Form factors of baryons within the framework of light-cone sum rules

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Abstract. We present the application of the method of light-cone sum rules to the determination of baryonic form factors at intermediate momentum transfer. After reviewing the current status of this field we give some outlook on possible future projects.

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INTRODUCTION

In this talk we present the current status of the determination of baryonic form factors and of transition form factors within QCD. Form factors are interesting quantities per se, since they encode information about the structure of the investigated baryon. This interest raised a lot in recent years, in particular because new data from JLAB [1, 2, 3, 4] for the well-known electromagnetic form factors of the nucleon contradict common textbook-wisdom! See [5] for a review and references therein.
In our approach - light-cone sum rules - we relate the form factors directly to the distribution amplitude of a baryon, mostly the nucleon. Therefore one can follow two different philosophies:

- Start with a non-perturbative model (lattice, sum rules,...) for the distribution amplitude and determine the physical interesting form factors.
- Determine the non-perturbative distribution amplitude by fitting the experimental numbers of the form factors to the light-cone prediction.

In the following we will present the current status of these investigations.

THE NUCLEON DISTRIBUTION AMPLITUDE

The nucleon distribution amplitude has been determined up to contributions of twist 6 in [6]. x^2-corrections to the leading twist distribution amplitudes were determined in [7, 8, 9]. In [9] there was unfortunately a sign error in the contribution of $A_1^{M(u)}$, the corrected plots are presented below and the corrected formulas can be found in the appendix.
Including next-to-leading terms in the conformal expansion the whole distribution am-

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plitude is expressed in terms of eight non-perturbative parameters. One can write

$$4\langle 0|\varepsilon_{ijk}u^i_\alpha(a_1x)u^j_\beta(a_2x)d^k_\gamma(a_3x)|N(P)\rangle = \sum \Gamma_3^{\alpha\beta}\Gamma_4^\gamma F\,, \tag{1}$$

where $\Gamma_{3/4}$ are certain Dirac structures, N describes the nucleon state, a_i are positive numbers with $a_1+a_2+a_3=1$ and the F are distribution amplitudes depending on the non-perturbative parameters f_N, λ_1, λ_2, V_1^d, A_1^u, f_1^d, f_1^u and f_2^d, for details see [6, 9]. As in the meson case these parameters can be estimated with QCD sum rules [10] see e.g. [11, 12, 13, 14] for some state of the art work in the meson case. QCD sum rule estimates for all eight parameters of the nucleon distribution amplitude were first presented in [7] and later on updated in [9]. The latter parameter set will be called *sum-rule* estimate in the following. Demanding that the next-to-leading conformal contributions vanish, fixes five of the eight parameters. This parameter set will be called *asymptotic*. In [9] we presented a third parameter set, called *BLW*: with the help of light-cone sum rules [15, 16] one can express the nucleon form factors in terms of the eight non-perturbative parameters. Choosing values in between the *asymptotic* and *sum-rule* ones, we got an astonishingly good agreement with the experimental numbers, see [9]. This procedure however does not replace the necessity of performing a real fit after α_s-corrections have been calculated to the light-cone sum rules. Finally a lattice determination of these eight non-perturbative parameters would be very desirable.

LIGHT-CONE SUM RULES FOR FORM FACTORS

The starting point for our analysis is a correlation function of the following form.

$$T(P,q) = \int d^4x e^{-ipx}\langle 0|T\{\eta(0)j(x)\}|N(P)\rangle\,, \tag{2}$$

which describes the transition of a baryon $B(P-q)$ to the nucleon $N(P)$ via the current j. The baryon B is created by the interpolating three-quark field η, e.g. the Ioffe-current for the nucleon

$$\eta_{\text{Ioffe}}(x) = \varepsilon^{ijk}\left[u^i(x)(C\gamma_\nu)u^j(x)\right](\gamma_5\gamma^\nu)d^k_\delta(x)\,. \tag{3}$$

A typical example for j is the electromagnetic current in the case of the electromagnetic form factors

$$j^{\text{em}}_\mu(x) = e_u\bar{u}(x)\gamma_\mu u(x) + e_d\bar{d}(x)\gamma_\mu d(x)\,. \tag{4}$$

The basic idea of the light-cone sum rule approach is to calculate this correlation function both on the hadron level (expressed in terms of form factors) and on the quark level (expressed in terms of the nucleon distribution amplitude). Equating both results and performing a Borel transformation to suppress higher mass states one can express the form factors in terms of the eight non-perturbative parameters of the nucleon distribution amplitude and in terms of the Borel parameter M_B and the continuum threshold s_0, for details see [7, 9].

We studied the electromagnetic nucleon form factors with the Chernyak-Zhitnitsky

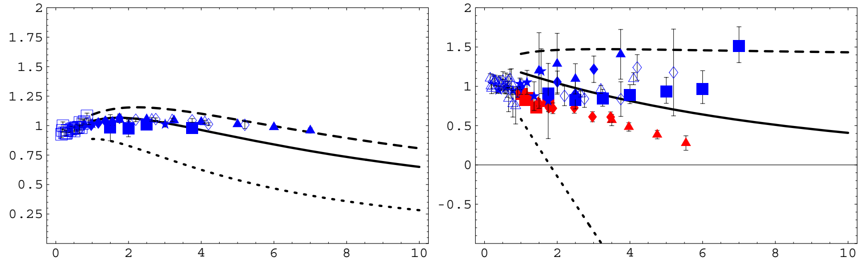

FIGURE 1. LCSR results for the electromagnetic form factors (left: $G_M/(\mu_p G_{Dipole})$ vs. Q^2; right: $\mu_p G_E/G_M$ vs. Q^2) of the proton, obtained using the BLW model (solid line), the asymptotic model (dashed line) and sum rule model (dotted line) of the nucleon DAs. The red data points on the right picture are JLAB data, while the blue ones are obtained via Rosenbluth separation. For the references to the experimental data see [9].

interpolating field in [7]. In [17] we found that η_{CZ} yields to large unphysical isospin violating effects, therefore we introduced a new isospin respecting CZ-like current to determine the electromagnetic form factors. In [9] we also studied the Ioffe current for the nucleon and extended our studies from the electromagnetic form factors to axial form factors, pseudoscalar form factors and the neutron to proton transition. It turned out that the Ioffe current yields the most reliable results. The $N \to \Delta$-transition was studied in this framework in [18] (for a similar approach for $Q^2 = 0$ see e.g. [19]), while in [20] we investigated pion-electroproduction. We present the numerical results of these calculations in the next section.
Moreover one can find the decay $\Lambda_b \to p l \nu$ within this approach in [8]. Using η_{CZ} the scalar form factor of the nucleon was considered in [21] and the axial and the pseudoscalar one in [22]. The authors of [23] considered $\Lambda_c \to \Lambda l \nu$ and therefore determined a part of the Λ distribution amplitude. In [24] the transition $\Sigma \to N$ was determined. Using a general form of the interpolating nucleon field the scalar form factor of the nucleon was considered again in [25]. Just recently the axial part of the $N \to \Delta$-transition was calculated in [26].

NUMERICAL RESULTS

The nucleon DAs provide the principal nonperturbative input to the LCSRs. We use here three models for the nucleon distribution amplitude: the *asymptotic* form (dashed lines), the *(QCD) sum-rule* estimate (dotted lines) and the *BLW* model (solid lines). The corresponding numerical values can be found in [9]. To our accuracy, the sum rules for the nucleon form factors do not depend on the parameters λ_2 and f_2^d; this dependence is present, however, in the transition form factors of $\gamma^* N \to \Delta$. One sees that the experimental data on the electromagnetic form factors are reproduced very well, and, most welcome, the unphysical tensor form factor G_T is consistent with zero. Also for the axial form factor there is a good agreement, both in shape and normalization. In particular for G_E^p/G_M^p we have a very strong dependence on the form of the nucleon

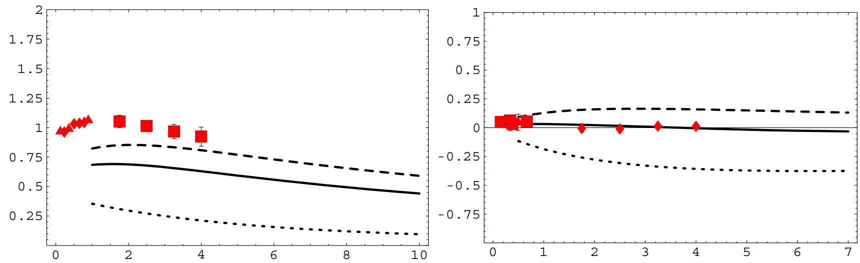

FIGURE 2. LCSR results for the electromagnetic form factors of the neutron (left: $G_M/(\mu_n G_{Dipole})$ vs. Q^2; right: G_E vs. Q^2), obtained using the BLW model (solid line), the asymptotic model (dashed line) and sum rule model (dotted line) of the nucleon DAs. For the references to the experimental data see [9].

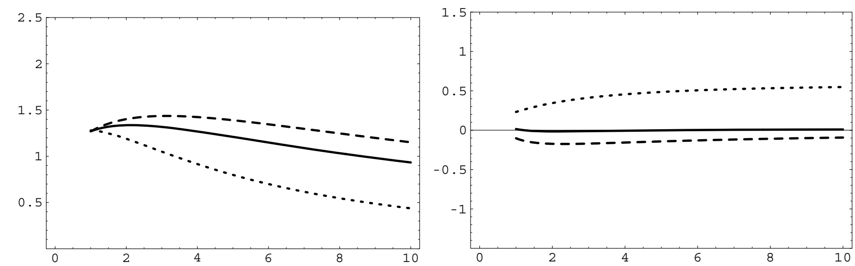

FIGURE 3. LCSR results (solid curves) for the axial form factor of the proton G_A^{CC} normalized to $G_D^{(a)} = g_A/(1+Q^2)^2$ vs. Q^2(left panel) and the tensor form factor G_T^{CC} normalized to G_A^{CC} vs. Q^2 (right panel), obtained using the BLW model (solid line), the asymptotic model (dashed line) and sum rule model (dotted line) of the nucleon DAs.

distribution amplitude. We also calculated the $\gamma^* N \to \Delta$ transition form factors within the LCSR approach, see [18]. The results are shown in Fig. 5. In this case we also get a relatively good agreement with the experimental data.

We should warn that the BLW model from [9] is not based on any systematic attempt to fit the data and in fact we believe that any fitting would be premature before the radiative corrections to the LCSR are calculated. In addition, one has to take into account the scale dependence of the parameters of the DAs and study in more detail the dependence of the sum rules on the Borel parameter. Still, the very possibility to describe many different form factors using the same set of DAs is nontrivial and indicates the selfconsistency of our approach.

OUTLOOK

In order to make our approach really quantitative one should also include α_s-corrections to the light-cone sum rules for the form factor. This work is currently in progress. Having the α_s-corrections at hand, one can fit the light-cone sum rules to the experimental values and therefore determine the nucleon distribution amplitude.

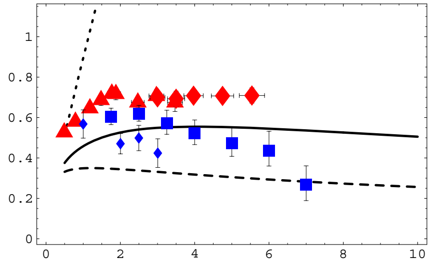

FIGURE 4. LCSR results (solid curves) for the ratio $\sqrt{Q^2}F_2^p/(F_1^p 1.79)$ obtained using the BLW model (solid line), the asymptotic model (dashed line) and sum rule model (dotted line) of the nucleon DAs. *Red symbols*: experimental values obtained via Polarization transfer: *Blue symbols*: experimental values obtained via Rosenbluth separation. For the references to the experimental data see [9].

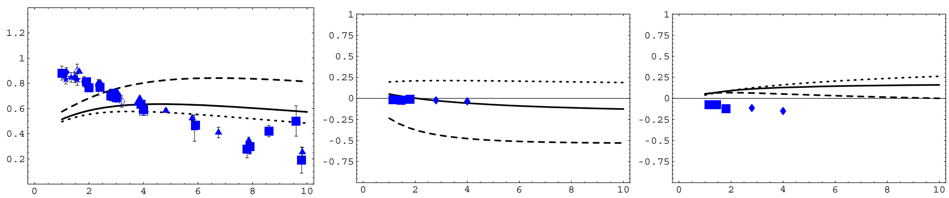

FIGURE 5. $\gamma * N \to \Delta$ transition form factors (left:$G_M^*/(3G_{Dipole})$ vs. Q^2, middle:R_{EM} vs. Q^2, right: R_{SM} vs. Q^2) in the LCSR approach [18] obtained using the BLW model (solid line), the asymptotic model (dashed line) and sum rule model (dotted line) of the nucleon DAs. For the references to the experimental data see [18]. (Color identification refers to the online version)

Moreover it would also be desirable to have some lattice determination of all non-perturbative parameters of the nucleon distribution amplitudes.
Finally from a phenomenological point of view it might be very interesting to apply this formalism to Λ_b decays. First steps in that direction have already been performed in [8, 23]. There are currently some data from TeVatron and there will be much more from the LHC for the heavy baryons. Investigating the Λ_b baryon is in particular interesting since there seems to be some problem with the lifetimes. Experimentally one always obtained values for $\tau(\Lambda_b)/\tau(B_d)$ smaller than one [27]. Inclusive decays of heavy hadrons, see e.g. [28, 29, 30] can be calculated within the framework of the Heavy Quark Expansion (HQE), e.g. [31] as an expansion in inverse power of the heavy b-quark mass. In that approach the dominant contributions to the lifetime ratios arise typically at the third order of the HQE, therefore the calculation is analogous to the mixing quantities in the neutral B-system, see e.g. [32, 33, 34, 35, 36, 37, 38]. The lifetime ratios of heavy hadrons have been theoretically considered e.g. in [39, 40, 41, 42, 43, 44, 45] and one obtains typically values for $\tau(\Lambda_b)/\tau(B_d)$ closer to one. Before speaking from a real discrepancy one has to keep in mind however, that $\tau(\Lambda_b)/\tau(B_d)$ is theoretically in a much worse shape than $\tau(B^+)/\tau(B_d)$, see e.g. [44]. Moreover there are now new measurements of $\tau(\Lambda_b)$ [46, 47] in non-leptonic channels on the market, that do not agree with each other. So the Λ_b-system is awaiting some theoretical and experimental

progress.

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APPENDIX

Unfortunately there were was a sign error in the x^2-correction $A_1^{M(u)}$ in appendix C of [9]. Eqs. (C23-C25) in [9] have to be replaced by

$$\begin{aligned}\mathscr{T}_1^{M(u)}(x) &= \frac{1}{2}\left[V_1^{M(d)}(x) + V_1^{M(u)}(x) + A_1^{M(u)}(x)\right], \\ \mathscr{T}_1^{M(d)}(x) &= V_1^{M(u)}(x) - A_1^{M(u)}(x).\end{aligned} \tag{5}$$

$$\begin{aligned}\mathscr{T}_1^{M(u)}(x) &= \frac{x^2}{48}\left(f_N E_f^u + \lambda_1 E_\lambda^u\right), \\ \mathscr{T}_1^{M(d)}(x) &= \frac{x^2(1-x)^4}{4}\left(f_N E_f^d + \lambda_1 E_\lambda^d\right)\end{aligned} \tag{6}$$

with

$$\begin{aligned}E_f^u &= -\Big[(1-x)\big(3(439+71x-621x^2+587x^3-184x^4) \\ &\quad + 4A_1^u(1-x)^2(59-483x+414x^2) - 4V_1^d(1301-619x-769x^2+1161x^3-414x^4)\big)\Big] \\ &\quad - 12(73-220V_1^d)\ln[x], \\ E_\lambda^u &= -\Big[(1-x)(5-211x+281x^2-111x^3 \\ &\quad + 10(1+61x-83x^2+33x^3)f_1^d - 40(1-x)^2(2-3x)f_1^u)\Big] - 12(3-10f_1^d)\ln[x], \\ E_f^d &= 17+92x+12(A_1^u+V_1^d)(3-23x), \\ E_\lambda^d &= -7+20f_1^d+10f_1^u.\end{aligned} \tag{7}$$

Note that the x^2-corrections do not depend on λ_2 and f_2^d.

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