

## **DISCRETE-EVENT SIMULATION OF PROCESS CONTROL IN LOW VOLUME HIGH VALUE INDUSTRIES**

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### **ABSTRACT**

This paper presents a new method of process control for set-up dominant processes. This new method known as Set-up Process Algorithm (SUPA) was compared with existing industrial practices and statistical techniques in the literature. To test the method's robustness, a generic discrete-event simulation model was built. This model was used to test four different statistical approaches to process control. It was concluded that SUPA offers a method of process control for set-up dominant processes, which is easier to apply than classically derived SPC approaches, by using simple rules and a traffic light system based on design specification. Simulation analysis shows that SUPA: is more sensitive, at detecting an incapable process as it will monitor more units when a process is less capable; is more sensitive than PRE-Control at detecting mean shifts in a process. SUPA is also a nonparametric methodology and therefore robust against processes with non-Gaussian distributions.

**Keywords:** pre-control, statistical process control, process set-up.

### **1 INTRODUCTION**

To improve the quality from high precision processes, many companies are implementing Six Sigma process improvement methods (Julien & Holmshaw, 2012). Six Sigma has been successfully used in high volume manufacturing (Bhuiyan & Baghel, 2005). However, the large amounts of data required to perform statistical tests associated with Six Sigma have led many practitioners, operating in low volume environments, to abandon objective analysis in favour of a subjective approach (Julien & Holmshaw, 2012). This is accomplished by utilizing the engineer's opinion over data driven improvement.

A current concern, within Six Sigma, is Control method implementation for low volume processes. Typically, low volume processes are high precision ones, capable with respect to Critical-to-Quality (CtQ) parameters for the production run duration. The dominant variation source between batches is linked to adjusting the process to the CtQ specification centre during set-up, resulting in set-up dominant processes (Juran & Gryna, 1988). It is common to use "rule of thumb" methods to control such processes. Otherwise, implementing no control methods, ignoring the problem.

This type of problem with "rule of thumb" methods was observed by (Carter & Butler, 1996). This case-study of precision turned components, reported that Control of small batch production had "a heavy reliance on the operator's experience". Instances of variability in a process being induced by the operators making unnecessary changes, were reported.

Two common “rule of thumb” methods identified are First-Off's and 100% Inspection (Pillet, 1996). First-Off's measure the CtQ of the first unit produced. However, little process information can be gained from the measurement of a single unit. 100% Inspection measures the CtQ of all units as they are produced. The process is then adjusted after every unit measured, either off-target or out of specification. This results in operators either hunting for the specification centre, or, even worse, allowing defects to occur. In either case, little statistical rigour is applied to the data captured.

## 2 REVIEW OF CURRENT PRACTICES

In high volume manufacture the application of Statistical Process Control (SPC) has been successful at maintaining the statistical stability of a process. A common SPC approach for a variable measure is the  $\bar{X}$  &  $R$  chart, plotting the mean ( $\bar{X}$ ) and the range ( $R$ ) of a subgroup of sampled units against capability based control limits. Then while a process is being monitored, if  $\bar{X}$  falls outside its control limits, it shows there has been a statistically significant change in the process mean. If  $R$  falls outside its control limits, it shows there has been a statistically significant change in the process variation.

In low volume processes there are issues with SPCs application. This is a result of few units being produced in a batch, making it impossible to gain sufficient data to estimate the process performance.

The process capability metrics  $C_{pk}$  and  $C_p$  are used to quantify process variation.  $C_p$  uses process standard deviation ( $\sigma$ ) to estimate process performance against the upper ( $U$ ) and lower ( $L$ ) specification, by

$$C_p = \frac{U - L}{6\sigma}. \quad (1)$$

$C_{pk}$  uses  $\sigma$  and process mean ( $\mu$ ) to estimate process performance against  $U$  and  $L$ , by

$$C_{pk} = \left[ \frac{U - \mu}{3\sigma}, \frac{\mu - L}{3\sigma} \right]. \quad (2)$$

A small batch  $\bar{X}$  &  $R$  chart using a five unit decision procedure is given in (Pillet, 1996). The  $\bar{X}$  &  $R$  charts control limits are recalculated as the subgroup size increases from one to five units. This overcomes the problem of waiting for a complete subgroup to indicate an issue. A decision can be made on the need to re-centred a process after one unit and whether there is too much process variation after two units. A disadvantage with this method is the necessity to calculate control limits on subjective assumptions, engineering knowledge, or possibly scarce surrogate/historical data.

Acceptance Control Charting (ACC) (ISO/TC 69/SC 4, 2010) use design target and specification limits to base control limits. They define acceptable process limits, which draw lines based on an acceptable risk of a false fail ( $\alpha$ -risk). Also defined are acceptance control limits, whereby any sample mean ( $\bar{X}$ ) above the upper or below the lower acceptance control limits, deems a process as non-acceptable. A version of ACC, from (ISO/TC 69/SC 4, 2010), bases upper and lower acceptance control limits on: the specification target ( $\mu$ ), process standard deviation ( $\sigma$ ) and the  $z$ -value statistical constant, retrieved from a standard normal table for the required  $\alpha$ -risk.

This method is slow to respond to off-target processes as it depends on the collection of samples in a subgroup. Derivation of its control limits also require an estimation of process variation. As with small-batch  $\bar{X}$  &  $R$ , ACC makes the assumption that the process conforms to a Gaussian distribution.

A different, less used approach to set-up dominant processes Control, is stage 1 of PRE-Control (PC). Unlike classic approaches to SPC, PC makes no assumption of the underlying distribution of a process (Juran & Gryna, 1988; Ledolter & Swersey, 1997; Steiner, 1998). Its objective is defect prevention, to ensure any validated set-up has a minimum  $C_{pk}$  of 1.33.

PC uses a traffic light system to divide the design specification of a measured CtQ. The central region covering 50% of the specified tolerance is designated as the Green Zone. The regions which cover 25% of the specified tolerance respectively, between the Green Zone and the specification limits are the Yellow Zones. The regions outside the specification limits are the Red Zones.

Units are sampled and their measured CtQ categorized as Green, Yellow or Red. To validate the new process set-up or an adjustment made to an existing process, stage 1 rules are applied, Table 1. Consecutive units are sampled from the process. If a sampled unit is Red it signals that the process is off-target. Two consecutive units in the same Yellow Zone signals that the process is off-target. Two

consecutive units in opposite Yellow Zones signals the process variation is too great. Five consecutive Green units demonstrate the process is capable and it is allowed to continue without further checks.

Table 1: Outline of Stage 1 (Validation) Decision Rules for PRE-Control

Sampled Units	Observation	Action
1	Red Unit	Stop and Adjust
1 2	Two Consecutive Yellow Units Same Side of Target	Stop and Adjust
1 2	Two Consecutive Yellow Units Opposite Sides of Target	Stop and Investigate
1 2 3 4 5	Five Consecutive Green Units	Continue Process

A significant issue with PC, is that as the process becomes more Capable, i.e.  $C_p$  increases, a greater percentage of off-target processes are qualified. In fact as the  $C_p$  of a process approaches 2.33, 98% of product will qualify the stage 1, even if the process mean is 2 standard deviations off-target.

As the bar has been risen with respect to precision processes, maintaining a  $C_{pk}$  of 1.33 is often not good enough. To overcome this, a new method called Set-Up Process Algorithm (SUPA) is described here. Subsequently, in order to assess the effectiveness of small-batch  $\bar{X}$  &  $R$  charts, ACC, PC and SUPA in a set-up dominant environment, a discrete-event simulation model is used. This model is able to test the respective methods efficiency against processes of varying performance.

### 3 SET-UP PROCESS ALGORITHM (SUPA)

Given the small run, low volume processes this paper focuses on, with production runs of five to ten units. A new algorithm for process adjustment and set-up qualification is proposed. The method will qualify or adjust a process within five units and will introduce a sliding scale Green Zone. This addition improves the centring of highly capable process within a CtQ specification maximising the limited data. It also provides a link between statistical tolerances and process capability.

There are two types of  $\alpha$ -risk: the chance of adjusting an on-target process (“hunting”) and the chance of signalling a capable process as incapable. SUPA achieves 98% confidence for the probability of qualifying a valid process, meaning a 2%  $\alpha$ -risk (probability of not qualifying a valid process), by the fact that sampling five consecutive Green units will validate a set-up and two consecutive Yellow units will initiate action to be taken (San Matias, Jabaloyes, & Carrion, 2004). To obtain a value of probability of qualifying,  $P(q)=0.98$ , for different values of  $C_{pk}$ , the probability of sampling a unit in the Green zone  $P(g)$  and the probability of sampling a unit in the Yellow zone  $P(y)$  need to be used according to

$$P(q) = P(g)^5 \frac{1 + P(y)}{1 - P(y) \sum_{i=1}^4 P(g)^i} \tag{3}$$

Based on Equation (3), the values of the look-up table shown in Table 2 are calculated.

Table 2: Look-up Table of Percentage Green Zone and Minimum  $C_{pk}$  at 98% Confidence

Green Zone	$C_{pk}$
0.471353	1.333333
0.418294	1.500000
0.376396	1.666666
0.313658	2.000000
0.268850	2.333333
0.250926	2.500000
0.235244	2.666666
0.209106	3.000000

Based on the values and data shown in Tables 1 and 2, the SUPA follows the sequence:

1. Select the Green Zone limits for the required minimum  $C_{pk}$ , using the look-up Table 2.
2. Sample, Measure and classify the CtQ of consecutive units as: Red, Yellow or Green.
3. Follow the PC rules to validate a process, i.e. a Red unit signals an adjustment is needed, five consecutive Green units signals the process is valid.

The Red zones are always set at the specification limits. The final SUPA chart will have zones and limits as in Figure 1, showing a SUPA chart monitoring a process requiring a minimum  $C_{pk} = 2.0$ . Given the application, where the cost of adjusting a process is not significant compared to the cost of producing out of specification units, allowing a 2%  $\alpha$ -risk is acceptable (Ledolter & Swersey, 1997).

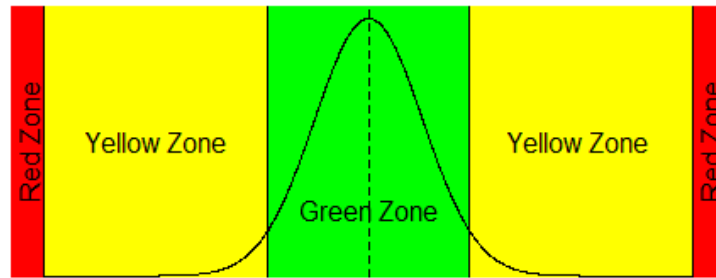


Figure 1: A SUPA Chart monitoring a process with minimum  $C_{pk} = 2.0$  using a Green Zone of 31%.

If the process is significantly off-target, SUPA will allow quick adjustments to be made after only one or two units. Whereas, classic SPC would require a subgroup of four or five units before a change. SUPA is a nonparametric method, deriving its limits based on the specification of the product’s CtQ being monitored. This allows the Green zone selection to be based on the minimum  $C_{pk}$  required from the process to protect the CtQ’s statistical tolerances and avoid tolerance stack-up in assemblies. Once five consecutive units are produced in the Green zone, SUPA signals the process is valid.

#### 4 GENERAL DISCRETE-EVENT SIMULATION MODEL

In order to further test the effectiveness of the different methods of validating the set-up of a process, a discrete-event simulation model was built using WITNESS 12. The model simulated a generic process applying a CtQ to a unit, which could represent a lathe machining the outer diameter of a gear. The process has a  $U$  and  $L$  of  $\pm 100$  and a process target,  $\mu_T$ , of 0. The current process mean,  $\mu$ , can be offset at the start of the simulation. The model will adjust  $\mu$  based on the decision rules of the Control method analysed. Capability is set prior to the simulation and remains constant throughout.

The simulation model applies adjustments by finding the mean of the units signalling an adjustment ( $\mu_A$ ). Then it subtracts the difference between  $\mu_A$  and the process target,  $\mu_T$ , from the current mean ( $\mu^t$ ) to find the new process mean ( $\mu^{t+1}$ ), i.e.

$$\mu^{t+1} = \mu^t - (\mu_A - \mu_T). \quad (4)$$

The general model can be seen in Figure 2. At the start the experimenter sets the initial parameters of capability and process mean (boxes 1 and 2). The model is allowed to run. Units enter the model (box 3) with a generic process applying a CtQ to each unit (box 4), based on parameters of capability and process mean. The model then samples consecutive units (box 5). Based on the decision rules of the Control method utilised, a decision is made on whether or not the process is valid (box 6). In the case of SUPA, if there were five consecutive units in the green zone the model will be validated. If a model is validated sampling immediately stops (box 7). If the model is not validated, the model decides whether an adjustment is needed to the current process mean (box 8). If an adjustment is not needed sampling continues (box 5). If an adjustment is needed, the mean is recalculated (box 9) by Equation (4). The adjustment is then applied to the process mean (box 2).

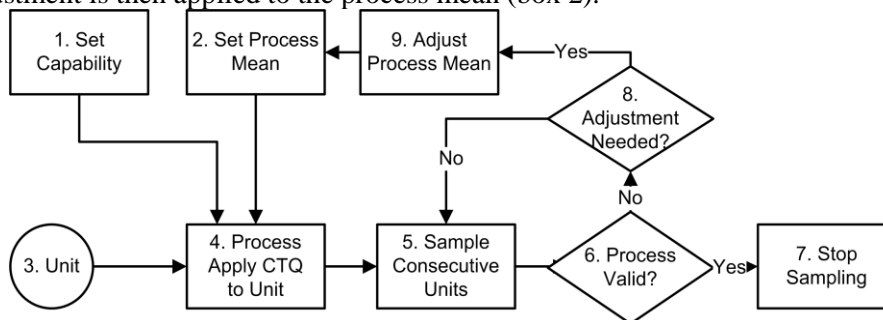


Figure 2: Process flow of general simulation model.

## 5 DISCRETE-EVENT SIMULATION RESULTS

There are two events which can cause a signal from a Control method: a) when a process is on-target but not capable and b) when a process is capable but off-target. Here, the results are presented from when the simulated Control methods are tested under these conditions.

The simulation will use a process which is producing parts with a Gaussian distribution. Since PC and SUPA are nonparametric methods, they do not make any distributional assumptions. However, small-batch  $\bar{X}$  &  $R$  and ACC do assume the process has a Gaussian distribution. Simulating a process with a Gaussian distribution offers a fair starting point for comparing the respective methods.

### 5.1 Process On-Target, but not Capable

The respective methods were tested against a process which was initially set on-target. However, each time the simulation was run it had a smaller  $C_p$  and therefore larger process variation. The control methods were all set to monitor a process with a  $C_{pk}$  of 2.0. The first run of the simulation had a  $C_p$  of 2.0. Each run decreased in the value of  $C_p$ , to a final value of 0.667. As these discrete-event simulation models are stochastic, each simulation run had 1,000 replications to minimise deterministic effects of the pseudo-random number generation.

Two questions are posed: a) what is the  $P(q)$ , despite any adjustments made? and b) how many units does it take to make a decision? These are answered in Figure 3 a) and b) respectively.

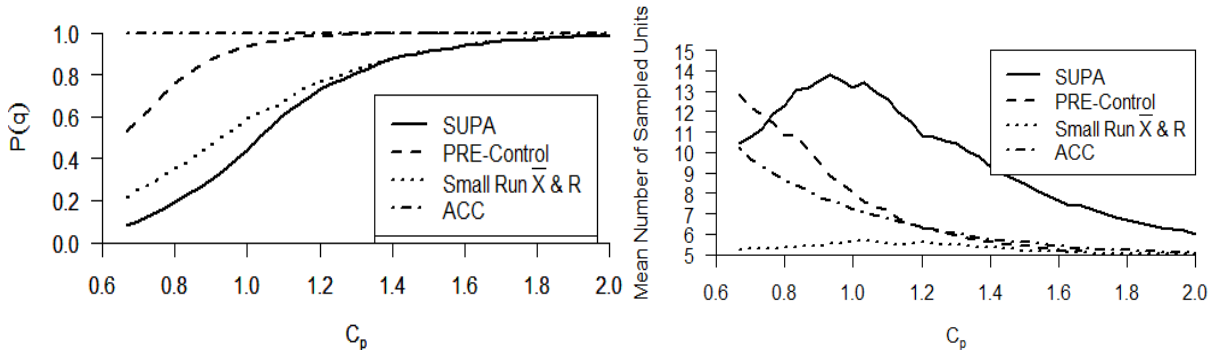


Figure 3: The Effect of Decreasing  $C_p$  against a) probability of qualifying ( $P(q)$ ) and b) number of units sampled for SUPA, PC, small-batch  $\bar{X}$  and  $R$  and ACC simulations.

Figure 3a demonstrates some important points. ACC has no mechanism to indicate that a process is incapable and will eventually validate all set-ups despite the capability; for CtQs which need to be made to a higher  $C_{pk}$  than 1.333, SUPA offers significantly improved performance than PC; SUPA has better performance than small-batch  $\bar{X}$  &  $R$  at detecting incapable processes, at a  $C_p$  of 0.667 small-batch  $\bar{X}$  and  $R$  still qualifies over 20% of set-ups whereas SUPA only qualifies 7%. These results indicate that SUPA is the most sensitive control method for detecting incapable processes.

Figure 3b highlights that as  $C_p$  decreases: small-batch  $\bar{X}$  and  $R$  consistently uses between 5-6 units; PC and ACC use more units, tending towards 13 and 10 units respectively, as  $C_p$  approaches 0.667; SUPA uses increasingly more units to make a decision, peaking at 14 units. On the face of these results small-batch  $\bar{X}$  &  $R$  looks the most efficient method. However, the increased monitoring of SUPA, when a process is less capable, is useful as it increases the sensitivity of detecting an incapable process and reduces the chance of an out-of-specification unit being produced unnoticed.

### 5.2 Process Capable, but not On-Target

This analysis tested the respective methods against a process which was initially set with a fixed  $C_p$ s of 1.333, 2.0 and 2.667. Each time the simulation was run at these settings it had a process mean further away from the on-target state. The Control methods were all set to monitor a process with a  $C_{pk}$  of 2.0. The first run of the simulation at each setting of  $C_p$  started on-target. Following runs began with the process mean further from the on target state by a factor of process standard deviation ( $\sigma$ ). Again, each simulation run had 1,000 replications.

The most important question answered was: what was the  $C_{pk}$  after a process was validated? This shows how effective the control methods are at centring an off-target process, presented in Figure 4.

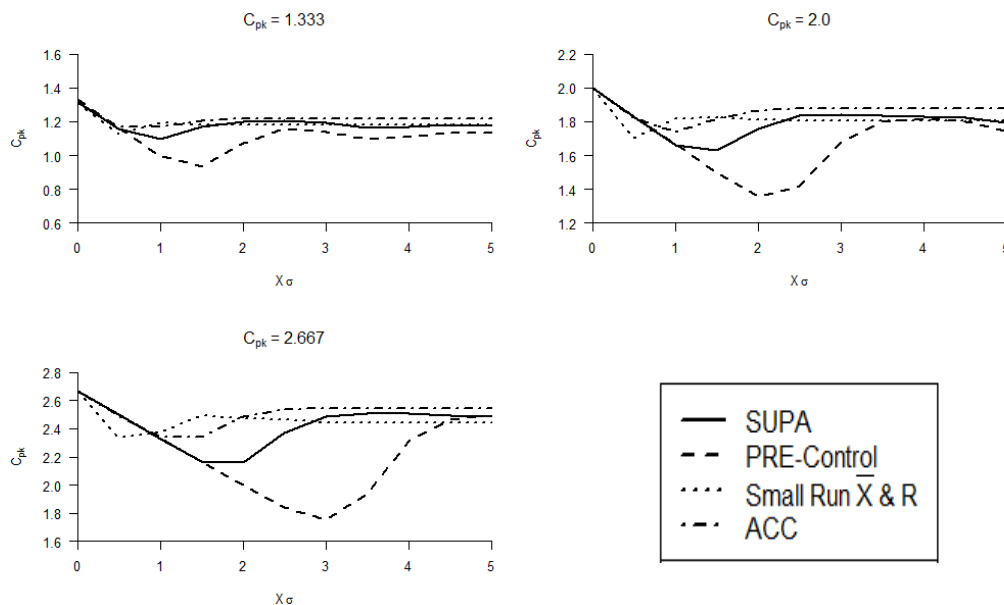


Figure 4: Graphs of final  $C_{pk}$  against start positions of process mean as a factor of  $\sigma$ , when the simulation has three different maximum values of  $C_{pk}$ , for SUPA, PC, small-batch  $X \& R$  and ACC.

Figure 4 highlights: that SUPA offers improved performance over PC; small-batch  $X \& R$  and ACC have steady performance profiles; SUPA has a dip in performance when a process is one to two standard deviations off-target, however, when a process is less than  $0.5\sigma$  or greater than  $2.5\sigma$ , SUPA performs marginally better than small-batch  $X \& R$ .

## 6 CONCLUSIONS

This paper has presented a new method of Process Control for set-up dominant processes. This new method known as SUPA was compared with existing industrial practices and statistical techniques in the literature. To test the method's robustness, a generic discrete-event simulation model was built. This model was used to test four different statistical approaches to process control of set-up dominant processes.

From this work it can be concluded that SUPA offers a method of process control for set-up dominant processes, which is easier to apply than classically derived SPC approaches. This is done by using simple rules and a traffic light system based on design specification, rather than control limits based on estimated process performance. Simulation analysis shows that SUPA: is more sensitive, than other approaches at detecting an incapable process as it will monitor more units when a process is less capable in its effort to reduce the risk of allowing a defect to occur; is more sensitive than PC at detecting mean shifts in a process. SUPA is also a nonparametric methodology and is therefore robust against processes with non-Gaussian distributions.

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