Fast Quasi-Synchronous Harmonic Algorithm Based on Weight Window Function - Mixed Radix FFT

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***Abstract*—According to the requirements of IEC61850-9-2LE, digital energy metering devices mainly adopt 80×*fr* fixed sampling rate. When the harmonic analysis is carried out under asynchronous sampling, it will produce large errors due to spectral leakage. Quasi-Synchronous Algorithm has high accuracy, but the calculation process is complicated and the hardware overheads are high. Based on the characteristics of digital energy metering devices, this paper puts forward a Fast Quasi-Synchronous Harmonic Algorithm using weight window function combined with Mixed Radix Fast Fourier Transform Algorithm. It will reduce the calculation by more than 94%. Compared with the Triangle/Hanning/Nuttall4(III)–Windowed Interpolated FFT Algorithm, the proposed algorithm will perform better in accuracy and has the feature that the more asynchronous of the sampling, the more obvious the error will be.**

Keywords—IEC61850-9-2LE; measuring; asynchronous sampling; harmonic

I. INTRODUCTION

With the continuous construction of digital substations, a large number of digital power metering systems composed by electronic transformer (ECT/EVT), merging unit (MU) and digital watt-hour meter have been put into use[1]. The construction of digital substations follows the series standard of IEC61850[2]. The IEC61850-9-2[3] standard states requirements on sampling rate, data bits and transmission frequency, etc. But the requirements are flexible. In practice, the lite edition of IEC61850-9-2LE is frequently adopted. It requires the voltage and current sampling rate of digital power metering devices to be 256×*fr* or 80×*fr* (*fr* is rated frequency 50Hz)[4]. The 80×*fr* sampling rate can meet the requirement of power system protection in harmonics monitoring, hence the adoption of 80×*fr* (4000Hz) sampling rates in most digital substations at present according to the survey results. However, the actual power system frequency always fluctuates around the *fr*, while digital energy metering devices mainly adopt 80×*fr* fixed sampling rates. As a result, the complete periodic sampling of the voltage and current cannot be guaranteed. When the harmonic analysis is carried out, spectral leakage will be produced due to asynchronous sampling. To address this problem, Professor Dai Xianzhong first put forward the Quasi-Synchronous Algorithm to improve the accuracy of current RMS voltage, active power, harmonic, frequency and other electrical parameters in measurement[5]. However, the actual application of the Quasi-Synchronous Algorithm is limited for its complicated computation, great time-consumption and difficult implementation in hardware. In addition, an alternative algorithm for harmonic analysis under asynchronous sampling based on Newton's interpolation successfully adjusts an actual sampling sequence to an ideal one synchronized with the original signal. The fundamental period is calculated through the zero-crossing method, and then the original sampling sequence is reconstructed by the time-domain Newton's interpolation polynomial and is similar to synchronous sampling[6][7]. However, in practice, the received data in digital watt-hour meters have already been interpolated in MU by the synchronous process. The re-interpolation of the data by Newton algorithm will result in additional errors and complicated calculation.

This paper is organized as follows. Section II presents the basic principles of applying the Quasi-Synchronous Algorithm in harmonic analysis. Section III expounds the combination of weight window function with Mixed Radix FFT Algorithm and introduces the method to simplify the algorithm operation structure while guaranteeing accuracy. Section IV gives the emulation and experiment results of harmonic amplitude and frequency deviation measurement under asynchronous sampling. A conclusion is given in Section V.

II. THE QUASI-SYNCHRONOUS ALGORITHM

Supposing $f(y)$ is periodic function, period is T and frequency is $f = \frac{1}{T}$. The function to calculate the average of $f(y)$ within period by integral is

$$\overline{f(y)} = \frac{1}{T}\int_{y_0}^{y_0+T} f(y)dy \qquad (1)$$

When $f(y)$ is sampled by a constant time interval ΔT, the number of sampling points is N per period, assuming sampling period is $T_s = N\Delta T$, then the discrete sequence of the signal is y_i ($i=0,1,\dots,LN+1$) in period number L. The average of $f(y)$ in L periods is

$$\overline{f(y_i)} = \frac{1}{LN+1}\sum_{i=1}^{LN+1} f(y_i) \qquad (2)$$

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When T=T_S, the signal is synchronously sampled, and the $\overline{f(y_i)}$ is a zero-error value. When the sampling is asynchronous, the $\overline{f(y_i)}$ has an error increasing with $|\Delta|$ (Δ=T-T_S).

The Quasi-Synchronous Algorithm provides an iterative operation to reduce error in asynchronous sampling. In the first iteration, an integral formula is used to calculate the averages of the corresponding functions of N+1 sampling points with beginning points changing from 1 to (L-1)N+1. The formula can be defined as

$$F_i^1 = \sum_{k=i}^{N+i} \rho_{k-i+1} f(y_k) / \sum_{k=i}^{N+i} \rho_{k-i+1} \qquad (i = 1, 2, \cdots, (L-1)N+1) \tag{3}$$

Here ρ refers to the weight coefficient sequence with a length of N+1. Compound rectangular or trapezoidal formulae is frequently adopted. The final result can be obtained after L times of iterations as follows,

$$F_i^M = \sum_{k=i}^{N+i} \rho_{k-i+1} F_k^{L-1} / \sum_{k=i}^{N+i} \rho_{k-i+1} \qquad (i = 1, 2, \cdots, (L-M)N+1)(2 \le M \le L) \tag{4}$$

Fig.1 presents an example of 3-time iteration. The iteration process is as follows [8].

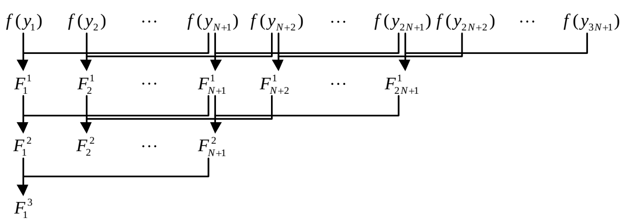

Fig. 1. Iteration computation of Quasi-Synchronization Algorithm

When the Quasi-Synchronous Algorithm is applied to conduct harmonic analysis of discrete sequence y_i (i=0,1,…,LN+1), $f(y_i)=y_i e^{-j\frac{2\pi mi}{N}}$. Assuming $W_N^{mi}=e^{-j\frac{2\pi mi}{N}}$, it can be inferred that the mth harmonic is

$$\begin{cases} F_{mi}^1 = \sum_{k=i}^{N+i} \rho_{k-i+1} y_k W_N^{km} / \sum_{k=i}^{N+i} \rho_{k-i+1} \qquad (i = 1, 2, \cdots, (L-1)N+1) \\ F_{mi}^M = \sum_{k=i}^{N+i} \rho_{k-i+1} F_{mk}^{M-1} / \sum_{k=i}^{N+i} \rho_{k-i+1} \qquad (i = 1, 2, \cdots, (L-M)N+1)(2 \le M \le L) \end{cases} \tag{5}$$

It can be inferred from article [5] that if $N \gg 1$ and $\frac{1}{NTs} \gg f_{m\Delta}$ ($f_{m\Delta}$ is the difference between the mth harmonic of sampling frequency and real frequency), then

$$\begin{cases} \left|F_{m1}^L\right| \approx (\gamma_m)^L A_m \\ \angle F_{m1}^L \approx \pi f_{m\Delta} LTs(N-1) + \varphi_m \\ \angle F_{m2}^L - \angle F_{m1}^L \approx f_{m\Delta} 2\pi Ts \\ \gamma_m = \dfrac{\sin(\pi f_{m\Delta} NT_S)}{N \sin(\pi f_{m\Delta} T_S)} \end{cases} \tag{6}$$

In order to get F_{m2}^L, only one more sampling point is needed (Sampling Point LN+2). When F_{m1}^L and F_{m2}^L are produced, the information of the mth harmonic can be obtained.

The most direct way to correct the error caused by spectral leakage is to increase the time of the window. It can keep the original information of the discrete sequence as much as possible and reduce the percentage of error caused by non-complete periodic sampling. The Quasi-Synchronous Algorithm in fact increases the window time by iterative operation and reduces the error from asynchronous sampling. The accurate electric parameter values can be obtained even without knowing the exact frequency. When the deviation between sampling period and the signal cycle deviation is less than half of a cycle, deviation can be reduced to a very low level within 3~5 cycles. Thus the Quasi-Synchronous Algorithm is suitable for digital power metering devices with a fixed sampling rate. However, the iterative operation requires a large amount of computation, which is not feasible in application. Therefore, the key to apply Quasi-Synchronous Algorithms in digital power metering is to ensure they can guarantee the accuracy while reducing the hardware overheads.

III. The Method of Weight Window Combined with Mixed Radix FFT

A. *Weight Window Function*

The mechanism of iteration computation in Quasi-Synchronous Algorithms is to assign different weights to the original data in different positions. As long as the sampling points N per period and period cycles L are determined, the weight parameter for each position is determined. So we can use the weight window function to get the final result F_{m1}^L directly.

Referring to (5),

$$\begin{aligned} F_{m1}^L &= \sum_{k=1}^{N+1} \rho_k F_k^{L-1} / \sum_{k=1}^{N+1} \rho_k \\ &= \frac{1}{\sum_{k=1}^{N+1} \rho_k} \left(\sum_{k=1}^{N+1} \rho_k \left(\frac{1}{\sum_{j=k}^{N+k} \rho_{j-k+1}} \sum_{j=k}^{N+k} \rho_{j-k+1} (\dots (y_i W_N^{im})) \right)\right) \\ &= \sum_{i=1}^{LN+1} \varphi_i y_i W_N^{im} \end{aligned} \tag{7}$$

Where the weight window function $\Psi = [\varphi_1, \varphi_2, \dots \varphi_{LN+1}]$, $\Upsilon = [y_1 W_N^m, y_2 W_N^{2m}, \dots y_{LN+1} W_N^{(LN+1)m}]$, and then

$$F_{m1}^L = \Psi \cdot \Upsilon' \tag{8}$$

Since the iteration computation is too complex, the exact expression of Ψ can't be written out. However, the whole window function sequence value can be obtained through computer operation. While Ψ is not related to Υ, it can be calculated in advance, cured to metering devices in the storage unit and be used directly.

Take sampling points from 5 cycles in the digital metering system and use trapezoidal integration formula to determine weight parameters as follows,

$$\begin{cases} f_s = 4000\text{Hz} \\ N = 80 \\ L = 5 \\ \rho_k = \begin{cases} 0.5 \quad (k = 1, 81) \\ 1 \quad (k = 2, 3 \ldots 80) \end{cases} \end{cases} \tag{9}$$

We obtained the image of the LN+1 points weight window function Ψ (Fig.2).

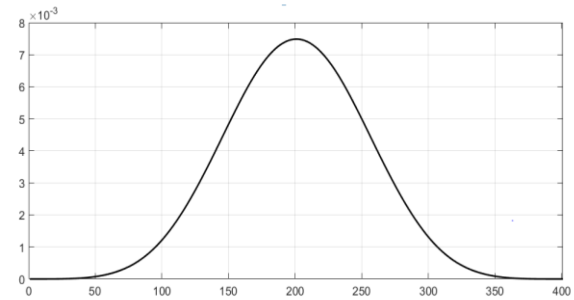

Fig. 2. weight window function Ψ (L=5,N=80)

When the iteration computation is replaced by the weight window function, and since $W_N^{im} = W_N^{(i+kN)m}$, i.e. W_N^{im} is N periodic, formula (8) can be further simplified as:

$$F_{m1}^{L} = \Upsilon_1 \cdot W' \tag{10}$$

Where $\Upsilon_1 = [\sum_{i=0}^{L} \varphi_{1+iN} y_{1+iN}, \sum_{i=0}^{L-1} \varphi_{2+iN} y_{2+iN}, \ldots, \sum_{i=0}^{L-1} \varphi_{N+iN} y_{N+iN}]$, and $W = [W_N^m, W_N^{2m}, \ldots, W_N^{Nm}]$.

Therefore, the Quasi-Synchronous Algorithm for LN+1 sampling points arithmetic operations F_{m1}^{L} can be converted to an ordinary N-point DFT operation. In this manner, F_{m2}^{L} in (6) can be calculated.

B. Radix-2 and Radix-5 Mixed FFT

The spectral analysis using DFT involves N^2 complex multiplications and $N(N$-1) complex additions, which are complex still. Therefore, further simplification of the computation with the fast algorithm is needed. It is commonly used in Radix-2 FFT, the basic idea of which is to halve the input sequence $y(i)$ according to the parity of position successively, so it applies only to cases where N is an integer power of 2. When $N \neq 2^M$, zero-padding or interpolation processing is needed. But this calculated spectrum represents the waveform after zero-padding and interpolation instead of the original signal spectrum, which not only increases the amount of calculation, but also leads to obvious error.

For the special case of a digital energy measurement, a specific mixed radix FFT algorithm is needed.

When p_i in $N=p_1p_2\ldots p_m$ are primes, assuming $q_1=p_2p_3\ldots p_m$, then $N=p_1q_1$. Divide $x(n)$ into p_1 groups and each group contains q_1 points, and then

$$\begin{aligned} X(k) = DFT[x(n)] &= \sum_{n=0}^{N-1} x(n) W_N^{nk} \\ &= \sum_{r=0}^{q_1-1} x(p_1 r) W_N^{p_1 rk} + \sum_{r=0}^{q_1-1} x(p_1 r + 1) W_N^{(p_1 r+1)rk} \\ &\quad + \cdots + \sum_{r=0}^{q_1-1} x(p_1 r + p_1 - 1) W_N^{(p_1 r + p_1 - 1)k} \\ &= \sum_{l=0}^{p_1-1} W_N^{lk} \sum_{r=0}^{q_1-1} x(p_1 r + l) W_N^{p_1 rk} \\ &= \sum_{l=0}^{p_1-1} W_N^{lk} \sum_{r=0}^{q_1-1} x(p_1 r + l) W_{q_1}^{rk} \\ &= \sum_{l=0}^{p_1-1} W_N^{lk} G_l(k) \end{aligned} \tag{11}$$

Where $G_l(k) = \sum_{r=0}^{q_1-1} x(p_1 r + l) W_{q_1}^{rk}$, and it is q_1-point DFT. Since q_1 is still a combinatorial number, it can be further decomposed. After this operation, the N-point DFT is gradually decomposed into smaller DFTs. With the strict equivalent in the process, the accuracy of results is maintained while the amount of calculation is greatly reduced.

There are some articles introducing Mixed radix FFT Algorithm which mainly concentrated in the usual sampling rates, such as 128 points/period, 256 points/period, etc. A kind of the structure of 128-point Mixed Radix FFT is given in [9], and [10] proposes a processor which can provide 128-point and 256-point FFT computations. In the digital metering system, N=80=2^4×5, so it can be calculated through the Radix-2 and 5-Radix Mixed FFT. Since the 80-point arithmetic structure diagram is too large, the smallest structure of the Radix-2 and Radix-5 Mixed FFT i.e. N=10 is adopted to demonstrate the schematic structure, as shown in Fig.3

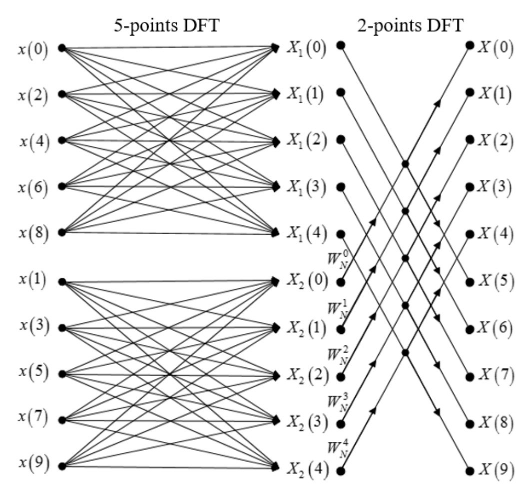

Fig. 3. Radix-2 and Radix-5 Mixed FFT structure

C. The computation analysis

For the case of (9), after operations in section III.*A* and section III.*B*, the Quasi-Synchronization Algorithm computation will be greatly simplified.

In III.*A*, the value of Y_1 in (10) is calculated with only 401 real multiplications and 321 real additions when the weight window function method is applied, which is actualized by the conversion of the Quasi-Synchronous Algorithm for 401 sampling points arithmetic operations to ordinary 80-point DFT operations.

In III.*B* the data of 80 sampling points are divided into 16 groups containing 5 points each to complete the 16 groups' 5-point DFT operation. The results are then processed by Radix-2 FFT computation. Since *N*-point DFT involves complex multiplications $m_D= N^2$ and complex additions $a_D=N(N-1)$, and *N*-point($N=2^M$) Radix-2 FFT involves $m_{2RF}=M\times N/2=(N/2)\log_2 N$ and $a_{2RF}=M\times N=N\log_2 N$, the amount of calculation for Mixed Radix FFT is

$$\begin{cases} m_{MRF}=16\times 5^2+5\times\left[\dfrac{16}{2}\times\log_2(16)\right]=560 \\ a_{MRF}=16\times 5\times 4+5\times\left[16\times\log_2(16)\right]=640 \end{cases}$$

Therefore, $F_{m1}^L(m=1,2\ldots 80)$ is obtained after 961 complex multiplications and 961 complex additions, and F_{m2}^L can be calculated in a similar manner. These operations completed, the state of each harmonic wave can be obtained by (6). The total amount of calculation is m=1922, a=1922.

By contrast, if the original iterative operation of Quasi-Synchronization Algorithm was adopted, it required $m=(LN+1)\times 80=32080$,$a=2L(LN+1)\times 80=320800$ to get $F_{m1}^L, F_{m2}^L(m=1,2\ldots 80)$ even if a recursive method was applied. If the Windowed Interpolated FFT Algorithm (hereafter abbreviated to WIFFTA) was adopted, 400 points sampling data were interpolated to 512 points, it required m=3840, a=5120 to calculate harmonics regardless of the refine process. Table I compares the amount of calculation for the three methods.

TABLE I. THE COMPARISON OF ALGORITHM COMPUTATION

Type	Original Algorithms	Proposed Algorithm	WIFFTA
Complex Multiplications	32080	1922	3840
Complex Additions	320800	1922	5120

In conclusion, with the application of a weight window function combined with a Mixed Radix FFT Algorithm, the amount of calculation is greatly reduced more than 94% compared with the original Quasi-Synchronous Algorithm, and it is also 50% smaller than WIFFTA. Besides, the proposed algorithm has the same accuracy with the original algorithms as the simplification of calculation process is essentially equivalent. Comparing with WIFFTA, the proposed algorithm has higher accuracy in digital watt-hour meters in theory, because WIFFTA need re-interpolation after the synchronous process in digital substation, this will result in additional errors, and the proposed algorithm is more suitable under asynchronous sampling. Therefore, the proposed Fast Quasi-Synchronous Harmonic Algorithm is quite practical and efficient for digital metering devices.

IV. RESULTS OF EMULATION AND EXPERIMENT

A. *Emulation of amplitude and frequency measurement*

To verify the validity of the proposed algorithm, an emulation test of harmonic amplitude and frequency deviation measurement under asynchronous sampling is carried out in MATLAB. The result is compared with that of the original Quasi-Synchronous Algorithm and the Triangle/Hanning/ Nuttall4(III)-Windowed Interpolated FFT Algorithm (hereafter abbreviated to TIFFTA, HIFFTA, N4(III)IFFTA). However, because of the spectral leakage and picket-fence effect, the accuracy is low when peak spectral lines are used directly to calculate parameter estimation through WIFFTAs. In order to improve the accuracy of the comparing algorithms, the double-spectrum-line interpolation[11] is adopted to refine the results.

Signals in (12) are analyzed through an emulation test, where the fundamental frequency f_1=49.8Hz. A_i, φ_i are assigned as table II. And sampling frequency is 4000Hz, the number of sampling points is 402.

$$y(t)=\sum A_i \sin(2\pi f_i t+\varphi_i) \quad (12)$$

TABLE II. THE HARMONIC COMPONENTS OF TEST SIGNAL

Emulation Signal	Order of Harmonics								
	1	2	3	4	5	6	7	8	9
A_i	380.0	20.0	50.0	5.0	20.0	6.0	10.0	5.0	1.0
ψ_i	-23.1	115.6	59.3	52.4	123.8	0	31.8	0	-63.7

The results are shown in table III, and the relative errors of amplitude measurement are demonstrated in Fig.4. It can be seen that the proposed algorithm has the same results with the original algorithm and leads to higher accuracy of both the harmonic amplitude and frequency measurement, which is 1-3 orders higher and 2-6 orders higher than comparing WIFFTAs respectively.

TABLE III. RESULT OF EMULATION TEST

Parameter	Algorithm	Order of Harmonics								
		1	2	3	4	5	6	7	8	9
Amplitude Ai/kV	Proposed	380.0000	20.0000	50.0000	5.0000	20.0000	6.0000	10.0000	5.0000	1.0000
	Original	380.0000	20.0000	50.0000	5.0000	20.0000	6.0000	10.0000	5.0000	1.0000
	TIFFTA	380.1880	24.3700	49.7823	4.1565	19.9661	5.8832	9.8849	4.8454	0.9913
	HIFFTA	379.9914	20.0504	49.9995	4.9895	19.9950	6.0138	10.0018	4.9968	0.9996

	N4(III)IFFTA	379.9992	19.9979	50.0001	5.0008	20.0002	5.9994	10.0001	5.0004	1.0001
Frequency f_i/Hz	Proposed	49.8000	99.6000	149.4000	199.2000	249.0000	298.8000	348.6000	398.4000	448.2000
	Original	49.8000	99.6000	149.4000	199.2000	249.0000	298.8000	348.6000	398.4000	448.2000
	TIFFTA	49.8396	100.5623	149.3309	197.9586	249.0943	298.6534	348.4444	398.1671	447.9067
	HIFFTA	49.7995	99.6812	149.3980	199.0661	248.9937	298.8708	348.5926	398.3490	448.1453
	N4(III)IFFTA	49.8000	99.6042	149.3998	199.1913	249.0001	298.8004	348.5990	398.3986	448.1983

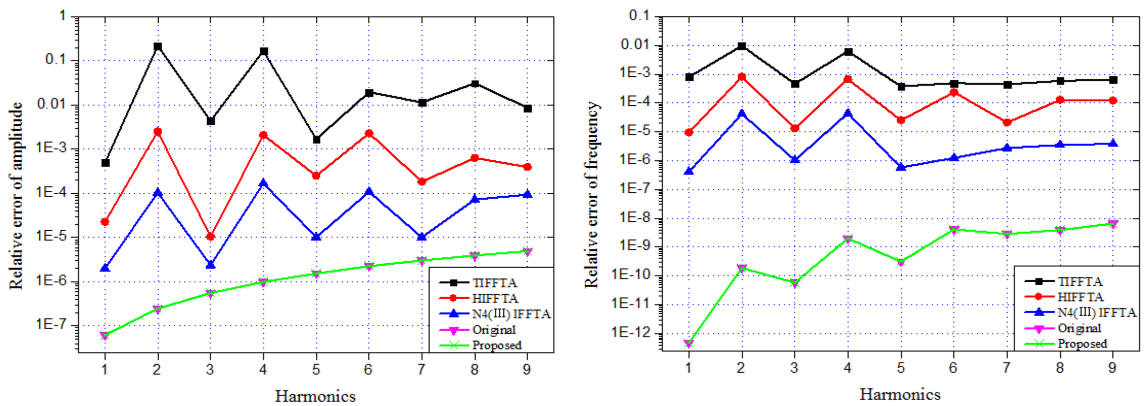

Fig. 4. The relative errors of amplitude and frequency

B. Emulation of sensitivity for asynchronous sampling,

To know the sensitivity of the proposed algorithm for asynchronous sampling, the emulation test of harmonic amplitude measurement was carried out with the fundamental frequency changing gradually from 49.5Hz to 50.5Hz. The relative errors of measurement are shown in Fig.5. The horizontal axis represents the fundamental frequency f_1 of the tested signal, and the degree of asynchronous sampling can be indicated by the distance between f_1 and f_r (50Hz). The vertical axis represents the number of harmonics, the height exhibiting the relative error of amplitude.

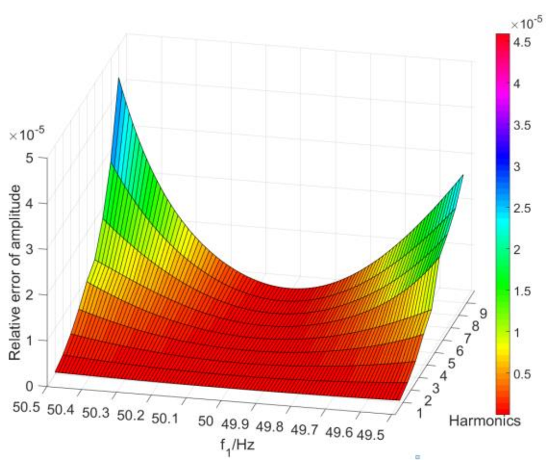

Fig. 5. The influnce of asynchronous sampling to amplitude measurement

As can be seen in Fig.5, the graph is high at the sides while low in the center along the horizontal axis. The vertical axis shows high insides and low outside, which means the error of amplitude measurement increases with the raising degree of the asynchronous sampling and the growing number of harmonics. However, within the range of 49.8Hz to 50.2Hz, the error grows slowly, mostly below 10^{-5}. In addition, as the vertical axis shows, the error of even order harmonics is not significantly higher than that of the odd harmonics, which is different from the windowed interpolated FFT algorithm.

C. Experimental Results

To specify the hardware overheads and its accuracy, the proposed algorithm is tested on experimental platform with the comparing algorithms. The experimental platform consists of digital power source(XL-828), MU simulator(XL-805) and digital watt-hour meter simulator(ADSP-BF609 integrated in XL-805) as Fig.6. Digital power source generates digital waveform in FT3 format like EVT in digital substation, the parameters are as the same as section IV.*A*. MU simulator converts the data format into IEC61850-9-2LE, digital watt-hour meter simulator processes the data with the algorithms.

Fig. 6. The experimental platform

The average computation burdens of ten repeated experiments are shown in Fig.7. As the sampling time of 402 points is 100.5ms, it's obvious that the original Quasi-Synchronous Algorithm is not feasible in application, but the proposed Fast Quasi-Synchronous Harmonic Algorithm is efficient. In addition, It can be seen that the time cost of the proposed algorithm is 96% smaller than the original algorithm, and nearly 50% smaller than the comparing WIFFTs. This is consistent with the theoretical analysis, the computation subtle difference is mainly caused by the process of results refinements and the hardware programming.

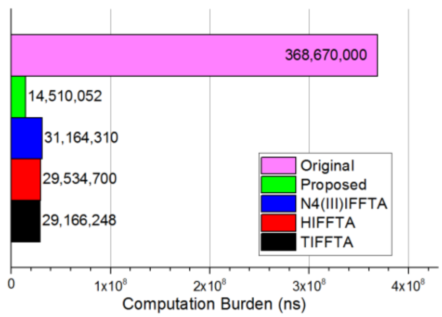

Fig. 7. The comparing of computation burden

The relative errors of amplitude measurement are shown in Fig.8. The general trend of different algorithms' accuracy is consistent with the emulation results.

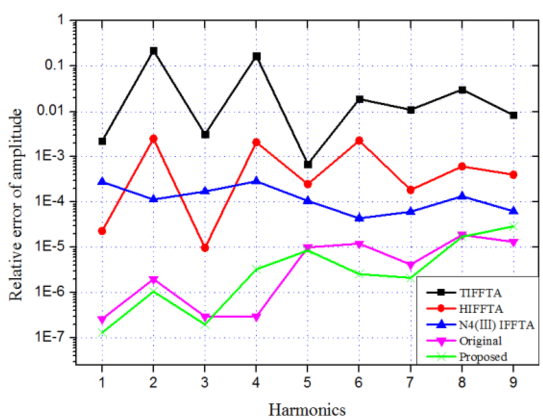

Fig. 8. Experiment resutls of relative errors of amplitude

However, there are two obvious differences. Firstly, the accuracy of the proposed algorithm, the original algorithm and N4(III)IFFTA is reduced nearly one order of magnitude. Due to the limitation of data bits and clock precision, the sampling data generated by digital power source is not exactly the same with theoretical value. In addition, IEC61850-9-2LE requires the least significant bit of voltage is 10mV, this brings truncation error. As a result, a high accuracy of amplitude measurement is difficult to realize. So the algorithms with high accuracy are impacted more obviously. Secondly, the results of the proposed algorithm and the original are not entirely same anymore. This is because of the truncation error caused by calculation.

V. Conclusions

Based on the characteristics of sampling data in digital power metering systems, this paper puts forward a Fast Quasi-Synchronous Algorithm, the core mechanism of which is to replace iteration computation with a weight window function, and convert a Quasi-Synchronization Algorithm to 80-point DFT operation that is then divided into short 5-point DFTs and Radix-2 FFTs. The calculation is reduced by more than 94%, and additional algorithm error caused by zero-padding or interpolation is also avoided. Compared with TIFFTA, HIFFTA, N4(III)IFFTA, the emulation test and experiment results show the proposed algorithm has the following features:

(1) The computation burden is 50% smaller and the accuracy is 1-3 orders higher in amplitude measurement than comparing WIFFTs.

(2) The error of measurement grows with the degree of asynchronous sampling and the number of harmonics.

(3) The accuracy of even-order harmonics measurement is basically the same with odd-order harmonics measurement.

Therefore, the proposed algorithm has the advantages of a simple operation structure, small amount of calculation, high accuracy and applicability to IEC850-9-2LE specified data. In practice, it helps to improve the performance of digital electric energy metering devices, and promotes their wide application. Future studies may analysis and test the influence of inter-harmonics to the proposed algorithm and research the effective method to reduce it.

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