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$B_s - \bar{B}_s$-mixing and lifetimes

Alexander Lenz*
IPPP Durham
E-mail: alexander.lenz@durham.ac.uk

We review the status of mixing of neutral B-mesons, including a discussion of the current precision of Standard Model (SM) predictions as well as the space that is left for effects of new physics. In that respect we present several observables, which are particularly sensitive to the remaining new physics (NP) parameter space. B-mixing can also be used to test the fundaments of quantum mechanics, here we suggest a new measurement of the ratio of like-sign dilepton events to opposite-sign dilepton events. Finally we summarise briefly the status of lifetimes of heavy hadrons. The corresponding theory predictions rely on the same tool - the Heavy Quark Expansion (HQE) - as some of the mixing quantities. New experimental data has recently proven the validity of the HQE to a high accuracy. However, the theoretical precision of lifetime predictions is strongly limited by a lack of non-perturbative evaluations of matrix elements of dimension-six operators.

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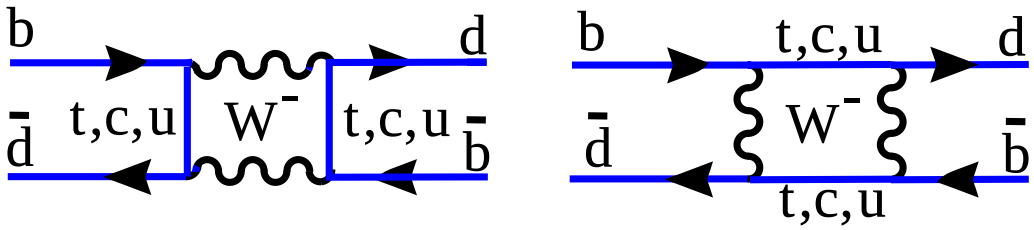

Figure 1: Box diagrams triggering the transition of a $\bar{B}_d$-meson into a B_d meson.

1. Introduction

Mixing of neutral mesons is a macroscopic quantum effect that is triggered by the so-called box diagrams shown in Fig. 1, see e.g. the reviews [1, 2, 3, 4] for a more detailed discussion and also some historical remarks. In the SM these transitions are suppressed by being a second order weak interaction process. NP contributions to mixing thus might be easily of a similar size as the SM contribution. Calculating the on-shell part of the box diagrams gives Γ_{12}^q ($q = d, s$) and the off-shell part gives M_{12}^q. Because of the CKM structure both Γ_{12}^q and M_{12}^q can be complex. The three quantities $|M_{12}^q|$, $|\Gamma_{12}^q|$ and $\phi_q = \arg(-M_{12}/\Gamma_{12})$ can be related to three observables:

1. The mass difference of the two mass eigenstates B_H and B_L:

$$\Delta M_q := M_H - M_L \approx 2|M_{12}^q| \,. \tag{1.1}$$

   As M_{12}^q is given by the off-shell intermediate states, it is sensitive to heavy internal particles. In the SM these are the W-boson and the top-quark; depending on your favourite model for NP, these might also be e.g. heavy SUSY-particles, see e.g. [5]. Hence ΔM_q is supposed to be sensitive to NP effects originating at a high scale.

2. The decay rate difference of the two mass eigenstates B_H and B_L:

$$\Delta\Gamma_q := \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}^q| \cos\phi_q \,. \tag{1.2}$$

   As Γ_{12}^q is given by on-shell intermediate states, it is sensitive to light internal particles, like the up- and charm-quark in the SM. At first sight it seems reasonable to assume almost no NP effects in Γ_{12}^q - later on we will challenge this assumption. $\Delta\Gamma_q$ can of course always be affected by new physics effects in the phase ϕ_q.

3. Flavour specific (or more specific semi-leptonic) CP asymmetries can also be expressed in terms of the three mixing quantities Γ_{12}^q, M_{12}^q and ϕ_q.

$$a_{sl}^q \equiv a_{fs}^q = \frac{\Gamma(\bar{B}_q(t) \to f) - \Gamma\left(B_q(t) \to \bar{f}\right)}{\Gamma(\bar{B}_q(t) \to f) + \Gamma\left(B_q(t) \to \bar{f}\right)} = \left|\frac{\Gamma_{12}^q}{M_{12}^q}\right| \sin\phi_q \,. \tag{1.3}$$

   Since both Γ_{12}/M_{12}^q and ϕ_q are small in the SM, the semi-leptonic CP asymmetries provide a powerful null test.

2. Standard model predictions

2.1 Mass difference

The SM expression for M_{12}^q is given as

$$M_{12,q} = \frac{G_F^2}{12\pi^2}(V_{tq}^* V_{tb})^2 M_W^2 S_0(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B \,. \tag{2.1}$$

The 1-loop result for the box-diagram is denoted by the Inami-Lim function $S_0(x_t)$ [6], NLO-QCD corrections to the box-diagrams by $\hat{\eta}_B$ [7] and non-perturbative contributions by the bag parameter B_{B_q} and the decay constant f_{B_q}. Taking the FLAG-average [8] for $f_{B_q}^2 B_{B_q}$ we obtain the SM prediction, which can be compared to the experimental averages given by HFAG [9]:

$$\Delta M_d^{\rm SM} = 0.543 \pm 0.091 \text{ ps}^{-1} \,, \quad \Delta M_d^{\rm Exp} = 0.510 \pm 0.003 \text{ ps}^{-1} \,, \tag{2.2}$$

$$\Delta M_s^{\rm SM} = 17.30 \pm 2.6 \text{ ps}^{-1} \,, \quad \Delta M_s^{\rm Exp} = 17.761 \pm 0.022 \text{ ps}^{-1} \,. \tag{2.3}$$

The measurements agree very nicely with the SM predictions, but the theoretical uncertainties are considerably larger than the experimental ones. Thus we still have quite some space for NP effects. The theoretical error is dominated by the non-perturbative uncertainties in B_{B_q} and f_{B_q}. Also some of the lattice predictions yield quite different values; compare e.g. the determinations from Fermilab/MILC [10] and the one from HPQCD [11]:

$$f_{B_s}^{\rm Fermilab/MILC} = 242.0 \pm 5.1 \pm 8.0 \text{ Mev} \,, \quad f_{B_s}^{\rm HPQCD} = 224 \pm 5 \text{ Mev} \,. \tag{2.4}$$

In view of the quadratic dependence of many observables on the decay constant further lattice studies would be very helpful.

2.2 Heavy Quark Expansion

The theoretical prediction of Γ_{12}^q is more involved than the one of M_{12}^q, here a second operator product expansion has to be performed, the so-called Heavy Quark Expansion (HQE), see e.g. [12] for a review of this theoretical tool. The HQE applies also for lifetimes and totally inclusive decays decay rates of heavy hadrons. Historically there had been several discrepancies between experiment and theory that questioned the validity of the HQE.

- In the mid-nineties there was the *missing charm puzzle* (see e.g. [13] for a brief review) - a disagreement between experiment and theory about the average number of charm-quarks produced per b-decay. This issue has been resolved, by both improved measurements and improved theory predictions [14].

- For a long time the Λ_b lifetime was measured to be considerably shorter than theoretically expected, this issue has been resolved experimentally, mostly by the LHCb Collaboration (e.g. [15, 16, 17]) but also from the TeVatron experiments [18]. The history of the *Λ_b-lifetime puzzle* and also attempts to obtain low theory values are discussed in detail in the review [12]. The current status of lifetimes is depicted in Fig. 2, taken from [19]. One finds a nice agreement between experiment and theory and no lifetime puzzle exists anymore. The theoretical precision is, however, strongly limited by a lack of up-to-date values for the arising non-perturbative parameters. For the Λ_b-baryon the most recent lattice numbers stem from 1999 [25] and for the B-mesons the most recent numbers are from 2001 [26].

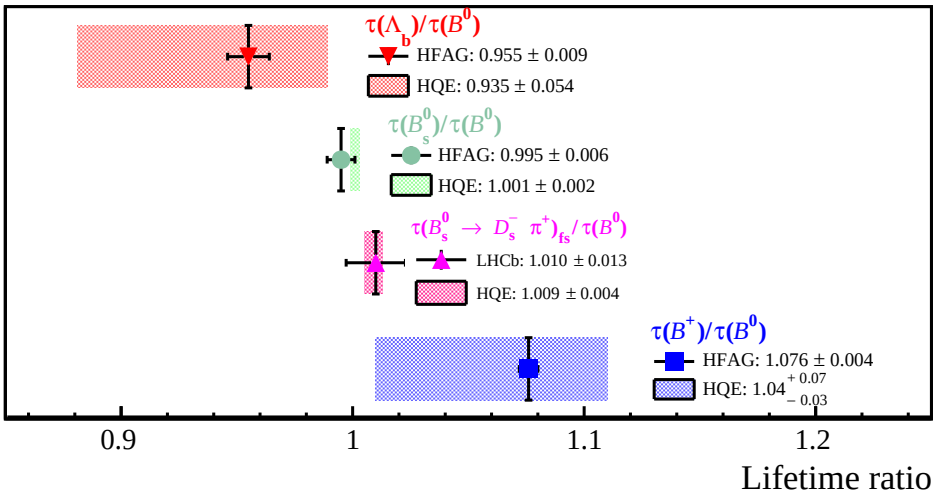

Figure 2: Comparison of HQE predictions for lifetime ratios of heavy hadrons with experimental values. The theory values are taken from [12], which is based on the calculations in [20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. Experimental numbers are taken from HFAG [9]. The figure is taken from [19].

- The applicability of the HQE was in particular questioned for $\Delta\Gamma_s$, see e.g. [30]. In the last years this was also related to the unexpected measurement of a large value of the di-muon asymmetry by the D0 collaboration [31, 32, 33, 34]. The issue of $\Delta\Gamma_s$ was solved experimentally - mostly by the LHC experiments LHCb, ATLAS and CMS and the current HFAG [9] average is in perfect agreement with the HQE prediction [35] based on [36, 37, 38, 39] - see [40] for a very early prediction with NLO-QCD effects.

$$\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\rm Exp} \Big/ \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\rm SM} = 1.02 \pm 0.09 \pm 0.19\,. \tag{2.5}$$

  Again an impressive confirmation of the HQE. The case of the di-muon asymmetry is still not settled yet. A new light was shed on it by the analysis of Borissov and Hoeneisen [41], who found that the measured asymmetry does not only have contributions proportional to $a_{sl}^{d,s}$, but also some that originate from interference between mixing and decay and that might be approximated by being proportional to $\Delta\Gamma_d$.

All in all the HQE has been experimentally proven to be very successful and one could try to test its applicability also for charm-physics, see e.g. [42, 43] for some first investigations, or one can apply the HQE now also to quantities that are sensitive to new physics, in particular to the semi-leptonic CP asymmetries. Their SM values are [35]:

$$a_{fs}^s = (1.9 \pm 0.3) \cdot 10^{-5}\,, \quad \phi_s = 0.22^\circ \pm 0.06^\circ\,, \tag{2.6}$$

$$a_{fs}^d = -(4.1 \pm 0.6) \cdot 10^{-4}\,, \quad \phi_d = -4.3^\circ \pm 1.4^\circ\,. \tag{2.7}$$

First measurements of these asymmetries [44, 45, 46, 47] are in agreement with the SM, but leave still some sizable space for NP effects.

$$a_{sl}^{s\,\rm LHCb} = -0.06 \pm 0.50 \pm 0.36\,, \quad a_{sl}^{s\,\rm D0} = -1.12 \pm 0.74 \pm 0.17\,, \tag{2.8}$$

$$a_{sl}^{d\,\rm D0} = 0.68 \pm 0.45 \pm 0.14\,, \quad a_{sl}^{d\,\rm BaBar} = 0.06 \pm 0.17^{+0.38}_{-0.32}\,. \tag{2.9}$$

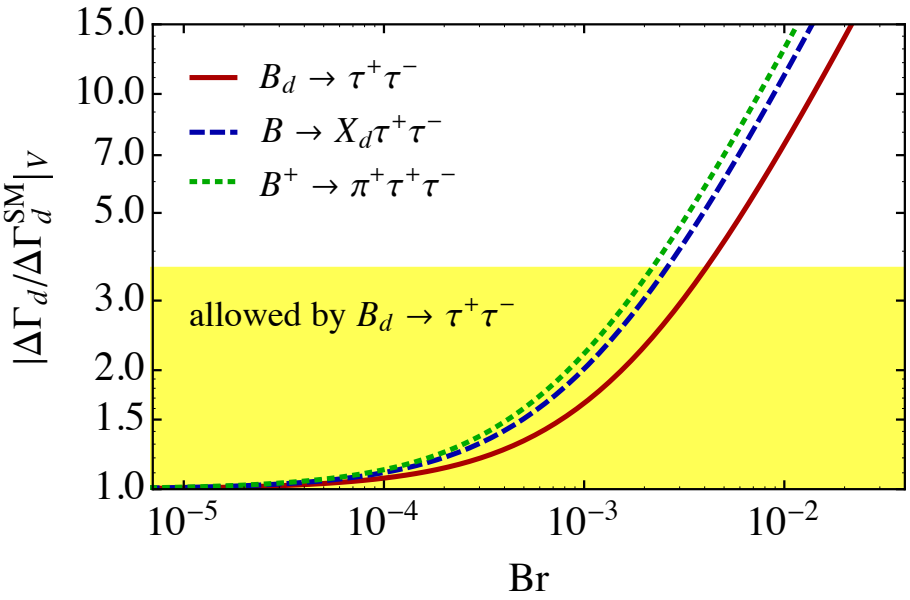

Figure 3: Required experimental precision in the decays $B_d \to \tau\tau$, $B \to X_d\tau\tau$ and $B^+ \to \pi^+\tau\tau$ in order to get a stronger bound on $\Delta\Gamma_d$ than currently available (yellow region).

3. New physics effects in mixing

A reasonable start to search model-independently for new physics effects in B-mixing is the assumption that new physics only arises in M_{12}^q, i.e. $M_{12}^q = \Delta_q \cdot M_{12}^{q\,\mathrm{SM}}$ and $\Gamma_{12}^q = \Gamma_{12}^{q\,\mathrm{SM}}$. All new effects are encoded in the complex parameter Δ_q. A corresponding strategy was suggested in [36] and worked out with real data in [48, 49]. It turns out again that everything is consistent with the SM and there are no huge NP effects, but there is still some space for sizable NP effects.
This results also implies the necessity of a higher precision in our theory investigations and in particular it might be reasonable to take smaller NP effects in $\Delta\Gamma_q$ into account. In [50] it was shown that for $\Delta\Gamma_s$ these effects can be at most of the order of 30%, because else other experimental constraints will be violated. This is not the case for $\Delta\Gamma_d$, which has a very small SM value [35] and is only weakly constrained by measurements

$$\left|\frac{\Delta\Gamma_d}{\Gamma_d}\right|^{\mathrm{SM}} = (4.2 \pm 0.8)\cdot 10^{-3}\,, \quad \left|\frac{\Delta\Gamma_d}{\Gamma_d}\right|^{\mathrm{HFAG}} = (1 \pm 10)\cdot 10^{-3}\,. \tag{3.1}$$

In [51] three general scenarios were investigated in order to show that a enhancement of $\Delta\Gamma_d$ of several hundred per cent is currently not excluded. These were a violation of CKM unitarity, new $bd\tau\tau$ operators and new physics effects on tree-level decays that act differently in the decays $b \to c\bar{c}d$, $b \to c\bar{u}d$, $b \to u\bar{c}d$ and $b \to c\bar{c}d$ [1]. Here first measurements in the $bd\tau\tau$ sector might yield some surprises, Fig.3 shows the required experimental precision; stronger constraints on the tree-level Wilson coefficients C_1 and C_2 would also be very helpful.

[1] Such non-universal, new tree-level effects can also affect the precision of the determination of the CKM angle γ.

4. Some very new physics effects

Mixing of heavy mesons can also be used to test the fundamentals of quantum mechanics, see e.g. [52, 53]. It was suggested to measure the ratio R of like-sign dilepton events and opposite-sign dilepton events and denote hypothetical deviations from the quantum mechanical coherence with the phenomenological parameter ζ.

$$R = \frac{N^{++}+N^{--}}{N^{+-}+N^{-+}} = \frac{1}{2}\left(\left|\frac{p}{q}\right|^2 + \left|\frac{q}{p}\right|^2\right) \frac{x^2+y^2+\zeta\left[y^2\frac{1+x^2}{1-y^2}+x^2\frac{1-y^2}{1+x^2}\right]}{2+x^2-y^2+\zeta\left[y^2\frac{1+x^2}{1-y^2}-x^2\frac{1-y^2}{1+x^2}\right]} \,. \tag{4.1}$$

Triggered by the 2013 paper of Alok and Banerjee [54], which found extreme precise limits for decoherence effects, I redid the analysis with six talented undergraduate students [55] and we found a flaw in the arguments of [54]. Using the most recent values for x and y from HFAG [9] and for R from ARGUS [56] (1994) and CLEO [57] (1993) we find that currently decoherence in B-mixing is only very loosely bounded

$$\zeta = -0.26^{+0.30}_{-0.28} \,. \tag{4.2}$$

Here future measurements would be very helpful to gain additional insights. To demonstrate the required experimental precision in R, we show how the error in R affects the uncertainty in ζ.

δR	$\pm 10\%$	$\pm 5\%$	$\pm 2\%$
$\delta\zeta$	$^{+45.2\%}_{-43.8\%}$	$^{+22.8\%}_{-22.4\%}$	$^{+10.0\%}_{-9.98\%}$

. (4.3)

5. Conclusion

The HQE has been successfully tested by many recent experiments, further more precise tests of the HQE demand non-perturbative input, mostly matrix elements of dimension six operators. Applying the HQE predictions to NP sensitive quantities one finds that everything is consistent with the SM, but there is still some space left for new effects. Promising observables in that respect are more precise values of $a_{sl}^{d,s}$ and $\Delta\Gamma_d$, first measurements of $bd\tau\tau$ and $bs\tau\tau$-transitions as well as further constraints on the tree-level Wilson coefficients $C_{1,2}$. Finally we suggest also a new measurement of the ratio R of like-sign dilepton events and opposite-sign dilepton events.

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