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A NON-ORDINARY STATE BASED PERIDYNAMICS IMPLEMENTATION FOR ANISOTROPIC MATERIALS

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ABSTRACT

In this work we present a peridynamics (PD) formulation for generally anisotropic materials. PD has been shown as a new approach for modelling fracture mechanics, and can easily model different dynamic effects, such as crack branching. The non-ordinary state-based framework allow us to model the material properties in the same way as in the classical continuum mechanics. The crack propagation path follows the same behaviour as seen in experimental results for unidirectional fibre composites. Damage is modelled through a deformation criterion.

Key Words: *peridynamics; non-ordinary state-based formulation; anisotropic materials; crack propagation*

1. Introduction

Fracture mechanics is a difficult problem to deal with since there are discontinuities arising from cracks. The continuum mechanics equilibrium equation presents a partial derivative, which becomes infinite at the discontinuities. Numerical models have different approaches in order to deal with this particular case, such as the quarter-point for the finite element method (FEM) and the dual formulation for the boundary element method (BEM).

Peridynamics (PD) is a novel numerical method developed by Silling [1] to treat fracture mechanics problems. The equilibrium equation is defined in an integral form, which eliminates the rising of discontinuities in fracture mechanics problems. For this reason, the same formulation of PD is used whether there is a discontinuity or not, being ideal for modelling crack initiation, branching and coalescence for example. These crack propagation behaviour are very difficult to model with conventional numerical methods.

PD is receiving increased attention over recent years, due to its potential for fracture mechanics. So far, most of the works considered only isotropic materials, but some works for PD in composite materials have been investigated (see [2] for instance). Nevertheless, a full anisotropic model for PD has not been developed to date, and it is shown in this work for the first time. A non-ordinary state-based formulation is used [3, 4], where elements of continuum mechanics such as the deformation gradient are employed in the PD framework. This formulation is a generalisation of the first PD proposed in [1].

2. Non-ordinary state-based peridynamics

In the classical continuum mechanics, the equation of motion is defined as

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{b}(\mathbf{x},t) = \rho \ddot{\mathbf{u}} \tag{1}$$

where $\boldsymbol{\sigma}$ represents the Cauchy stresses, $\mathbf{b}(\mathbf{x},t)$ stands for the body forces, ρ is the mass density and $\ddot{\mathbf{u}}$ is the acceleration.

The equation of motion in state-based peridynamics (PD) is defined as

$$\int_{\mathscr{H}} \{\underline{\mathbf{T}}[\mathbf{x},t]\langle \mathbf{x}' - \mathbf{x}\rangle - \underline{\mathbf{T}}[\mathbf{x}',t]\langle \mathbf{x} - \mathbf{x}'\rangle\} dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x},t) = \rho \ddot{\mathbf{u}}(\mathbf{x},t) \tag{2}$$

with $\underline{\mathbf{T}}$ as the force vector state field, and square brackets denote that the variables are taken in the state vector framework.

In order to Eq. (2) to be valid, it must satisfy both balance of linear and angular momentum. The proofs can be found in [5] for instance.

Figure 1 illustrates the reference (or initial) configuration, and the deformed configuration after a displacement $\mathbf{u}$ and $\mathbf{u}'$ has been imposed on particles $\mathbf{x}$ and $\mathbf{x}'$, respectively. In the original bond-based formulation, the particles inside the horizon of $\mathbf{x}$ make a contribution for the displacement solution. However, the fact that these other particles $\mathbf{x}'$ also possess their own horizon is not taken into account. This leads to a restriction of the material properties [1]. The state-based theory [3] removes this limitation, allowing to model any material properties.

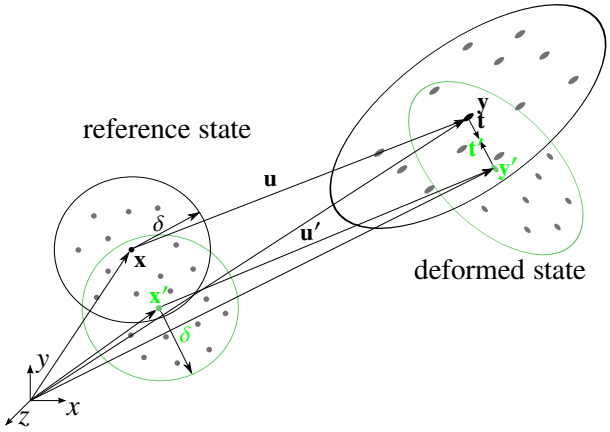


Figure 1: : Reference and deformed configuration in state-based PD.

There are 2 types of state-based formulation: ordinary and non-ordinary. In the ordinary theory, the forces in the bonds are defined in the direction of the bonds, in the same way as in the bond-based formulation. The main issue is how to obtain the equivalent material properties from the classical continuum mechanics. An energy equivalent approach can be used, as detailed in Madenci and Oterkus [6].

In the non-ordinary framework, the forces between particles are not constrained to the bond direction, which provides a more general formulation. In this case, some of the main parameters in continuum mechanics, such as the deformation gradient, are expressed in terms of the PD formulation. Additionally, the material constitutive matrix can be used, overcoming this limitation in the ordinary state-based PD theory.

The peridynamics state-based formulation consists in the use of the so called state fields. They are fully detailed in [3]. The deformation vector state field is stated as

$$\underline{\mathbf{Y}}[\mathbf{x},t]\langle\boldsymbol{\xi}\rangle = \mathbf{y}(\mathbf{x}+\boldsymbol{\xi},t) - \mathbf{y}(\mathbf{x},t) \tag{3}$$

The non-local deformation gradient $\mathbf{F}(\mathbf{x})$ for each particle is given by

$$\mathbf{B}(\mathbf{x}) = \left[\int_{\mathscr{H}} \omega(|\xi|)(\xi\otimes\xi)dV_\xi\right]^{-1} \tag{4}$$

$$\mathbf{F}(\mathbf{x}) = \left[\int_{\mathscr{H}} \omega(|\xi|)(\underline{\mathbf{Y}}(\xi)\otimes\xi)dV_\xi\right].\mathbf{B}(\mathbf{x}) \tag{5}$$

where $\mathbf{B}(\mathbf{x})$ is the shape tensor, $\otimes$ denotes the dyadic product of two vectors, and $\omega(|\xi|)$ is a dimensionless weight function, used to increase the influence of the nodes closes to $\mathbf{x}$. In this work, we assumed that $\omega(|\xi|) = 1$.

To incorporate the kinematic stress into the PD model, the transpose of the first Piola-Kirchhoff stress is equivalent to [7]

$$\mathbf{P}(\mathbf{x})^T = \frac{\partial W}{\partial \mathbf{F}} \tag{6}$$

with W being the strain energy density function.

The force vector at time t is finally stated as [7]

$$\underline{\mathbf{T}}[\mathbf{x},t]\langle \mathbf{x}' - \mathbf{x} \rangle = \omega(|\mathbf{x}' - \mathbf{x}|)\mathbf{P}(\mathbf{x})^T.\mathbf{B}(\mathbf{x}).(\mathbf{x}' - \mathbf{x}) \tag{7}$$

The processing of mapping a stress tensor as a peridynamic force state is the inverse of the process of approximating the deformation state by a deformation gradient tensor. A peridynamic constitutive model that uses stress as an intermediate quantity results in general in bond forces which are not parallel to the deformed bonds.

3. Numerical discretisation

In order to obtain the values for the acceleration, velocity and displacements, we need to integrate the PD solution using an explicit approach. The values of acceleration are calculated directly from Eq. (1). The velocities are integrated using a forward difference approach, while the displacements are obtained through a backward scheme. The numerical integration is done by

$$\ddot{\mathbf{u}}(\mathbf{x},t) = \frac{1}{\rho}\left(\int_{\mathscr{H}} \{\underline{\mathbf{T}}[\mathbf{x},t]\langle \mathbf{x}' - \mathbf{x} \rangle - \underline{\mathbf{T}}[\mathbf{x}',t]\langle \mathbf{x} - \mathbf{x}' \rangle\} dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x},t)\right) \tag{8}$$

$$\dot{\mathbf{u}}(\mathbf{x},\Delta t + t) = \dot{\mathbf{u}}(\mathbf{x},t) + \ddot{\mathbf{u}}(\mathbf{x},t)\Delta t \tag{9}$$

$$\mathbf{u}(\mathbf{x},\Delta t + t) = \mathbf{u}(\mathbf{x},t) + \dot{\mathbf{u}}(\mathbf{x},t)\Delta t \tag{10}$$

where Δt is the time step, $\dot{\mathbf{u}}(\mathbf{x},t)$ are the velocities and $\mathbf{u}(\mathbf{x},t)$ are the displacements. Since we are using an explicit approach, the time step must be smaller than a certain threshold in order to the analysis to be valid. In this work we considered that the $\Delta t \propto \delta/c_p$, where $c_p = \sqrt{C_{22}/\rho}$ and c_p is the dilatational wave speed and C_{22} comes from the material properties. The time step was obtained by $\Delta t = 0.01\frac{\delta}{c_p}$.

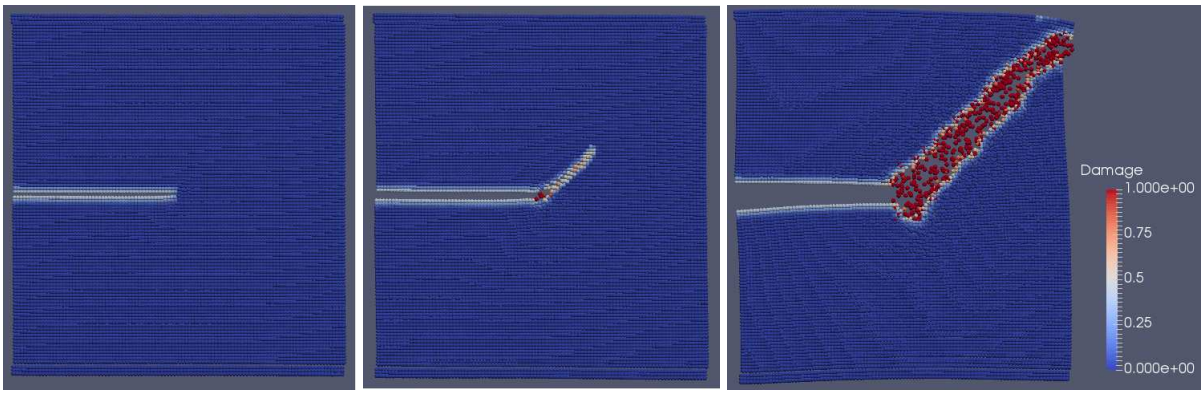
4. Results

We investigate the dynamic effects in an anisotropic square plate containing an edge crack. The plate has dimensions 100 mm ×100 mm, the crack tip is located at the centre of the plate, and the length of the crack is 50 mm. The dimensions of the plate were chosen just to assess the formulation. The plate is a graphite-epoxy where the fibre forms an angle of 45° with respect to the horizontal axis. The material properties in global coordinates system are given by (in Voigt notation): C_{11} = 5.01 GPa, C_{12} = 3.08 GPa, C_{16} = 3.34 GPa, C_{22} = 5.01 GPa, C_{26} = 3.34 GPa, C_{66} = 3.80 GPa. The plate is subjected to an initial velocity gradient $\mathbf{L}$ given by

$$\mathbf{L} = \begin{bmatrix} 0 & 0 \\ 0 & 350 \end{bmatrix} \frac{1}{s} \tag{11}$$

Figure 2 depicts some stages of the crack propagation with respect to a damage parameter. If damage is 0, all the bonds are active, while if damage is 1, all the bonds are broken. In Figure 2(a), the crack is about to start propagating, while in Figure 2(c) the crack has propagated through the entire plate. One can note that the angle of crack propagation is 45°, which is consistent with experimental results for a unidirectional composite fibre [8].

The time step of the analysis is $\Delta t = 6.0 \times 10^{-9} s$, and the total time until complete failure is $t = 1.50\ \mu s$, resulting in 5000 steps of the integration. The bond breaking criterion is defined in the same way as in [4], and it consists in using a deviatoric deformation component. If the deformation between two particles exceeds a critical deformation, the bond breaks. In this work we assumed $\epsilon_{crit} = 0.0042$.



(a) $t = 0.368\ \mu s$ (b) $t = 0.603\ \mu s$ (c) $t = 1.50\ \mu s$

Figure 2: : Anisotropic crack propagation - Edge crack.

5. Conclusions

A peridynamics model for fully anisotropic materials has been presented for the first time in the literature. The non-ordinary state-based framework has been used to model a general anisotropic material. The crack propagation path is consistent to the one find in experiments in the literature for an unidirectional fibre composite. Future work include using different horizon sizes and other problems such as anisotropic bimaterials or inclusions.

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