Modelling tertiary creep in geomaterials using a continuum damage mechanics approach

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ABSTRACT: Tertiary creep is often observed in soft rocks and it represents a problem in a mining environment. Tertiary creep behaviour appears due to progressive micro cracking of the material and would result in a loss of strength and stiffness, which may eventually lead to failure and a complete loss of load carrying capability of the material. In this paper, the authors combined the continuum damage mechanics within the framework of hyperplasticity, thus encompassing viscoplasticity and damage within a single theory. The authors present a family of models which obeys the laws of thermodynamics. The entire constitutive behaviour is derived from two scalar potentials; a free energy potential which provides the elasticity law, and a dissipation potential which provides the yield function, the direction of plastic flow and the evolution of a damage variable. No additional assumptions are required. These new models require only few parameters which have physical meanings and are capable of capturing tertiary creep observed in soft rocks.

1 INTRODUCTION

Creep behaviour is prominent in soft rocks such as salt rock and potash, and has even been observed in hard rocks, highlighting the importance of designing for such behaviour. In this paper we present a family of time-dependent constitutive models which take into account both the secondary and tertiary stages of creep deformation. Three stages of creep are typically observed in laboratory creep tests: a primary stage considered to be viscoelastic, a secondary stage considered to be viscoplastic and a tertiary stage considered to be caused by material damage through the formation of cracks. Thus here we develop visco-plastic-damage constitutive models. These models are novel as they are derived using the hyperplastic framework of Houlsby and Puzrin (2000) and therefore obey the laws of thermodynamics. In this approach the entire constitutive behaviour can be derived from two scalar potentials; a free energy potential which provides the elasticity law, and a dissipation potential which provides the yield function, the direction of plastic flow and the evolution of a damage variable. No additional assumptions are required. In these models, we assume that the visco-plasticity and damage are coupled, therefore they occur simultaneously. The classical sign convention in soil mechanics is considered here where compressive stresses are taken to be positive.

2 FORMULATION OF VISCOPLASTIC-DAMAGE MODELS

2.1 *Von-Mises type material*

The free energy potential ψ can also be written as follows:

$$\psi = \frac{1}{2}\left(1-\alpha_d\right)\boldsymbol{\varepsilon}^{\mathbf{e}} : \mathbf{D}^{\mathbf{e}} : \boldsymbol{\varepsilon}^{\mathbf{e}} \tag{1}$$

where $\boldsymbol{\varepsilon}^{\mathbf{e}}$ is the elastic strain tensor, $\mathbf{D}^{\mathbf{e}}$ is the standard isotropic elasticity tensor and α_d is a damage parameter. In the case of isotropic damage α_d is a scalar internal variable starting from 0 and increasing to a maximum value of 1.0 and can be defined as:

$$\alpha_d = \frac{A-A_s}{A} \tag{2}$$

where A is the total cross-section area of a surface within the unit cell in one of the three perpendicular directions and A_s is the solid matrix area within A.

Differentiating the free energy potential with respect to the elastic strain vector produces the elasticity law:

$$\boldsymbol{\sigma} = \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}^{\mathbf{e}}} = \frac{1}{2}(1-\alpha_d)\boldsymbol{\varepsilon}^{\mathbf{e}} : \mathbf{D}^{\mathbf{e}} : \boldsymbol{\varepsilon}^{\mathbf{e}} \qquad (3)$$

where $\boldsymbol{\sigma}$ is the stress tensor.

The thermodynamical force χ_d associated with the damage can be derived from the free energy potential as follows:

$$\chi_d = \frac{\partial \psi}{\partial \alpha_d} = \frac{q^2}{6G(1-\alpha_d)^2} + \frac{p^2}{2K(1-\alpha_d)^2} \qquad (4)$$

where q is the von Mises equivalent stress, p is the mean stress and G and K are the shear modulus and bulk modulus respectively.

The dissipation potential is defined as:

$$\phi = \bar{\boldsymbol{\chi}} : \dot{\boldsymbol{\alpha}} \qquad (5)$$

where $\bar{\boldsymbol{\chi}}$ is the dissipative stress tensor and $\dot{\boldsymbol{\alpha}}$ is internal variables tensor.

For a coupled viscoplastic-damage von Mises material the dissipation potential ϕ can be written in the form:

$$\phi = \left(\dot{\alpha}_p \frac{\left(r_p \sigma_y \left(1 + \eta^{n-1} \dot{\alpha}_p^{n-1}\right)\right)^2}{\Phi} \right) \dot{\alpha}_p + \left(\frac{\dot{\alpha}_d (\sigma_y r_d \rho)^2}{\Phi} \right) \dot{\alpha}_d \qquad (6)$$

where

$$\Phi = \sigma_y \sqrt{\left(r_p \dot{\alpha}_p + r_p \eta^{n-1} \dot{\alpha}_p^n\right)^2 + (r_d \rho \dot{\alpha}_d)^2} \qquad (7)$$

where $\dot{\alpha}_p$ is an internal variable representing the plastic strain rate and $\dot{\alpha}_d$ is an internal variable representing the damage parameter rate. η and n *are* material constants govern the viscoplastic behavior, whilst r_p and r_d are constants governing the ratio of viscoplasticity and damage. If the damage is assumed to start with the plastic behavior, ρ can be shown to take the form:

$$\rho = \frac{q}{6G(1-\alpha_d)^2} + \frac{p^2}{2Kq(1-\alpha_d)^2} \qquad (8)$$

It follows from equation (6) that the dissipative stresses are given by:

$$\bar{\chi}_q = \frac{\left(r_p \sigma_y + r_p \sigma_y \eta^{n-1} \dot{\alpha}_p^{n-1}\right)^2 \dot{\alpha}_p}{\Phi}$$
$$\bar{\chi}_d = \frac{\dot{\alpha}_d (\sigma_y r_d \rho)^2}{\Phi} \qquad (9)$$

Assuming Ziegler's orthogonality condition (Ziegler, 1983) and no kinematic hardening, the true stresses and the dissipative stresses are identical. From equations (9):

$$\left(\frac{q/\sigma_y}{r_p + r_p \eta^{n-1} \dot{\alpha}_p^{n-1}} \right)^2 + \left(\frac{q/\sigma_y}{r_d} \right)^2 = 1.0 \qquad (10)$$

From Equation (10), it can be shown that the internal variable representing the plastic deviatoric strain rate $\dot{\alpha}_p$ is given by:

$$\dot{\alpha}_p = \frac{1}{\eta} \left(\frac{1}{\sqrt{\left(\frac{\sigma_y r_p}{q}\right)^2 - \left(\frac{r_p}{r_d}\right)^2}} - 1 \right)^{1/(n-1)} \qquad (11)$$

In the absence of damage, the above relationship reduces to the well-known Perzyna model (Perzyna 1966) which is widely used in computational applications of viscoplasticity.

Following Ziegler's orthogonality and using equations (4) and (9), the rate of the damage parameter $\dot{\alpha}_d$ can be related to the plastic strain $\dot{\alpha}_p$ by:

$$\dot{\alpha}_d = \frac{\dot{\alpha}_p \left(1 + (\eta \dot{\alpha}_p)^{n-1}\right)^2}{\rho} \left(\frac{r_p}{r_d}\right)^2 \qquad (12)$$

The plastic strain rate $\dot{\alpha}_p$ is a non-negative quantity, therefore the expression inside the bracket in equation (11) must be greater than or equal to zero. Therefore the yield criterion is given by:

$$\left(\frac{q}{\sigma_y}\right)^2\left(\frac{1}{r_p^2}+\frac{1}{r_d^2}\right)=1.0 \tag{13}$$

Thus:

$$\frac{1}{r_p^2}+\frac{1}{r_d^2}=1.0 \tag{14}$$

which is consistent with the yield function suggested by Einav et al. (2007) for a rate-independent damage model.

2.2 *Drucker-Prager type material*

For a frictional material, the dissipation function Φ can be assumed to take the form:

$$\Phi=\sqrt{\begin{array}{l}\left(r_p\left(c_1+c_2p\right)\dot{\alpha}_p+r_p\eta^{n-1}\dot{\alpha}_p^n\right)^2\\+\left(r_d\left(c_1+c_2p\right)\rho\dot{\alpha}_d\right)^2\end{array}} \tag{15}$$

where c_1 and c_2 are material parameters related to the cohesion C and the angle of friction φ, respectively. ρ can be shown to take the form:

$$\begin{aligned}\rho=&\frac{q^2}{6G\left(1-\alpha_d\right)^2\left(c_1+c_2p\right)}\\&+\frac{p^2}{2Kq\left(1-\alpha_d\right)^2\left(c_1+c_2p\right)}\end{aligned} \tag{16}$$

Starting by rewriting the dissipation function in form of equation (5), assuming a linear relationship between the volumetric and shear strain rates and following the procedure illustrated in the previous section, it can be shown that:

$$\dot{\alpha}_p=\frac{1}{\eta}\left(\left(c_1+c_2p\right)\left(\frac{1}{\sqrt{\left(\frac{r_p}{c_k}\right)^2-\left(\frac{r_p}{r_d}\right)^2}}-1\right)\right)^{1/(n-1)} \tag{17}$$

where

$$c_k=\frac{\left(q-c_3p\right)}{\left(c_1+c_2p\right)}$$

For the full mathematical details, the readers can refer to Osman et al. (2014) and Birchall (2013).

3 VALIDATION WITH TRIAXIAL TESTS

The Drucker-Prager viscoplastic-damage constitutive model, which is applicable to geomaterials, has been validated using data from triaxial compression testing of sandstone carried out by Yang and Jiang (2010). The experimental data was obtained from short-term loading tests and long-term creep tests. The short-term loading tests were conducted by under different confining pressures. We simulated these triaxial tests by using a single material point in MATLAB 7.9.0. We apply the initial confining pressure in the first time step and then apply an increment of strain over each subsequent time step, ensuring that the rate of applied strain corresponds to a rate of applied stress of 0.127 MPa/s, which is consistent with the rate adopted in the triaxial experiments. This rate of stress is applied over the full time period of the simulation (prior to and during material softening). Yang and Jiang (2010) conducted a creep test under multi levels of stress. When a required level of stress was obtained it was held constant and the sandstone was allowed to creep for 48 hours. The final and highest level of stress used was q = 160 MPa. Under this constant level of stress, the three typical stages of creep were observed. This stress was below the peak strength of the material, but corresponded to the point where yield was initiated.

Figure 1 compares the short-term stress-strain curves obtained by Yang and Jiang (2010) and those obtained using the new frictional constitutive model. The material parameters used in the simulation is listed in Table 1. The constitutive model provides a good prediction of the experimental data for two confining pressures of 3MPa and 5MPa. The experimental data obtained from the long-term tests is shown in Figure 2, along with the response of the new constitutive model. Here we observe that the constitutive model underestimates the strain shown in the experimental data. However the overall shape of the curves is very similar, therefore the constitutive model provides a useful approximation.

4 LIMITATIONS OF THE MODELS

The models presented here deal with isotropic damage only. Therefore, they could only be applicable to the cases of monotonic loading. The models are formulated in term of triaxial stress space and validations are carried out against triaxial data. These models could be generalised by incorporating an appropriate Lode angle dependency function such as that suggested by Eekelen (1980). Further development is needed to extend these models to simulate different loading conditions. It should also be noted that the effect of variation of the temperature on creep response is not modelled and therefore these models should be used to simulate triaxial tests under constant temperature.

5 CONCLUSIONS

Novel viscoplastic-damage constitutive models have been derived within the framework of hyper-plasticity. These models are able to describe both the secondary and tertiary stages of creep behaviour. The new frictional viscoplastic-damage model is validated with experimental data obtained by Yang and Jiang (2010) for samples of sandstones.

This model have the potential to can be used for a variety of applications such as the creep analysis of underground tunnels and storage caverns as well as applications outside the field of geomechanics. The model can be generalised by incorporating an appropriate lode angle dependency function such as that suggested by Eekelen (1980).

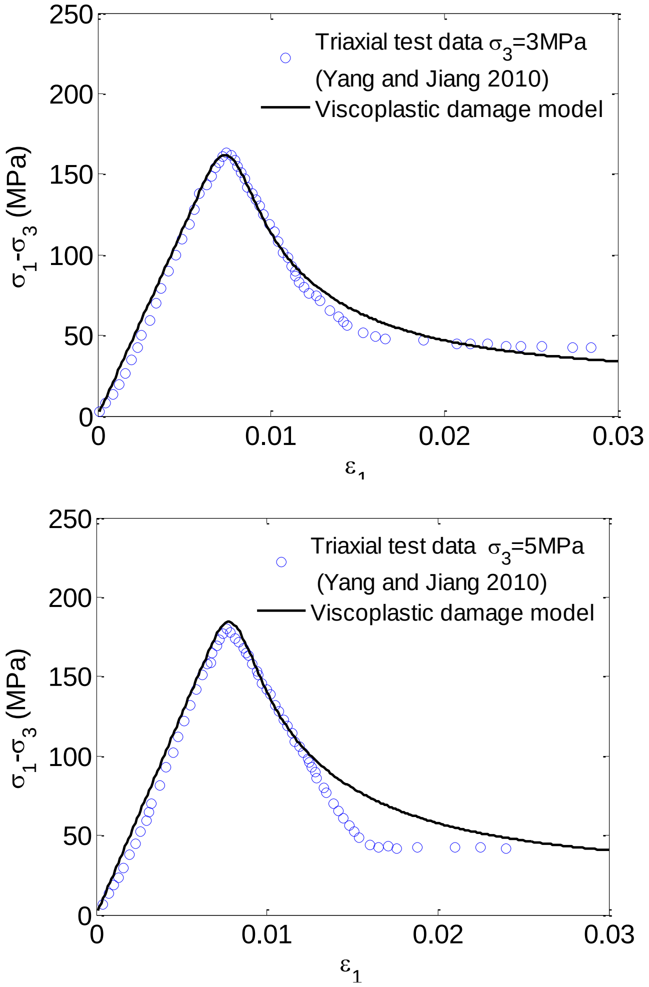

Figure 1. Validating constitutive model using short-term stress-strain triaxial data

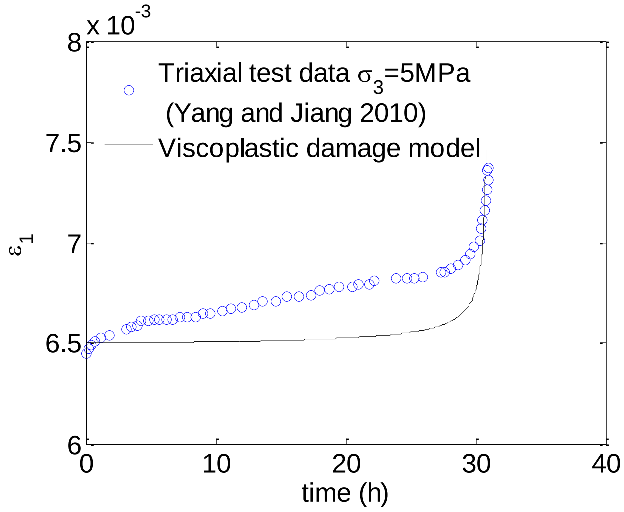

Figure 2. Creep response of model under yield stress

φ	C	η	n	r_p
58.4°	15 MPa	5000	1.42	1.05

Table 1. Parameters for Drucker-Prager Visco-plastic damage model

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