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Branched crack modelling with the Cracking Particle Method

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ABSTRACT

Multiple crack simulation is of great importance in failure analysis since fracture in brittle materials in practice usually comprises multiple cracks. Traditional numerical methods such as the extended finite element and the element free Galerkin methods meet dilemmas when solving this kind of problem, as the computational expense increases with the number of level set functions used for crack descriptions. The cracking particle method (CPM) developed by Rabczuk, by which crack patterns are simplified and discretized through a set of cracking segments, has shown to be a promising alternative. The branched crack problem, as a representative of multiple crack problems, is studied here to demonstrate the advantages of the CPM. Cracking particles can be split multiply due to the use of bilinear cracking lines and then the discontinuity at the intersection is fulfilled easily. An adaptivity strategy is adopted to control the size of cracking segments and the number of degrees of freedom. Stress intensity factors at the crack tips are calculated and show good agreement with previous results.

Key Words: *branched cracks; cracking particle method; meshfree; adaptivity*

1. Introduction

Multiple-crack problems are common in natural brittle fracture, but are still challenging to model with current numerical methods, since crack propagation is a highly non-linear phenomenon. The finite element method (FEM) is the most common numerical method that has been tried on crack problems, however in its basic form requires that a crack edge propagates along element edges, otherwise there will be an overestimation of fracture energy [1]. The extended finite element method (XFEM) was proposed primarily to model cracks without reference to mesh arrangements but via enrichments [2]. A major issue, however, is that it requires an explicit description of crack surfaces which is usually through level set functions and this process becomes more complex with an increase in numbers of cracks [3]. Meshless methods only require nodal data to discretise a problem domain and can avoid many problems associated with element-based methods, such as mesh distortion and volumetric locking [4]. These methods have also been applied to crack simulation, combined in some cases with the level set method [5], then leading to the same issues with the XFEM as mentioned above. The numerical manifold method (NMM) has also been tried for multiple crack problems by separating the problem domain into “mathematical covers" and “physical covers", but it is still a bottleneck to generate these covers effectively [6]. The cracking particle method (CPM) was proposed to approximate a 2D crack path by the use of nodes with discontinuous lines representing crack paths [7, 8], a method in which it is relatively easy to update cracks through adding or deleting cracking particles, which makes it a promising way to handle cracks with complex patterns. In this work, the branched crack is studied as a representative of multiple-crack problems. A modified CPM is presented in which cracking particles can be split multiply. Numerical results are provided to demonstrate the performance of the proposed methodology.

2. Formulations

Consider a branched crack problem in two dimensions (2D) as shown in Fig. 1(a). The equilibrium equation in the domain Ω is

$$\nabla \cdot \boldsymbol{\sigma} = \mathbf{0} \qquad \text{in } \Omega \tag{1}$$

with boundary conditions

$$\boldsymbol{\sigma} \cdot \mathbf{n} = \bar{\mathbf{t}} \qquad \text{on } \Gamma_t, \tag{2}$$

$$\mathbf{u} = \bar{\mathbf{u}} \qquad \text{on } \Gamma_u, \tag{3}$$

where $\boldsymbol{\sigma}$ and $\mathbf{u}$ are the Cauchy stress tensor and displacement vector respectively, and body forces are not considered. $\mathbf{n}$ is the unit normal to the domain Ω, $\bar{\mathbf{t}}$ and $\bar{\mathbf{u}}$ are traction and displacement constraints on the boundary Γ with $\Gamma = \Gamma_t \bigcup \Gamma_u$.

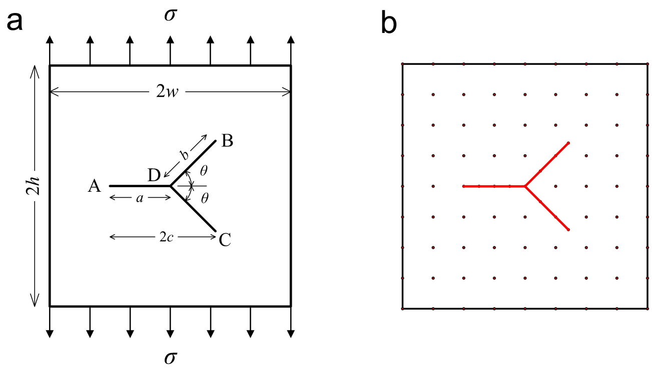

Figure 1: Problem statement: (a) sketch; (b) initial nodes.

In this work, the Element Free Galerkin method is used where Eqns 1 to 3 are discretised into a weak form and the essential boundary conditions are imposed with Lagrange multipliers. The approximate displacement at location $\mathbf{x}$ in Ω, $u^h(\mathbf{x})$ is given by a linear combination of shape functions $\Phi_i(\mathbf{x})$ and nodal displacements u_i as

$$u^h(\mathbf{x}) = \sum_{i=1}^{n} \Phi_i(\mathbf{x}) u_i = \boldsymbol{\Phi}^{\mathrm{T}} \mathbf{u}, \tag{4}$$

where i is the node index with coordinate $\mathbf{x}_i$ and n is the number of nodes. The shape functions $\Phi_i(\mathbf{x})$ are determined via a moving least squares (MLS) approximation and a quartic spline weight function is used, more details can be found in [4].

3. Crack description

In the CPM, crack surfaces are described by a set of cracking particles with discontinuous lines cutting the support. The discontinuity at cracks was firstly achieved through enrichments in the original CPM paper [7] and it has been shown that these enrichments can be removed by splitting cracking particles into two parts [8]. However, it is very hard to describe a branched crack in that way. Here a multiple split strategy is presented which is much simpler than approaches using level set functions,e.g. in [3, 5].

A point D at the branch point with a circular support as in Fig. 2 is split into three nodes D_1, D_2 and D_3 and the support of node D is cut by three crack branches. Point D_1 belongs to crack AD and DB, Point D_2 belongs to crack DB and DC and Point D_3 belongs to crack DC and DA. Nodes D_1, D_2 and D_3 are not connected to each other. The visibility method [4] is used to handle relationships among all nodes, through which discontinuity at crack surfaces is achieved.

4. Adaptivity

An *a posteriori* adaptivity strategy, as found in [10], is employed here to handle the contradiction between accuracy and calculating efficiency. Sound nodal arrangements with a high density of nodes around crack surfaces and fewer nodes away from cracks can be set up automatically. An error estimator for each cell (with subdomain Ω_i) is based on the difference between the “projected” stress $\boldsymbol{\sigma}$ (to approximate the unknown exact stresses [9]) and the calculated stress $\boldsymbol{\sigma}^h$ using the error energy norm

$$\|E_i\| = \left\{ \frac{1}{2} \int_{\Omega_i} (\boldsymbol{\sigma}^p - \boldsymbol{\sigma}^h)^{\mathrm{T}} \mathbf{D}^{-1} (\boldsymbol{\sigma}^p - \boldsymbol{\sigma}^h) \mathrm{d}\Omega_i \right\}^{1/2}, \tag{5}$$

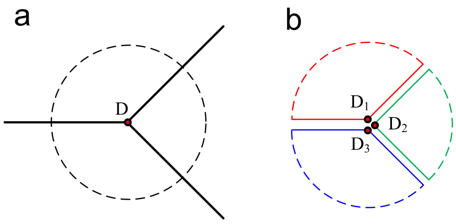

Figure 2: Multiply split cracking particle: (a) original support; (b) supports after being split.

with the elastic constitutive matrix being

$$\mathbf{D} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \quad \text{for plane stress}\,. \tag{6}$$

Cells with large errors are divided into four sub-cells by a quadtree structure and four sub-cells with small errors are combined together to recover an original cell.

5. Numerical results

The dimensions of the branched crack problem in Fig. 1 are $h = w = 50$mm, $\theta = 45°$ and $a = b = 25$mm with initial node arrangements in Fig. 1 (b). The material properties are Young's modulus $E = 200$GPa and the Poisson's ratio $\nu = 0.3$. A pair of uniform tensions are applied with $\sigma = 200$N/mm as shown under plane stress conditions.

The adaptive steps of the branched crack problem are shown in Fig. 3. During this process, three groups of nodes generate automatically around three crack tips, and the crack opening is achieved as in Fig. 3 (d). Global errors during the adaptive steps are shown in Fig. 4 (a) and the convergence rate achieved using the adaptive refinement is shown to be higher than that using uniform refinement. From Fig. 4 (b, c and d), it can be seen that calculated stress intensity factors around crack tips agree well with [3], and the difference with [5] is reasonable since those results were calculated without enrichments.

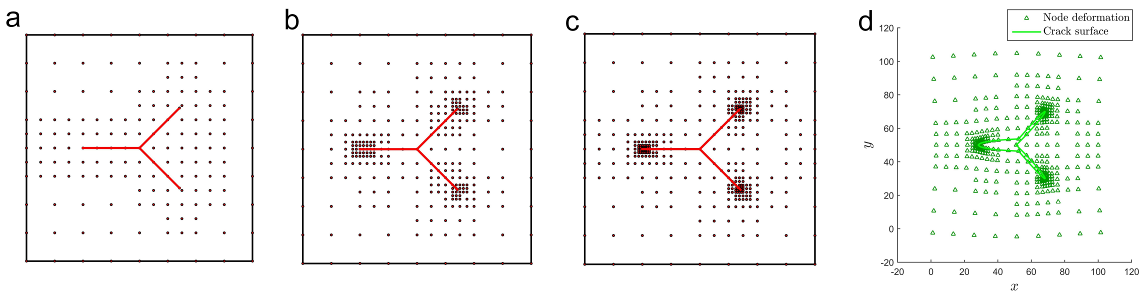

Figure 3: Adaptive steps of branched crack problem: (a) step 1; (b) step 3; (c) step 5; (d) final deformation enlarged by 100 times.

6. Conclusions

A new CPM is presented here which is able to handle multiple crack problems. A cracking particle belonging to more than one crack is split multiply and then the discontinuity at crack surfaces can be achieved through the visibility criterion, which is much simpler than through several groups of level set functions. An adaptivity strategy is introduced into the CPM to improve efficiency. A branched crack problem is studied here to demonstrate the performance of the new CPM and good agreement can be obtained compared with previous results.

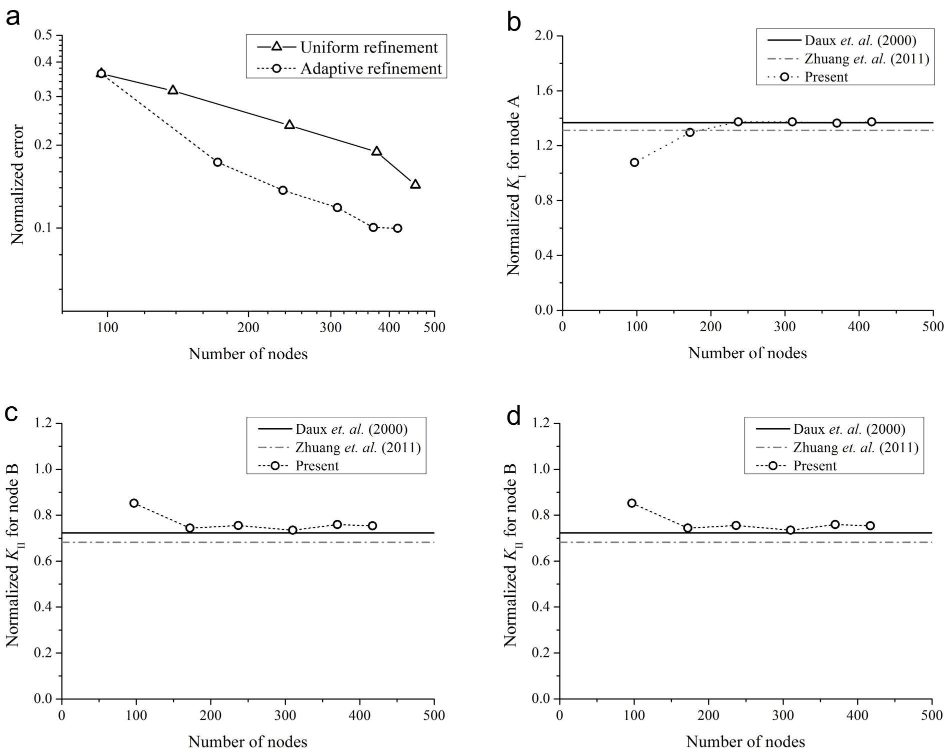

Figure 4: Verification of results: (a) global error; (b) K_{I} of node A; (c) K_{I} of node B; (d) K_{II} of node B.

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