model is relatively untested under these close conditions as the near wake is often considered a task for Computational Fluid Dynamics (CFD) software [10].

6. RESULTS AND DISCUSSION

A. Case 1: Two Turbine Array

Table 2 shows the results of the Jensen model's two-turbine array calculations. The observed wind speeds for the first and second turbines in each row are shown against the model's computed value for wake speed at the second turbines. The wake decay constant shown was the value required to achieve this output.

Table 2: Observed and Computed Wind speed values for the first two turbines in each row

	Wind Speed (m/s)				*Wake Decay Constant* , k
	1st Turbine	2nd Turbine			
		Observed	Comp.	% error	
A1	5.6	4.48	4.52	0.9	0.13
A2	5.88	4.66	4.65	0.2	0.17
A3	5.4	4.6	4.61	0.2	0.28
A4	5.7	4.65	4.65	0	0.135
B1	5.9	4.3	4.29	0.2	0.12

With the locally calibrated values of *k,* the model output is within 0.05m/s (~1%) of the target value for every row.

The Jensen model is therefore able to accurately replicate a two-turbine array for multiple cases. However, two decimal places is an unnecessary level of precision; by the nature of the wind, the local speed will not remain at one value for more than a second.

The local wake decay constant figures are roughly double the general value recommended for flat land in commercial programmes; however, using the surface roughness value of 0.03m (appropriate for flat terrain with grass or very low vegetation [17]) in (5), a global wake decay constant of 0.15 was calculated.

Incorporating *k*=0.15 into the model proved effective as the computational outputs for four out of the five rows were within 0.5m/s (10%) of the observed wind speeds. As the standard deviation of each turbine was approximately 1m/s this level of accuracy is high enough to be considered of practical use when estimating the average wind speed.

The *k* value for row A3 appears anomalous, being almost four times the recommended default value and twice the calculated value. On inspection, the aforementioned cliff edge runs in front of the first turbine within the row. The rapid change in terrain is very likely to cause more turbulent conditions, which are accounted for in the higher value of *k*.

Case 1 showed that the Jensen model could be accurately calibrated to estimate the average wind flow within a two-turbine array. Furthermore, if a surface roughness value is known equation (5) allows a global wake decay constant to be calculated. However, Row A3 demonstrated that close examination of terrain is required to minimise estimation errors, as values of z_0 and, therefore, *k* can vary drastically within a wind farm.

B. Case 2: Wake Accumulation

Figure 3 shows the wind speed at each turbine for the first five turbines in rows A2-A4 and the first six turbines in row A1. It is important to note, firstly, that Figure 3 does not show the wake responses between turbines and therefore any wind speed not directly on a turbine number is inaccurate. Secondly, that A1, A2 and A3 are continuous rows, meaning that the turbines are equally spaced along their length. Alternatively, A4 consists of three different spacing lengths, as seen in Table 1.

i. Continuous Rows- Jensen Model

As Rows A1 - A3 are continuous their Jensen model outputs consistently contain two clear stages: a large drop in speed from turbine 1 to 2 and then a very gradual decline along the rest of the row. Contrastingly, although the 2 stages are still present, the observed data exhibits a unique response for each row. This suggests that the Jensen model is unable to map the specific response of an individual turbine row.

Having said this, at no point is the margin of error between the observed and computed wind speeds greater than 10%. As discussed in Case 1, this is a high enough level of accuracy for the Jensen model to be applied in a practical system. It can be concluded that the model is capable of estimating mean wake speeds along the length of a turbine row situated in an area with high values of terrain roughness.

There is potential to improve the accuracy of the Jensen model, for an individual row by implementing turbine specific values for thrust coefficient (see Case 3) and wake decay constant.

Calculation of an individual wake decay constant for each turbine would attune the Jensen model in a similar way to Case 1. However, the lack of significant changes in terrain roughness makes estimation of varied *k* values through site observation difficult. Another option is to calibrate each value by using observed wind speed data. Although possible, this method is impractical as it requires detailed historic wind data for every wind farm the Jensen model is applied to. Furthermore, it is impossible to apply such levels of accuracy to future wind farms (as the data does not exist).

A final option, and area for future work, is incorporating a turbulence estimation model into the Jensen.

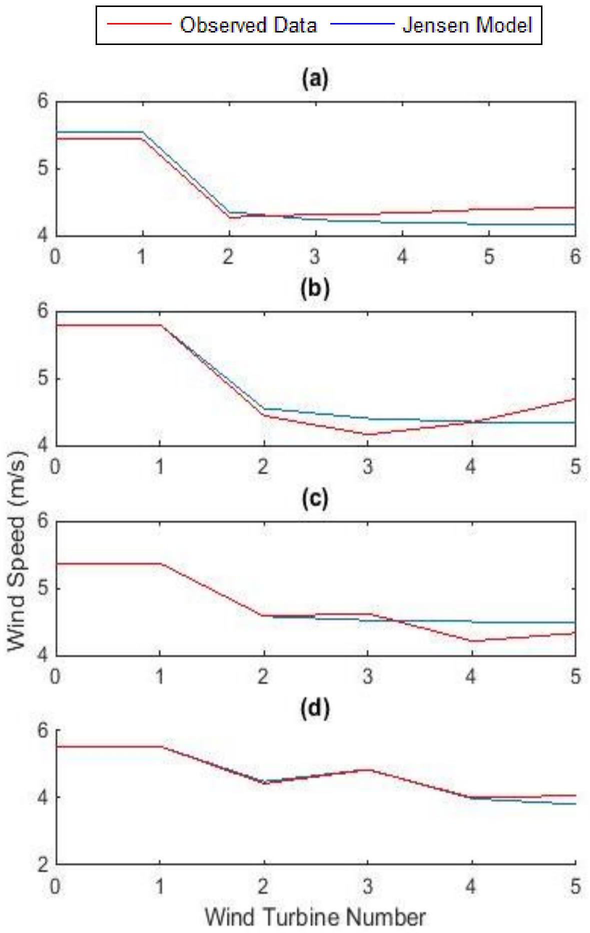

Figure 3: Observed and Computed wind speed along the Turbine Row (a)A1 (b)A2 (c)A3 (d)A4

ii. Continuous Rows- Observations

As previously stated, each rows output is slightly different, however, none of the observed wake responses match the anticipated pattern. The expected response was a gradual and continuous reduction in wind speed along the length of the row. A more visible trend is an abrupt 'levelling off' of the wind speed after the initial drop; with the potential for a slight increase in speed along the array as seen predominantly in Rows A1 and A2.

References [18] and [19] provide an explanation for this phenomenon. Using experimental data from Horns Rev I and Nysted wind farms the reports demonstrate how the anticipated gradual decrease in speed along the row's length occurs in the majority of cases. However, this response does not occur when the wind is flowing from within ±2.5° of parallel with the turbine row. In this unique case, after the initial drop in wind speed from turbine 1 to 2 all trailing turbines experience similar conditions to turbine 2. This explains the wake responses for rows A1 - A3 as the observed data was restricted to similar flow conditions as those explained above.

To further corroborate this hypothesis, the observed data was analysed under conditions with the direction of wind flow varied between 5°-10° away from the parallel. However, these experiments proved inconclusive with the results showing greater variety between measured speeds and no apparent correlation along the row. This outcome is likely due to site specific conditions and terrain irregularities. The predominant factors are the surface roughness and turbulence conditions. Horns Rev I and Lillgrund are both offshore wind farms resulting in low turbulence and roughness values; enabling a smooth transition between the "level" and gradual responses. Contrastingly, this report has already highlighted how the Brazos wind farms have varying roughness values throughout, as well as small but obstructive terrain features, such as crevasses, capable of affecting individual turbines. In this way it would be possible for the surface roughness and terrain features to have a greater effect on an individual turbine's wind speed than the wake accumulation effects; especially when the wind turbines are only experiencing partial wake effects.

To conclude, it has been shown that, the Jensen model can estimate to a practical accuracy the wind speed response along a continuous turbines row. It should be noted that the onshore conditions of the wind farm appear to negate partial wake effects. Furthermore, the only response that matched offshore wind farms was the unique "levelling off" that occurs with parallel flow.

iii. Varied Row Spacing

Row A4 is unique in this report as the turbines are separated by varying distances. Figure 3(d) shows that the Jensen model can accurately estimate the wind speed response under these conditions as well as, if not better than, for a continuous row. Notably in this case the wind speed at turbine 3 is greater than that at turbine 2. As stated in Section 4.D the Jensen model is only capable of modelling this outcome with the accumulation (6), and this experiment clearly demonstrates the accuracy of this method.

C. Case 3: Varying Thrust Coefficient

Figure 4 shows the wind speeds, calculated by the adjusted Jensen model with an independently calculated value of thrust coefficient at each turbine, for Rows A1 and A2. The adjusted Jensen model's estimated values are shown against the observed data and original Jensen model responses.

Figure 4 shows how the only disagreement between the two versions of the Jensen model occurs at turbine 3. The adjusted model approximates the wake speed to be 0.4m/s (~10%) greater than the original model at this point. This division is due to the adjusted Jensen model estimating a rise in wake speed between turbine 2 and 3, which is not apparent in the original model. Other than this, both models follow a similar pattern of a large initial drop after the lead turbine and a gradual decrease in speed further down the row (after turbine 3).

If turbines 1 and 2 are both operating with a greedy approach (attempting to extract the maximum energy from the wind flow) then there is no explanation for an increase in speed at turbine 3. Therefore, from a theoretical point of view the original Jensen model should have the most accurate response.

Having said this, as previously mentioned, increases in speed along the row are apparent within the observed data. Progressing along the row also shows a clear shift in error measurements between the two models. For turbines 2-4 the original model is by far the most accurate, however, from turbine 5 the observed data has risen in speed to be closer to the adjusted model.

In conclusion the adjusted Jensen model was unable to improve the accuracy of the wind speed estimation. However, this case did demonstrate that the unpredictable responses of the different turbine rows were not solely dependent on varying thrust coefficients between individual turbines. It can be assumed that these unique responses are also dependent upon changes in wake turbulence values and localised terrain conditions.

D. Case 4: Theoretical Control System, Two Turbine Array

The thrust coefficient was varied between 0 and 1 for the first turbine in each row and the resulting Jensen estimation for wake speed at the second turbine was recorded. The most interesting results can be seen in Figure 5 where wind speed at the second turbine is plotted against thrust coefficient at the first turbine. As row A3 did not experience the same ambient wind speed as rows A2 and B1, an example simulation was run with similar parameters to A3 but in higher wind conditions.

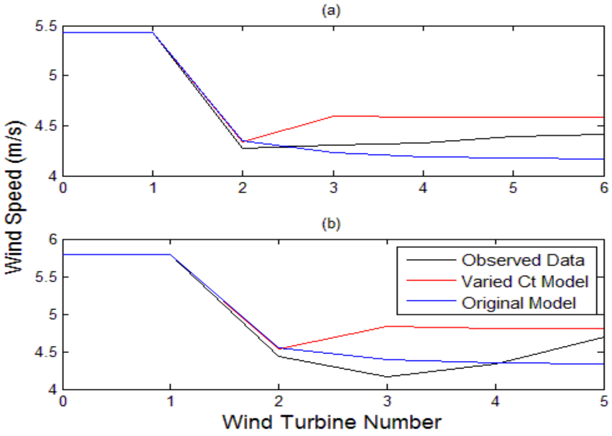

Figure 5: Jensen Model with varying Thrust Coefficient compared to observed data (a)A1 (b)A2

The thrust coefficient and resulting wind speed have a clear negative correlation. Thus demonstrating that as the first turbine is 'powered back' (reduces it thrust coefficient) it leaves a greater quantity of kinetic energy in the air flow. This proves how a control system could be beneficial to a farm's total power production by increasing the second turbine's output by more than 'powering back' restricts the first turbine.

It is important to note at this time that if the lead turbine has a thrust coefficient of zero then the modelled second turbine experiences ambient conditions.

The strong negative correlation between thrust coefficient and wind speed at the second turbine demonstrates that the Jensen model is capable of replicating the response of 'powering back' a lead turbine. Therefore, the Jensen model can be considered an appropriate tool to be incorporated into a coordinated control system.

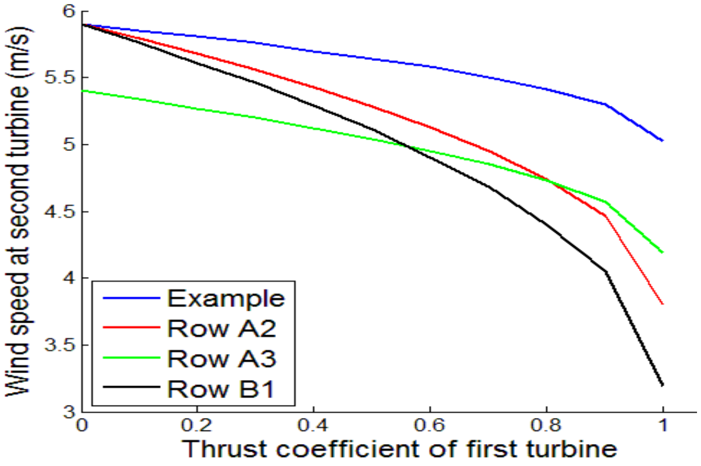

Figure 4Wind Speed at second turbine against Thrust coefficient at the Lead turbine

The second objective of this case was to consider any effects local surface parameters may have on the outcome of a coordinated control system. Figure 5 highlights the wake decay constant as an influential parameter. Row A3 has a value of 0.28 whilst the example curve has a value of 0.4, both of which can be considered high values of k. The lower values for Row A2 and B1- 0.17 and 0.12 respectively- produce a much steeper gradient than the higher values. This is derived from the wake decay constant's effect on the wake recovery time. The faster the wake recovers (the greater the value of *k*) the less of an effect the initial deficit has on wind speeds.

Offshore wind farms will, therefore, benefit more greatly from 'powering back' wind turbines as they have lower *k* values than onshore wind farms. It is also possible that a control system would be of little or no benefit for an onshore wind farm in excessively rough or turbulent conditions [17].

E. Case 5: Theoretical Control System, Wake Accumulation

Figure 6, shows the effect 'powering back' the lead turbine has on the whole turbine row. To highlight the system benefits the 'powered back' model is compared with the original Jensen model.

As shown in Case 4 the reduction in output of the lead turbine causes a notable increase in wind speed at turbine 2, designated Δ*u* in Figure 6.

To further explore the response of individual downwind turbines, Figure 7 shows the change in wind speed for each turbine along the row as the lead turbine's value of C_T changes. The outcomes are congruent with Figure 6; there is a large difference between the beneficial effect experienced by turbine 2 and the rest of the turbines in the row, which appears to receive only minimal increases in local wind speed.

Although the changes in wind speed for downwind turbines are minimal there is still a noticeable negative correlation with the C_T value of the lead turbine. This suggests the Jensen model is successfully transferring the control system effects downstream and is therefore capable of integration into such a system operating on a row containing larger number of turbines.

The explanation for this response is derived from the unusual circumstances concerning the row alignment and wind angle. Case 2 demonstrated that, after the lead turbine, the observed wind speeds along the row were very consistent; [18] and [19] suggested this was due to the row angle being parallel with the wind direction.

Case 4 demonstrated that if the first turbine has 0 thrust coefficient then the second would effectively replace it as the lead turbine and all the following turbines would remain under the same conditions. This scenario creates an identical turbine array with one fewer turbines and is therefore not beneficial. Further research should focus on "powering back" multiple turbines along the row in an attempt to raise as many turbine outputs as possible above the level.

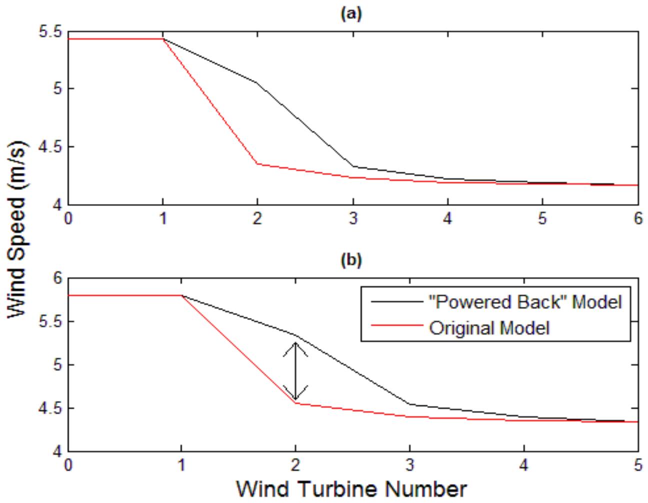

Figure 7: Jensen Model with 'Powered back' lead turbine compared to the Original Jensen Model (a)A1 (b)A2

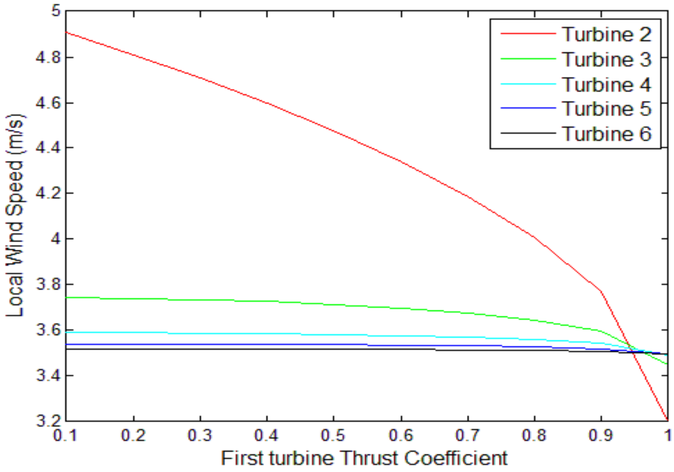

Figure 7: Wind Speed at each turbine in Row A1 as the Thrust coefficient of the Lead Turbine is varied

This case demonstrated that the Jensen model was capable of integration into a wind farm control system.

7. CONCLUSION

The Jensen model was successfully used to estimate the average wake speeds experienced within a number of onshore turbine arrays. This showed the model was capable of being applied to areas with high terrain roughness values.

The Jensen model was successfully used to estimate the average wake speeds experienced within both a two-turbine and extended-turbine row. Calculations calibrated to two-turbine arrays achieved the highest levels of accuracy (<1% error) whilst the estimations run using global parameters achieved lower accuracy (<10% error) but with improved computation time.

The global parameters demonstrated the time saving potential for the application of a general estimation model to several rows within a wind farm. However, row A3 highlighted the impact of rapid changes in surface roughness and how prior site knowledge is necessary to confidently apply the model.

Applying the calibrated parameters to the extended rows showed the Jensen model achieved a practical level of accuracy (<10% error) when estimating the average wind speed for every turbine in an array.

However, the Jensen model was unable to predict unique patterns within row responses, even after a varied thrust coefficient equation was incorporated. Furthermore, as all wake estimations have been compared to average values the Jensen model should not be considered capable of predicting the wind speed at a specific time. It is thought for further accuracy, varied wake decay constants and turbulence models should be considered.

These negatives do not invalidate the model as a practical tool. Instead, the Jensen model has demonstrated its ability to accurately estimate the wakes of onshore wind turbines.

The Jensen model highlighted the potential benefits for the incorporation of a coordinated control system into a turbine array. In so doing the model demonstrated its own capability for application within a control system. The speed, simplicity and versatility of the model also serve to increase the value of its integration.

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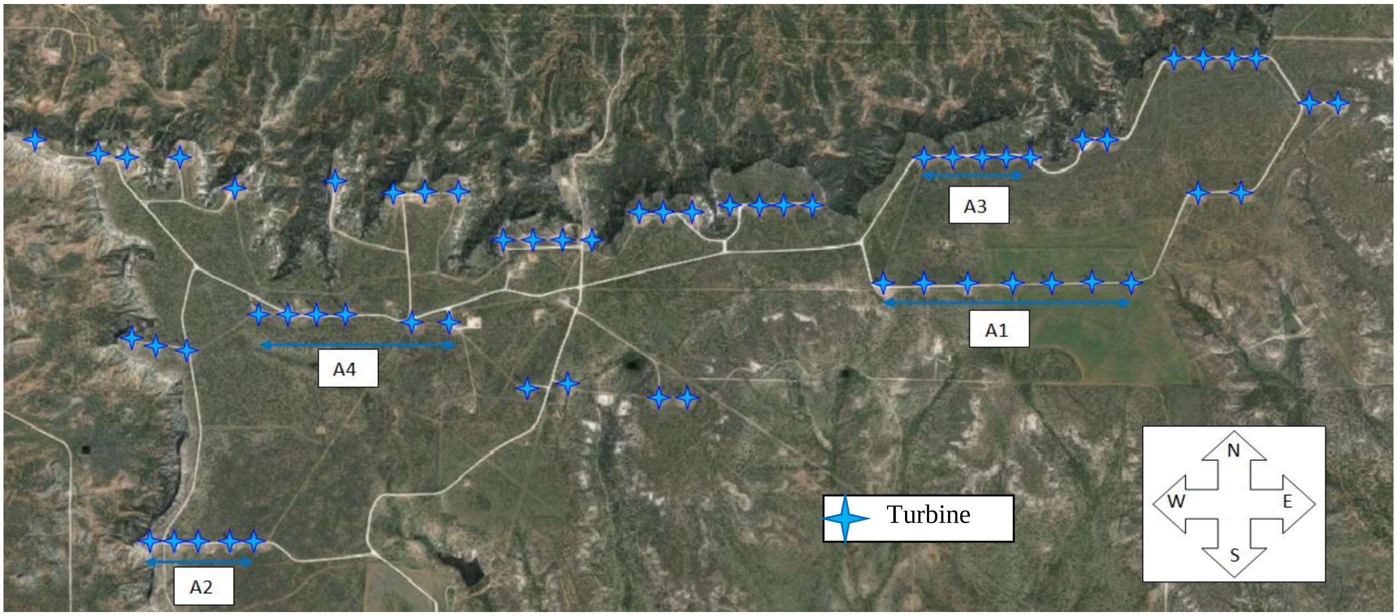
APPENDIX



Appendix: Brazos A wind farm with marked rows and turbines