

# Dynamic Time and Power Allocation for Opportunistic Energy Efficient Cooperative Relay

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**Abstract**—Exponential growth in power consumption of wireless communication devices and lack of progress in battery capacity are increasing pressure for more energy efficient (EE) wireless networks. This paper presents an algorithm for optimum EE time allocation for two cooperative relay selection schemes: opportunistic decode-and-forward (ODF) and opportunistic energy efficiency (OEE) with and without rate constraint. By dynamically optimising transmission time between source and relay it is possible to simultaneously improve EE and minimise capacity loss. Simulation in a multi-user scenario with randomly distributed number and location of cooperative nodes demonstrates the algorithm’s effectiveness for improving network performance and applicability to both dynamic and static networks. Results imply a unique globally optimum time and power allocation dependent on relay position.

## I. INTRODUCTION

Over recent decades, information and communications technology has evolved at a fast and alarming rate. Silicon technology continues to improve, doubling roughly every two years [1], with corresponding increase in processor energy consumption of 150% [2]. Advances in silicon technology have permitted emergence of widespread data-hungry handheld mobile devices, and growth rate of data volume transmitted through mobile cellular systems mirrors that of semiconductors, increasing by factor of 10 roughly every five years [3]. Cellular network energy consumption contributes over 3% of worldwide energy demand [4].

Reducing the energy needed for wireless data transmission makes economic sense. The energy bill, depending on country, may reach 32% of total operational expenditure in cellular markets [5]. From the user side, battery technology is falling sharply behind processor power consumption. The result is an exponentially increasing gap between the energy demand of mobile devices and the battery capacity to supply it. The past decade has seen dramatic reductions in battery life of mobile devices [6], and with wireless data transmission consuming roughly 60% of mobile battery usage [7], the incentive to reduce energy consumption is strong.

Growing demand for data-hungry applications has meant much work towards improving network throughput. But high throughput tends to mean high energy demand, and recently, curbing the energy rise of wireless communications has drawn increasing attention. Relays have been shown to save power in

two ways: reducing path loss due to shorter transmission range and reducing interference due to lower required transmission power ([8], [9]). Cooperative networks are a special case of relay network where each node is both information source and relay. If users closer to the destination can exploit surplus resources and act as relay for distant users, the benefits of relay networks can be achieved with minimal changes to existing infrastructure and hardware.

Cooperative systems have been studied extensively in recent years ([10]–[16]). Opportunistic decode-and-forward (ODF) is a popular technique ([17], [18]), where the transmission link switches dynamically between direct transmission (DT) and decode-and-forward (DF) relay based on channel quality. In opportunistic energy efficient (OEE) relay, selection is made based on energy efficiency (EE). Half-duplex (HD) DF requires that transmission time is divided into two phases, with proportion strongly affecting performance. Traditionally transmission time is divided equally [18], however [19] showed that time allocation according to link quality brings significant improvement, especially for rate-constrained OEE.

This paper improves the time allocation for rate-constrained OEE over that in the literature by identifying an additional criterion for optimum performance, and presents a computationally simple algorithm for better dynamic switching between the best available relay and DT. Simulation is also extended to multiple relays randomly distributed in number and location around a central destination node, demonstrating scheme applicability in mobile, ad hoc and vehicular networks as well as for static nodes. The algorithm’s effectiveness for improving network performance is demonstrated, and analysis of transmission power shows a globally optimum energy efficient time and power allocation dependent on relay position.

The rest of this paper is laid out as follows. Section II introduces the system model. Section III solves the dynamic time allocation problem. Section IV presents simulation results. Section V concludes the topic.

## II. SYSTEM MODEL

The model in Fig. 1 consists of a source  $S$ , destination  $D$ , and  $N$  relays  $R_1, \dots, R_N$  scattered randomly within the  $D$  cell radius. Relays are users with idle communication resources who are able and willing to relay information for

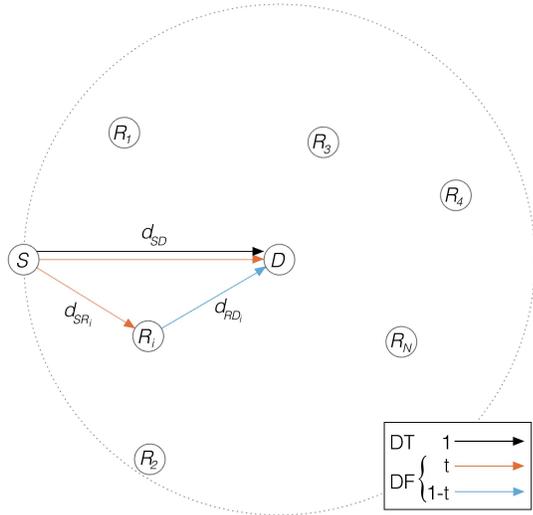


Fig. 1. Cooperative Relay Network with Multiple Users

*S.* Any one may be chosen for cooperation. All nodes use HD communication.

Links between terminals are modelled with a simplified path loss model as well as Rayleigh fading. Path loss (combining free-space loss and ray tracing [20]) between points  $X$  and  $Y$  separated by distance  $d_{XY}$  is

$$G_{XY} = \left( \frac{\lambda}{4\pi d_0} \right)^2 \left( \frac{d_{XY}}{d_0} \right)^{-\gamma} \quad (1)$$

where  $\lambda$  is signal wavelength,  $d_0$  is reference distance and  $\gamma$  is path loss exponent. The received signal amplitude  $h_{XY}$  for a transmitted symbol block is modelled by independent, quasi-static, frequency non-selective Rayleigh fading with coherence time larger than the time to transmit a block of symbols.

Both ODF and OEE schemes select either DT or DF relay depending on an expected end-to-end performance metric. ODF maximises capacity ( $C$ ), whereas OEE maximises EE. In DT only the SD link is used and  $S$  transmits to  $D$  for the entire transmission time. In DF, the best available relay is chosen and the schedule described as Protocol II in [21] is used. Here, total transmission time is divided into two phases: broadcasting phase  $t$  and relay phase  $(1-t)$ . In  $t$ ,  $S$  communicates with both  $R_i$  and  $D$ . In  $(1-t)$ , only  $R_i$  communicates with  $D$ , relaying the message received in the first time slot via independent Gaussian codebook.  $D$  tries to decode the message by combining the signal received from both  $S$  and  $R_i$ . The time slot  $t$  is optimised dynamically depending on channel quality.

The Shannon channel  $C$  of an  $XY$  link,  $X, Y \in \{S, R_i, D\}$

$$C_{XY} = B \log_2 \left( 1 + \frac{G_{XY} h_{XY}^2 P_X}{N_0 N_f B} \right) \quad (2)$$

where  $B$  is transmission bandwidth,  $P_X$  is transmission power,  $N_0$  is noise power spectral density and  $N_f$  is receiver noise figure. Assuming fixed  $B$ , a node can control link capacity

only by changing  $P_X$ . For DF with  $R_i$  at point  $(d_{SR_i}, d_{R_i D})$  around the SD pair,  $t_i$ ,  $P_S$  and  $P_R$  must be optimised.

The available DT and DF channels between  $S$  and  $D$  are denoted  $i = 0, 1, \dots, N$ , where  $i = 0$  is the DT link with  $C_0 = C_{SD}$ . For constant  $d_{SD}$ ,  $C_0$  is a function of  $P_S$  only. DF capacity  $C_i$  using  $R_i$  and time  $t_i$  is derived in [18]

$$C_i = \min \left[ t_i C_{SR_i}, t_i C_0 + (1-t_i) C_{R_i D} \right] \quad (3)$$

The minimisation occurs from the need for signal decoding at both  $R_i$  and  $D$ . DT link EE in bits/joule is

$$EE_0 = \frac{C_0}{(1+\alpha)P_S + P_{ct} + P_{cr}} = \frac{C_0}{P_0} \quad (4)$$

where  $\alpha$  is a constant depending on inefficiencies in the power amplifier [22],  $P_{ct}$  and  $P_{cr}$  are the transmitter and receiver circuit power, modelled constant, independent of data rate and equal for all nodes,  $P_0$  is SD link power consumption. EE for DF channel  $R_i$  is

$$EE_i = \frac{C_i}{t_i P_0 + (1-t_i)[(1+\alpha)P_{R_i} + P_{ct}] + P_{cr}} \quad (5)$$

With (2)-(5) it is possible to formulate a time allocation optimisation problem for ODF and OEE to be solved for all available transmission channels  $i = 0, 1, \dots, N$ .

$$\max_{t_i} \quad Q_i = \begin{cases} C_i & \text{for ODF} \\ EE_i & \text{for OEE} \end{cases} \quad (6a)$$

$$\text{s. t.} \quad 0 < t_i < 1, \quad (6b)$$

$$C_i > \rho C_0, \quad (6c)$$

$$0 \leq \rho \leq 1 \quad (6d)$$

The channel  $i$  with highest performance metric  $Q_i$  is chosen for transmission. Because  $t_i$  is defined only for  $i \geq 1$ , the DT time phase  $t_0 = 1$ , since  $S$  will transmit for all the time available. Constraints (6c) and (6d) are only relevant to OEE, implemented to ensure the increase in EE does not incur unacceptable degradation in end-to-end rate.

### III. OPTIMUM TIME ALLOCATION

The optimum time allocation policies for ODF and OEE with and without the rate constraint are given by three theorems explained below and summarised in Fig. 2.

1) **ODF**: The ODF scheme requires that relay  $R_i$  is chosen only if  $C_i > C_0$ . Because of the minimisation in (3), for the relay to be used both the SR and RD links must be 'better' than SD. This gives two conditions for DF mode to be chosen:

$$\underline{C1}: \quad C_{SR_i} > C_0 \quad (7)$$

$$\underline{C2}_{odf}: \quad C_{R_i D} > C_0 \quad (8)$$

The optimum time phase  $t'_i$  will then occur at the junction  $t_{J,i}$

$$t_{J,i} = \frac{C_{R_i D}}{C_{SR_i} + C_{R_i D} - C_0} \quad (9)$$

If no relays meet both conditions,  $t'_i = 1$  and DT is used.

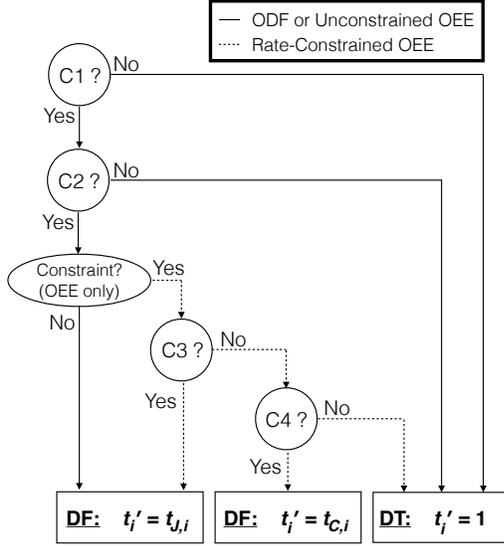


Fig. 2. Optimum time allocation algorithm for ODF ( $C2 = C2_{odf}$ ) and for OEE ( $C2 = C2_{oee}$ ) with and without rate-constraint.

2) **Unconstrained OEE:** For relay to be chosen over DT, both  $0 < t_i < 1$  and  $EE_i > EE_0$  are required. The former is given by (7), so condition C1 is shared. The latter occurs only if

$$\frac{C2_{oee}}{C_0} : C_{SR_i} > \left[ 1 + \frac{P_{cr}}{P_0} + \left( \frac{C_{SR_i} - C_0}{C_{R_iD}} \right) \left( 1 + \frac{(1 + \alpha)(P_{R_i} - P_S)}{P_0} \right) \right] \quad (10)$$

If (7) and (10) are met, the relay is preferred over DT and (9) gives the optimum OEE time allocation  $t'_i = t_{J,i}$ . For all relays in a cooperative network that meet these conditions, the relay with highest EE will be used. If none meet both conditions,  $t'_i = 1$  and DT is chosen.

It will be shown in III-A that  $t'_i$  for ODF and OEE schemes do not always coincide. ODF chooses better C and OEE better EE, however improvement in one comes at the price of a drop in the other. Unconstrained OEE may be acceptable for delay-tolerant applications, e.g. emails or text messages, where drop in capacity is of little consequence to user-perceived performance. However, for delay-sensitive applications like real-time voice and video communication, maintaining a minimum transmission rate is essential. Introducing rate constraint  $\rho$  as in (6c) and (6d) ensures  $C_i$  will not drop below  $\rho C_0$ , permitting use of OEE in delay-sensitive transmissions.

3) **Rate-Constrained OEE:** The rate constraint brings two further conditions to optimum time allocation:

$$\underline{C3}: C_i(t_{J,i}) > \rho C_0 \quad (11)$$

$$\underline{C4}: \underbrace{\frac{\rho C_0 - C_{R_iD}}{C_0 - C_{R_iD}}}_{t_{C,i}} < 1 - \underbrace{\frac{C_0 P_{cr}}{C_{R_iD} P_0 - C_0 [(1 + \alpha) P_{R_i} + P_{ct}]}_{t_{H,i}}} \quad (12)$$

Once a relay is found meeting (7) and (10) the rate constraint imposes that  $t'_i = t_{J,i}$  only if (6c) is met, giving condition C3

in (11). If not, a fourth condition C4 in (12) defines  $t'_i$ . If true,  $t'_i = t_{C,i}$ , otherwise  $t'_i = 1$  and DT is chosen.

C1-4 form a computationally simple algorithm for optimum time allocation ensuring  $t'_i$  is chosen for maximum EE and maintaining C constraint, allowing for rapid switching between cooperative relays in a dynamically changing network. Graphical significance of  $t_{J,i}, t_{C,i}, t_{H,i}$  is explained in III-A, III-B.

#### A. ODF vs Unconstrained OEE

Without rate constraint, the algorithm in Fig. 2 is the same for ODF and OEE, except for ODF  $C2 = C2_{odf}$  and for OEE  $C2 = C2_{oee}$ . The three conditions C1,  $C2_{odf}$  and  $C2_{oee}$  provide four notable cases for comparison. Fig. 3 shows plots of  $t_i$  with  $C_i$  and  $EE_i$  for each.

**Case 1:** ( $\overline{C1 + C1C2}$ ). Fig. 3a) Here,  $t'_i = 1$  and both schemes chose DT. For  $R_1$ , C1 fails, giving  $t_{J,1} > 1$ .  $R_2$  and  $R_3$  have C1 true, but both  $C2_{odf}$  and  $C2_{oee}$  fail. Here,  $0 < t_{J,i} < 1$  but  $C_i(t_{J,i}) < C_0$  and  $EE_i(t_{J,i}) < EE_0$ .

**Case 2:** ( $\overline{C1C2_{odf}C2_{oee}}$ ). Fig. 3b) ODF chooses DF, but OEE chooses DT. OEE foregoes a C rise with the relay for the reward of greater EE with DT. ODF does the opposite.

**Case 3:** ( $\overline{C1C2_{odf}C2_{oee}}$ ). Fig. 3c) OEE chooses DF, but ODF chooses DT. OEE transmits with more EE via relay, however also incurs a C loss. This case is relevant to the rate constraint discussed in III-B.

**Case 4:** ( $\overline{C1C2_{odf}C2_{oee}}$ ). Fig. 3d) Both schemes agree on DF mode, improving both C and EE over DT performance.

#### B. Rate-Constrained OEE

Case 3 shows that the OEE scheme can incur a capacity loss compared to DT, which may be unacceptable depending on the nature of the transmitted message. Introducing a rate constraint  $\rho$  can serve to limit this capacity loss. Since this analysis concerns OEE only,  $C2 = C2_{oee}$  unless otherwise stated.

The constraint  $\rho$  only affects case 3, since only here is a rate loss incurred. The range of  $t_i$  for which  $EE_i(t_i) > EE_0$  gives an upper bound  $t_{H,i}$ . For  $t_i < t_{H,i}$ , DF is preferred, otherwise DF is chosen. The constraint time  $t_{C,i}$  such that  $C_i(t_{C,i}) = \rho C_0$  gives the minimum time allocation for  $R_i$  to be chosen. It follows that for DF mode to be chosen  $t_{C,i} < t_{H,i}$ , else DT is used. Respectively,  $t_{C,i}$  and  $t_{H,i}$  are given by the LHS and RHS of the inequality in (12).

Instance of  $\overline{C1C2C3}$  leads to two possible sub-cases 3A and 3B, shown in Fig. 4.

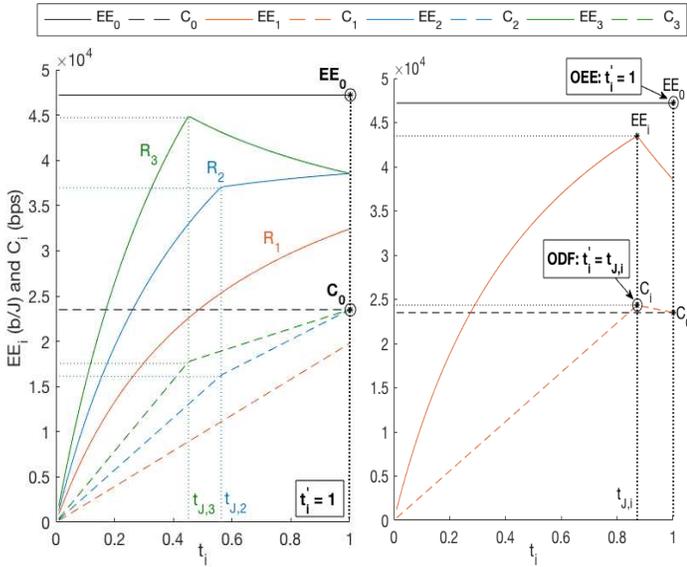
**Case 3A:** Fig. 4a)  $t'_i = t_{C,i}$ , since capacity lost is set to the constrained value defined by  $\rho$ , and the loss in EE from unconstrained OEE (at  $t_{J,i}$ ) is minimised.

**Case 3B:** Fig. 4b) Here  $t_{C,i} > t_{H,i}$  and C4 fails. DT is preferred with  $t'_i = 1$ , since operating at  $t_{C,i}$  results in degradation of both C and EE.

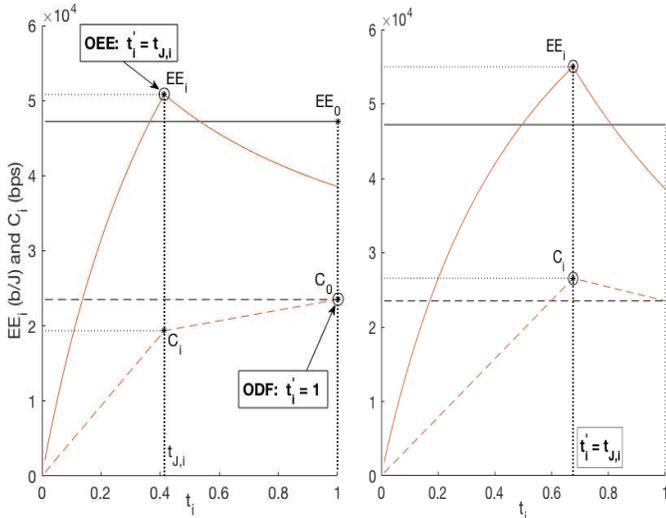
## IV. SIMULATIONS

The algorithm in Fig. 2 was tested in simulation to evaluate effect on system performance. The model from II was used with parameters from the 2.5GHz system in [19], [22].

Clearly, relay location plays a critical role in transmission scheme performance due to path loss. To gauge this effect,



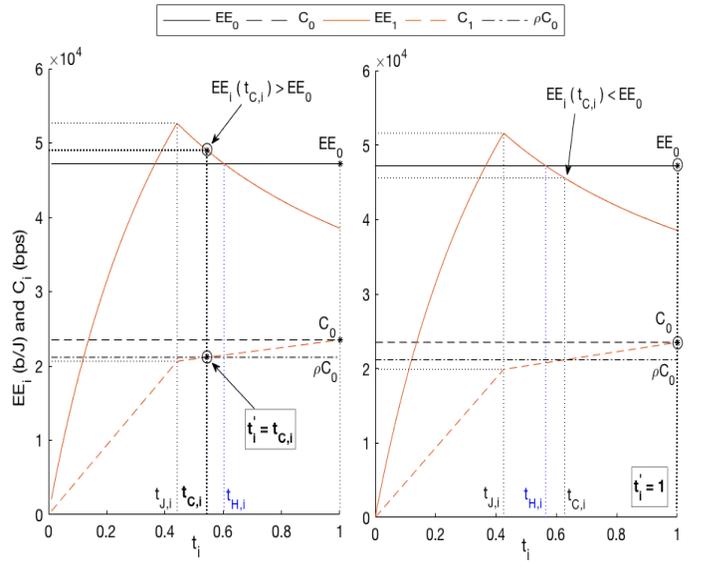
(a) Case 1: DT is optimum,  $t'_i = 1$  (b) Case 2: DT for OEE, DF for ODF



(c) Case 3: DF for OEE, DT for ODF (d) Case 4: DF is optimum,  $t'_i = t_{J,i}$   
 Fig. 3. Four notable cases for optimum time allocation depending on  $C_1$ ,  $C_{2_odef}$ ,  $C_{2_oeo}$ . Y-axis units are all equal.

ODF, unconstrained OEE ( $OEE_{UC}$ ) and constrained OEE ( $OEE_C$ ) with  $\rho = 0.9$  were tested over  $10^6$  iterations for four single-relay situations. It was found that if  $d_{SR}$  or  $d_{RD} > d_{SD}$  the effect of a relay on EE and C is negligible, with less than 0.05% relay use. However, when the relay is located on the direct line between S and D, DF transmission may compose over 85% of iterations for OEE and almost 95% for ODF.

The system was also simulated where the number of relays (from 0 to 10) and their location within the destination cell radius followed a uniform distribution for each iteration, simulating variation in availability and location of cooperative users.  $10^6$  iterations showed that EE improvements of  $OEE_{UC}$  are inevitably accompanied by marked C degradation compared to ODF, with EE gains directly offset by the same proportion of C loss. However, the rate constraint brings down C loss by



(a) Case 3A: DF chosen,  $t'_i = t_{C,i}$  (b) Case 3B: DT chosen,  $t'_i = 1$

Fig. 4. If C3 fails,  $t'_i$  is dependent on C4. Y-axes in each plot are the same.

over 50%, for only 25% less EE. Thus  $OEE_C$  maintains the largest combined system performance improvement.

If  $P_S$  and  $P_R$  are variable, the system can be optimised further. Consider a relay on direct line between S and D at distance ratio  $d_r = d_{SR}/d_{SD}$ . Ignoring DT and the rate constraint, OEE time allocation is  $t'_i = t_{J,i}$ , giving  $C_i(t_{J,i})$  and  $EE_i(t_{J,i})$ . Fig. 5 shows curves for  $EE_i, C_i$  vs  $d_r$  as  $P_R$  increases from 3 to 30dBm, with constant  $P_S$ . Note that  $C_i$  continuously rises with  $P_R$ , but by a lesser amount the further  $R_i$  is from S. However, the  $EE_i$  curves rise to a maximum before falling again. Increasing  $P_R$  improves EE only while the resulting C improvement outweighs the rise in power. Varying  $P_S$  at constant  $P_R$  shows a similar relation, except the effect is stronger when  $R_i$  is near D. Fig. 6 shows  $EE_i(P_S, P_R)$  for an arbitrary non-direct-line relay location. A roughly bell-shaped surface results with a single clearly visible maximum. Including  $EE_0$ , shown in transparent black, a range of useable  $(P_S, P_R)$  pairs for which  $EE_{opt} > EE_0$  is visible. Note  $EE_0$  has its own optimum  $P_S$  coinciding with that for  $EE_{opt}$ .

These relationships show there is a single optimum  $P_S, P_R$  and time allocation for EE dependent on relay position. A useable  $(P_S, P_R)$  range is especially useful for power-limited devices. If a cooperative user cannot commit the optimum  $P_R$  towards transmission, the power that it can commit may still suffice to improve EE. It is also possible that a reduction in  $P_R$  will bring greater EE. Ultimately, a power and time-optimised rate-constrained OEE scheme incorporating maximum relay power is anticipated, with view towards a fully energy efficient cooperative protocol.

## V. CONCLUSION

This paper presents a simple algorithm to find the optimum dynamic time allocation for ODF and OEE with an applied

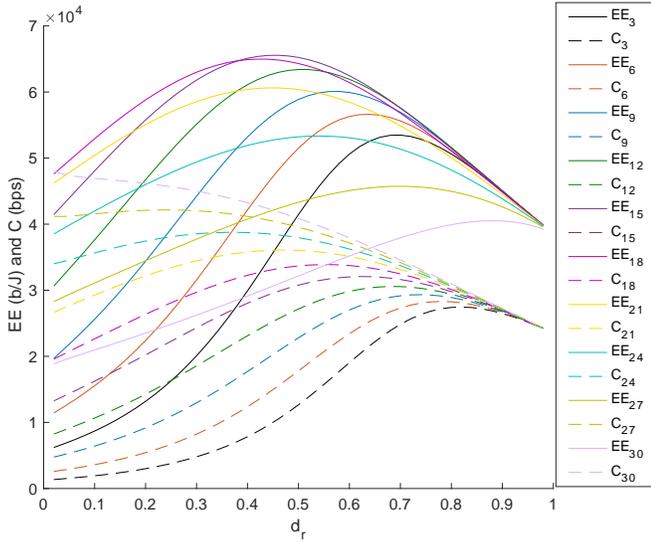


Fig. 5. OOE-optimised EE and C at distance  $d_r$  between source and relay, for  $P_R$  from 3 to 30dBm (notation  $EE_{P_R}, C_{P_R}$  is used).

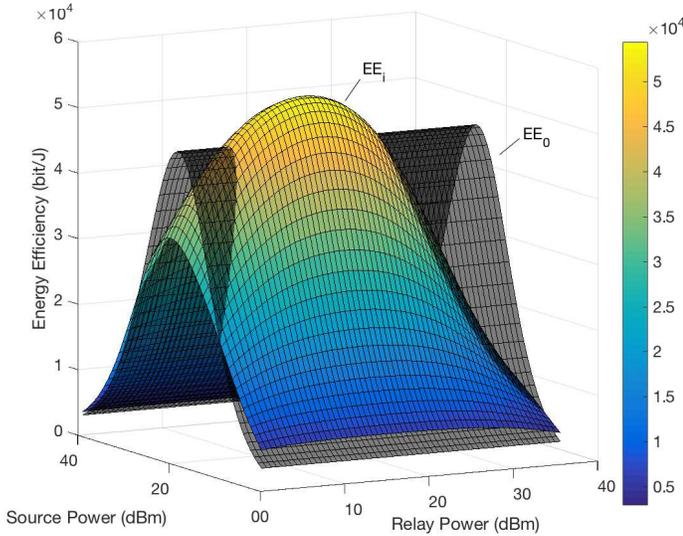


Fig. 6.  $EE_i$  as a function of  $P_S$  and  $P_R$  for a non-direct-line relay location.

rate constraint, allowing for opportunistic choice between DT and DF in a dynamically changing cooperative network. The algorithm's effectiveness at improving network performance was demonstrated in simulation extended to multiple relays randomly distributed in number and location around a destination node, illustrating its applicability to mobile, ad hoc, vehicular and other dynamic network types as well as traditional static networks. Finally, analysis of transmission power showed a globally optimum energy-efficient source and relay power allocation, along with a useable region of energy efficient ( $P_S, P_R$ ) pairs.

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